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Shear lag effects on wide U-section pre-stressed concrete light rail bridges

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Abstract. Recently, U-section decks have been more and more used in metro and light rail bridges as an innovative concept in bridge deck design and a successful alternative to conventional box girders because of their potential advantages. U-section may be viewed as a single vent box girder eliminating the top slab connecting the webs, with the moving vehicles travelling on the lower deck. U-section bridges thus solve many problems like limited vertical clearance underneath the bridge lowest point, besides providing built-in noise barriers. Beam theory in mechanics assumes that plane section remains plane after bending, but it was found that shearing forces produce shear deformations and the plane section does not remain plane. This phenomenon leads to distortion of the cross section. For a box or a U section, this distortion makes the central part of the slab lagging behind those parts closer to the webs and this is known as shear lag effect. A sample real-world double-track U-section metro bridge is modelled in this paper using a commercial finite element analysis program and is analysed under various loading conditions and for different geometric variations. The three-dimensional finite element analysis is used to demonstrate variations in the transverse bending moments in the deck as well as variations in the longitudinal normal stresses induced in the cross section along the U-girder's span thus capturing warping and shear lag effects which are then compared to the stresses calculated using conventional beam theory. This comparison is performed not only to locate the distortion, warping and shear lag effects typically induced in U-section bridges but also to assess the main parameters influencing them the most.

Keywords: U-section bridge; metro bridge; shear lag; pre-stressed concrete; finite element analysis; beam theory

1. Introduction

Bridges help much to overcome many natural obstacles like rivers and mountains and other manmade structures. Over ages, the shapes of bridges have changed to satisfy structural, architectural and circulation needs. Building new bridges can be limited by existing geographical roads or railways if using a bridge with a considerable construction depth. In densely populated regions the big depth will eventually increase the bridge approaches or ramps and consequently increase the mounts of landfills. Many land resources will then be wasted and cost a lot of money. So, a small construction depth design for bridges is desired to solve this issue especially in highly dense regions.

Recently, U-section girder bridges also sometimes known as "channel bridges" came to light (Shepherd and Gibbens 2004, Gibbens *et al.* 2004, Staquet *et al.* 2004). It is a competitive solution of bridge decks that is commonly used in metro railway projects. U-section girder bridges may be viewed as a single vent box girder bridge without the top slab connecting the webs as shown in Fig. 1. They have many advantages like: increase in the vertical clearance under the bridge lowest point as shown in Fig. 2, decrease of the traffic noise pollution, reduction of construction time, slim and attractive aesthetical appearance, besides offering a cost competitive solution. Shear lag effect was identified as a structural mechanics phenomenon since a long period of time and it appears in many applications both in building (e.g., Mazinani *et al.* 2014) and bridge arenas. In bridge engineering, shear lag effect should be considered in design and a relevant example has been illustrated by Dezi *et al.* (2003) for a composite bridge deck. It appears in slender elements and occurs when some parts of the cross section are not directly or rigidly connected. For a box or a U-section girder, all parts of the flange/slab are not fully connected to the webs; so, the closely and/or directly connected parts experience larger stresses than those away from the webs.

The beam theory assumes that a plane section remains plane after bending. This assumption leads to a linear distribution of bending stresses in the studied cross section. This assumption is only valid when the shear stiffness of the cross section is infinite or when there is no shearing force acting on the cross section, according to Timoshenko and Goodier (1969) who introduced the shear deformations in the beam theory. As it is known that any closed section is stiffer than any open section, U-section girders are somehow less stiff than box section girders. Also, box section bridges commonly use diaphragms at any section to resist warping and distortion while, inherently, U-section bridge cannot use any diaphragms as they will act as obstacles in the direction of the traffic flow. Therefore, a U-section bridge experiences much more stresses from

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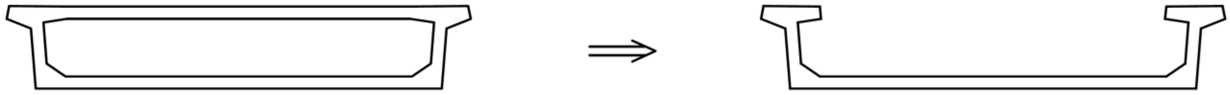


Fig. 1 Transforming a box section bridge to a U-section bridge

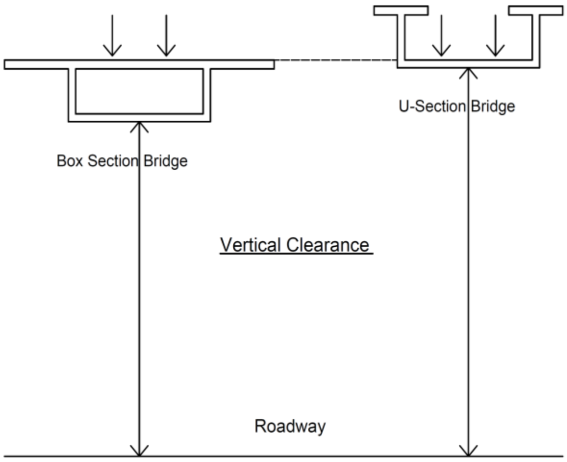


Fig. 2 Vertical clearance below Box section bridge and U-section bridge

warping, distortion and shear lag effects than the box section bridge, especially for wide U-sections (as for two-track railway bridges studied herein) and/or larger bridge width-to-span ratios. Under these considerations, the current paper studies the effects of shear lag on wide double-track U-section light rail bridge decks considering some variables such as various loading types in addition to changes in cross-section thicknesses, girder span, and boundary conditions.

The main objectives of this research are to:

- Perform a finite shell element analysis of the U-section girder bridge and get the values of the induced longitudinal normal stresses and transverse bending moments with a high degree of accuracy;
- Study the consequences of variations in span, thickness and loading types on likely shear lag and warping effects;
- Compare stresses retrieved from finite element analysis with stresses obtained from conventional Bernoulli's beam equation; and finally
- Scrutinize the locations within the cross-section with high deviation in the values of longitudinal stresses determined from the two analysis techniques (finite element analysis versus Bernoulli's beam equation) which may serve as indicator to the presence and severity of shear lag effects and warping phenomenon.

Among the outcomes of this work is to present and assess the output longitudinal normal stresses developed in the U-section metro bridge using finite element analysis technique and Euler-Bernoulli's beam equation as well as to show variations in the transverse bending moments in the deck slab along the girder's span. The U-section metro bridge is analyzed under the effect of vertical gravity loads (self-weight and moving train loads) as well as pre-stressing loads. The following assumptions are used throughout the paper:

- The material of the cross section is isotropic with linear stress-strain relationship.
- The vertical train loads are applied at the actual position of the tracks for the double-track bridge deck considered herein.

2. Literature review

2.1 Topic overview

For years many industries sought stronger and lighter structures which could optimize the performance and the cost of structures. Civil engineering is one of them. This led to a growing utilization of thin walled structures like: cold formed steel sections and concrete box girders where one dimension is small relative to other dimensions.

2.2 Classical beam theory

The classical beam theory (Euler-Bernoulli's theory) is based on Bernoulli's assumption, where plane section remains plane after bending, and it is also assumed that the axis of the beam remains perpendicular to the in-plane cross section according to Timoshenko and Goodier (1969). Also, Hooke's physical assumption on linear elasticity is made, this means that the distribution of the strain varies linearly over the cross-section. This further means that the forces may be divided into three independent components: normal force, bending moment about the major axis and bending moment about the minor axis of the cross section. The shearing forces result from the variation of moment, and the shear stress distribution is deduced from the formula of Grashof which is based on static assumptions of equilibrium by Timoshenko and Gere (2009). The classical beam theory is adequate for many practical purposes. Nonetheless, this adequacy cannot be interpreted in a simple way and it relies upon the cross section and span properties like: the length of beam elements, the thickness relative to the height and the structural application.

