Structural Engineering and Mechanics, *Vol. 67, No. 6 (2018) 671-682*
DOI: https://doi.org/10.12989/sem.2018.67.6.671

The effect of mass eccentricity on the torsional response of building structures

George K. Georgoussis* and Anna Mamou[a]

Department of Civil Engineering Educators, School of Pedagogical and Technological Education (ASPETE), N. Heraklion 14121, Attica, Greece

(Received April 4, 2018, Revised May 25, 2018, Accepted July 10, 2018)

Abstract. The effect of earthquake induced torsion, due to mass eccentricities, is investigated with the objective of providing practical design guidelines for minimizing the torsional response of building structures. Current code provisions recommend performing three dimensional static or dynamic analyses, which involve shifting the centers of the floor masses from their nominal positions to what is called an accidental eccentricity. This procedure however may significantly increase the design cost of multistory buildings, due to the numerous possible spatial combinations of mass eccentricities and it is doubtful whether such a cost would be justifiable. This paper addresses this issue on a theoretical basis and investigates the torsional response of asymmetric multistory buildings in relation to their behavior when all floor masses lie on the same vertical line. This approach provides an insight on the overall seismic response of buildings and reveals how the torsional response of a structure is influenced by an arbitrary spatial combination of mass eccentricities. It also provides practical guidelines of how a structural configuration may be designed to sustain minor torsion, which is the main objective of any practicing engineer. A parametric study is presented on 9-story common building types having a mixed-type lateral load resisting system (frames, walls, coupled wall bents) and representative heightwise variations of accidental eccentricities.

Keywords: earthquake engineering; structural design; dynamics; mass eccentricity; modal analysis; optimum torsion axis

1. Introduction

Previous earthquakes have revealed that translational-torsional coupling may be a significant cause for severe damage in building structures. In the majority of seismic codes, in the cases in which the equivalent lateral load (static) analysis is recommended, the induced torsion is introduced by the following pair of design eccentricities

$$e_{d1} = \alpha e_s + \beta b \quad (1a)$$

$$e_{d2} = \gamma e_s - \beta b \quad (1b)$$

where e_s is the static (or inherent) eccentricity, which is defined as the distance between the centre of rigidity (CR) and the centre of mass (CM) and the term $\pm\beta b$ represents the accidental eccentricity, where b is the building plan dimension perpendicular to the direction of the ground motion. The coefficients α and γ are dynamic amplification factors, which are specified by individual country codes. In general, these factors vary from code to code (for example, the Greek seismic code EAK-2000 requires α=1.5 and γ=0.5, Eurocode 8 (EC-8/2004) requires α=γ=1.0, but in ASCE/SEI-7, α=1.0 and γ may take values less than unity when the static eccentricity exceeds the accidental one), while the term of the accidental eccentricity is usually taken as a percentage (5% to 10%) of the horizontal dimension b. The accidental eccentricity originates from many uncertainties:

(i) the variability of mass distribution, in plan and in elevation, which shifts the CM away from the geometrical center of the floor plan,

(ii) unforeseen variations in the dimensions and material properties of structural elements which, in combination with the contribution of non-structural elements (i.e., infill walls), may lead to stiffness variations

(iii) by possible rotational effects of the ground motion.

All these issues, along with the possible effect of strength unbalance, when the structure is expected to enter the inelastic phase of deformation, have been investigated during the last two decades with the objective of establishing code provisions that may adequately predict the torsional behavior of building structures. In the mid '90s De-La-Llera and Chopra (e.g., 1994a, 1994b, 1995) conducted extensive research on the effects of the accidental eccentricity on the static and dynamic response of single and multistory elastic structures, subjected to a purely translational ground motion. The main structural response parameter under investigation was the ratio of the uncoupled torsional to translational frequencies and it was demonstrated in De-La-Llera and Chopra (1994a) that stiffness uncertainties increased the structural deformations, to 10 and 5 per cent, for reinforced concrete and steel buildings respectively. In another study conducted by De-La-Llera and Chopra (1994b), the effect of mass

*Corresponding author, Professor
E-mail: ggeorgo@tee.gr
[a]Ph.D.
E-mail: a.p.mamou@gmail.com

http://www.techno-press.com/journals/sem&subpage=7
ISSN: 1225-4568 (Print), 1598-6217 (Online)

eccentricity on the torsional response of structures was examined and the results suggest, that the dynamic edge displacements and member forces were significantly higher that the equivalent static ones. It was demonstrated that the results of single-story systems may be also applicable to a specific class of multistory buildings in which the stiffness matrices of the lateral load resisting bents are proportional to each other (proportionate building). A new methodology for including the effects of all sources of accidental torsion was presented in De-La-Llera and Chopra (1995), which demonstrates significant advantages over the current code guidelines, by avoiding additional dynamic and static analyses to be performed to account for the effects of accidental torsion.

Experimental studies to assess the torsional component of ground motions have been conducted by De-La-Colina *et al.* (2013) and Wolf *et al.* (2014), while Sheikhabadi (2014) proposed a new equation to account for the effects of torsional earthquake components. An alternative approach for using accidental eccentricity concepts to account for torsional ground effects was proposed by Basu *et al.* (2014) and verified in simple linear and no-linear systems.

Questions about the benefits of using accidental eccentricities have been raised in the past, since their numerous spatial combinations in the structural design of multistory buildings, may substantially increase the computational analyses and the overall costs for practicing engineers. Detailed studies conducted by Stathopoulos and Anagnostopoulos (2010), Anagnostopoulos *et al.* (2015a, 2015b) and Bosco *et al.* (2015, 2017) on inelastic systems, based on the most recent design codes, indicated that the benefits of the inelastic analyses may be small, compared to designs where the accidental eccentricity was not taken into account. It was therefore suggested that accidental design eccentricities may not be taken into account or perhaps replaced by simpler design guidelines.

The main feature of Eqs. (1a), (1b) is that a reduced, or even negligible static eccentricity, implies lower values of design eccentricities and minor torsional effects, and it is well known that practicing engineers prefer rather symmetrical structural configurations, where the CM and the CR lie as close as possible. It is worth noting that when e_s=0, the design eccentricities of Eqs (1a), (1b) appear as static eccentricities, since the dynamic factors α and γ only act as amplifiers of the static eccentricity. In such cases (e_s=0), when the code provisions are based on Eqs. (1a), (1b), as for example in EAK-2000 and EC-8/2004, the dynamic effects are not taken into account. For this reason, it may be more reasonable to apply the dynamic amplification factor, α, to the accidental eccentricity only, as for example in ASCE/SEI-7, where this factor, denoted as A_x, may take values between 1 and 3. It should be noted that in all the aforementioned codes, β=0.05. Nevertheless, the reason for defining the design eccentricities in relation to the CR, is that in single-story systems with a rigid floor diaphragm, any lateral load passing through the CR causes only a translation of the slab and any torque applied on the slab causes a rotation about CR. This attribute of the CR enables the assessment of the severity of the torsional effects and for this reason, the CR is usually taken as the reference point to quantify these effects. Defining the CR in one-story systems may be a relatively straight forward procedure, as it represents the center of lateral stiffness, but its definition, in the various floors of a multistory building is not an equally straightforward task. Cheung and Tso (1986) defined the centers of rigidity (CRs) as a set of points located at the floor levels, such that when a given distribution of lateral loading passes through them only translational movement of the floors will occur. The centers of rigidity, are generally not located on the same vertical line and they are significantly scattered over the height of the building, even in the case of uniform systems with minor stiffness variations of the lateral load resisting elements. The centers of rigidity (CRs) are also load dependent and may therefore not be used as reference points to assess the torsional response of structures.

