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The role of micromechanical models in the mechanical response of elastic foundation FG sandwich thick beams

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Abstract. This paper presents an analysis of the bending, buckling and free vibration of functionally graded sandwich beams resting on elastic foundation by using a refined quasi-3D theory in which both shear deformation and thickness stretching effects are included. The displacement field contains only three unknowns, which is less than the number of parameters of many other shear deformation theories. In order to homogenize the micromechanical properties of the FGM sandwich beam, the material properties are derived on the basis of several micromechanical models such as Tamura, Voigt, Reuss and many others. The principle of virtual works is used to obtain the equilibrium equations. The elastic foundation is modeled using the Pasternak mathematical model. The governing equations are obtained through the Hamilton's principle and then are solved via Navier solution for the simply supported beam. The accuracy of the proposed theory can be noticed by comparing it with other 3D solution available in the literature. A detailed parametric study is presented to show the influence of the micromechanical models on the general behavior of FG sandwich beams on elastic foundation.

Keywords: FG sandwich beam, micromechanical model; quasi 3D shear deformation theory; stretching effect; bending; buckling; free vibration; Pasternak foundation

1. Introduction

Functionally graded materials (FGMs) are a type of heterogeneous composite material in which the properties change gradually over one or more directions (Mantari 2015). They have been first proposed by Japanese scientists to decrease the thermal stresses in propulsion and airframe structural systems of astronautical flight vehicles (Koizumi 1997).

In recent years, there is a rapid increase in the use of functionally graded (FG) sandwich structures in various engineering applications such as aerospace, biomedical and civil engineering this is due to the main characteristic offered by these materials namely high strength-to-weight ratio. With the wide application of these structures, understanding behaviors of FG sandwich beams becomes an important task and more accurate theories are required to predict their bending, buckling and free vibration response.
Since the shear deformation influences are more found in thicker FGBs (functionally graded beams) three main theories that are first-order shear deformation theory, higher-order shear deformation theory and quasi-3D shear deformation theory have been employed by the researchers during the last decade (Armagan 2017).

Chakraborty *et al.* (2003) used the first order shear deformation theory to study the thermoelastic behaviour of FGM beam structures. Li (2008) gave a unified approach for analyzing static and dynamic behaviours of FGM Timoshenko and Euler–Bernoulli beams.

For a better accuracy and in order to take into account the transverse shear deformation, studies on FGM beams were performed based on the higher-order shear deformation beam theory (Wang and Li 2016). Based on this theory Tounsi and his co-worker have developed several models for studying the static and dynamic behavior of FGM structures (Bachir Bouiadjra *et al.* 2013, Fekrar *et al.* 2014, Zidi *et al.* 2014, Bousahla *et al.* 2014, Ait Yahia *et al.* 2015, Hamidi *et al.* 2015, Bourada *et al.* 2015, Bounouara *et al.* 2016).

Using an efficient third-order zigzag theory, Kapuria *et al.* (2008) have presented a finite element model for static and free vibration responses of layered FG beams and they estimated the effective modulus of elasticity, and its experimental validation for two different FGM systems under various boundary conditions.

Kadoli *et al.* (2008) have analyzed the static behavior of an FG beam by using higher-order shear deformation theory and finite element method. Sina *et al.* (2009) used a new

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beam theory different from the traditional first-order shear deformation beam theory to analyze the free vibration of FG beams. Simsek *et al.* (2009) studied the static analysis of an FG beam under uniformly distributed load within the framework of the higher-order shear deformation beam theory by Ritz method.

Recently, Tounsi and his co-workers (Hadji *et al.* 2011, Houari *et al.* 2011, Abdelaziz *et al.* 2011, Merdaci *et al.* 2011, Bourada *et al.* 2012, Bouberda *et al.* 2013, Taibi *et al.* 2015, Tounsi *et al.* 2013, Bousahla *et al.* 2016, Attia et al 2018, Belabed *et al.* 2018, Bellifa *et al.* 2017, Bennoun *et al.* 2016, Abdelaziz *et al.* 2017, Fourn *et al.* 2018) developed a refined theory for both plate and beam which considers only a few unknown variables and yet takes into consideration shear deformations. These theories are based on the idea of partitioning the vertical displacements into the bending and shear components. This theory was used for the study of bending response, thermo-mechanical, buckling and free vibration of simply supported FGM plate and sandwich plate.

Moreover, Bennai *et al.* (2015) presented a hyperbolic shear and normal deformation beam theory to study the vibration and buckling responses of FG sandwich beams under boundary conditions.

Vo *et al.* (2014) studied vibration and buckling of sandwich beams with FG skins - homogeneous core using a refined shear deformation theory.

It should be noted that the studies detailed above neglect the effect of stretching. Its effect becomes very important for thick beams. In order to include this effect, quasi-3D theories have been proposed.

By using this theory, many works have been developed but practically concern only the case of plates. We cite as an example the references (Benahmed *et al.* 2017, Mahmoudi *et al.* 2017, Hebali *et al.* 2014). The research on FG sandwich beams is limited.

Carrera *et al.* (2011) developed Carrera Unified Formulation (CUF) using various refined beam theories (polynomial, trigonometric, exponential and zig-zag), in which non-classical effects including the stretching effect were automatically taken into account.

On the other hand, Beam structures are often found to be resting on earth in various engineering applications. These include railway lines, geotechnical areas, building structures, offshore structures, and transversely supported pipe lines (Yas *et al.* 2017).

To describe the interactions of the beam and its foundation as appropriately as possible, scientists have proposed various kinds of foundation models (A.D. Kerr 1964). The simplest one is that of Winkler. Winkler (1867) presented a one-parameter model to describe the mechanical behavior of elastic foundations. This model does not take into account the coupling effects between the separated springs which was corrected later by Pasternak (1954) who added a shear layer as a parameter.

The manufacture of FGMs can be envisaged by mixing two discrete phases of materials, for example a separate mixture of a metal and a ceramic. Often, accurate information on the shape and distribution of particles may not be available. Thus, the properties of effective materials, such as modulus of elasticity, shear modulus; density, etc. are evaluated solely on the basis of the distribution of the volume fractions and the approximate shape of the dispersed phase. Several micromechanical models have been developed over the years to deduce the effective properties of macroscopically homogeneous composite materials (Jha *et al.* 2013). Among them, we can cite the models of Mori-Tanaka, Tamura, Reuss, Voigt, etc.

As far as authors known, in literature there is no work available using the quasi-3D theories to study bending, buckling and free vibration of FG sandwich beams resting on elastic foundation using various micromechanical models.

Thus, the present paper presents an analysis of the bending, buckling and free vibration of rectangular thick sandwich FG beam resting on elastic foundation by using a new quasi 3D shear deformation theory. The number of the unknowns evoked by the present theory is only three. The material properties of the beam through its thickness will be calculated using several micro-mechanical models.
The effect of these models on the overall response of the beam will be discussed in detail via a parametric study.

2. Effective properties of FGMs

FGMs are manufactured by mixing different material phases continuously through a specific spatial direction. A number of micromechanics models have been proposed for the determination of effective properties of FGMs. In the following, we present some micromechanical models to calculate the effective properties of the FG plate.

2.1 Voigt model

The simplest micromechanical model for obtaining equivalent macroscopic material properties is the mixing rule formulated by Voigt (1889). Voigt's idea was to determine the properties of the materials by averaging the stresses on all phases with the uniformity of deformation assumption in the material. The Voigt model, frequently used in most FGM analysis, estimates the Young's modulus as

$$E^{(n)}(z) = E_c V^{(n)}(z) + E_m\left(1 - V^{(n)}(z)\right) \tag{1}$$

E_c and E_m are respectively the Young modulus of the ceramic and the metal. $V^{(n)}$ is the volume fraction of each layer composing the sandwich beam.

2.2 Reuss model

Reuss assumed the stress uniformity through the material and obtained the effective properties as (Mishnaevsky 2007, Zimmerman 1994)

$$E^{(n)}(z) = \frac{E_c\, E_m}{E_c\left(1 - V^{(n)}(z)\right) + E_m V^{(n)}(z)} \tag{2}$$

2.3 Tamura model

The method of Tamura *et al.* (Tamura *et al.* 1973, Williamson *et al.* 1993) assumes a linear rule of mixing for the effective Poisson's ratio of a two-phase composite while incorporating an empirical adjustment parameter q_T (stress to strain transfer) in the formulation of the effective Young's modulus. The empirical parameter relates the stress and strain in the matrix and the particle phases (Akbarzadeh *et al.* 2015).

$$q_T = \frac{\sigma_1 - \sigma_2}{\varepsilon_1 - \varepsilon_2} \tag{3}$$

Estimate Estimate for $q_T = 0$ correspond to Reuss rule and with $q_T = \pm\infty$ to the Voigt rule, being invariant to the consideration of which phase is matrix and which is particulate. The effective Young's modulus is found as

$$E^{(n)}(z) = \frac{(1-V^{(n)}(z))E_m(q_T - E_i) + V^{(n)}(z)E_c(q_T - E_m)}{(1-V^{(n)}(z))(q_T - E_c) + V^{(n)}(z)E_c(q_T - E_m)} \tag{4}$$

2.4 Description by a representative volume element (LRVE)

The local representative volume element (LRVE) is based on a "mesoscopic" length scale which is much larger than the characteristic length scale of particles (inhomogeneities) but smaller than the characteristic length scale of a macroscopic specimen (Ju and Chen 1994). The LRVE is developed based on the assumption that the microstructure of the heterogeneous material is known. The input for the LRVE for the deterministic micromechanical framework is usually volume average or ensemble average of the descriptors of the microstructures.