2.3 Thin walled sections theory

A thin walled structure is made from thin plates joined along their edges. The thickness of the plate is small when compared to other dimensions of the cross section which are often small when compared with the length of the member according to Murray (1986). Thin walled sections are used in concrete and steel bridges. They are found in the form of plate girder, box columns, box girders and purlins. Thin walled structures can be designed to withstand high torsional loading like box girders, or can be designed to

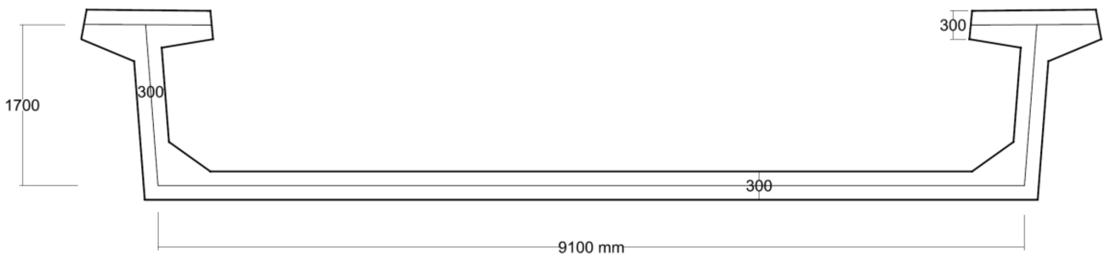


Fig. 3 Railway/metro bridge cross section at the mid-span (courtesy of Ayoub *et al.* 2017)

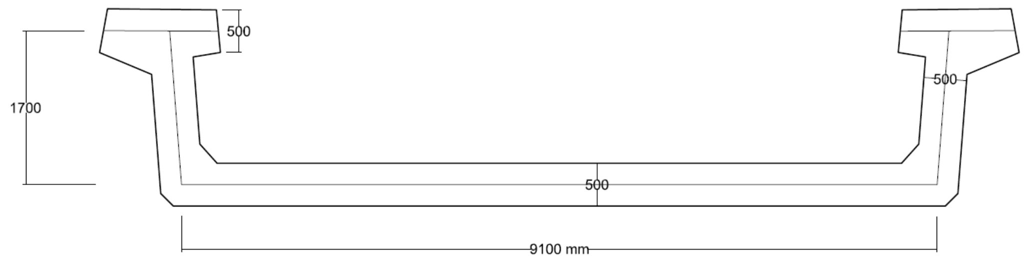


Fig. 4 Railway/metro bridge cross section at the supports (courtesy of Ayoub *et al.* 2017)

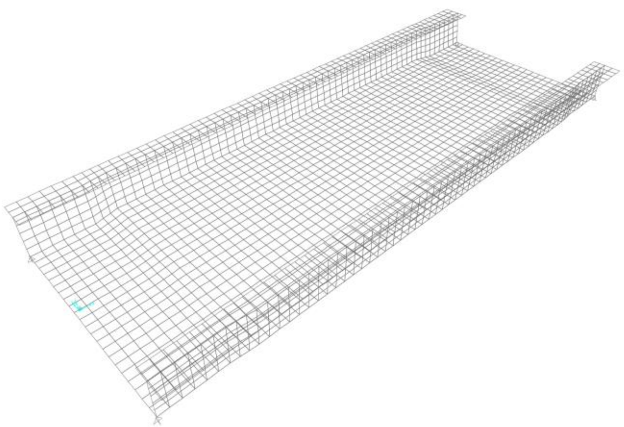


Fig. 5 A sample 3D shell element model of the 25 m span U-section bridge deck using Sap2000 software

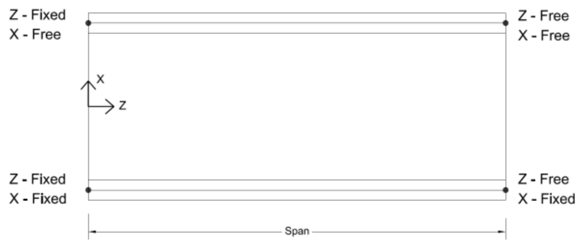


Fig. 6 Boundary conditions of the supports

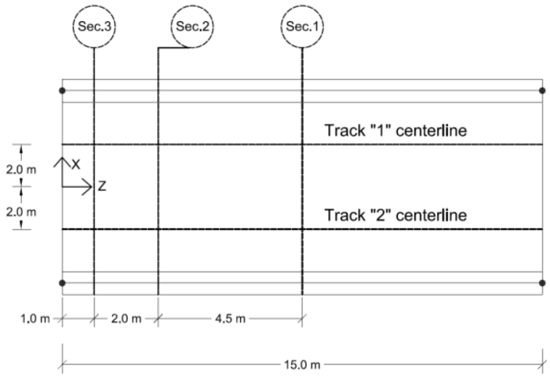
have low torsional rigidity like plate girder. Thin walled structures are characterized with being very light compared with other alternative structures, so they are used in long span bridges and other structures where cost and weight are of main considerations. Thin walled structures are not always steel structures, a box column or a box girder can be made from concrete material also. In 1961 a new theory was developed by Vlasov (1961) to describe the combined effects of torsion and bending of thin walled sections leading to the well-known theory of thin walled sections.

The wall of the bridge cross section is considered thin when its thickness is very small, typically 10% or less, when compared to the overall dimensions of the cross section. The U-section is usually classified as a thin walled open section and the stresses induced in its components should be studied to account for the excessive deformations that could occur in such components.

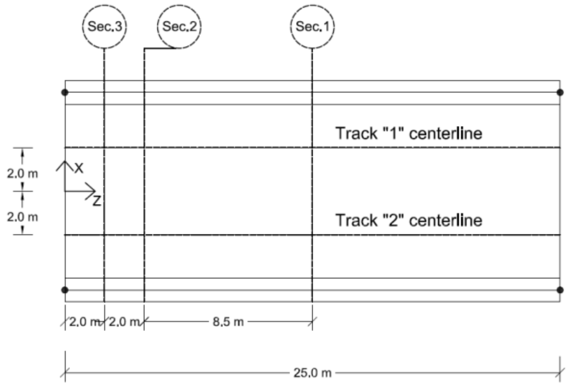
2.4 Shear lag phenomenon

The shear lag phenomenon was identified long ago. In the 1930s, researches started to study the effect of the shear lag in box beams. It is related to any slender box element which is laterally loaded such as box girder bridges and structural elements of buildings like shear walls. Shear lag effect can arise in a box girder when subjected to torsion and also when subjected to bending without torsion, as in case of symmetrical loading, and is called "shear lag in bending". When the box girder is subjected to torsion from one end and the other end of the girder is restrained against warping, the end restrained against warping creates longitudinal stresses in that region. Shear stresses at that area are redistributed due to shear deformations (Shehata 1994, Bakhoum and Shehata 1995).

Shear lag effect generally increases the deflections of box (Yamagushi *et al.* 2008) and U-section (Hu and Wang 2015) girders. The shear lag phenomenon increases with the increase in the width of the section. So, it is essential to be considered for the design of modern bridges which have wide single cell box or U sections. The effect of the shear lag is obvious with increasing the ratio of girder width-to-span length, which happens in the side spans of box and U girder bridges. The non-uniformity of the distribution of longitudinal stresses is specifically evident in the region of big concentrated loads. Therefore, the shear lag effect on the distribution of stresses in the flange of the box girder or the slab of the U-section is appreciable when compared to



(a) 15 meters span model



(b) 25 meters span model

Fig. 7 The positions of a few critical sections along the U-girder span where results are reported

the prediction made by the beam theory.

The literature has some good recent research on the shear lag effects on conventional pre-stressed box girder concrete bridges (Zhou 2014, Bazant *et al.* 2012a, b, Zhang 2012, Zhou 2011, Chang 2004). Nonetheless, on the other hand, U-section bridges are quite recently adopted and hence there are no wealth of data in the literature on this currently evolving cross-section in the light rail bridge design/construction arena.

Based on the theory of elasticity and the variation principle, a theoretical solution (Hu and Wang 2015) was derived for simply supported idealistic U-section bridges which are subjected to uniform surface loading. The theoretical solution showed that the shear lag effects increase the maximum deflection due to global bending by a factor of about 1.2. Moreover, it confirmed that this amplification factor to deflection increases as the width-to-span ratio of the bridge deck increases for the case where the ratio between the moment of inertia of the deck slab and the total moment of inertia of the U-section increases.

In further efforts towards understanding the behavior of U-girder bridges but from a design perspective, Raju and Menon (2011, 2013) stated that a simplified longitudinal analysis through conventional beam theory underestimates the maximum stress in the web-deck slab junction by about 12%, because it is not able to capture the effects arising from shear lag and transverse bending. They further mentioned that a simplified transverse analysis overestimates the sagging moments in the deck slab by about 9%, but fails to capture the hogging moments near the webs and the transverse bending in the webs. However, it is worth reporting that they only considered in their research relatively narrow single track U-section decks rather than the more challenging-currently emerging in practice-wide double track U-sections with likely much more significant shear lag, warping, torsion and distortion effects. The present paper investigating wide U-section metro bridges could be thus considered a step towards filling the gap along this front.