Acknowledging the deficiencies of using the CRs as a basis for assessing the torsional response of building structures, a number of researches (i.e., Makarios and Anastassiadis (1998a, 1998b), Marino and Rossi (2004), Basu and Jain (2007)) proposed alternative methods to define the reference points for implementing the torsional code provisions. The main objective of these studies was to determine the location of a vertical axis such that any in-plane lateral loading passing through this axis would minimize the torsional distortion on the structure. This, optimum torsion axis (OTA) may be defined as the axis passing through the stiffness centre of an equivalent single story system (Georgoussis 2016). This point is defined as the modal centre of rigidity (m-CR) and its derivation is based on the approximate method of the continuous medium, which assumes uniform over the height building systems. It was demonstrated that in systems where the centers of mass of the various floors lie on the same vertical line (mass axis), their dynamic response is essentially translational when the mass axis passes through the m-CR point (Georgoussis 2010, 2014, 2015). In a recent paper (Georgoussis 2017) it was also shown that the same equation for predicting the location of OTA can be derived by means of the discrete element approach (stiffness method), which is familiar to practicing engineers. The methodology, for defining the OTA, is applicable to, either regular (as specified for example by the current Eurocode-8 (EC-8/2004)) or irregular structures, provided that they have a single mass axis.

The objective of this study was to investigate the effect of mass eccentricities on the torsional response of medium height multistory buildings with in plan and in elevation irregularities, as defined in clauses 4.2.3.3 and 4.2.3.3 of EC-8/2004. Georgoussis (2017) showed that when the mass axis coincides with the OTA, this results in an essentially translational response, and such a structural configuration may be easily attained by a suitable arrangement of the lateral load resisting bents. This paper examines how such an arrangement can also be achieved when the centers of floor masses are shifted, in a random spatial way, from their nominal positions, but within the limits of the code ($\pm\beta b$). The effect of mass eccentricities on the torsional response of a building structure is studied using an analytical (mathematical) approach, based on the stiffness matrix methodology and, the derived analytical solution is compared with the results from parametric structural analyses on 9-story buildings with heightwise variations of the accidental eccentricities.

2. Systems considered and mathematical analysis

As discussed in the previous section, the effect of induced seismic torsion on the response of structures and the objective of mitigating the torsional effects during a strong ground motion have been researched extensively during the last decades. When designing structural configurations to sustain minimum torsional effects in the linear phase of deformation, the concept of the optimum torsion axis (OTA) may be a useful tool, but this concept applies only to multistory systems where all floor masses lie on the same vertical line (mass axis). The principle of the OTA design concept is that when the locations of the lateral load resisting bents define an OTA which coincides with the mass axis, the torsional response during a translational ground excitation is mitigated. In the case of multistory buildings, where the centers of masses are not aligned on a vertical axis, the concept of the OTA is not valid anymore. In this case in order to design a structural configuration with minimum torsional response a different approach needs to be applied, and the same requirement applies when the accidental mass eccentricity, as defined in the current design codes (e.g., EC-8/2004), is taken into account.

Assuming a N-story building with orthogonal framing along the global x and y directions and rigid floor slabs, which is subjected to the lateral load vector

$$\mathbf{F} = \langle \mathbf{f}_x \quad \mathbf{f}_y \quad \mathbf{f}_\theta \rangle^{\mathrm{T}} \tag{2a}$$

where the load sub-vectors $\mathbf{f}_x$, $\mathbf{f}_y$ and $\mathbf{f}_\theta$ are of N order. The equilibrium equation between forces and displacements is expressed by the following matrix equation

$$\begin{bmatrix} \mathbf{K}_{xx} & 0 & \mathbf{K}_{xz} \\ 0 & \mathbf{K}_{yy} & \mathbf{K}_{yz} \\ \mathbf{K}_{zx} & \mathbf{K}_{zy} & \mathbf{K}_{zz} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_x \\ \mathbf{u}_y \\ \mathbf{u}_\theta \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_x \\ \mathbf{f}_y \\ \mathbf{f}_\theta \end{Bmatrix} \tag{2b}$$

which displays a set of $3N$ equations for the displacement vectors $\mathbf{u}_x$ and $\mathbf{u}_y$, and the rotation vector $\mathbf{u}_\theta$ (all of order N) in an arbitrary coordination system Oxyz. For buildings with the lateral load resisting elements in two orthogonal directions, the sub-matrices of the above stiffness matrix are given as

$$\begin{gathered} \mathbf{K}_{xx}=\Sigma\mathbf{K}_i \\ \mathbf{K}_{yy}=\Sigma\mathbf{K}_j \\ \mathbf{K}_{xz}=\mathbf{K}_{zx}^{\mathrm{T}}=-\Sigma y_i\mathbf{K}_i \\ \mathbf{K}_{yz}=\mathbf{K}_{zy}^{\mathrm{T}}=\Sigma x_j\mathbf{K}_j \\ \mathbf{K}_{zz} = \Sigma x_j^2\mathbf{K}_j + \Sigma y_i^2\mathbf{K}_i \end{gathered} \tag{3}$$

where the element sub-matrices $\mathbf{K}_i$ and $\mathbf{K}_j$ (of order NxN) represent the stiffness matrices of the i-bent (oriented along the x-direction at a distance y_i from the x-reference axis) and j-bent (oriented along the y-direction at a distance x_j from y-reference axis) respectively. If the assumed structural building is subjected to a translational ground excitation along the y-direction, the requirement of a practically translational response implies that the applied lateral loads should be proportional to the first mode vector of the uncoupled structure, which is the dominant mode of vibration of a medium height building. That is

$$\mathbf{f}_y = \mathbf{M}\mathbf{\Phi}_{y1} = \frac{1}{\omega_{y1}^2}\mathbf{K}_{yy}\mathbf{\Phi}_{y1} \tag{4a}$$

where

$$\mathbf{M} = \begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & . & \\ & & & m_N \end{bmatrix} \tag{4b}$$

$\mathbf{M}$ is the diagonal mass matrix, m_1, m_2...m_N are the floor masses, numbered from the base upwards, and $\mathbf{\Phi}_{y1}$ and $\boldsymbol{\omega}_{y1}$ are the first modal shape and frequency of the uncoupled structure. In such a case, the floor components of the lateral load vector $\mathbf{f}_y$ are applied at the centers of mass of the various floors, which are located at a distance e_{mx1}, e_{mx2},... e_{mxN} from the vertical reference axis respectively. Therefore, since $\mathbf{f}_x=\mathbf{0}$, the torsional moment vector about this axis will be equal to

$$\mathbf{f}_\theta = \mathbf{E}_m\mathbf{f}_y \tag{5}$$

where

$$\mathbf{E}_m = \begin{bmatrix} e_{mx1} & & & \\ & e_{mx2} & & \\ & & . & \\ & & & e_{mxN} \end{bmatrix} \tag{6}$$

is the mass eccentricity matrix.

The displacement vectors $\mathbf{u}_x$, $\mathbf{u}_y$ and $\mathbf{u}_\theta$, derived from Eqs. (2b), are equal to

$$\mathbf{u}_x = -\mathbf{K}_{xx}^{-1}\mathbf{K}_{xz}\mathbf{u}_\theta \tag{7a}$$

$$\mathbf{u}_y = \mathbf{K}_{yy}^{-1}\mathbf{f}_y - \mathbf{K}_{yy}^{-1}\mathbf{K}_{yz}\mathbf{u}_\theta \tag{7b}$$

$$\mathbf{u}_\theta = \mathbf{K}_{\theta\theta}^{-1}(\mathbf{E}_m\mathbf{M}\mathbf{\Phi}_{y1} - \mathbf{K}_{zy}\mathbf{K}_{yy}^{-1}\mathbf{M}\mathbf{\Phi}_{y1}) \tag{7c}$$

where

$$\mathbf{K}_{\theta\theta} = \mathbf{K}_{zz} - \mathbf{K}_{zx}\mathbf{K}_{xx}^{-1}\mathbf{K}_{xz} - \mathbf{K}_{zy}\mathbf{K}_{yy}^{-1}\mathbf{K}_{yz} \tag{8}$$