Young's modulus is expressed as follows by the LRVE method (Akbarzadeh *et al.* 2015)

$$E^{(n)}(z) = E_m\left(1 + \frac{V^{(n)}(z)}{FE - \sqrt[3]{V^{(n)}(z)}}\right), \quad FE = \frac{1}{1 - \dfrac{E_m}{E_c}} \tag{5}$$

2.5 Mori-Tanaka model

According to Mori-Tanaka homogenization scheme, the Young's modulus is given by (Benveniste 1987, Mori and Tanaka 1973)

$$E^{(n)}(z) = E_m + (E_c - E_m)\left(\frac{V^{(n)}(z)}{1 + (1 - V^{(n)}(z))(E_c/E_m - 1)(1+\upsilon)/(3-3\upsilon)}\right) \tag{6}$$

The bottom and top sheets of the sandwich plate are composed of FG metal/ceramic material, which are graded from the metal at the bottom and top surfaces to the ceramic at the interfaces (see Fig. 1). While the core layer is assumed to be fully ceramic (hardcore). In this case, the volume fraction $V^{(n)}$ can be given

$$\begin{aligned} V^{(1)} &= \left(\frac{1+2z}{1+2h_1}\right)^p, \quad -h/2 \le z \le h_2, \\ V^{(2)} &= 1, \qquad\qquad h_2 \le z \le h_3, \\ V^{(3)} &= \left(\frac{1-2z}{1-2h_2}\right)^p, \quad h_3 \le z \le +h/2, \end{aligned} \tag{7}$$

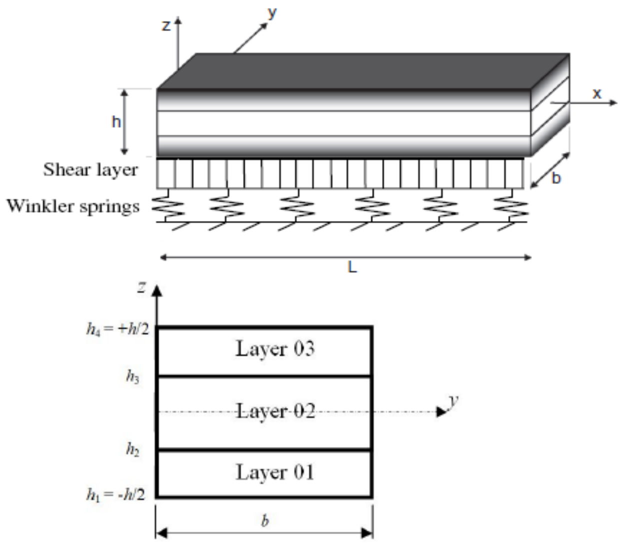

Fig. 1 Coordinate and geometry of a FG sandwich beam resting on elastic foundation

Where "*p*" is the power law index. Note that, when *p*=0, one obtains a fully homogeneous ceramic plate. Whereas, if $p \approx \infty$, a metal-ceramic-metal sandwich plate (m–c–m) is obtained.

The effective mass density ρ is given by the rule mixtures as (Yaghoobi and Torabi 2013, Bessaim *et al.* 2013, Benachour *et al.* 2011), regardless of the utilized micromechanical models

$$\rho^{(n)}(z) = \rho_m + (\rho_c - \rho_m)V^{(n)}(z) \tag{8}$$

3. Kinematics

The displacement field satisfying the conditions of transverse shear stresses (and hence strains) vanishing at (x, y, ±h/2) on the outer (top) and inner (bottom) surfaces of the beam, is given as follows (Zenkour 2013, Bachir Bouiadjra *et al.* 2018)

$$\begin{aligned} u\ (x,y,z,t) &= u_0 - z\frac{\partial w_b}{\partial x} + f(z)\frac{\partial w_s}{\partial x} \\ w(x,y,z,t) &= w_b + g(z)w_s \end{aligned} \qquad (9$$

With

$$\begin{aligned} f(z) &= z - \left(\frac{h}{\pi}\sinh\left(\frac{\pi z}{h}\right) - z\right)\left(\cosh\left(\frac{1}{2} - \pi\right) - 1\right) \\ g(z) &= rf'(z) \qquad (r = \frac{1.675}{15}) \end{aligned} \tag{10}$$

The value of the parameter "*r*" was properly selected to provide accurate results of deflection and stresses in a static medium.

Where u_0, w_b and w_s are the three unknown displacement functions of the middle surface of the beam.

The kinematic relations can be obtained as follows

$$\varepsilon_x = \varepsilon_x^0 + zk_x + f(z)\eta_x \quad , \quad \varepsilon_z = g'(z)\varepsilon_z^0$$
$$\gamma_{xz} = f'(z)\gamma_{xz}^0 + g(z)\gamma_{xz}^0 \tag{11}$$

Where

$$\varepsilon_x^0 = u_{0,x}, \quad \varepsilon_z^0 = w_s, \quad k_x = -w_{b,xx},$$
$$\eta_x = w_{s,xx}, \quad \gamma_{xz}^0 = w_{s,x} \tag{12}$$

It should be noted that the comma subscript is used for space derivative.

4. Constitutive relations

The linear constitutive relations are

$$\sigma_x = Q_{11}\varepsilon_x + Q_{13}\varepsilon_z,$$
$$\sigma_z = Q_{13}\varepsilon_x + Q_{33}\varepsilon_z, \tag{13}$$
$$\tau_{xz} = Q_{55}\gamma_{xz}$$

where ($\sigma_x, \sigma_z, \tau_{xz}$) and ($\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{yx}$) are the stress and strain components, respectively. The stiffness coefficients, Q_{ij} can be expressed as

$$Q_{11} = Q_{33} = \frac{E(z)}{1-\nu^2}, \tag{14a}$$
$$Q_{13} = \frac{\nu E(z)}{1-\nu^2}, \tag{14b}$$
$$Q_{55} = \frac{E(z)}{2(1-\nu)} \tag{14c}$$

5. Equations of motion

Hamilton's principle is used herein to derive the equations of motion. The principle can be stated in analytical form as

$$\int_{t_1}^{t_2} (\delta U + \delta U_F - \delta K + \delta W)dt = 0 \tag{15}$$

Where δU is the variation of strain energy; δK is the variation of kinetic energy; δW is the variation of work done, and δU_F is the variation of strain energy of foundation.

The variation of strain energy of the beam stated as

$$\delta U = \int\int (\sigma_x\, \delta\varepsilon_x + \sigma_z\, \delta\varepsilon_z + \tau_{xz}\, \delta\gamma_{xz})dx\, dz \tag{16}$$

Substituting Eqs. (11) and (13) into Eq. (16) and integrating through the thickness of the beam, we can obtain

$$\delta U = \int\left\{N_1\delta\varepsilon_x^0 + M_1\,\delta k_x + P_1\delta\eta_x + R_3\delta\varepsilon_z^0 + Q_5\delta\gamma_{xz}^0 + K_5\delta\gamma_{xz}^0\right\}dx \tag{17}$$

The stress resultants N, M, P, Q and R are defined by

$$(N_i, M_i, P_i) = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n} \sigma_i\,(1, z, f(z))dz, \quad (i=1) \tag{18a}$$
$$(K_i, Q_i) = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n} \sigma_i\left(f'(z), g(z)\right)dz, \quad (i=5) \tag{18b}$$
$$R_i = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n} \sigma_i g'(z)dz, \quad (i=3) \tag{18c}$$

Where h_n and h_{n-1} are the top and the bottom z-coordinates of the nth layer.