3. Modeling and analysis procedures

3.1 Finite element analysis

The Finite Element Method (FEM) is the most prevailing discretization technique used in structural mechanics. The fundamental concept in the explanation of the FEM is the subdivision of the mathematical model into components of simple geometry called finite elements. In the present research, a finite shell element model of the U-section girder bridge is generated using Sap2000 commercial software where the material used is isotropic with linear stress-strain relationship.

3.2 Euler-Bernoulli's beam theory

The governing differential equation for modeling the Euler-Bernoulli's beam is based on the assumption that plane sections remain plane after deformation. The straining actions (i.e., internal forces) at any given section along the span are calculated by hand from first principles of mechanics or are retrieved from the finite element model through integration of results across a given pre-defined section cut. The section properties are hence calculated and then used in Eq. (1) to get longitudinal normal stresses in the cross section of the bridge. It is worth noting that such technique is unable to capture any possible shear lag and/or warping effects since it is based on the simplistic assumption of "plane sections remain plane after deformation" that inherently (unrealistically) assumes infinite shear stiffness of the cross section (Bakhoum 2010).

$$\sigma = \frac{P}{A} \mp \frac{M_x\, Y}{I_x} \mp \frac{M_y\, X}{I_y} \tag{1}$$

To conclude, three different quantities/forms for the longitudinal normal stresses, σ, are computed and presented in the current research, viz. (i) $\sigma_{\mathrm{B,Mx\ only}}$ (as per Eq. (1) and the conventional beam theory) that is based on the hand calculated longitudinal bending moment M_x for any loading be it symmetric or eccentric; (ii) $\sigma_{\mathrm{B,Mx\&My}}$ still as per Eq. (1) but based on software integrated results at a particular "section cut" only for eccentric loading configuration; and finally (iii) $\sigma_{\mathrm{F.E.,warping}}$ that is fully retrieved from the 3D finite element analysis thus capturing shear lag and warping effects for any loading configuration either symmetric or

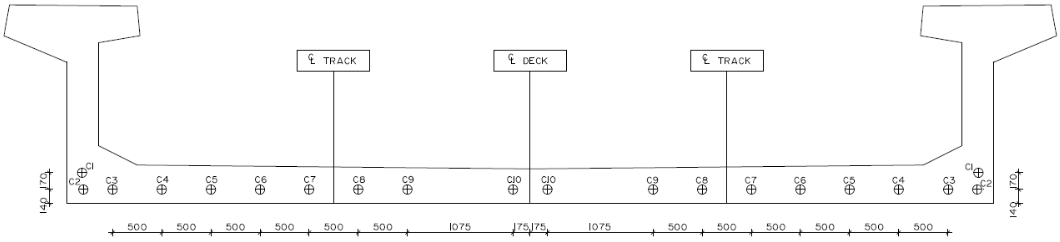


Fig. 8 Pre-stressing cables layout at mid-span

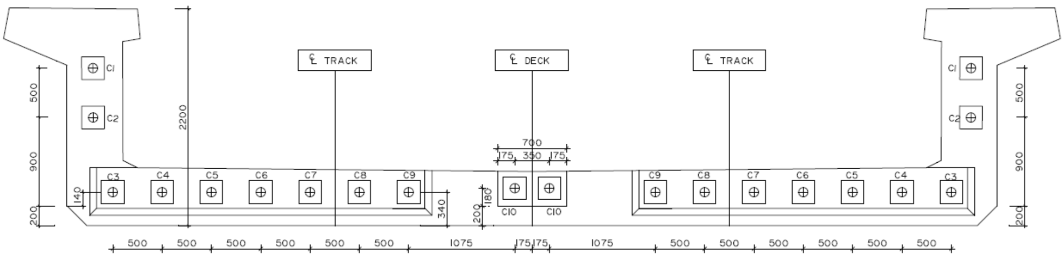


Fig. 9 Pre-stressing cables layout at supports

eccentric.

3.3 Case-study bridge cross section

The case study bridge considered in the analysis is a U-section light rail (namely, metro) bridge. The cross section is for the "Doha Metro Green line" as fully detailed in Ayoub *et al.* (2017). The bridge has two different cross sections along the span. The first prevailing section, shown in Fig. 3 is used in the mid-span of the bridge and up to a position close to the supports where shear forces start to get more significant and pre-stressing tendons are anchored. The second cross section, shown in Fig. 4, is used near and at the supports. The second cross section is thicker in order to compensate for the absence of the diaphragm at the supports. The bridge cross section hosts two tracks for moving metro trains.

3.4 Analysis model

The U-section bridge of Doha metro Green line is typically composed of a series of simple spans along most of its alignment. For the purpose of the current study, a single simply supported typical span 25 m long is considered. An explicitly shorter span of 15 m has been also considered for comparison purposes. The bridge deck and webs across the full span are modeled at their centerlines using shell elements with various thicknesses as applicable. Four-node shell elements of Sap2000 library with a formulation that combines membrane and plate-bending behavior have been used. A four-point numerical integration formulation is used for the shell stiffness. Stresses and internal forces and moments are evaluated at the 2-by-2 Gauss integration points and extrapolated to the joints/nodes of the element. A sample 3D model of the U-section bridge deck with an appropriate mesh size based on mesh sensitivity analysis is shown in Fig. 5. The boundary conditions at the supports are defined as per Fig. 6. For more details reference is made to Boules (2017).

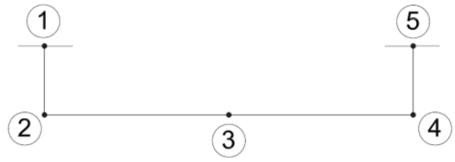


Fig. 10 Critical points within the bridge U-section

3.5 Loading types

The U-section bridge is studied herein under different loading conditions. The assigned vertical moving train loads are allocated at the actual position of the tracks of the moving train vehicles. The bridge is analyzed under the separate effects of its own weight, moving load (as a moving train vehicle) and pre-stressing forces.

3.6 Results manipulation

Longitudinal normal stresses induced in the U-section are retrieved from two analysis techniques: finite element analysis and hand calculations as illustrated in a previous section of the manuscript. To analyze and assess the shear lag effect on the U-section, a comparison is made between the finite element analysis results and Euler-Bernoulli's beam equation results. In order to enhance comparison, results are illustrated graphically and a diagram is hence drawn which gives some ratio between the results of the finite element analysis and Bernoulli's beam equation at critical points of the bridge cross section. It has to be noted that the longitudinal normal stresses calculated based on Euler-Bernoulli's beam theory consider both the in-plane, M_x, as well as the out-of-plane, M_y, bending moments. The latter takes place for eccentric (i.e., non-symmetric relative to the cross section center line) loading generally arising from single track loading of the two-track deck. Transverse bending moments in the deck slab are also shown for some studied schemes of the U-section bridge.