The displacement vectors $\mathbf{u}_x$, $\mathbf{u}_y$ and $\mathbf{u}_\theta$ shown above are obtained by superpositioning two loading conditions. The first is the response of the uncoupled structure (i.e., the system in which the floors are restrained against rotations) when it is subjected to a lateral force vector equal to $\mathbf{f}_y$. This loading case provides the first term of the second part of Eq. (7b). The second loading case constitutes a purely torsional moment vector, equal to

$$\mathbf{T}_\theta = \mathbf{E}_m\mathbf{M}\mathbf{\Phi}_{y1} - \mathbf{K}_{zy}\mathbf{K}_{yy}^{-1}\mathbf{M}\mathbf{\Phi}_{y1} \tag{9}$$

and its effect on the response of the uncoupled structure may be accounted for as a superposition of the effects of its modal components, $\mathbf{T}_{\theta n}$ (n=1,2,...). This modal expansion of $\mathbf{T}_\theta$ is obtained as follows (Chopra 2008)

$$\mathbf{T}_\theta = \Sigma\mathbf{T}_{\theta n} = \Sigma\Gamma_n\mathbf{M}\mathbf{\Phi}_{yn}\text{ , }(n=1,2,..) \tag{10}$$

where $\mathbf{\Phi}_{yn}$ is the n-mode shape of the uncoupled structure and the corresponding modal participation factor Γ_n is equal to

$$\Gamma_n = \frac{\mathbf{\Phi}_{yn}^T\mathbf{T}_\theta}{\mathbf{\Phi}_{yn}^T\mathbf{M}\mathbf{\Phi}_{yn}} \tag{11}$$

If the first modal participation factor is set equal to zero (Γ_1=0), this implies that the first modal component of $\mathbf{T}_\theta$, which represents its major contribution on the response of the assumed structure, is nullified and therefore the overall torsional effect is minimized. The condition of Γ_1=0, in combination with Eqs. (4a) and (9), implies that

$$\mathbf{\Phi}_{y1}^T\mathbf{E}_m\mathbf{M}\mathbf{\Phi}_{y1} = \mathbf{\Phi}_{y1}^T\mathbf{K}_{zy}\mathbf{K}_{yy}^{-1}\mathbf{M}\mathbf{\Phi}_{y1} = \frac{1}{\omega_{y1}^2}\Sigma x_j\mathbf{\Phi}_{y1}^T\mathbf{K}_j\mathbf{\Phi}_{y1} \tag{12a}$$

and any structural configuration (any arrangement of the lateral load resisting bents) that satisfies the above equation is expected to undergo minimum torsional response. Dividing all parts of Eq. (12a) by the first mode generalized mass $\mathbf{\Phi}_{y1}^T\mathbf{M}\mathbf{\Phi}_{y1}$, the following expression is obtained

$$\frac{\mathbf{\Phi}_{y1}^T\mathbf{E}_m\mathbf{M}\mathbf{\Phi}_{y1}}{\mathbf{\Phi}_{y1}^T\mathbf{M}\mathbf{\Phi}_{y1}} = \frac{1}{\omega_{y1}^2}\Sigma x_j\frac{\mathbf{\Phi}_{y1}^T\mathbf{K}_j\mathbf{\Phi}_{y1}}{\mathbf{\Phi}_{y1}^T\mathbf{M}\mathbf{\Phi}_{y1}} \tag{12b}$$

An approximate expression of Eq. (12b) may be given by using the concept of the element frequency of the lateral load resisting elements (Georgoussis 2016). For example, for the j-element (bent), its element frequency, ω_{j1}, is defined as

$$\omega_{j1}^2 = \frac{\mathbf{\Phi}_{j1}^T\mathbf{K}_j\mathbf{\Phi}_{j1}}{\mathbf{\Phi}_{j1}^T\mathbf{M}\mathbf{\Phi}_{j1}} \tag{13}$$

and represents the first mode frequency of the j-lateral load resisting element, when it is assumed to carry, as a plane frame, the mass per floor of the real structure ($\mathbf{\Phi}_{j1}$ is the first mode vector of the particular j-bent). It is worth noting that a lower bound of the first mode frequency of the uncouple structure, ω_{y1}, may be evaluated by means of the element frequencies, according to Southwell's formula (Newmark and Rosenblueth 1971). For example, if the lateral stiffness in the y-direction is composed by a number of k-elements, then

$$\omega_{y1}^2 \approx \sum_{j=1}^{k}\omega_{j1}^2 \tag{14}$$

The equation above is based on the potential of the Rayleigh's quotients, according to which any approximate first mode shape vector may provide a reasonable estimate of the first mode frequency. This is particularly true in building structures, which belong to the same class of shear-flexural cantilever systems and have similar mode shapes. Replacing each modal vector $\mathbf{\Phi}_{y1}$with $\mathbf{\Phi}_{j1}$, in the second part of Eq. (12b), this equation takes the form

$$\frac{\mathbf{\Phi}_{y1}^T\mathbf{E}_m\mathbf{M}\mathbf{\Phi}_{y1}}{\mathbf{\Phi}_{y1}^T\mathbf{M}\mathbf{\Phi}_{y1}} = \frac{1}{\omega_{y1}^2}\Sigma x_j\omega_{j1}^2 \tag{15}$$

where the fundamental frequency of the uncoupled system, ω_{y1}, may also be assessed from Eq. (14).

When all the floor masses are aligned on a vertical (mass) axis, i.e.,

$$e_{mx1} = e_{mx2} = \ldots = e_{mxN} = e_{mx} \tag{16a}$$

which suggests that the eccentricity matrix of Eq. (6) is equal to

$$\mathbf{E}_m = e_{mx}\mathbf{I} \tag{16b}$$

where $\mathbf{I}$ is the unit matrix, Eq. (15) takes the form

$$e_{mx} = \frac{\Sigma x_j\omega_{j1}^2}{\omega_{y1}^2} \tag{17a}$$

The last part of Eq. (17a) defines the OTA (Georgoussis 2017) and, when the OTA coincides with the mass axis, the torsional response is minimized. By definition, the coordinates of the OTA depend on the structural element frequencies and different structural configurations may result in OTA axes that coincide with the mass axis. When the mass axis is taken as the reference vertical axis, the condition of minimum torsional response requires that the arrangement of the various bents should satisfy the following equation

$$\Sigma x_j\omega_{j1}^2 = 0 \tag{17b}$$

In structural applications, however, where the arrangement of most of the lateral load resisting bents is determined by architectural or functional considerations, the structural engineer needs to relocate one or two of such bents in order to obtain a structural configuration, where the OTA coincides with the mass axis. In practice, it is convenient to construct such a system by locating a particular element (denoted as key element and assumed to be the k-element, when the lateral resistance is provided by a number of k bents), in a way that

$$\sum_{j=1}^{k}x_j\omega_{j1}^2 = \sum_{j=1}^{k-1}x_j\omega_{j1}^2 + x_k\omega_{k1}^2 = 0 \tag{18}$$

In buildings where the centers of floor masses are not aligned on a vertical line, the vertical axis of the reference system can be conveniently taken as the axis passing through the center of the total mass of the building structure. That is, with respect to the initially assumed Oxyz coordination system, the x-coordinate, e_{mx}, of the vertical reference axis may be defined as

$$e_{mx}M_{tot} = \Sigma e_{mxn}m_n\text{ , }n=1,2,...\text{N} \tag{19}$$

where M_{tot} is the total mass of the building ($=m_1+m_2+...+m_N$). The new vertical (reference) axis, defined by Eq. (19), has the advantage that when the gravity loads are uniformly distributed on all the floors and when their centroids are located on the same vertical line, the aforementioned axis is the mass axis of the system. In such a case, when the code accidental eccentricities are neglected, any structural configuration that satisfies Eq. (17b) defines a system of minimum torsional response. When however, the accidental eccentricities are taken into account, the optimum location of the k-element may be determined by evaluating the first term of Eq. (15). A convenient method to evaluate this part of Eq. (15) is to express the matrix product $\mathbf{E}_m\mathbf{M}$ as

$$\mathbf{E}_m\mathbf{M} = \begin{bmatrix} e_{mx1}m_1 & & & \\ & e_{mx2}m_2 & & \\ & & \cdot & \\ & & & e_{mxN}m_N \end{bmatrix} = e_{mxo}\left(\mathbf{M} - \mathbf{M}_f\right) \quad (20a)$$

where $\mathbf{M}_f$ is a matrix defined as

$$\mathbf{M}_f = \begin{bmatrix} (1-\frac{e_{mx1}}{e_{mxo}})m_1 & & & \\ & (1-\frac{e_{mx2}}{e_{mxo}})m_2 & & \\ & & \cdot & \\ & & & (1-\frac{e_{mx1}}{e_{mxo}})m_N \end{bmatrix} \quad (20b)$$

$$= \begin{bmatrix} m_{f1} & & & \\ & m_{f2} & & \\ & & \cdot & \\ & & & m_{fN} \end{bmatrix}$$