Using Eq. (13) in Eq. (18), the stress resultants of the FG beam can be related to the total strains by

$$N_i = A_{ij}\varepsilon_j^0 + B_{ij}k_j + C_{ij}\eta_j + F_{ij}\varepsilon_z^0, \quad (i=1)$$
$$M_i = B_{ij}\varepsilon_j^0 + G_{ij}k_j + H_{ij}\eta_j + K'_{ij}\varepsilon_z^0, \quad (i=1)$$
$$P_i = C_{ij}\varepsilon_j^0 + H_{ij}k_j + L_{ij}\eta_j + O_{ij}\varepsilon_z^0, \quad (i=1)$$
$$Q_i = Q'_{ij}\gamma_j^0 + P'_{ij}\gamma_j^0, \quad (i=5) \tag{19}$$
$$K_i = Q'_{ij}\gamma_j^0 + S_{ij}\gamma_j^0, \quad (i=5)$$
$$R_i = F_{ij}\varepsilon_j^0 + K'_{ij}k_j + O_{ij}\eta_j + U_{ij}\varepsilon_z^0, \quad (i=3)$$

where A_{ij}, B_{ij}, C_{ij}, etc., are the beam stiffness, defined by

$$(A_{ij}, B_{ij}, C_{ij}, F_{ij}) = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n} Q_{ij}\left(1, z, f(z), g'(z)\right)dz,$$
$$(G_{ij}, H_{ij}, K'_{ij}, L_{ij}) = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n} Q_{ij}\left(z^2, zf(z), zg'(z), f^2(z)\right)dz,$$
$$(O_{ij}, P'_{ij}, Q'_{ij}, S_{ij}) = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n} Q_{ij}\left(f(z)g'(z), g^2(z), g(z)f'(z), f'^2(z)\right)dz, \tag{20}$$
$$U_{ij} = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n} Q_{ij}\left(g'^2(z)\right)dz,$$

The variation of kinetic energy is expressed as

$$\delta K = \int\int (\rho(\dot{u}\,\delta\dot{u} + \dot{w}\,\delta\dot{w}))dx\, dz \tag{21}$$

$$\begin{aligned}\delta K = \int (&I_1\ddot{u}_0\delta u_0 + I_1\ddot{w}_b\delta w_b - I_6\ddot{w}_{s,xx}\delta w_s \\ &+ I_2(\ddot{u}_{0,x}\delta w_b - \ddot{w}_{b,x}\delta u_0) - I_3(\ddot{w}_{b,xx} + \ddot{w}_{b,xx})\delta w \\ &- I_4(\ddot{u}_{0,x}\delta w_s - \ddot{w}_{s,xx}\delta u_0) + I_5(\ddot{w}_{b,xx}\delta w_s + \ddot{w}_{s,xx}\delta w_b) \\ &+ I_7(\ddot{w}_b\delta w_s + \ddot{w}_s\delta w_b) + I_8\ddot{w}_s\delta w_s)\ \}dx\end{aligned} \tag{22}$$

Where the inertia term are defined by the following equations

$$(I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8) = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n} \rho(z)\left(1, z, z^2, f(z), z\,f(z), f^2(z), g(z), g^2(z)\right)dz \tag{23}$$

It should be noted that the dot subscript is used for time derivative.

The variation of work done can be expressed as

$$\delta W = -\int q\delta w dx + \int N_0 \left(\frac{\partial w}{\partial x}\frac{\partial \delta w}{\partial x}\right) dx \tag{24}$$

Where q is the transverse load and N_0 is the axial force.

The variation of strain energy of foundation is expressed as

$$\delta U_F = \int f_e \delta w dx \tag{25}$$

Where f_e is the density of reaction force of foundation. For the Pasternak foundation model, it is given by

$$f_e = k_w w - k_p \frac{\partial^2 w}{\partial x^2} \tag{26}$$

Where k_w is the Winkler foundation parameter and k_p is the Pasternak foundation parameter.

If the foundation is modeled as the linear Winkler foundation, the coefficient kp in Eq. (26) is zero.

Substituting the expressions for δU; δW; δK *and* δU_F from Eqs. (17), (22), (24) and (25) into Eq. (15) and integrating by parts and collecting the coefficients of u_0, w_b and w_s, the following equations of motion of the beam are obtained

$$\begin{aligned}
&\delta u_0 : N_{1,x} = I_1\ddot{u}_0 - I_2\ddot{w}_{b,x} + I_4\ddot{w}_{s,x} \\
&\delta w_b : M_{1,xx} + q + N_0\delta w_{,xx} - k_w(w_b - y^* w_s) \\
&\quad + k_p(w_{b,xx} - y^* w_{s,xx}) = I_1\ddot{w}_b + I_2\ddot{u}_{0,x} \\
&\quad - I_3\ddot{w}_{b,xx} + I_5\ddot{w}_{s,xx} + I_7\ddot{w}_s \\
&\delta w_s : -P_{1,xx} + Q_{5,x} + K_{5,x} - R_3 - y^* q + N_0\delta w_{,xx} \\
&\quad + y^* k_w(w_b - y^* w_s) - y^* k_p(w_{b,xx} - y^* w_{s,xx}) = -I_4\ddot{u}_{0,x} \\
&\quad + I_5\ddot{w}_{b,xx} - I_6\ddot{w}_{s,xx} + I_7\ddot{w}_b + I_8\ddot{w}_s
\end{aligned} \tag{27}$$

Where $y^*=-g(h/2)$.

6. Solution procedure

For the analytical solution of Eqs. (27), the Navier method is used under the specified boundary conditions. The displacement functions that satisfy the equations of simply supported FGM beam are selected as the following Fourier series

$$\begin{Bmatrix} u_0 \\ w_b \\ w_s \end{Bmatrix} = \sum_{m=1}^{\infty} \begin{Bmatrix} U_m \cos(\alpha x)e^{i\omega t} \\ W_{bm} \sin(\alpha x)e^{i\omega t} \\ W_{sm} \sin(\alpha x)e^{i\omega t} \end{Bmatrix} \tag{28}$$

Where U_m, W_{bm} and W_{sm} are arbitrary parameters to be determined, ω is the eigen frequency associated with m^{th} eigen mode, and $\alpha=m\pi/l$. The transverse load q is also expanded in Fourier series as

$$q = \sum_{m=1}^{\infty} q_m \sin(\alpha x) \tag{29}$$

Where $q_m=4q_0/m\pi$ (m=1, 3, 5, etc) for uniformly distributed load with density q_0.

Substituting Eqs. (28) and (29) into Eq. (27), the following problem is obtained

$$\left(\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{12} & k_{22} & k_{23} \\ k_{13} & k_{23} & k_{33} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{12} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{bmatrix}\right)\begin{Bmatrix} U_m \\ W_{bm} \\ W_{sm} \end{Bmatrix} = \begin{Bmatrix} 0 \\ q_m \\ -y^* q_m \end{Bmatrix} \tag{30}$$

Where $y^*=-g(h/2)$.

In which

$$\begin{aligned}
&k_{11} = -A_{11}\alpha^2, \\
&k_{12} = B_{11}\alpha^3, \\
&k_{13} = -C_{11}\alpha^3 + F_{13}\alpha, \\
&k_{22} = -G_{11}\alpha^4 - k_w - k_p\alpha^2 + N_0\alpha^2 \\
&k_{23} = H_{11}\alpha^4 - K'_{13}\alpha^2 + y^* k_w + y^* k_p\alpha^2 + g_0 N_0\alpha^2, \\
&k_{33} = -L_{11}\alpha^4 + 2O_{13}\alpha^2 - \left(Q'_{55} + P'_{55}\right)\alpha^2 - U_{33} \\
&\quad - y^{*2}(k_w + k_p\alpha^2) + g_0^2 N_0\alpha^2 \\
&m_{11} = -I_1, \quad m_{12} = I_2\alpha, \quad m_{13} = -I_4\alpha, \\
&m_{22} = -I_1 - I_3\alpha^2, \quad m_{23} = I_5\alpha^2 - I_7, \quad m_{33} = -I_6\alpha^2 - I_8
\end{aligned} \tag{31}$$

7. Numerical results and discussion

First, the results of the new refined quasi-3D method for different micromechanical models are compared with previous published works (Nguyen T.K. and Nguyen B.D. 2017) to check the accuracy of the model. After that, the effects of the different micromechanical models on the stress, central deflection and critical buckling load are investigated and discussed in detail.

The sandwich beam adopted here is considered to be Hardcore type with homogeneous core Alumina (AL2O3) and FG faces with top and bottom surfaces made from Aluminum (Al).

The material properties used are:

- Alumina, Al_2O_3: $E_c = 380\times10^9$ N/m^2; $\nu = 0.3$; $\rho_c = 3960$ kg/m^3.
- Aluminium, Al: $E_m = 70\times10^9$ N/m^2; $\nu = 0.3$; $\rho_m = 2702$ kg/m^3.

For convenience, the following non-dimensional parameters are used

$$\begin{aligned}
&\bar{\omega} = \frac{\omega l^2}{h}\sqrt{\frac{\rho_m}{E_m}}, \quad \bar{w} = w(l/2)\frac{384h^3E_m}{60q_0l^4}, \\
&\bar{\sigma}_x = \sigma_x(\frac{l}{2}, z)\frac{h}{q_0 l}, \quad \bar{\sigma}_{xz} = \sigma_{xz}(0, z)\frac{h}{q_0 l}, \\
&\bar{N}_{cr} = N_0\frac{12l^2}{E_m h^3}, \quad K_w = k_w\frac{12l^4}{E_m h^3}, \quad K_p = k_p\frac{12l^2}{E_m h^3}
\end{aligned} \tag{32}$$