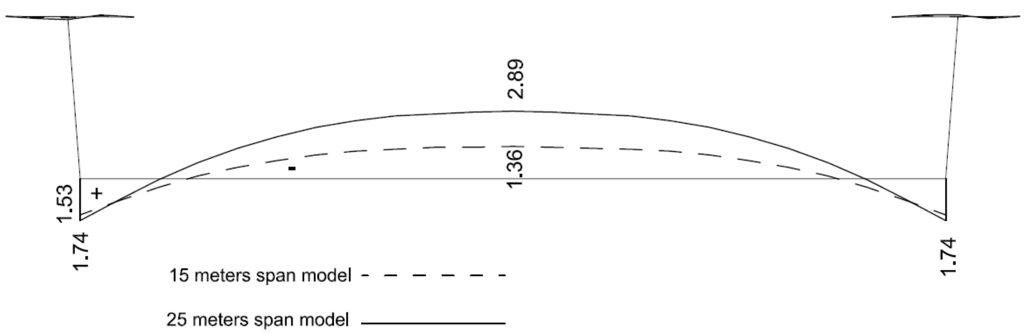


Fig. 11 Transverse moments (kN.m) in the deck slab at mid-span sections under pre-stressing forces

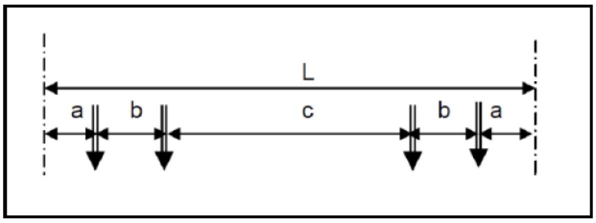


Fig. 12 Schematic of the vehicle of the considered train

Table 1 Values of the "normalized difference" in stresses for certain critical points under deck own weight and pre-stressing forces for a 25 m U-girder span

Point	Loading type	Mid-span section		4 m from support		2 m from support	
		F.E.	Beam Equ.	F.E.	Beam Equ.	F.E.	Beam Equ.
2 & 3	Own weight	0.30	0.0*	1.51	0.0*	5.68	0.0*
	Prestressing	0.06	0.0*	0.02	0.0*	0.10	0.0*

*Due to load symmetry with respect to the cross-section centerline

4. Results and data analysis of shear lag phenomenon on U-section bridge decks

The analysis is performed on different schemes of the case study U-section bridge considering various loading types and geometric parameters. The results presented in what follows address the difference between the stresses due to Bernoulli's beam theory and the stresses due to the finite element analysis. An index is also proposed to evaluate/quantify the extent of the shear lag effects on the U-section bridge. This index is called "Normalized Difference" and it is computed by calculating the difference between the values of the longitudinal normal stresses at the web-slab junction and at the middle of the slab spanning between webs hence normalizing this difference through dividing it by the value of the longitudinal stresses at the middle of the slab. This index is then used to assess the presence and the extent of shear lag effects under either eccentric or symmetric loading conditions.

4.1 Base case under symmetric loading conditions: deck own weight and pre-stressing forces

The overall shape of the U-section is defined as shown in (Figs. 3 and 4). The study of the shear lag effect is conducted on three transverse sections of the bridge as shown in Figs. 7(a)-(b) in terms of longitudinal normal stresses under two symmetric loading conditions: deck own weight and pre-stressing forces. The distribution of the pre-stressing cables within the cross section is as shown in Fig. 8 at mid-span and Fig. 9 at supports.
The pre-stressing data is as follows:

- Pre-stressing strands area is A_p =140 mm^2
- Pre-stressing force is 182 kN
- Elasticity modulus 195 GPa
- Friction coefficient (μ) 0.2
- Wobble coefficient (k) 0.005/m
- Anchorage slip is assumed 6 mm

Table 1 shows the values of the "normalized difference" for the longitudinal normal stresses at the point connecting the web with the deck slab (Point 2) and the point in the middle of the deck of the U-section (Point 3) as identified in Fig. 10.

By studying the "normalized difference" values in Table 1, it is clear that the own weight results in higher effects of shear lag than the pre-stressing forces. This means that the U-section is more prone to shear lag under own weight loading condition than under symmetric pre-stressing forces. Moreover, the shear lag effects increase by moving from mid-span sections to sections near supports.

4.2 Effect of span variation of the U-section bridge for different loading conditions

The U-section bridge is analyzed under different types of loads (symmetric versus eccentric loading configurations) for two span lengths: an actual medium span of 25 meters in Doha Metro project (Ayoub *et al.* (2017)) and a short span of 15 meters that could be used for specific requirements along the alignment of these railway bridges.

4.2.2 Symmetric loading (pre-stressing) and shear lag severity

Relevant pre-stressing forces are applied in a symmetric configuration to each of the two models spanning 15 meters and 25 meters.

By referring to Table 2, the "normalized difference" for the girder spanning 15 meters is 0.18 while for the girder spanning 25 meters is only 0.06 at points (2&3) identified in Fig. 10. So, the U-shape deck of the 15 meters span

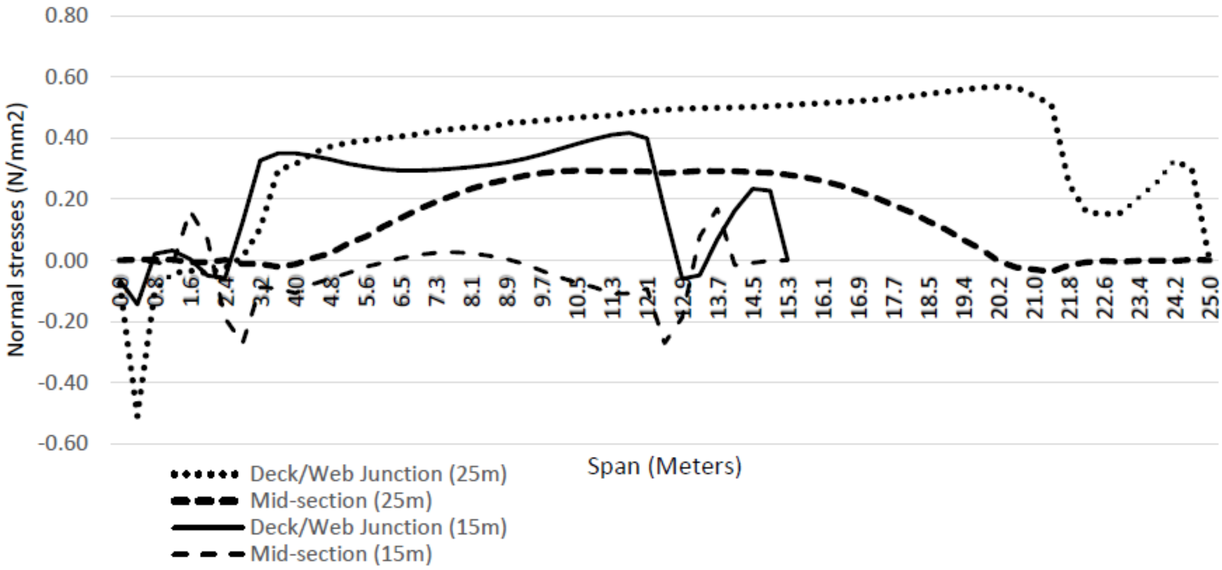


Fig. 13 Variation of longitudinal normal stresses along the span under eccentric moving vehicle

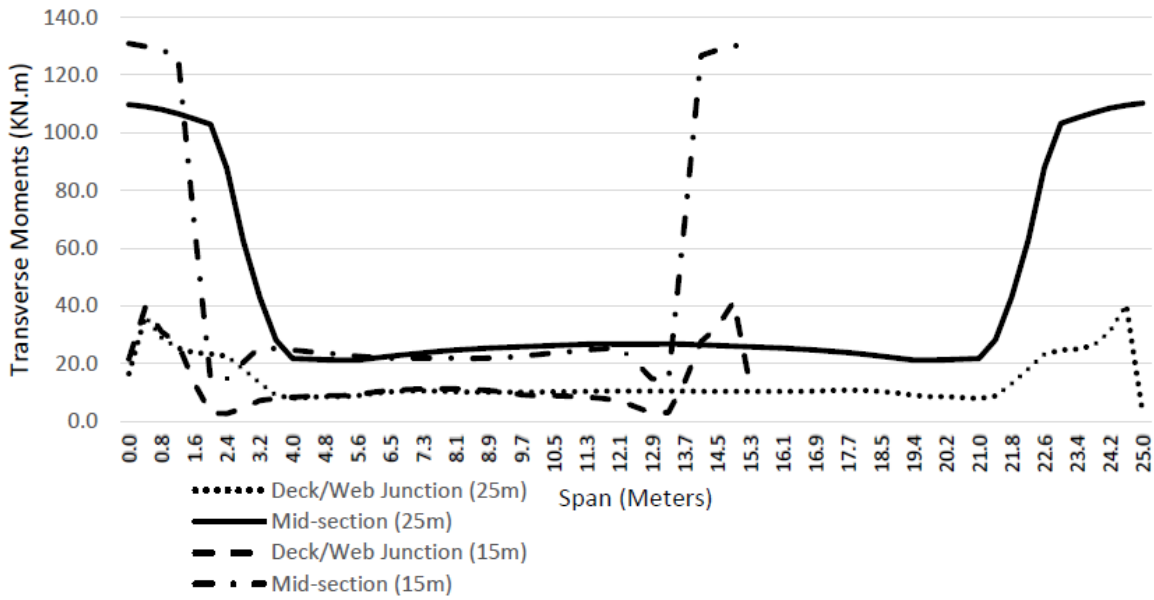


Fig. 14 Variation of transverse bending moments along the span under eccentric moving vehicle

Table 2 Values of the "normalized difference" in longitudinal normal stresses for certain critical points due to pre-stressing forces for 15 m and 25 m spans

Point	Loading type	Mid-span section		(3-4) m from support		(1-2) m from support	
		F.E.	Beam Equ.	F.E.	Beam Equ.	F.E.	Beam Equ.
2 & 3	Pre-stressing (15 m)	0.18	0.00	0.06	0.00	0.32	0.00
	Pre-stressing (25 m)	0.06	0.00	0.02	0.00	0.10	0.00

experiences more pronounced shear lag effects than the 25 meters span at the mid-span section. Also, studying the sections very close to supports (at a distance of 1-2 m) reveal that the 15 meters span is still more prone to shear lag effects than the 25 meters span.