In this equation, all eccentricities have been defined in relation to the mass axis (e_{mx}=0 in Eq. (19)) and e_{mxo} is the largest of them (in absolute value), i.e.,

$$\left|e_{mxo}\right| \geq \left|e_{mxn}\right|,\ n=1,2,..N \quad (20c)$$

It may be seen that none of the elements of $\mathbf{M}_f$ registers a negative value and therefore this matrix may be considered as a fictitious mass matrix. The sum of the fictitious floor masses of $\mathbf{M}_f$ is equal to the total mass M_{tot} of the assumed building, since the location of the vertical reference axis satisfies Eq. (19).

Rearranging Eq. (15), in combination with Eqs. (20a) and (20b), the following is obtained

$$e_{xmo}\left(1-RGM\right) = \frac{1}{\omega_{y1}^2}\left(\sum_{j=1}^{k-1} x_j\omega_{j1}^2 + x_k\omega_{k1}^2\right) \quad (21a)$$

where, RGM represents a ratio of generalized masses and is equal to

$$RGM = \frac{\mathbf{\Phi}_{y1}^T\mathbf{M}_f\mathbf{\Phi}_{y1}}{\mathbf{\Phi}_{y1}^T\mathbf{M}\mathbf{\Phi}_{y1}} \quad (21b)$$

3. Implications on the design procedure

It may be of interest to examine the optimum location of the aforementioned k-element when all the accidental eccentricities of Eq. (6) change their algebraic sign. By definition the accidental mass eccentricity may occur on either side of the nominal location of the center of mass, but in systems with an asymmetric structural configuration the effects of such a reversed heightwise mass eccentricity is not obvious. Assuming that the nominal locations of all the centers of mass lie on the same vertical axis, this represents the vertical reference axis in all equations following Eq. (19). Based on this assumption the following conclusions may be drawn:

When all the accidental eccentricities of Eq. (6) reverse their algebraic sign, the first term of Eq. (15) also changes its algebraic sign. Assuming that e_{mxo} in Eq. (21a), which is another expression of Eq. (15), registers a positive value, let the corresponding optimum location of the k-element be defined as x_k^+. When e_{mxo} registers a negative value, which implies that all the accidental eccentricities are symmetrically aligned across the mass axis, let the corresponding optimum location of the k-element be defined as x_k^-. Taking into account that the first term of Eq. (21a) is equal to zero when no accidental eccentricities are applied (all floor masses are aligned on the mass axis) let the corresponding optimum (nominal) location of the k-element be defined as x_k^0. It follows that the new location of the k-element (when e_{mxo}<0) is symmetrical to its initial location (when e_{mxo}>0) with respect to its nominal location, x_k^0. In other words, within the limits of the outlined procedure which is based on the dominant (first) modal shape, the distance x_k^0 is the mean distance of x_k^+ and x_k^-. As a result, in the case of multistory buildings which have a mass axis, the effect of any spatial variation of mass eccentricities may be investigated without examining the reversed heightwise variation as it may be sufficiently accurate to investigate the variation of the location shift (length difference) of the key element $x_k^+ - x_k^0$, since this difference is equal to $x_k^0 - x_k^-$.

The absolute value of the Δx_k shift is equal to

$$\Delta x_k = \left|x_k^+ - x_k^0\right| = \frac{\omega_{y1}^2}{\omega_{k1}^2}\left|\left(1-RGM\right)e_{mxo}\right| \quad (22)$$

It follows from Eq. (22) that the location shift reduces when the stiffest lateral load resisting bent is the key element (i.e., the selected k-element) which specifies a structural configuration of minimum torsional response. In this case the key element provides the largest element frequency (i.e.,: ω_{k1}) and, as noted above, the square value of the fundamental frequency of the uncoupled structure, ω_{y1}, may be reasonably assessed by the sum of the square values of the element frequencies according to Southwell's formula (Eq. (14)). The location shift Δx_k is further reduced when the RGM ratio approaches unity. In general, accidental eccentricities introduced in the lower floors are likely to have a minor effect on the geometric configuration which is designed to respond in a predominantly

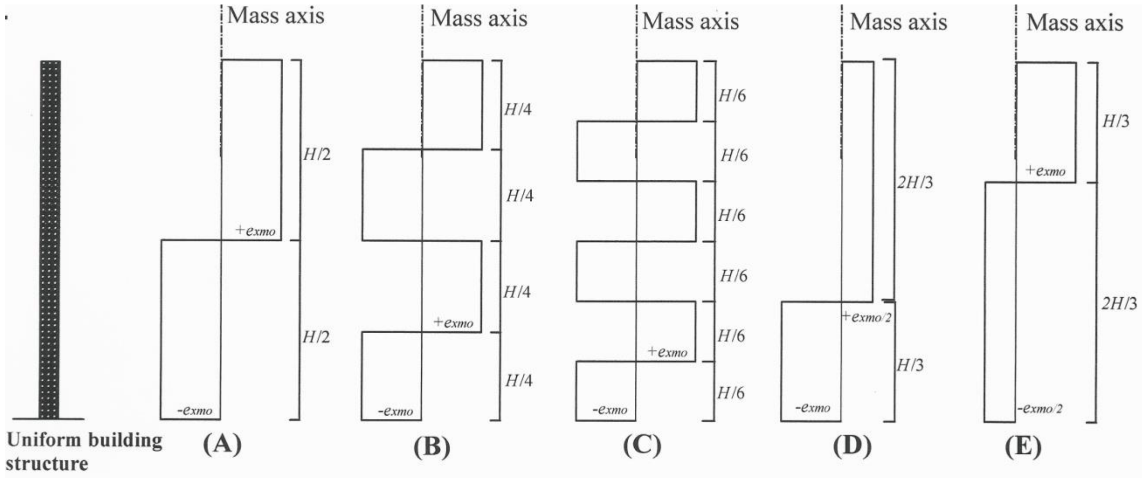

Fig. 1 Five different mass eccentricity variations of a uniform building system

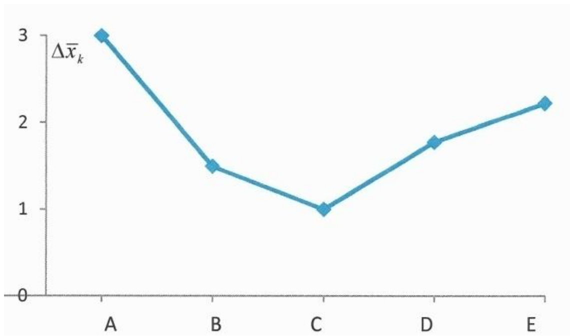

Fig. 2 The effect of 5 different mass eccentricity variations on the required shift of the key element

translational manner (i.e.,: when the k-element is located at the nominal coordinate x_k^0).