Table 1 Non-dimensional central deflection $\overline{w}$ of Al/Al_2O_3 sandwich beam with homogenous Hardcore and without elastic foundation

l/h	*p*	Theory		Scheme					
				1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
5	0	Nguyen T.K. and Nguyen B.D (2017)		0.2026	0.2026	0.2026	0.2026	0.2026	0.2026
		Present	Voigt	0.2007	0.2007	0.2007	0.2007	0.2007	0.2007
			Reuss	0.2007	0.2007	0.2007	0.2007	0.2007	0.2007
			LRVE	0.2007	0.2007	0.2007	0.2007	0.2007	0.2007
			Tamura	0.2007	0.2007	0.2007	0.2007	0.2007	0.2007
			Mori-Tanaka	0.2007	0.2007	0.2007	0.2007	0.2007	0.2007
	0.5	Nguyen T.K. and Nguyen B.D (2017)		0.3539	0.3282	0.3172	0.3092	0.2964	0.2834
		Present	Voigt	0.3475	0.3224	0.3117	0.3039	0.2915	0.2790
			Reuss	0.5949	0.5245	0.4883	0.4682	0.4288	0.3923
			LRVE	0.4813	0.4296	0.4063	0.3909	0.3650	0.3398
			Tamura	0.4821	0.4317	0.4082	0.3933	0.3671	0.3420
			Mori-Tanaka	0.5287	0.4698	0.4412	0.4241	0.3926	0.3629
	1	Nguyen T.K. and Nguyen B.D (2017)		0.5014	0.4437	0.4189	0.4012	0.3738	0.3464
		Present	Voigt	0.4893	0.4327	0.4090	0.3918	0.3655	0.3392
			Reuss	0.7739	0.6717	0.6128	0.5841	0.5216	0.4652
			LRVE	0.6710	0.5789	0.5344	0.5073	0.4598	0.4148
			Tamura	0.6620	0.5743	0.5306	0.5052	0.4583	0.4144
			Mori-Tanaka	0.7112	0.6164	0.5663	0.5391	0.4856	0.4363
	2	Nguyen T.K. and Nguyen B.D (2017)		0.7258	0.6194	0.5689	0.5369	0.4837	0.4325
		Present	Voigt	0.7044	0.5993	0.5513	0.5199	0.4695	0.4206
			Reuss	0.9269	0.8078	0.7262	0.6940	0.6078	0.5327
			LRVE	0.8662	0.7433	0.6720	0.6363	0.5619	0.4942
			Tamura	0.8536	0.7347	0.6650	0.6309	0.5579	0.4919
			Mori-Tanaka	0.8875	0.7679	0.6928	0.6592	0.5803	0.5102
	5	Nguyen T.K. and Nguyen B.D (2017)		0.9714	0.8450	0.7568	0.7185	0.6267	0.5449
		Present	Voigt	0.9462	0.8139	0.7230	0.6912	0.6043	0.5262
			Reuss	1.0313	0.9144	0.8148	0.7887	0.6820	0.5934
			LRVE	1.0103	0.8887	0.7928	0.7613	0.6599	0.5734
			Tamura	1.0053	0.8838	0.7888	0.7576	0.6571	0.5714
			Mori-Tanaka	1.0174	0.8992	0.8011	0.7720	0.6686	0.5815
	10	Nguyen T.K. and Nguyen B.D (2017)		1.0425	0.9359	0.8329	0.8042	0.6943	0.6019
		Present	Voigt	1.0231	0.9022	0.8037	0.7725	0.6682	0.5798
			Reuss	1.0613	0.9473	0.8429	0.8228	0.7093	0.6176
			LRVE	1.0507	0.9360	0.8328	0.8092	0.6981	0.6069
			Tamura	1.0490	0.9340	0.8312	0.8074	0.6967	0.6058
			Mori-Tanaka	1.0547	0.9403	0.8367	0.8146	0.7026	0.6113

Several kinds of sandwich beam are presented according to the thickness of the core layer

-The (1-0-1) FGM sandwich beam: The beam is symmetric and made of only two equal-thickness FGM layers, that is, there is no core layer. Thus, $h_2 = h_3 = 0$.

-The (1-1-1) FGM sandwich beam: Here the beam is symmetric and made of three equal thickness layers. In this case, we have, $h_2 = -h/6$, $h_3 = h/6$.

-The (2-1-2) FGM sandwich beam: The beam is symmetric and we have: $h_2 = -h/10$, $h_3 = h/10$.

-The (2-2-1) FGM sandwich beam: The beam is non-symmetric and we have: $h_2 = -h/10$, $h_3 = 3h/10$.

-The (1-2-1) FGM sandwich beam: The beam is symmetric and we have $h_2 = -h/4$, $h_3 = h/4$.

7.1 Comparison studies

In order to demonstrate the validity of the present new

Table 2 Non-dimensional axial stress $\bar{\sigma}_x$ of Al/Al_2O_3 sandwich beam with homogenous Hardcore and without elastic foundation

l/h	p	Theory		Scheme					
				1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
5	0	Nguyen T.K. and Nguyen B.D (2017)		3.8022	3.8022	3.8022	3.8022	3.8022	3.8022
		Present	Voigt	3.7492	3.7492	3.7492	3.7492	3.7492	3.7492
			Reuss	3.7492	3.7492	3.7492	3.7492	3.7492	3.7492
			LRVE	3.7492	3.7492	3.7492	3.7492	3.7492	3.7492
			Tamura	3.7492	3.7492	3.7492	3.7492	3.7492	3.7492
			Mori-Tanaka	3.7492	3.7492	3.7492	3.7492	3.7492	3.7492
	0.5	Nguyen T.K. and Nguyen B.D (2017)		1.2547	1.1632	1.0699	1.0939	1.0036	0.9995
		Present	Voigt	1.2399	1.1496	1.1604	1.0809	0.9905	0.9870
			Reuss	2.1661	1.9252	1.9415	1.7165	1.4380	1.4262
			LRVE	1.7451	1.5610	1.5767	1.4165	1.2313	1.2216
			Tamura	1.7457	1.5681	1.5835	1.4254	1.2379	1.2301
			Mori-Tanaka	1.9198	1.7144	1.7305	1.5448	1.3208	1.3114
	1	Nguyen T.K. and Nguyen B.D (2017)		1.7967	1.5898	1.3885	1.4349	1.2475	1.2330
		Present	Voigt	1.7799	1.5748	1.5910	1.4206	1.2334	1.2119
			Reuss	2.8295	2.4956	2.5094	2.1706	1.7368	1.7126
			LRVE	2.4633	2.1403	2.1564	1.8713	1.5396	1.5147
			Tamura	2.4245	2.1208	2.1369	1.8620	1.5342	1.5129
			Mori-Tanaka	2.6043	2.2830	2.2983	1.9946	1.6219	1.5991
	2	Nguyen T.K. and Nguyen B.D (2017)		2.6195	2.2400	1.8476	1.9383	1.5874	1.5528
		Present	Voigt	2.5996	2.2223	2.2386	1.9222	1.5716	1.5377
			Reuss	3.3712	3.0212	3.0302	2.6024	2.0111	1.9792
			LRVE	3.1836	2.7788	2.7892	2.3781	1.8669	1.8276
			Tamura	3.1313	2.7434	2.7545	2.3562	1.8539	1.8183
			Mori-Tanaka	3.2449	2.8701	2.8802	2.4669	1.9247	1.8903
	5	Nguyen T.K. and Nguyen B.D (2017)		3.5001	3.0730	2.4070	2.6124	2.0195	1.9706
		Present	Voigt	3.4758	3.0550	3.0615	2.5954	2.0021	1.9543
			Reuss	3.6838	3.4235	3.4306	2.9729	2.2432	2.2194
			LRVE	3.6502	3.3333	3.3392	2.8681	2.1754	2.1406
			Tamura	3.6316	3.3136	3.3199	2.8530	2.1664	2.1326
			Mori-Tanaka	3.6574	3.3656	3.3722	2.9088	2.2021	2.1727
	10	Nguyen T.K. and Nguyen B.D (2017)		3.7235	3.4044	2.6296	2.9294	2.2200	2.1827
		Present	Voigt	3.6933	3.3859	3.3912	2.9124	2.0851	2.1661
			Reuss	3.7373	3.5412	3.5496	3.1046	2.3270	2.3151
			LRVE	3.7313	3.5040	3.5112	3.0532	2.2930	2.2728
			Tamura	3.7269	3.4963	3.5036	3.0459	2.2886	2.2683
			Mori-Tanaka	3.7321	3.5177	3.5255	3.0735	2.3066	2.2901

3D shear deformation theory, the results of FG sandwich beams computed by the present theory are compared with those obtained by Nguyen T.K. and Nguyen B.D (2017) using higher-order shear deformation without the thickness stretching effect $(\varepsilon_z = 0)$.

Tables 1-5 contain respectively dimensionless central deflection, dimensionless axial stress, dimensionless transverse stress, buckling load and fundamental frequencies for simply supported FG sandwich beams with hardcore and without elastic foundation.

The results of the present method are presented for five micromechanical models which are Voigt, Reuss, LRVE, Tamura and Mori-Tanaka.