For completeness, referring to Fig. 11 that reports results at the mid span of studied girders, the transverse bending moment in the deck slab of the bridge U-girder spanning 25 meters at the mid-slab section (2.89 kN.m) is significantly larger than that at the point close to the webs (1.74 kN.m). On the other hand, in the case of the U-girder spanning 15 meters, the transverse moment in the mid-slab section (1.36 kN.m) is lower than its value at the slab-web junction (1.53 kN.m). It may be thus concluded that increasing deck width-to-span ratio (comparing the 15m to the 25m schemes) leads to further concentration of transverse moments at the web-slab junction relative to the section at mid-slab.

4.2.3 Eccentric loading (moving train loads) and shear lag severity

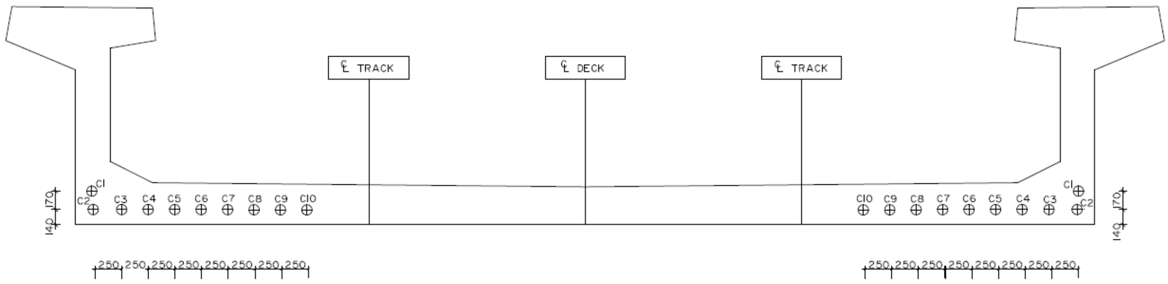


Fig. 15 Alternative pre-stressing cables layout at the mid-span

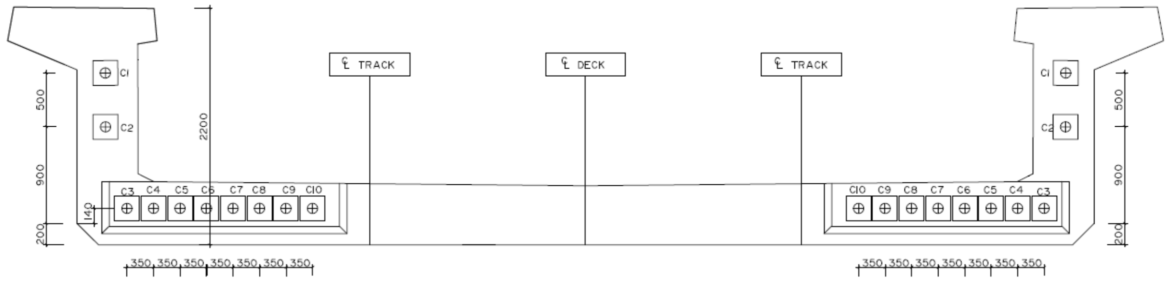


Fig. 16 Alternative pre-stressing cables layout at the supports

The bridge deck is subjected to a moving load as a single train vehicle (shown in Fig. 12) running on a single track (Track ‘1’ in Fig. 7) in an eccentric configuration. The same medium (25 m) and short (15 m) spans are again studied herein to assess the effect of span variation-if any-on the severity of shear lag under eccentric loading conditions.

Fig. 12 represents the dimensions notation for a moving train vehicle used in the present study. Specific loading and geometric data are as follows:

- Axle load = 160 kN
- L = 20 m (length of one train’s car/vehicle)
- a = 2.65m
- b = 2.1 m (wheel base in a bogie)
- c = 10.5 m (distance between Axle-2 and Axle-3 in the car)

This is representative of actual train vehicles recently used worldwide in various metro projects.

The diagram in Fig. 13 displays the longitudinal normal stresses in the U-shape double-track deck for the models spanning 25 meters and 15 meters subjected to an eccentric moving vehicle (single track loaded model). The studied points are: point 2 (at the web-deck slab junction) and point 3 (at mid-slab). The diagram shows that for the mid-span section (i.e., at 7.5 meters from supports), the model spanning 15.0 meters experiences large difference between the longitudinal normal stresses at the deck/web junction and the mid-deck slab section. The longitudinal stresses at the deck-web junction (0.276 N/mm^2) are significantly greater than those at mid-deck slab (0.027 N/mm^2). On the other hand, the U-girder spanning 25.0 meters at its mid span (i.e., 12.5 m from supports) has much smaller difference between the longitudinal normal stresses at the deck-web junction (0.453 N/mm^2) and mid-deck slab (0.287 N/mm^2).

In addition, by studying the section located at 1.0 meter from supports for the 15.0 meters span girder, the longitudinal normal stresses at the deck-web junction changed from tension of (+0.160 N/mm^2) to compression of (-0.013 N/mm^2) at mid-slab. Similarly, in the scheme spanning 25.0 meters, the stresses also changed from a tension value of (+0.161 N/mm^2) at the deck-web junction to compression at mid-slab but with a smaller value (-0.004 N/mm^2).

The transverse bending moments induced in the U-section bridge under the effect of eccentric moving vehicle are also captured along the span for the two studied girders (15 and 25 meters long) at the middle of the slab (point 3) and at the web/deck junction point (point 2) and are shown superimposed in Fig. 14 for comparison purposes. For the girder spanning 15 meters, the transverse moments at the mid-span section (i.e., at 7.5 meters from supports) were 21.8 kN.m at the middle of the slab and 11.2 kN.m at the web/deck junction; while for the 25 meters span model, the transverse moments at the middle span of the bridge (i.e., 12.5m from supports) changed from 25.8 kN.m at the middle of the slab to 10.3 kN.m at the web/deck junction. It is however worth reporting that at a given critical point within the U cross-section the variation in the transverse moment is very minor along most of the girder span (namely, about the middle 70% of the girder length).

By moving to the sections closer to the supports, for instance at 1.0 meter from the support in case of the girder spanning 15.0 meters, the moments changed from 127.6 kN.m at the middle of the slab to 28.9 kN.m at Point (2) which is the web-slab junction. On the other hand, the section at 2.0 meters from the supports for the girder spanning 25.0 meters, the transverse moments were 102.9 kN.m and 23.4 kN.m at points (3) and (2), respectively.

It could be hence noticed that under the effect of an eccentric moving load, the shear lag effect increases near the supports.