A practical example, of the effect of mass eccentricities on the normalized location shift $\Delta\bar{x}_k = \Delta x_k / e_{mxo}$ is the case of uniform multistory buildings (i.e.,: m_1=m_2=...=m_N in Eq. (4b)) by assuming a linear first mode shape, $\mathbf{\Phi}_{y1}$, of the uncoupled structure. Consider for example the five representative mass eccentricity variations of Fig. 1. Using the formulation of the distributed mass systems (Chopra 2008) and defining $m(x)=m$ as the mass per unit height of the system, the *RGM* ratio of Eq. (21c) may be expressed as follows

$$RGM = \frac{\int_0^H \phi^2(x)\overline{m}(x)dx}{\int_0^H \phi^2(x)dx} \tag{23a}$$

where H is the height of the building structure, $\Phi=x/H$ and

$$\overline{m}(x) = m_f(x)/m(x) = m_f(x)/m \tag{23b}$$

The quantity $m_f(x)$ is the (distributed) mass per unit height of the uncoupled system, (which, in the discrete formulation would be expressed by the mass matrix of Eq. (20b)) and, for the five mass eccentricity variations of Fig. 1 (shown as subscripts in the following equation), the ratio $\overline{m}(x)$ is respectively equal to

$$\overline{m}(x) = \begin{Bmatrix} 0, & \bar{x} \in (0 \div 1/2) \\ 2, & \bar{x} \in (1/2 \div 1) \end{Bmatrix}_{\mathbf{A}} = \begin{Bmatrix} 0, & \bar{x} \in (0 \div 1/4), (1/2 \div 3/4) \\ 2, & \bar{x} \in (1/4 \div 1/2), (3/4 \div 1) \end{Bmatrix}_{\mathbf{B}}$$
$$= \begin{Bmatrix} 0, & \bar{x} \in (0 \div 1/6), (1/3 \div 1/2), (2/3 \div 5/6) \\ 2, & \bar{x} \in (1/6 \div 1/3), (1/2 \div 2/3), (5/6 \div 1) \end{Bmatrix}_{\mathbf{C}}$$
$$= \begin{Bmatrix} 0, & \bar{x} \in (0 \div 1/3) \\ 1.5, & \bar{x} \in (1/3 \div 1) \end{Bmatrix}_{\mathbf{D}}, ... = \begin{Bmatrix} 1.5, & \bar{x} \in (0 \div 2/3) \\ 0, & \bar{x} \in (2/3 \div 1) \end{Bmatrix}_{\mathbf{E}} \tag{23c}$$

where $\bar{x} = x/H$. The corresponding values of the normalized location shift $\Delta\bar{x}_k = \Delta x_k / e_{mxo}$ are shown in Fig. 2, for the case in which the k-element contributes 50% of the fundamental frequency of the uncoupled structure (i.e.,: $\omega_{k1}=0.5\omega_{y1}$). The values represent the required shift of the k-element, from its nominal position x_k^0, for the system to sustain minimum torsional response. The results of Fig. 2, are qualitative and of no practical value for the structural design of real buildings, since an, approximate, linear distribution has been assumed for the first mode of vibration and the building structure has been analyzed with the assumption of the continuous medium, which is applicable to uniform buildings. In reality, structural buildings may have significant in plan and in elevation irregularities, and due to this fact an alternative, but practical, method for determining the location of the k-element, and of the required shift for optimum torsional response in Eq. (22), is presented below.

4. Locating the key element for optimum torsional response - a practical procedure

Generally, the *RGM* ratio of Eq. (21b) may be estimated by the following equation

$$RGM = \frac{\mathbf{\Phi}_{y1}^T \mathbf{K}_{yy} \mathbf{\Phi}_{y1} / \mathbf{\Phi}_{y1}^T \mathbf{M} \mathbf{\Phi}_{y1}}{\mathbf{\Phi}_{y1}^T \mathbf{K}_{yy} \mathbf{\Phi}_{y1} / \mathbf{\Phi}_{y1}^T \mathbf{M}_f \mathbf{\Phi}_{y1}} \tag{24}$$

In this equation, the numerator defines the square value of the first mode frequency, ω_{y1}, of the uncoupled structure, while the denominator, defined as

$$RQ = \frac{\mathbf{\Phi}_{y1}^T \mathbf{K}_{yy} \mathbf{\Phi}_{y1}}{\mathbf{\Phi}_{y1}^T \mathbf{M}_f \mathbf{\Phi}_{y1}} \tag{25}$$

is a Rayleigh's quotient of the fundamental frequency, ω_{fy1}, of the uncoupled system which has the floor masses shown in the main diagonal of the matrix of Eq. (20b). Therefore, since it is easy to evaluate ω_{fy1} by any structural analysis software, an approximate equation which provides the required location of the k-element, x_k, is as follows

$$e_{xmo}\left(1 - \frac{\omega_{y1}^2}{\omega_{fy1}^2}\right) = \frac{1}{\omega_{y1}^2}\left(\sum_{j=1}^{k-1} x_j \omega_{j1}^2 + x_k \omega_{k1}^2\right) \tag{26}$$

It should be noted that when the maximum mass eccentricity appears in two floors with opposite algebraic sign, the e_{mxo} eccentricity in Eqs. (20a) and (21a) should be taken as that of the lower floor. A typical example is the eccentricity variations of case A in Fig. 1. To explain this suggestion, it may be worth recalling that the sum of the fictitious floor masses of Eq. (20b) is equal to the total mass M_{tot}. Therefore, when e_{mxo} is taken as that of the floors in the upper half of its height, this means that the fictitious uncoupled system, which specifies ω_{fy1}, has masses (of double value) only in lower half of its height. The value of ω_{fy1}, thus produced, is equal to

$$\omega_{fy1}^2 = \frac{\mathbf{\Phi}_{fy1}^T \mathbf{K}_{yy} \mathbf{\Phi}_{fy1}}{\mathbf{\Phi}_{fy1}^T \mathbf{M}_f \mathbf{\Phi}_{fy1}} \tag{27}$$

where $\mathbf{\Phi}_{fy1}$ is the first mode vector of the aforesaid fictitious uncoupled system. There is a clear difference between the shape functions $\mathbf{\Phi}_{y1}$ (in Eq. (25)) and $\mathbf{\Phi}_{fy1}$ (in Eq. (27). Both of them satisfy the displacement boundary conditions at the base of the structure, but $\mathbf{\Phi}_{y1}$ satisfies the force boundary conditions at the top of the system, while $\mathbf{\Phi}_{fy1}$ satisfies the corresponding conditions at the mid height of the system, since there are no masses above this point. Therefore, the procedure of approximating the quotient RQ of Eq. (25), by the frequency of Eq. (27) may not be reasonable. On the other hand, when e_{mxo} is taken as that of the lower floors (case A of Fig. 1), the corresponding fictitious uncoupled system, which has (double) masses only in its top part, provides the first mode frequency ω_{yf1} by an expression similar to Eq. (27), but the corresponding mode vector $\mathbf{\Phi}_{fy1}$ satisfies now the force boundary conditions at the top of the system, as the vector $\mathbf{\Phi}_{y1}$. Therefore, it is reasonable to approximate the quotient RQ of Eq. (25), by the square value of the first mode frequency of this uncoupled fictitious system.