In general, and for all cases studied, the results of the present method calculated by the Voigt model are in very good agreement with those of Nguyen T.K. and Nguyen

Table 3 Non-dimensional transverse shear stress $\bar{\sigma}_{xz}$ of Al/Al_2O_3 sandwich beam with homogenous Hardcore and without elastic foundation

l/h	p	Theory		Scheme					
				1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
5	0	Nguyen T.K. and Nguyen B.D (2017)		0.7350	0.7350	0.7350	0.7350	0.7350	0.7350
		Present	Voigt	0.7005	0.7005	0.7005	0.7005	0.7005	0.7005
			Reuss	0.7005	0.7005	0.7005	0.7005	0.7005	0.7005
			LRVE	0.7005	0.7005	0.7005	0.7005	0.7005	0.7005
			Tamura	0.7005	0.7005	0.7005	0.7005	0.7005	0.7005
			Mori-Tanaka	0.7005	0.7005	0.7005	0.7005	0.7005	0.7005
	0.5	Nguyen T.K. and Nguyen B.D (2017)		0.8959	0.8371	0.8354	0.8087	0.8032	0.7830
		Present	Voigt	0.9666	0.9010	0.8883	0.8635	0.8466	0.8218
			Reuss	1.4258	1.1765	1.1407	1.0610	1.0180	0.9514
			LRVE	1.1918	1.0465	1.0224	0.9718	0.9414	0.8955
			Tamura	1.2054	1.0539	1.0293	0.9768	0.9458	0.8985
			Mori-Tanaka	1.2930	1.1046	1.0756	1.0122	0.9764	0.9212
	1	Nguyen T.K. and Nguyen B.D (2017)		1.0349	0.9139	0.9106	0.8602	0.8496	0.8141
		Present	Voigt	1.1799	1.0407	1.0169	0.9685	0.9383	0.8938
			Reuss	1.7929	1.3512	1.2965	1.1727	1.1119	1.0171
			LRVE	1.5027	1.2186	1.1785	1.0903	1.0428	0.9702
			Tamura	1.5118	1.2222	1.1817	1.0921	1.0443	0.9709
			Mori-Tanaka	1.6262	1.2771	1.2308	1.1271	1.0738	0.9913
	2	Nguyen T.K. and Nguyen B.D (2017)		1.2664	1.0208	1.0152	0.9263	0.9091	0.8511
		Present	Voigt	1.5057	1.2223	1.1817	1.0939	1.0457	0.9733
			Reuss	2.2405	1.5233	1.4464	1.2727	1.1941	1.0712
			LRVE	1.9222	1.4070	1.3459	1.2076	1.1409	1.0376
			Tamura	1.9233	1.4069	1.3456	1.2071	1.1404	1.0370
			Mori-Tanaka	2.0562	1.4578	1.3899	1.2363	1.1644	1.0524
	5	Nguyen T.K. and Nguyen B.D (2017)		1.7725	1.1854	1.1755	1.0133	0.9873	0.8940
		Present	Voigt	2.1083	1.4782	1.4086	1.2493	1.1754	1.0607
			Reuss	2.8500	1.7056	1.6006	1.3683	1.2710	1.1182
			LRVE	2.5726	1.6286	1.5368	1.3296	1.2403	1.1003
			Tamura	2.5623	1.6257	1.5341	1.3281	1.2390	1.0994
			Mori-Tanaka	2.6880	1.6618	1.5643	1.3464	1.2536	1.1081
	10	Nguyen T.K. and Nguyen B.D (2017)		2.3128	1.3065	1.2888	1.0670	1.0347	0.9165
		Present	Voigt	2.6222	1.6426	1.5504	1.3371	1.1658	1.1050
			Reuss	3.2233	1.7967	1.6752	1.4124	1.3056	1.1383
			LRVE	3.0213	1.7485	1.6364	1.3895	1.2877	1.1281
			Tamura	3.0087	1.7456	1.6339	1.3880	1.2866	1.1275
			Mori-Tanaka	3.1051	1.7690	1.6529	1.3993	1.2954	1.1325

B.D. (2017). The slight difference which exists between the two methods for this case can be justified by the fact that the method developed by Nguyen T.K. and Nguyen B.D (2017) does not take into account the stretching effect. The latter becomes important in the case of thick beams. And since all results presented in these tables are for a ratio of "*l* / *h* = 5" (case of a thick beam) this explains this slight difference.

As for the other results calculated by the other micromechanical models, the difference can be explained by the way in which the Young's modulus is estimated.

7.3 Parametric studies

The impact of the micromechanical models on the estimated buckling load, the out-of-plane displacement and

Table 4 Non-dimensional critical buckling load $\bar{N}_{cr}$ of Al/Al_2O_3 sandwich beam with homogenous Hardcore and without elastic foundation

l/h	*p*	Theory		Scheme					
				1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
5	0	Nguyen T.K. and Nguyen B.D (2017)		48.5960	48.5960	48.5960	48.5960	48.5960	48.5960
			Voigt	48.4529	48.4529	48.4529	48.4529	48.4529	48.4529
			Reuss	48.4529	48.4529	48.4529	48.4529	48.4529	48.4529
		Present	LRVE	48.4529	48.4529	48.4529	48.4529	48.4529	48.4529
			Tamura	48.4529	48.4529	48.4529	48.4529	48.4529	48.4529
			Mori-Tanaka	48.4529	48.4529	48.4529	48.4529	48.4529	48.4529
	0.5	Nguyen T.K. and Nguyen B.D (2017)		27.8374	30.0141	31.0576	31.8649	33.2339	34.7551
			Voigt	28.0714	30.2564	31.2845	32.0940	33.4473	34.9399
			Reuss	16.4242	18.6409	20.0145	20.8796	22.7898	24.9016
		Present	LRVE	20.2938	22.7386	24.0345	24.9868	26.7501	28.7269
			Tamura	20.2598	22.6296	23.9263	24.8343	26.5970	28.5423
			Mori-Tanaka	18.4790	20.8039	22.1425	23.0414	24.8789	26.9084
	1	Nguyen T.K. and Nguyen B.D (2017)		19.6531	22.2113	23.5246	24.5598	26.3609	28.4444
			Voigt	19.9698	22.5798	23.8834	24.9350	26.7167	28.7824
			Reuss	12.6307	14.5706	15.9626	16.7553	18.7545	21.0206
		Present	LRVE	14.5716	16.8988	18.2998	19.2791	21.2598	23.5619
			Tamura	14.7680	17.0323	18.4280	19.3614	21.3304	23.5850
			Mori-Tanaka	13.7464	15.8729	17.2712	18.1476	20.1363	22.4043
	2	Nguyen T.K. and Nguyen B.D (2017)		13.5808	15.9158	17.3248	18.3591	20.3748	22.7862
			Voigt	13.8881	16.3311	17.7439	18.8171	20.8293	23.2434
			Reuss	10.5417	12.1220	13.4786	14.1132	16.1058	18.3715
		Present	LRVE	11.2887	13.1737	14.5651	15.3886	17.4166	19.7958
			Tamura	11.4547	13.3271	14.7157	15.5181	17.5385	19.8869
			Mori-Tanaka	11.0142	12.7508	14.1261	14.8504	16.8641	19.1788
	5	Nguyen T.K. and Nguyen B.D (2017)		10.1473	11.6685	13.0272	13.7218	15.7307	18.0914
			Voigt	10.3347	12.0349	13.4124	14.1725	16.2006	18.6006
			Reuss	9.4608	10.7103	12.0133	12.4241	14.3596	16.5010
		Present	LRVE	9.6659	11.0206	12.3493	12.8694	14.8390	17.0753
			Tamura	9.7139	11.0817	12.4104	12.9330	14.9021	17.1337
			Mori-Tanaka	9.5945	10.9037	12.2207	12.6914	14.6453	16.8361
	10	Nguyen T.K. and Nguyen B.D (2017)		9.4526	10.5356	11.8372	12.2611	14.1995	16.3787
			Voigt	9.5444	10.8567	12.1825	12.6852	14.6556	16.8872
			Reuss	9.1831	10.3363	11.6122	11.9095	13.8084	15.8563
		Present	LRVE	9.2818	10.4628	11.7544	12.1096	14.0297	16.1364
			Tamura	9.2967	10.4852	11.7772	12.1370	14.0579	16.1663
			Mori-Tanaka	9.2440	10.4143	11.6988	12.0293	13.9400	16.0204

the fundamental frequency of sandwich beams resting on elastic foundation is studied in this section.

In Fig. 2, the relative percentage difference of the buckling load between micromechanical models versus power law index *p* is presented.

The discrepancy between the estimated buckling load of sandwich beams by the Voigt, Reuss and other micromechanical models depends considerably on the power law index *p*.

The discrepancy between the Voigt model and other micromechanical models for the estimated values of the buckling load reaches a maximum of 32% between Voigt and Reuss and it is 27% between Voigt and Mori-Tanaka.

While between Voigt and other models namely LRVE and Tamura it exceeds 20%.