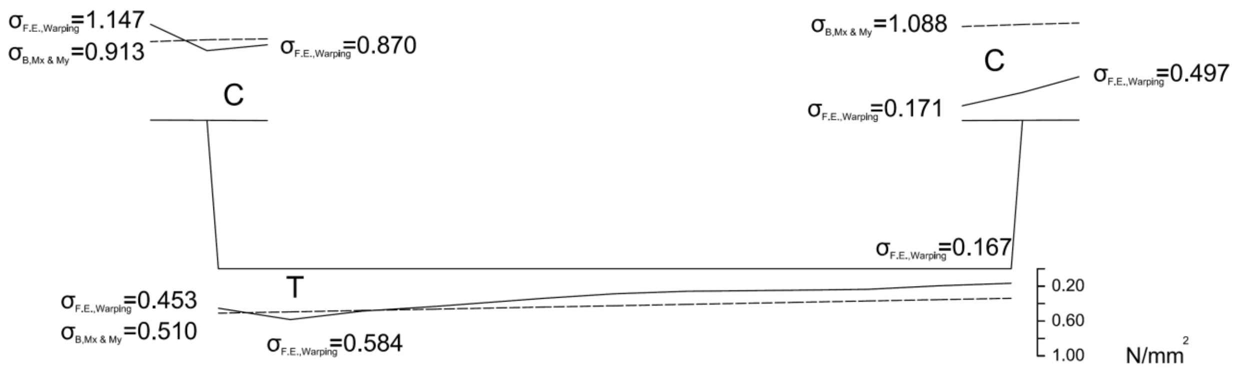


Fig. 17 Longitudinal normal stresses at mid-span due to eccentric moving train for span = 25 meters (base cross section)

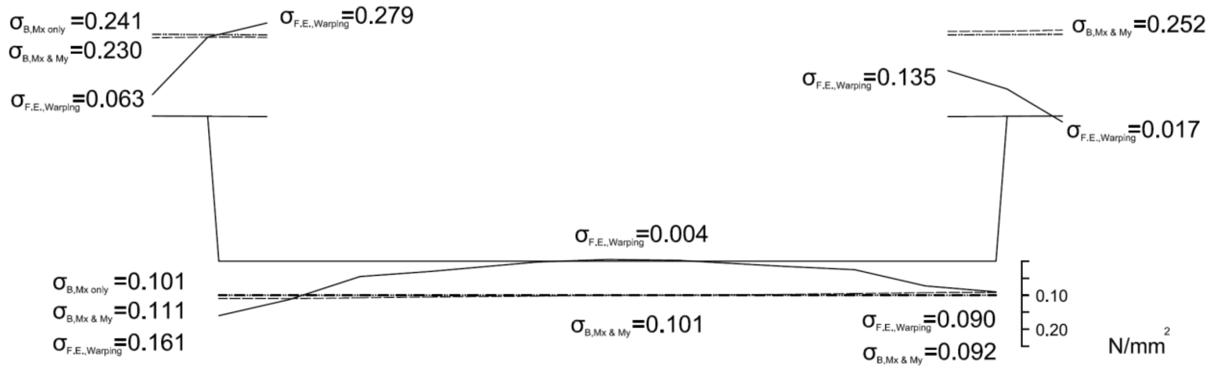


Fig. 18 Longitudinal normal stresses at supports due to eccentric moving train for span = 25 meters (base cross section)

Table 3 Values of the normalized difference in stresses for various pre-stressing cables layouts

Point	Loading type	Mid-span section		4 m from support		2 m from support	
		F.E.	Beam Equ.	F.E.	Beam Equ.	F.E.	Beam Equ.
2 & 3	Cables evenly distributed	0.06	0.0	0.02	0.0	0.09	0.0
	Cables closely spaced near webs	0.08	0.0	1.22	0.0	5.28	0.0

4.3 Effect of pre-stressing cables closely placed near webs

The bridge is investigated under symmetric pre-stressing forces taking into account the various layouts for the pre-stressing cables. Two different models are hence considered both having same cross section properties with respect to thickness of webs and slab and same pre-stressing forces and data. The two models also have the same span (namely, 25 meters) and same boundary conditions as per Fig. 6. Figs. 15 and 16 show the distribution of pre-stressing cables for the two different cables layouts. In Figs. 8 (section at mid span) and 9 (section at supports), the pre-stressing cables are quite evenly distributed in such a way that is covering most of the deck slab, while in Fig. 15 and Fig. 16, the pre-stressing cables are placed closer to the webs of the bridge U-section.

Changing the position of the pre-stressing cables changed the distribution of the resulting longitudinal normal stresses especially at the points near the web and hence influenced the shear lag effects.

The values of the "normalized difference" for the mid-span section for the two studied cases are nearly the same as per Table 3.

There is therefore almost no (or slight) difference in the response to the shear lag phenomenon at the mid-span section by changing the position of the pre-stressing cables (namely, by intensifying the cables in the deck slab near the webs). By moving outwards to the section at 4 meters from the supports, the "normalized difference" for the cables closely placed near the webs (1.22) is remarkably larger than that for the cables evenly distributed across the deck slab (0.02). Moreover, the "normalized difference" values become much significantly larger when further approaching the supports (5.28 versus 0.09 for cables layout closely spaced near webs and cables layout evenly distributed, respectively, at a section located 2m from the supports) as viewed in Table 3.

To conclude, and as anticipated by intuition, the U-section is more prone to shear lag effects near supports and the influence decreases by moving to the mid-span of the bridge. Moreover, the shear lag effects evolve and are much more pronounced by placing the pre-stressing cables in the U-section close to the webs than evenly distributing them across the bridge deck slab.

4.4 Effect of changing web and slab thicknesses of the bridge cross section on shear lag phenomenon

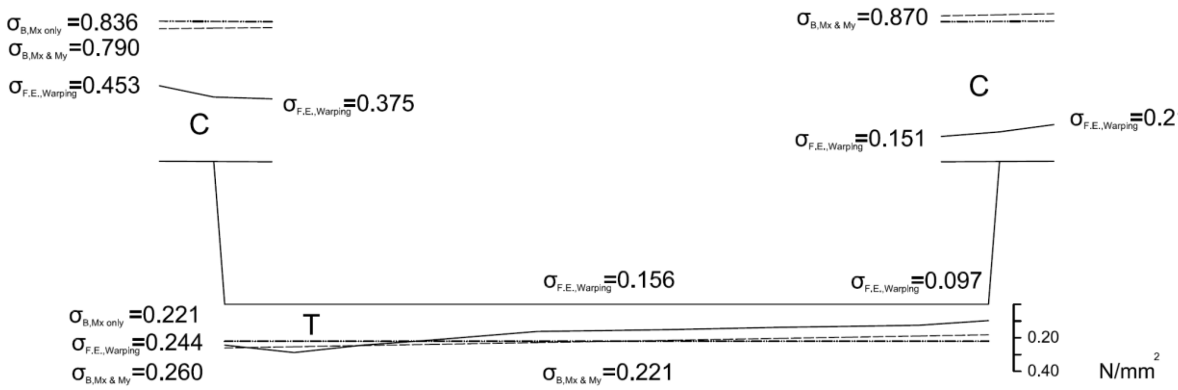


Fig. 19 Longitudinal normal stresses at mid-span due to eccentric moving train for span = 25 meters (thicker cross section)

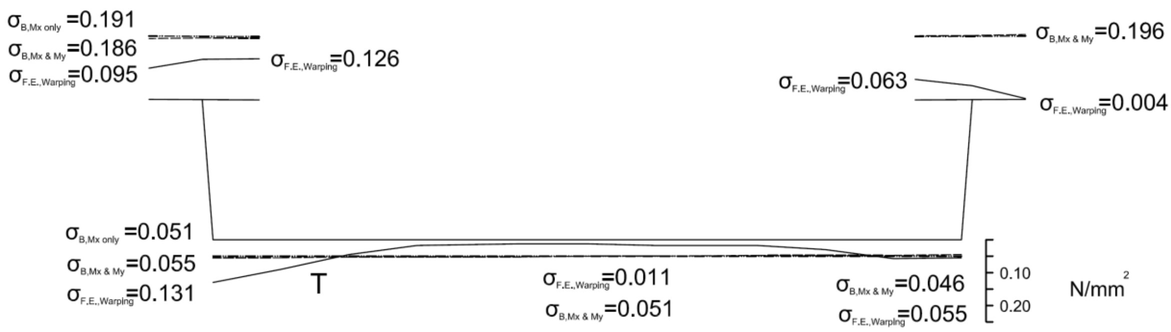


Fig. 20 Longitudinal normal stresses at supports due to eccentric moving train for span = 25 meters (thicker cross section)

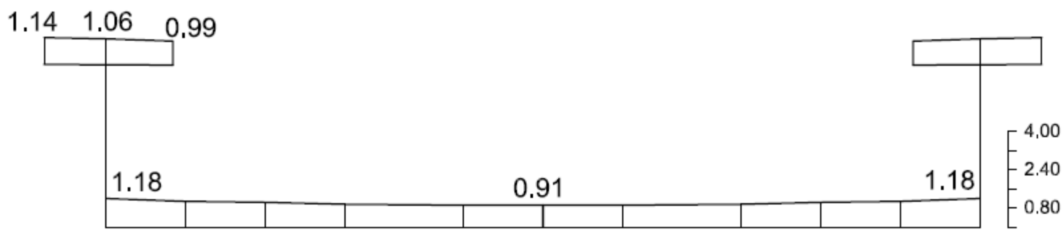


Fig. 21 Ratio between results of the two analysis methods at girder mid span due to own weight for U-section (Finite element results / Bernoulli's beam theory results)