5. Case study

Three different models of the building structure shown in Fig. 3 are analyzed under an unidirectional excitation along the y-direction. The same models, but without accidental mass eccentricities, have been discussed in an earlier paper

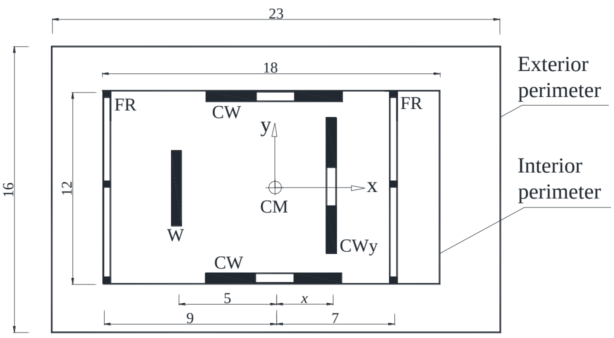

Fig. 3 Plan configuration of example model structures (all dimensions in meters)

(Georgoussis 2017) and comprise 9-storey mono-symmetrical building systems with in plan and in elevation irregularities. Model T0/B9 consists of nine identical orthogonal floors of 23×16 m, as shown in Fig. 3 by the exterior perimeter. The mass per floor is equal to m_b=338 t, corresponding to a total uniformly distributed gravity load of 9 kN/m^2 and the radius of gyration r_b about the CM is 8.088 m. The story height is 3.5 m and the modulus of elasticity (E) is assumed equal to 20×10^6 kN/m^2, typical for concrete structures. The nominal centers of mass (CM) of the floor slabs lie on the same vertical axis, when a uniformly distributed gravity load is assumed. The lateral load resisting system along the y-direction consists of a wall, W, of lateral dimensions 35×400 cm, a coupled wall bent, CWy, composed of two walls of 35×300 cm at a distance of 6 m, connected by lintel beams 30×80 cm at the floor levels and, also, by two moment resisting frames, FR, composed by three columns of 70×70 cm, 6 meters apart, connected by beams of a cross section 40×70 cm. The lateral load resisting system along the x-direction consists of two CW bents, identical to CWy, located symmetrically to the axis of symmetry at distances $\pm 6\mathrm{m}$. Model T3/B6 is a setback building, which consists of a base structure with six floor plans identical to those of model T0/B9, and a top structure composed by three floors of a reduced size of 18×12 m, as shown in Fig. 3 by the interior perimeter. The mass of the latter floors is equal to 154 t (corresponding to a total uniformly distributed gravity load of 7 kN/m^2) and the radius of gyration is equal to r_t=6.24 5 m.

Because of the reduced gravity load applied at the top three floors, the dimensions of the vertical members of all the bents at these levels are reduced accordingly: the size of W is taken as 35×350 cm, the cross-sections of the coupled wall bents (CWy and CW) are 35×270 cm and the columns of frames FR are 60×60 cm. Model T6/B3 is also a setback building, composed of a base structure with three floors identical to those of Model T0/B9, and a top structure composed of six floors similar to those of the top floors of Model T3/B6. The assumed three building models are symmetric along the x-direction, but structurally asymmetric along the y-direction, as the location of the various bents oriented in this direction is illustrated in Fig. 3: the frames FR are located at distances equal to -9 m and +7 m respectively from the CM, while the flexural wall W is located at a distance of 5 m on the left of CM. CWy bent, is located, between -9 m to+9 m, along the x-axis in order to assess the optimum coordinate, that minimizes

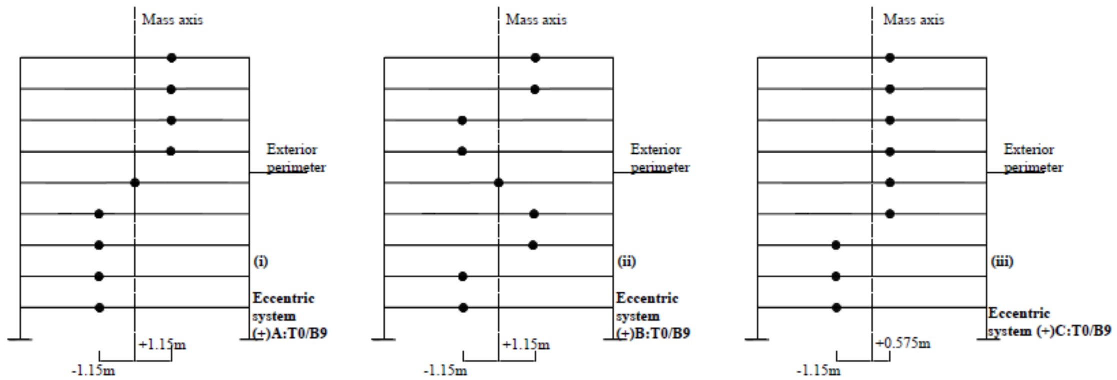

Fig. 4(a) The first three analyzed mass eccentric systems of model T0/B9: (i) (+)A:T0/B9; (ii) (+)B:T0/B9 and (iii) (+)C:T0/B9

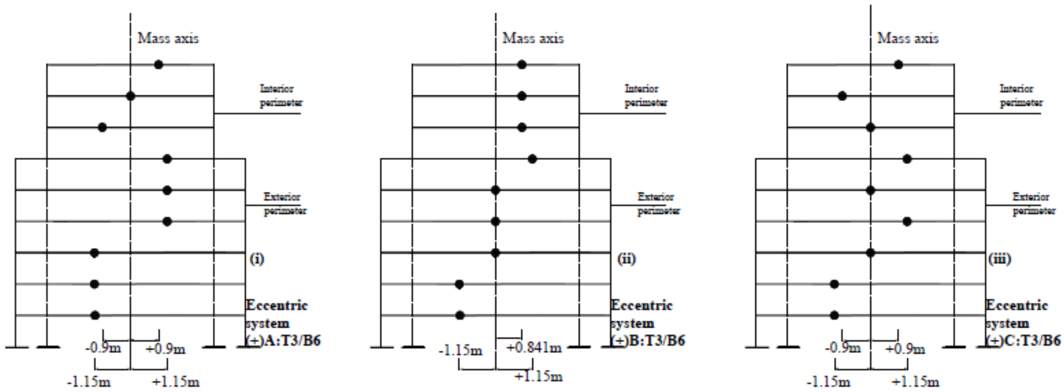

Fig. 4(b) The first three analyzed mass eccentric systems of model T3/B6: (i) (+)A:T3/B6; (ii) (+)B:T3/B6 and (iii) (+)C:T3/B6

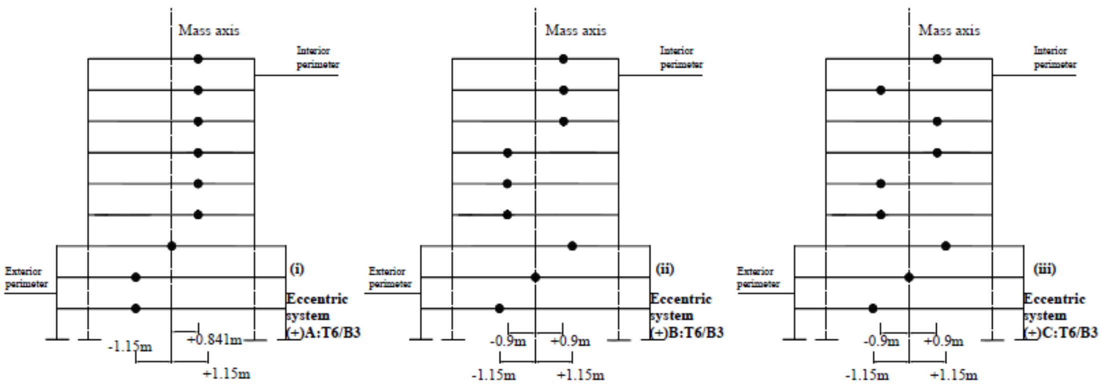

Fig. 4(c) The first three analyzed mass eccentric systems of model T6/B3: (i) (+)A:T6/B3; (ii) (+)B:T6/B3 and (iii) (+)C:T6/B3

the system's torsional response in relation to the following accidental mass eccentricities.

For each model building, the response of six different systems, with varying mass eccentricities, are investigated against a ground excitation along the y-direction, defined by the acceleration spectrum of EC8-2004 (type 1, ground type B, soil factor 1, horizontal ground acceleration 0.40 g).). It is worth noting here that torsional distortions may arise from torsional ground motions (Basu *et al.* (2014)), stiffness uncertainties, bi-directional excitations etc., but these issues are beyond the objectives of this study. The first three mass eccentricities analyzed for the case of model T0/B9 are shown in Fig. 4(a).