The second comparison shown in this figure is the discrepancy between the values of the buckling load

Table 5 Non-dimensional fundamental frequency $\bar{\omega}$ of Al/Al_2O_3 sandwich beam with homogenous Hardcore and without elastic foundation

l/h	p	Theory		1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
5	0	Nguyen T.K. and Nguyen B.D (2017)		5.1528	5.1528	5.1528	5.1528	5.1528	5.1528
		Present	Voigt	5.1646	5.1646	5.1646	5.1646	5.1646	5.1646
			Reuss	5.1646	5.1646	5.1646	5.1646	5.1646	5.1646
			LRVE	5.1646	5.1646	5.1646	5.1646	5.1646	5.1646
			Tamura	5.1646	5.1646	5.1646	5.1646	5.1646	5.1646
			Mori-Tanaka	5.1646	5.1646	5.1646	5.1646	5.1646	5.1646
	0.5	Nguyen T.K. and Nguyen B.D (2017)		4.1254	4.2340	4.2943	4.3294	4.4045	4.4791
		Present	Voigt	4.1557	4.2644	4.3237	4.3587	4.4329	4.5057
			Reuss	3.1767	3.3441	3.4548	3.5123	3.6554	3.8004
			LRVE	3.5316	3.6946	3.7873	3.8436	3.9619	4.0834
			Tamura	3.5287	3.6858	3.7788	3.8319	3.9505	4.0702
			Mori-Tanaka	3.3698	3.5334	3.6346	3.6904	3.8201	3.9514
	1	Nguyen T.K. and Nguyen B.D (2017)		3.5735	3.7298	3.8206	3.8756	3.9911	4.1105
		Present	Voigt	3.6123	3.7709	3.8606	3.9160	4.0296	4.1470
			Reuss	2.8723	3.0272	3.1536	3.2076	3.3732	3.5414
			LRVE	3.0848	3.2606	3.3773	3.4415	3.5925	3.7503
			Tamura	3.1057	3.2736	3.3893	3.4489	3.5985	3.7522
			Mori-Tanaka	2.9964	3.1599	3.2808	3.3386	3.4959	3.6566
	2	Nguyen T.K. and Nguyen B.D (2017)		3.0680	3.2365	3.3546	3.4190	3.5718	3.7334
		Present	Voigt	3.1105	3.2863	3.4034	3.4698	3.6207	3.7805
			Reuss	2.7112	2.8306	2.9651	3.0036	3.1820	3.3594
			LRVE	2.8047	2.9509	3.0825	3.1367	3.3095	3.4877
			Tamura	2.8254	2.9681	3.0985	3.1500	3.3211	3.4957
			Mori-Tanaka	2.7709	2.9031	3.0357	3.0816	3.2564	3.4327
	5	Nguyen T.K. and Nguyen B.D (2017)		2.7448	2.8440	2.9789	3.0181	3.1965	3.3771
		Present	Voigt	2.7776	2.8947	3.0297	3.0738	3.2514	3.4321
			Reuss	2.6600	2.7309	2.8674	2.8777	3.0604	3.2319
			LRVE	2.6877	2.7701	2.9071	2.9288	3.1112	3.2879
			Tamura	2.6944	2.7778	2.9144	2.9361	3.1179	3.2935
			Mori-Tanaka	2.6782	2.7554	2.8920	2.9085	3.0908	3.2647
	10	Nguyen T.K. and Nguyen B.D (2017)		2.6934	2.7356	2.8715	2.8809	3.0629	3.2357
		Present	Voigt	2.7149	2.7832	2.9200	2.9363	3.1186	3.2927
			Reuss	2.6651	2.7160	2.8512	2.8451	3.0270	3.1903
			LRVE	2.6787	2.7324	2.8685	2.8689	3.0512	3.2184
			Tamura	2.6808	2.7353	2.8712	2.8721	3.0543	3.2214
			Mori-Tanaka	2.6735	2.7261	2.8617	2.8594	3.0414	3.2068

between the Reuss model and other micromechanical models. The difference is insignificant between Reuss and Mori-Tanaka and it reached a maximum of 5% between Reuss and other models. The difference between Reuss and Mori-Tanaka is close to 9%. While between Reuss and Tamura on one side and Reuss and LRVE on the other side exceeds 15%.

It is also observed that the difference between the results obtained by the Reuss model and the other models becomes insignificant from a value of $p = 6$ and tends to 0 with the increase of p.

The relative Percentage difference of the out-of-plane displacement between micromechanical models versus power law index p is presented in Fig. 3.

The discrepancy between the Reuss model on one side and the Tamura and LRVE models on the other side is really insignificant and reaches a maximum of 14% for a material index value just under 1. Exceeding this value, the

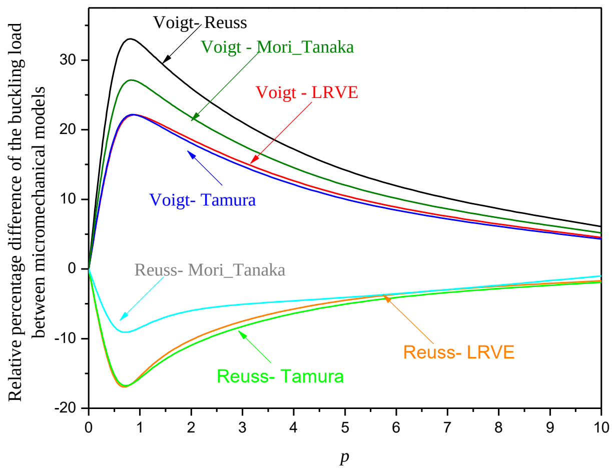

Fig. 2 Relative percentage difference of the buckling load between micromechanical models

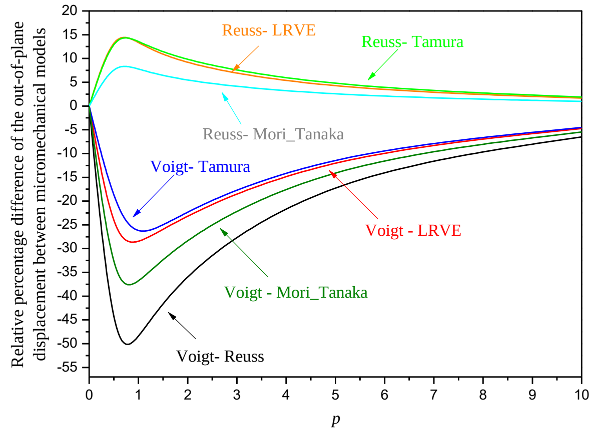

Fig. 3 Relative percentage difference of the out-of-plane displacement between micromechanical models

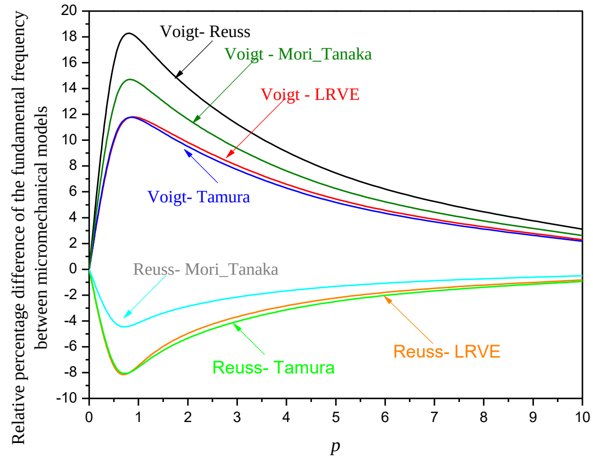

Fig. 4 Relative percentage difference of the fundamental frequency between micromechanical models

difference between these three models tends to 0 and they give practically the same results.

The second comparison between the Voigt model and the other models reveals that there is a big difference between the results of the different models, especially between Voigt and Reuss. Indeed, the difference between Voigt and Reuss reaches a maximum of 50% for a value of *p* just below unity. Then, exceeding this value, there is a rapid reduction in the difference in results between these two models. Also, the same observation is found between Voigt and Mori-Tanaka models, where the gap reaches 37%. For the other two remaining models (Tamura and LRVE), the maximum of the difference is also considerable but reaches a peak of 28%.

Fig. 4 shows the relative Percentage difference of the fundamental frequency between the different micromechanical models. For this case, the difference found between the different models is not considerable enough as in the case of displacement.

Indeed, the difference between Voigt model is the other models reaches a maximum of 18% for the case of comparison with Reuss, 14% for the difference with Mori-Tanaka and 11% with LRVE and Tamura.

The comparison between the Reuss model and the other models namely Mori-Tanaka, Tamura and LRVE is not considerable. It reaches a maximum of 8%.

Therefore, the need for appropriate micromechanical modeling of FGM is evident to accurately estimate mechanical properties.

The variation of out-of-plane displacement as a function of the power index "*p*" of a sandwich beam type 1-1-1 is shown in Fig. 5 for different micromechanical models and for both cases with and without elastic foundation. The first observation that can be drawn from this figure is that the presence of an elastic foundation type Pasternak greatly reduces the value of the maximum displacement. The second finding is that the Voigt model gives the smallest displacement values compared to the other models and that the Reuss model gives the highest results. In addition, there is a rapid variation of the displacement values for the low values of "*p*". But this variation tends to stabilize with the increase of "*p*". This observation is valid for both cases of the sandwich beam with and without foundation.

Fig. 6 depicts the variation of the fundamental frequency versus the power law index for different micromechanical models. The presence of an elastic foundation increases the values of the frequencies. It is to be noted that, in contrast to displacements, for this case the Voigt model gives the highest values of the fundamental frequency and that of Reuss the smallest. In addition, increasing the values of the power index "*p*" reduces the fundamental frequencies.

Fig. 7 contains plots of the buckling load of a FG sandwich beam (1-1-1) versus the power law index for different micromechanical models. The same statement established for Fig. 6 remains valid. Namely the model of Voigt presents the highest values of buckling load and that of Reuss gives the weakest.

In Fig. 8, we present the variation of the out-of-plane displacement $\overline{w}$ through the thickness direction for different values of the power index "*p*". It is found that the out-of-plane displacement $\overline{w}$ of metal plates is larger than the corresponding one of ceramic beam and in general, the transverse displacement increases as the power index "*p*" increases.