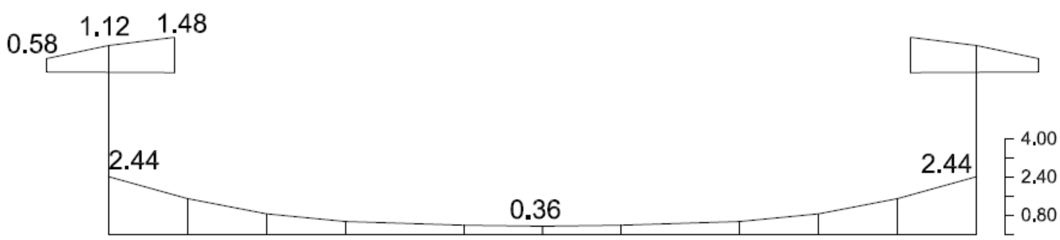


Fig. 22 Ratio between results of the two analysis methods at supports due to own weight for U-section (Finite element results / Bernoulli's beam theory results)

The cross section thicknesses of the base case U-girder under consideration have been changed so that the shear lag effect is studied under the effect of thickness variation. Two models are accordingly investigated both having the same span of 25 meters. The first cross section is the base case as shown in Fig. 3 for section at mid-span and Fig. 4 for section at supports. The second cross section is achieved by increasing the base U-section thicknesses (mid-span and at supports) by 300 mm. The typical thickness of the mid-span section is hence 600 mm all over the cross section while the

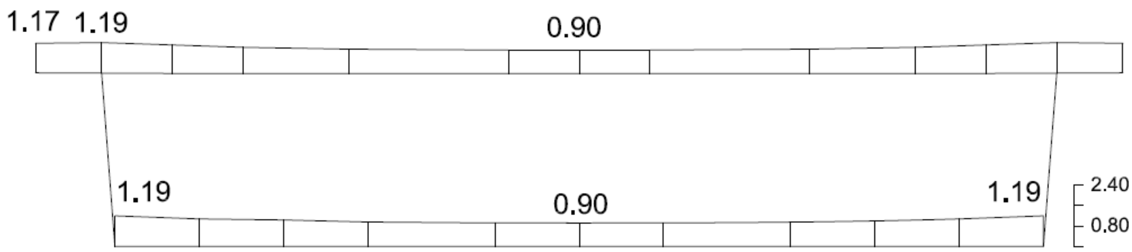


Fig. 23 Ratio between results of the two analysis methods at girder mid span due to own weight for box section (Finite element results / Bernoulli's beam theory results)

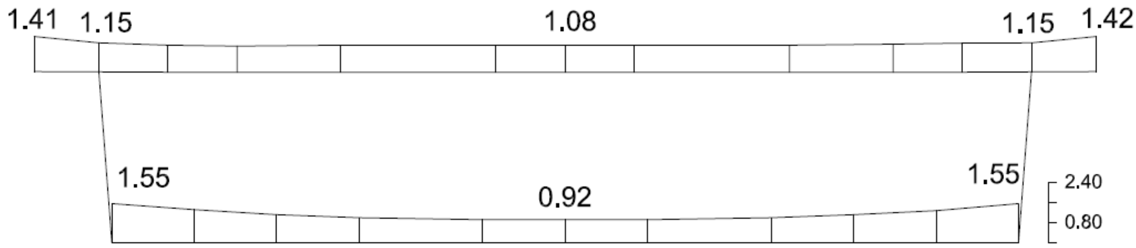


Fig. 24 Ratio between results of the two analysis methods at supports due to own weight for box section (Finite element results / Bernoulli's beam theory results)

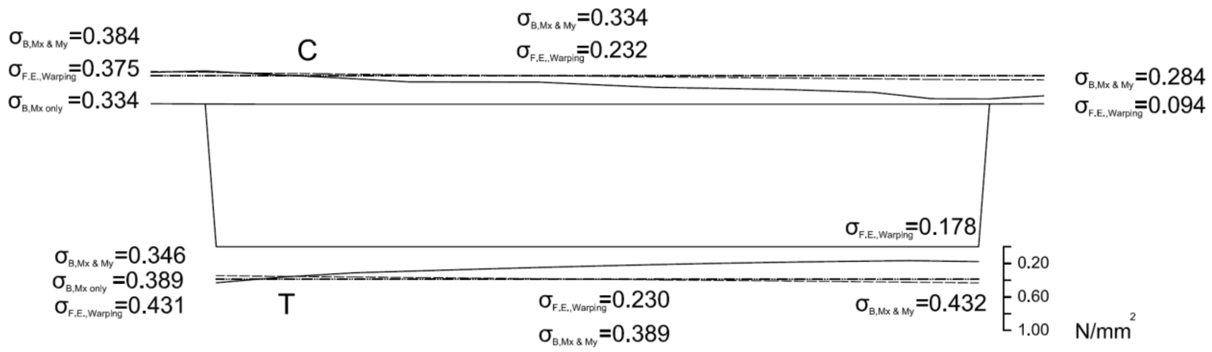


Fig. 25 Longitudinal normal stresses at mid-span due to eccentric moving train for the box section bridge

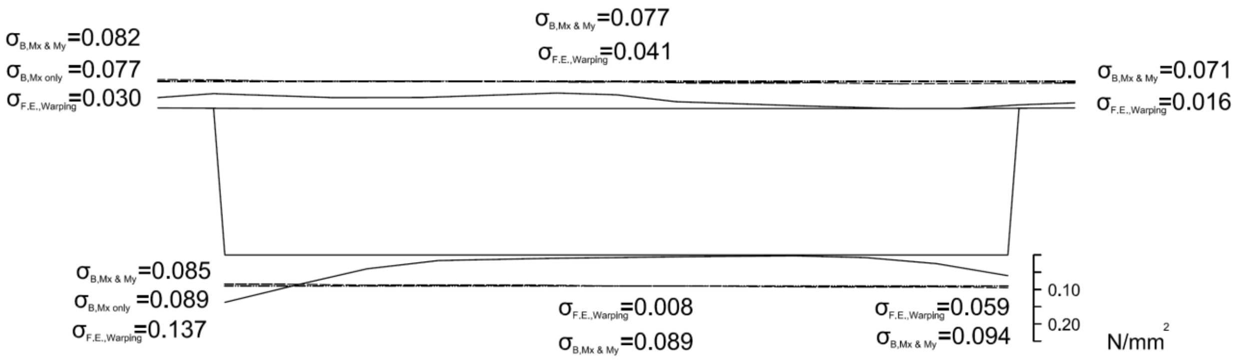


Fig. 26 Longitudinal normal stresses at supports due to eccentric moving train for the box section bridge

thickness for the U-section at supports is 800 mm for the whole section. The two schemes (the base and the thicker sections) are studied under the effect of an eccentric moving train vehicle on a single track (namely, train running on left track).

For the mid-span section of the base U-section as shown in Fig. 17, the longitudinal normal stresses due to the finite element analysis change from 0.453 N/mm^2 at Point (2) to 0.287 N/mm^2 at Point (3) with a "normalized difference" of 0.58 [=(0.453-0.287)/0.287], while the beam theory shows that the stresses change from 0.510 N/mm^2 at Point (2) to 0.425 N/mm^2 at Point (3) with a "normalized difference" of only 0.18. Such values of the "normalized difference" index imply the presence of shear lag effects within the U-section at mid-span of the base section scheme. Similarly, the analysis of the thicker U-section as per Figs. 19 shows that the longitudinal normal stress due to finite element analysis automatically capturing warping effects-if any-at Point (2) is 0.244 N/mm^2 and the stress value at Point (3) is 0.156 N/mm^2 with a "normalized difference" of 0.56 for the mid-span section. However, the beam theory shows that the stresses slightly changed from 0.260 N/mm^2 at Point (2) to 0.221 N/mm^2 at Point (3) with a "normalized difference" of 0.18. By inspecting these values, it is observed that the variation in response between the two models to the eccentric moving train vehicle on a single track at the two points under consideration within the U-section (namely, Points 2 and 3) is nearly the same for the mid span cross section. By moving to the section close to the supports, the finite element analysis of the base cross section as per Fig. 18 shows that the longitudinal normal stresses change from 0.161 N/mm^2 at Point (2) to -0.004 N/mm^2 at Point (3).