In system (+)A:T0/B9, the CM of the first four floors is shifted to the left of the reference axis, along the x-direction, to a distance equal to the code accidental eccentricity: 0.05×23=1.15 m. On the top four floors, the CM is shifted to the opposite direction, at an equal distance, while the location of the CM on the fifth floor remains the same. In all cases the mass polar moment of inertia remained unchanged and equal to that of the system when no mass eccentricities are taken into account. This is equivalent to assuming that all storey masses are lumped, and equal to m_b with a polar moment of inertia equal to $m_b * r_b^2$ (De la Llera and Chopra 1994). In system (+)B:T0/B9, the centers of mass of the 1st, 2nd, 6th and 7th floor are shifted to the left of the reference axis by the code eccentricity, while these centers of the 3rd, 4th, 8th and 9th floors are symmetrically located on the right side of the reference axis (the location of the CM on the fifth floor remains unchanged). In system (+)C:T0/B9, the CM of the three floors at the bottom of the structure is shifted to the left of the reference axis by the code eccentricity, while in the top six floors their CM is located on the right of the reference axis at a distance equal to half of the code accidental eccentricity (0.5*1.15= 0.575 m). Since the model structures are structurally asymmetric along the y-direction, all the aforementioned eccentric systems are also examined when the various floor eccentricities are reversed (i.e.,: reversing their algebraic sign). The corresponding eccentric systems are defined as (-)A:T0/B9, (-)B:T0/B9 and (-)C:T0/B9. It should be noted that according to the analysis presented in the previous section, the optimum location of the CWy bent in the latter systems, is expected to be in a more or less symmetrical location of that in the former (+)A:T0/B9, (+)B:T0/B9 and (+)C:T0/B9 systems, with respect to the nominal location of CWy, which is defined when no mass eccentricities are taken into account (model T0/B9).

Similarly, the first three eccentric systems for the case of model T3/B6, denoted as (+)A:T3/B6, (+)B:T3/B6 and (+)C:T3/B6, are shown in Fig. 4(b). Their response is also examined when the mass eccentricities are reversed and the corresponding systems are defined as (-)A:T3/B6, (-)B:T3/B6 and (-)C:T3/B6. Similar are the eccentric systems of Model T6/B3 (Fig. 4(c)).

6. Discussion of results

The structural configurations in Fig. 4(a) are analyzed against a ground excitation along the y-direction, defined by the acceleration spectrum of EC8-2004 (type 1, ground type B, soil factor 1, horizontal ground acceleration 0.40 g), and their torsional response, in terms of the sustained base torque and top floor rotation is presented by the red lines in Fig. 5(a). Normalized base torques, $\overline{T} = T/r_b V_o$ (where V_o is the base shear of the corresponding uncoupled building model) and top rotations, Θ, were calculated by the structural analysis program SAP2000-V16 for different locations (indicated by the normalized coordinate $\bar{x} = x/r_b$) of the coupled wall bent CWy, are shown by red lines in Fig. 5(a). The aforementioned data have been calculated on the basis of the first 12 peak modal values combined according to the CQC rule (the damping ratio in each mode of vibration was taken as 5%). The blue lines show the torsional response of the systems with reversed mass eccentricities (systems (-)A:T0/B9, (-)B:T0/B9 and (-)C:T0/B9), while the black lines show the response of these systems when no mass eccentricities are taken into account (as shown in Georgoussis, 2017). Note here that the value of the base shear of the uncoupled structures V_o (i.e., the systems in which the floors are restrained against rotations), used to normalize the base torque, amounted to 12532, 12300 and 9772 kN for models T0/B9, T3/B6 and T6/B3 respectively.

Similar observations apply to the data presented in Fig. 5(b), which show (in red lines) the torsional response of the structural configurations in Fig. 4(b), along with the response of the corresponding systems with reversed eccentricities (in blue lines) and the system without mass eccentricities (black lines). The results in Fig. 5(c) show, in a similar manner, the response of the structural configurations of Fig. 4(c), together with the response of the systems with reversed and no mass eccentricities respectively.

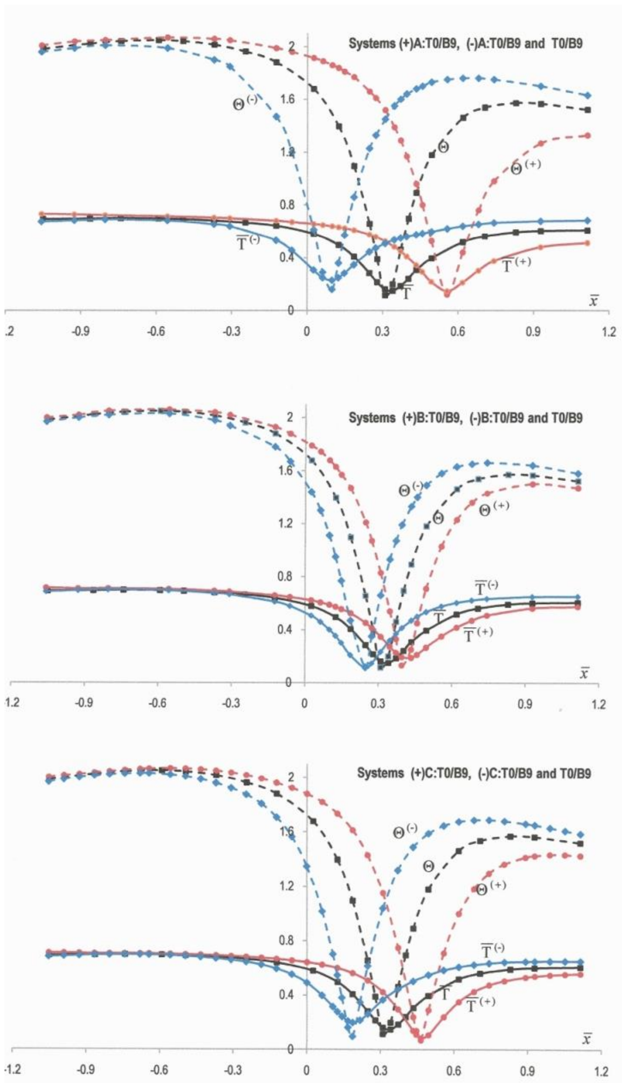

Fig. 5(a) Top rotations (x10^{-2} rads) and normalized base torques of the eccentric systems (+)A:T0/B9, (+)B:T0/B9 and (+)C:T0/B9 (red lines of $\Theta^{(+)}, \overline{T}^{(+)}$) and the systems (-)A:T0/B9, (-)B:T0/B9 and (-)C:T0/B9 (blue lines of $\Theta^{(-)}, \overline{T}^{(-)}$), together with the corresponding quantities of the system with no mass eccentricities (model T0/B9, black lines $\Theta, \overline{T}$)

The first observation is that the results illustrated in Figs. 5(a)- 5(c), show that the variation of base torques is smoother than those of the top rotations, which suggests that the mostly affected response parameter is the top rotation and not the base torque. The variation of the base torque appears to be decreasing in the setback model with a rather low base structure (Fig. 5(c)).