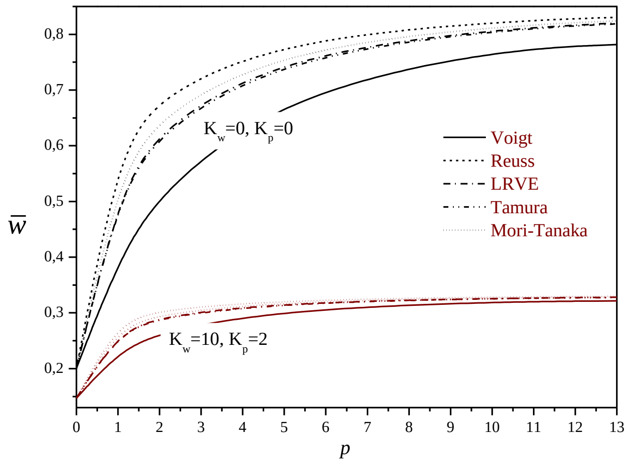

Fig. 5 Variation of the out-of-plane displacement versus the power law index for different micromechanical models (beam sandwich type 1-1-1, l/h=5)

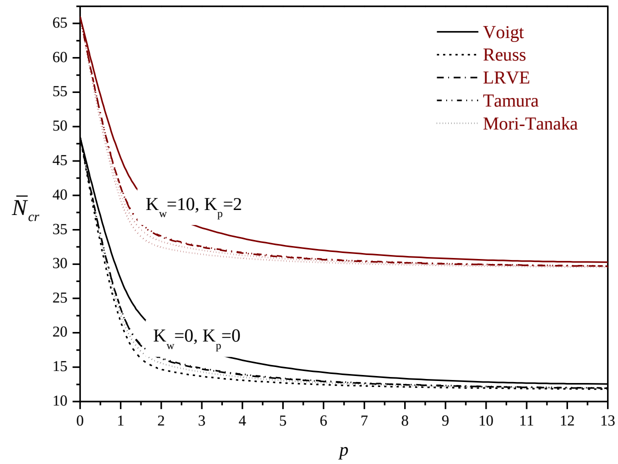

Fig. 7 Variation of the buckling load versus the power law index for different micromechanical models (beam sandwich type 1-1-1, l/h=5)

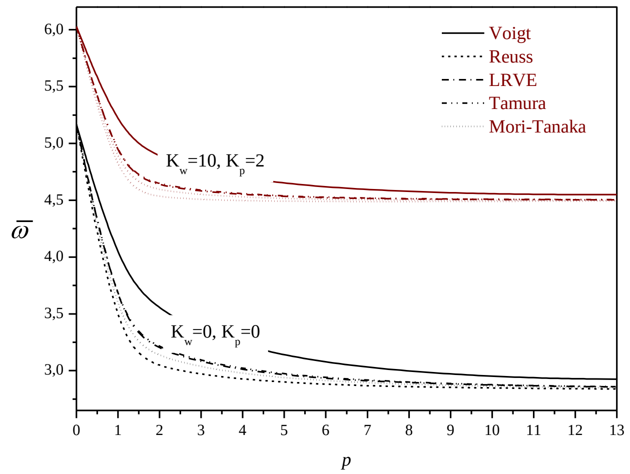

Fig. 6 Variation of the fundamental frequency versus the power law index for different micromechanical models (beam sandwich type 1-1-1, l/h=5)

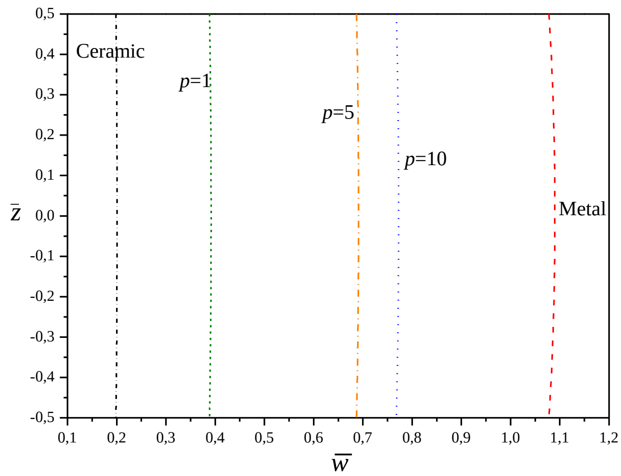

Fig. 8 The transverse displacement $\bar{w}$ through the thickness of FG sandwich beam

8. Conclusions

A new quasi 3D shear deformation theory was proposed to analyze the buckling, the bending and the free vibration of functionally graded sandwich thick beam resting on Winkler-Pasternak elastic foundations.

The theory accounts for parabolic distribution of the transverse shear strains and satisfies the zero traction boundary conditions on the surfaces of the functionally graded sandwich beam without using shear correction factors. The highlight of this theory is that, in addition to including the thickness stretching effect, the displacement field is modeled with only three unknowns, which is even less than the other shear and normal deformation theories where we find four or more variables.

Different micromechanical models were used to determine the effective properties of such sandwich FG beam. The equilibrium equations and associated boundary conditions of the beam are obtained using Hamilton's principle. The Navier method is used for the analytical solutions of the FG sandwich beam with simply supported boundary conditions. The results obtained using this new theory, are found to be in excellent agreement with previous studies.

Furthermore, the influences of micromechanical models on the bending, buckling and free vibration of sandwich thick beam have been comprehensively investigated. From these results and comparisons between different micromechanical models, it has been found significant differences between some models. This proves the need for a proper micromechanical modeling of FGMs to accurately estimate the general response of FG sandwich beam.

References

Abdelaziz, H.H., Atmane, H.A., Mechab, I., Boumia, L., Tounsi, A. and Adda Bedia, E.A. (2011), "Static analysis of functionally graded sandwich plates using an efficient and simple refined theory", *Chin. J. Aeronaut.*, **24**, 434-448.

Abdelaziz, H.H., Ait Amar Meziane, M., Bousahla, A.A., Tounsi, A., Mahmoud, S.R. and Alwabli, A.S. (2017), "An efficient hyperbolic shear deformation theory for bending, buckling and free vibration of FGM sandwich plates with various boundary conditions", *Steel Compos. Struct.*, **25**(6), 693-704.

Ait Yahia, S., Ait Atmane, H., Houari, M.S.A. and Tounsi, A. (2015), "Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories", *Struct. Eng. Mech, Int. J.*, **53**(6), 1143-1165.

Akbarzadeh, A.H., Abedini, A., Chen, Z.T. (2015), "Effect of micromechanical models on structural responses of functionally graded plates", *Compos. Struct.*, **119**, 598-609.

Armağan Karamanlı. (2017), "Bending behaviour of two directional functionally graded sandwich beams by using a quasi-3d shear deformation theory", *Compos. Struct.*, **174**, 70-68.

Attia, A., Bousahla, A.A., Tounsi, A., Mahmoud, S.R., Alwabli, A.S. (2018), "A refined four variable plate theory for thermoelastic analysis of FGM plates resting on variable elastic foundations", *Struct. Eng. Mech.*, **65**(4), 453-464.

Bachir Bouiadjra, R., Adda Bedia, E.A. and Tounsi, A. (2013), "Nonlinear thermal buckling behavior of functionally graded plates using an efficient sinusoidal shear deformation theory", *Struct. Eng. Mech.*, **48**(4), 547-567.

Bachir Bouiadjra, R., Mahmoudi, A., Benyoucef, S., Tounsi, A. and Bernard, F. (2018), "Analytical investigation of bending response of FGM plate using a new quasi 3D shear deformation theory: Effect of the micromechanical models", *Struct. Eng. Mech.*, **66**(3), 317-328.

Belabed, Z., Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2018), "A new 3-unknown hyperbolic shear deformation theory for vibration of functionally graded sandwich plate", *Earthq. Struct.*, **14**(2), 103-115.

Bellifa, H., Bakora, A., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2017), "An efficient and simple four variable refined plate theory for buckling analysis of functionally graded plates", *Steel Compos. Struct.*, **25**(3), 257-270.

Benachour, A., Daouadji, H. T., Ait Atmane, H., Tounsi, A. and Meftah, S.A. (2011), "A four variable refined plate theory for free vibrations of functionally graded plates with arbitrary gradient", *Compos. Part B-Eng.*, **42**, 1386-1394.

Benahmed, A., Houari, M.S.A., Benyoucef, S., Belakhdar, K. and Tounsi, A. (2017), "A novel quasi-3D hyperbolic shear deformation theory for functionally graded thick rectangular plates on elastic foundation", *Geomech. Eng.*, **12**(1), 9-34.

Bennai, R., Ait Atmane, H. and Tounsi, A. (2015), "A new higher-order shear and normal deformation theory for functionally graded sandwich beams", *Steel Compos. Struct.*, **19**(3), 521-546.

Bennoun, M., Houari, M.S.A. and Tounsi, A. (2016), "A novel five variable refined plate theory for vibration analysis of functionally graded sandwich plates", *Mech. Adv. Mater. Struct.*, **23**(4), 423-431.

Benveniste, Y. (1987), "A new approach to the application of Mori-Tanaka's theory in composite materials", *Mech. Mater.*, **6**, 147-157.