So, the stresses change from tension at the web-deck slab junction to compression at the middle of the deck slab. On the other hand, the beam theory assumes that the stresses slightly change only from 0.111 N/mm^2 at the web junction close to the moving load to 0.101 N/mm^2 at Point (3).

On the other hand, for the thicker cross section at the section near supports, the finite element analysis gives the longitudinal stresses at the web-deck slab junction and at the middle of deck slab as 0.131 N/mm^2 and 0.011 N/mm^2, respectively, as per Fig. 20; while the beam theory shows that the corresponding values are 0.055 N/mm^2 and 0.051 N/mm^2 at Points (2) and (3), respectively.

To conclude increasing the thickness of the cross section has a slight influence on the shear lag effects on the section at the mid-span of the bridge. Nonetheless, increasing the thickness led to an observable influence on decreasing the shear lag effects for the section close to the bridge girder supports.

4.5 Comparative response of the U-section bridge and the corresponding box section to shear lag effects

The previously defined and investigated U-section is modified-for comparative purposes-to a box section by connecting the compression flanges atop of the two webs through an upper deck slab as illustrated in Fig. 1. The model is then studied under the same loads formerly applied to the U-section deck; however, the moving train loads in the box section girder are applied to the top slab while in the U-section the loads are applied to the deck slab located at bottom of the U-section. This study is used to identify the difference in the response to the shear lag effects between the U-section bridge and a corresponding box section bridge. It is nevertheless worth mentioning that in real world cases a box section with the current dimensions (namely, the rather large span of the top slab between webs) shall be composed of two vents by introducing an inner web and not a single vent, which in turn would further reduce shear lag effects and distortion in the box girder relative to the U-girder with "open" (viz. distortion/warping-prone) section.

The two cross sections are studied first under their own weight then under eccentric moving train load in order to investigate the shear lag phenomenon.

By referring to Figs. 21 and 23, under the effect of the cross-section own weight, the value of the longitudinal normal stresses at the web-lower slab junction for the box section bridge model is slightly larger than that for the U-section bridge model by about 1% and the value at middle of the lower slab for the box section bridge is also very slightly smaller than that of the U-section bridge by 1%. So, there is practically no difference in the response to the shear lag phenomenon at the mid-span section by changing the bridge cross section from U to box under the effect of the girder's own weight. Moreover, by studying the web-top slab junction for the box section and the middle of the top slab of the box section, it is noted that the shear lag effect is exactly the same for the top slab and the bottom slab of the box section bridge under this uniform/symmetric loading condition.

For the section close to the supports with results illustrated in Figs. 22 and 24, the longitudinal normal stresses at Point (2), i.e., at web-deck slab junction, from the finite element analysis for the U-section bridge model are larger than those from the beam theory analysis and the ratio between them is 2.44. Also for the box section bridge, the stresses at the corresponding location (i.e., web-bottom slab junction) from the finite element analysis are still larger than those from the beam theory analysis, however assuming a smaller increase of only 1.55. Adopting the "normalized difference" in stresses criterion previously proposed, and focusing on the bottom slab of the box section, it could be noted that the normalized difference in longitudinal stresses between Points (2) and (3) is 0.69, while this normalized difference in longitudinal stresses for the U-section is remarkably much larger (5.69) which ascertains the explicitly more pronounced effect of the shear lag and warping on the "open" U-section than on the "closed" box section. If you apply the same "normalized difference" in stresses index to the top slab of the box section focusing on the web-top slab junction and the middle of the top slab, the value is about 0.31 showing that the shear lag effect is less pronounced in the top slab than at the bottom slab of the box section under this uniform and symmetric own weight loading. Thus, it has been demonstrated that under the effect of symmetric and uniformly distributed loading effects the open U-section is more prone to shear lag effects than the closed box section especially at sections closer to girder's supports.

The U-section bridge is then compared to its corresponding box section bridge under the effect of eccentric moving train vehicle but with a special focus on the behavior of the slab directly carrying the load (viz., the top slab of the box section). The values of the longitudinal normal stresses induced in the U-section bridge at the mid-span section and at the supports are previously illustrated in Figs 17 and 18, respectively, while the corresponding values for the box section bridge are presented in Figs 25 and 26.

The results of the longitudinal normal stresses from the finite element analysis at mid span of the U-girder bridge show that the "normalized difference" in stresses between

Points (2) and (3) is 0.58, while the beam theory shows that the corresponding value is only 0.20. For a better/suitable comparison, results are monitored at two other points for the box section bridge as the moving train load is applied to the top slab. These two points are: Point (1) at the web-top slab junction as per Fig. 10 and another point located at the middle of the top slab spanning between webs identified as Point (6) in what follows. Note that Point (6) is not shown in Fig. 10.

The results from the finite element analysis shows that the "normalized difference" in stresses of Points (1) (6) is 0.70, while the beam theory shows that the corresponding "normalized difference" is 0.13. So, there are more concentrated longitudinal stresses at the mid-span near the webs of the box section at top slab than near the webs of the U-section under eccentric moving train vehicles on the slab of the U-section bridge and on the top slab of the box section bridge. This conclusion would however be reversed if the box section under consideration has an inner web (i.e., a two-vent box as previously mentioned) to represent a real world design for such distance between webs which in turn will significantly reduce warping, distortion and shear lag effects.

For the section at the supports, the longitudinal normal stresses in the U-section bridge changed from tension at the web (Point 2) to compression at middle of the section (Point 3) and the "normalized difference" in stresses between them is about 41.25, while the beam theory shows that the normalized difference between the two points is only 0.10. On the other hand, for the box section bridge, the results from the finite element analysis shows that the "normalized difference" between the stresses of Points (1) and (6) is only 0.02, while the beam theory shows this "normalized difference" between the two points as 0.06. It is thus very obvious that for the section close to the supports, the "open" U-section bridge is subjected to larger longitudinal normal stresses at the webs due to the warping of the section (plane section does not remain plane after bending) relative to the "closed" box section even for the single-vent box with unrealistically large span of its top slab.

5. Conclusions

U-section bridges are used recently in light railway and metro lines as they offer many advantages. U-section bridges are typically composed of adjacent simply supported spans (such as the sample spans studied herein) except at few locations along the alignment where switch zones of the rail are located that necessitates having continuous spans. The material of the bridge in the current research is pre-stressed concrete and it is assumed to be linearly elastic. The bridge cross section is subjected to variations in load and geometric parameters. The geometric parameters used are: variation in the thickness of the cross section components and variation in span length. The loading parameters are: pre-stressing forces with different cables distribution, bridge own weight, and eccentric moving train loads. Models are defined as shell elements with different thicknesses.

In the present research, the study of the longitudinal and transverse behavior of the double-track U-section metro bridge composed of adjacent simply supported spans revealed the existence of distortion, warping and shear lag effects at different locations across the span of the bridge under symmetric and eccentric loading conditions. Among major conclusions of the present study are:

(i) Increasing width-to-span ratio leads to evolving shear lag effects, especially under eccentric loading conditions and at sections close to supports where warping effects are more pronounced;

(ii) Increasing the thickness of the cross section (namely, webs and deck slab) improves the properties of this primarily open (and accordingly deformation sensitive) U-section and hence shear lag effects decrease;

(iii) Shear lag effects increase when the pre-stressing cables are placed close to the webs of the U-section rather than evenly distributing them across the bridge bottom deck;

(iv) U-section is generally more prone to shear lag effects, warping and excessive deformations than its equivalent box section for various symmetric and eccentric load configurations especially at sections near girder's supports; and

(v) The use of three dimensional finite element analysis is highly recommended to accurately capture likely significant effects of shear lag, torsion and distortion (not accounted for in simplified beam analysis), especially under eccentric loading that occurs in wide U-girders such as in double track railway/metro bridges.

The current study thus offers to both researchers and practitioners in the field of railway bridge engineering some good insight into the longitudinal and transverse behavior of wide double-track U-section light rail bridges which have recently emerged due to their various potential advantages.

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