Bessaim, A., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Adda Bedia, E.A. (2013), "A new higher-order shear and normal deformation theory for the static and free vibration analysis of sandwich plates with functionally graded isotropic face sheets", *Sandw. Struct. Mater.*, **15**(6), 671-703.

Bouderba, B., Houari, M.S.A. and Tounsi, A. (2013), "Thermomechanical bending response of FGM thick plates resting on Winkler-Pasternak elastic foundations", *Steel. Compos. Struct.*, **14**(1), 85-104.

Bounouara, F., Benrahou, K.H., Belkorissat, I. and Tounsi, A. (2016), "A nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation", *Steel Compos. Struct.*, **20**(2), 227-249.

Bourada, M., Kaci, A., Houari, M.S.A. and Tounsi, A. (2015), "A new simple shear and normal deformations theory for functionally graded beams", *Compos. Struct.*, **18**(2), 409-423.

Bourada, M., Tounsi, A., Houari, M.S.A. and Adda Bedia, E.A. (2012), "A new four-variable refined plate theory for thermal buckling analysis of functionally graded sandwich plates", *J. Sandw. Struct. Mater.*, **14**(1), 5-33.

Bousahla, A.A., Benyoucef, S., Tounsi, A. and Mahmoud, S.R. (2016), "On thermal stability of plates with functionally graded coefficient of thermal expansion", *Struct. Eng. Mech.*, **60**(2), 313-335.

Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Adda Bedia, E.A. (2014), "A novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates", *Int. J. Computat. Meth.*, **11**(6), 1350082.

Carrera, E., Giunta, G. and Petrolo, M. (2011), *Beam Structures: Classical and Advanced Theories*, John Wiley & Sons.

Chakraborty, A., Gopalakrishnan, S. and Reddy, J.N. (2003), "A new beam finite element for the analysis of functionally graded materials", *Int. J. Mech. Sci.*, **45**(3), 519-539.

Fekrar, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2014), "A new five-unknown refined theory based on neutral surface position for bending analysis of exponential graded plates", *Meccan.*, **49**, 795-810.

Fourn, H., Ait Atmane, H., Bourada, M., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2018), "A novel four variable refined plate theory for wave propagation in functionally graded material plates|", *Steel Compos. Struct.*, **27**(1), 109-122.

Hadji, L., Atmane, H. A., Tounsi, A., Mechab, I. and Adda Bedia, E.A. (2011), "Free vibration of functionally graded sandwich plates using four variable refined plate theory", *Appl. Math. Mech.*, **32**, 925-942.

Hamidi, A., Houari, M.S.A., Mahmoud, S.R. and Tounsi, A. (2015), "A sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates", *Steel Compos. Struct.*, **18**(1), 235-253.

Hebali, H., Tounsi, A., Houari, M.S.A., Bessaim, A. and Adda Bedia, E.A. (2014), "New quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates", *J. Eng. Mech.*, **140**, 374-383.

Houari, M.S.A., Benyoucef, S., Mechab, I., Tounsi, A. and Adda Bedia, E.A. (2011), "Two variable refined plate theory for thermoelastic bending analysis of functionally graded sandwich plates", *J. Therm. Stress.*, **34**, 315-334.

Jha, D.K., Kant, T. and Singh, R.K. (2013), "critical review of recent research on functionally graded plates", *Compos. Struct.*, **96**, 833-849.

Ju, J. and Chen, T.M. (1994), "Micromechanics and effective moduli of elastic composites containing randomly dispersed ellipsoidal inhomogeneities", *Acta. Mech.*, **103**, 103-121.

Kadoli, R., Akhtar, K. and Ganesan, N. (2008), "Static analysis of functionally graded beams using higher order shear deformation theory", *Appl. Math. Mod.*, **32**(12), 2509-2525.

Kapuria, S., Bhattacharyya, M. and Kumar, A.N. (2008), "Bending and free vibration response of layered functionally graded beams: A theoretical model and its experimental validation", *Compos. Struct.*, **82**(3), 390-402.

Kerr, A.D. (1964), "Elastic and viscoelastic foundation models", *ASME J. Appl. Mech.*, **31**(3), 491-498.

Koizumi, M. (1993), "The concept of FGM", *Ceram. Trans. Funct. Gradient Mater.*, **34**, 3-10.

Li, X.F. (2008), "A unified approach for analyzing static and dynamic behaviors of functionally graded Timoshenko and Euler– Bernoulli beams", *J. Sound Vibr.*, **318**(4-5), 1210-1229.

Mahmoudi, A., Benyoucef, S., Tounsi, A., Benachour, A., Adda

Bedia, E.A. and Mahmoud, S.R. (2017), "A refined quasi-3D shear deformation theory for thermo-mechanical behavior of functionally graded sandwich plates on elastic foundations", *J. Sandw. Struct. Mater*., In Press.

Mantari, J.L. (2015), "A refined theory with stretching effect for the dynamics analysis of advanced composites on elastic foundation", *Mech. Mater*., **86**, 31-43.

Merdaci, S., Tounsi, A., Houari, M.S.A., Mechab, I., Hebali, H. and Benyoucef, S. (2011), "Two new refined shear displacement models for functionally graded sandwich plates", *Arch. Appl. Mech*., **81**, 1507-1522.

Mishnaevsky, Jr. L. (2007), *Computational Mesomechanics of Composites*, John Wiley & Sons, U.K.

Mori, T. and Tanaka, K. (1973), "Average stress in matrix and average elastic energy of materials with misfitting inclusions", *Acta. Metall.*, **21**, 571-574.

Nguyen, T.K. and Nguyen, B.D. (2017), "A new higher-order shear deformation theory for static, buckling and free vibration analysis of functionally graded sandwich beams", *J. Sandw. Struct. Mater.*, **17**(6), 613-631.

Pasternak, P.L. (1954), *On a New Method of Analysis of an Elastic Foundation by Means of Two Foundation Constants*, Gosudarstvennoe Izdatelstvo Literaturi po Stroitelstvu i Arkhitekture, Moscow, USSR, 1-56.

Simsek, M. and Kocaturk, T. (2009), "Free and forced vibration of a functionally graded beam subjected to a concentrated moving harmonic load", *Compos. Struct*., **90**(4), 465-473.

Sina, S.A., Navazi, H.M. and Haddadpour, H. (2009), "An analytical method for free vibration analysis of functionally graded beams", *Mater. Des*., **30**(3), 741-747.

Taibi, F.Z., Benyoucef, S., Tounsi, A., Bachir Bouiadjra, R., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "A simple shear deformation theory for thermo-mechanical behaviour of functionally graded sandwich plates on elastic foundations". *J. Sandw. Struct. Mater*., **17**(2) ,99-129.

Tamura, I., Tomota, Y. and Ozawa, M. (1973), "Strength and ductility of Fe-Ni-C alloys composed of austenite and martensite with various strength", *Proceedings of the 3rd International Conference on Strength of Metals and Alloys*, *Cambridge*, **1**, 611-615.

Tounsi, A., Houari, M.S.A., Benyoucef, S. and Adda Bedia, E.A. (2013), "A refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates", *Aerosp. Sci. Technol*., **24**(1), 209-220.

Vo, T.P., Thai, H.T., Nguyen, T.K., Maheri, A. and Lee, J. (2014), "Finite element model for vibration and buckling of functionally graded sandwich beams based on a refined shear deformation theory", *Eng. Struct.,* **64**, 12-22.

Voigt, W. (1889), "Über die beziehung zwischen den beiden elastizitätskonstanten isotroper körper", *Wied. Ann. Phys*., **38**, 573-587.

WANG, X., LI, S., (2016),"Free vibration analysis of functionally graded material beams based on Levinson beam theory", *Appl. Math. Mech. -Engl. Ed*., **37**(7), 861–878.

Williamson, RL., Rabin, BH., Drake, JT. (1993), "Finite element analysis of thermal residual stresses at graded ceramic–metal interfaces. Part I. Model description and geometrical effects", *J. Appl. Phys*. **74**: 1310–20.

Winkler, E. (1867), *Die Lehre von der Elasticitaet und Festigkeit*, *Prag, Dominicus*.

Yaghoobi, H. and Torabi, M. (2013), "Post-buckling and nonlinear free vibration analysis of geometrically imperfect functionally graded beams resting on nonlinear elastic foundation", *Appl. Math. Model*., **37**(18-19), 8324-8340.

Yas, M.H., Kamarian, S. and Pourasghar, A. (2017), "Free vibration analysis of functionally graded beams resting on variable elastic foundations using a generalized power-law distribution and GDQ method", *Ann. Sol. Struct. Mech.*

Zenkour, A.M. (2013), "Bending analysis of functionally graded sandwich plates using a simple four-unknown shear and normal deformations theory", *J. Sandw. Struct. Mater*, **15**(6), 629-659.

Zidi, M., Tounsi, A., Houari M.S.A., Adda Bedia, E.A. and Anwar Bég, O. (2014), "Bending analysis of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory", *Aerosp. Sci. Technol.*, **34**, 24-34.

Zimmerman, R.W. (1994), "Behavior of the Poisson ratio of a two-phase composite material in the high-concentration limit", *Appl. Mech. Rev*., **47**(1),38-44.

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