The second observation is that the inverted peaks in Figs. 5(a)-5(c), mainly of the dotted lines which represent the top rotation and, to a lesser degree, of the solid lines which represent the base torques, clearly indicate an optimum location of the coupled wall bent CWy. In Figs. 5(a) and 5(b)

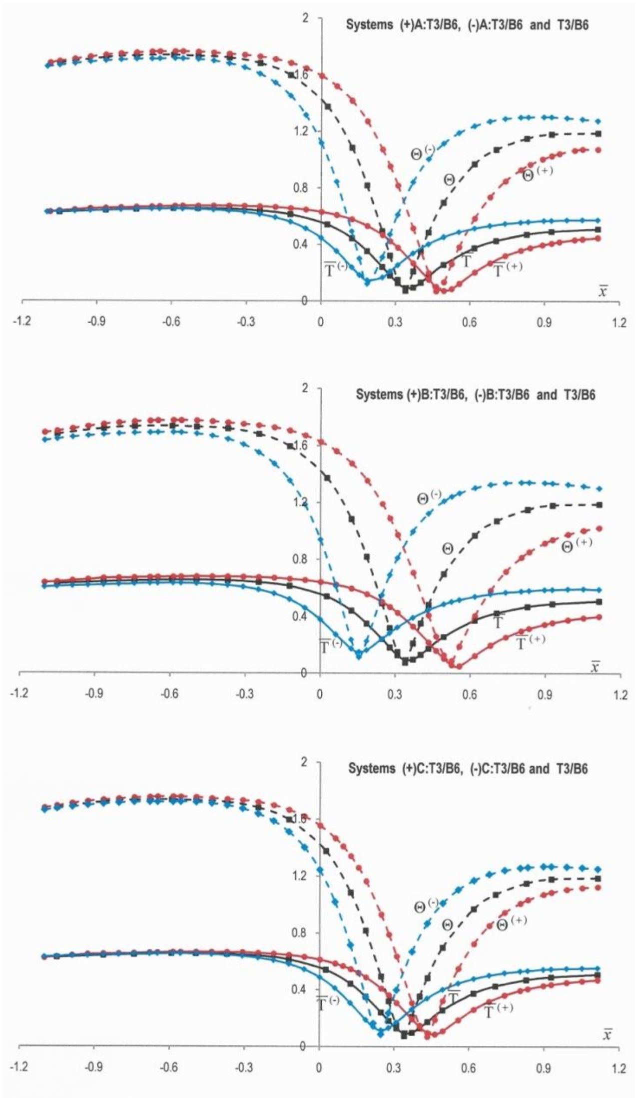

Fig. 5(b) Top rotations (x10^{-2} rads) and normalized base torques of the eccentric systems (+)A:T3/B6, (+)B:T3/B6 and (+)C:T3/B6 (red lines of $\Theta^{(+)}, \overline{T}^{(+)}$) and the systems (-)A:T3/B6, (-)B:T3/B6 and (-)C:T3/B6 (blue lines of $\Theta^{(-)}, \overline{T}^{(-)}$), together with the corresponding quantities of the system with no mass eccentricities (model T3/B6, black lines $\Theta, \overline{T}$)

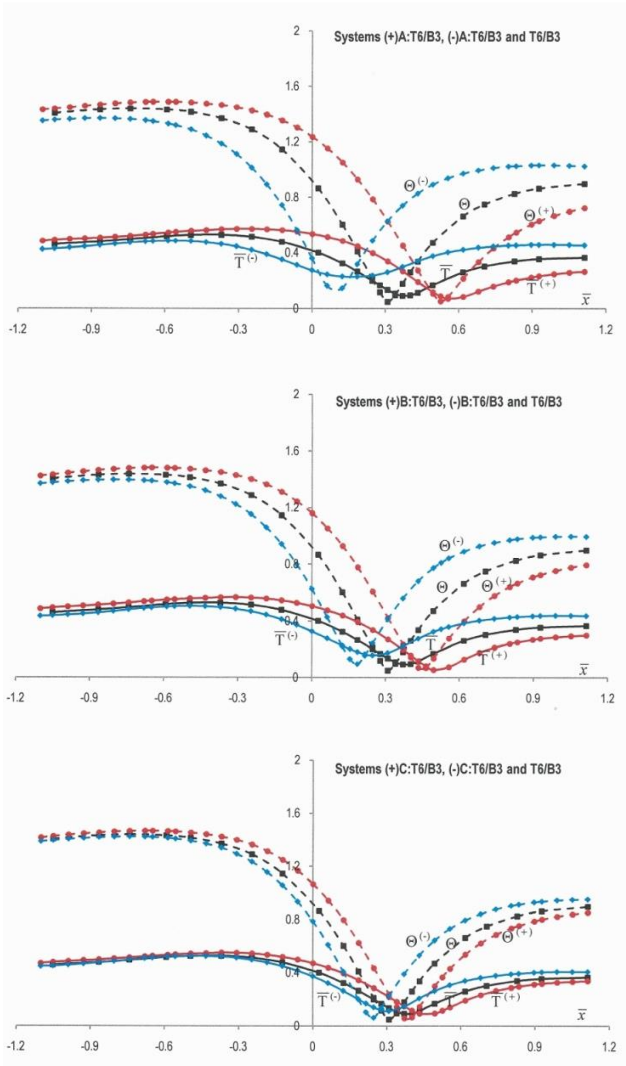

Fig. 5(c) Top rotations (x10^{-2} rads) and normalized base torques of the eccentric systems (+)A:T6/B3, (+)B:T6/B3 and (+)C:T6/B3 (red lines of $\Theta^{(+)}, \overline{T}^{(+)}$) and the systems (-)A:T6/B3, (-)B:T6/B3 and (-)C:T6/B3 (blue lines of $\Theta^{(-)}, \overline{T}^{(-)}$), together with the corresponding quantities of the system with no mass eccentricities (model T6/B3, black lines $\Theta, \overline{T}$)

the minimum values of the rotations and base torque (of the same color) point to almost the same value of $\overline{x}$. This is less noticeable by the torques curves (solid lines) in Fig. 5(c). They show a rather extended range of locations of the coupled wall bent CWy where the base torque registers small values. The third observation is that the inverted peaks of the red and blue lines (either solid, which represent base torques, or dotted, which represent top rotations) are pointing to almost symmetrical locations with respect to those indicated by the black lines. It is interesting to note here that, a small shift of CWy from its optimum location results in large torsional distorsions, but the effects of this distorsion reduce as the CWy bent moves further away from its optimum location (all curves in Figs. 5(a) to (c) have a steep dip near the optimum location of CWy, but they become smoother, or even flat, at larger shifts of this bent).

The curves indicated by the black lines, show the response of the systems with no mass eccentricities and their inverted peaks indicate the optimum location of CWy where the torsional response is minimized (nominal location of the CWy bent). The observation that the inverted peaks of the colored lines are pointing to symmetrical locations with respect to the nominal location of the CWy, is in agreement with the analysis outlined in section 3. The results in Figs. 5(a)-5(c) demonstrate, that, for any spatial variation of mass eccentricities, the optimum location of the key element, which minimizes the torsional effect of the structure has an almost symmetrical location, with respect to its nominal location, when the mass eccentricities are reversed.

The normalized locations of the CWy bent for minimum

Table 1 Predicted normalized locations of the key element (CWy bent) for optimum torsional response of the systems of Figs. 4(a)-4(c) and the models with no mass eccentricity

Model with no mass ecc.	Min torsional response for CWy at x^0	Mass eccentric systems	Min torsional response for CWy at x^+/x^-
T0/B9	$x^0=0.248$	(+)A:T0/B9 (-)A:T0/B9	$x^+=0.492$ $x^-=0.004$
		(+)B:T0/B9 (-)B:T0/B9	$x^+=0.341$ $x^-=0.155$
		(+)C:T0/B9 (-)C:T0/B9	$x^+=0.391$ $x^-=0.106$
T3/B6	$x^0=0.256$	(+)A:T3/B6 (-)A:T3/B6	$x^+=0.390$ $x^-=0.120$
		(+)B:T3/B6 (-)B:T3/B6	$x^+=0.437$ $x^-=0.074$
		(+)C:T3/B6 (-)C:T3/B6	$x^+=0.350$ $x^-=0.160$
T6/B3	$x^0=0.246$	(+)A:T6/B3 (-)A:T6/B3	$x^+=0.456$ $x^-=0.037$
		(+)B:T6/B3 (-)B:T6/B3	$x^+=0.374$ $x^-=0.120$
		(+)C:T6/B3 (-)C:T6/B3	$x^+=0.319$ $x^-=0.174$

torsional distortion of the mass eccentric systems of Figs. 4(a)-4(c), derived by means of Eq. (26), are shown in Table 1. The corresponding locations of the models without mass eccentricities, as derived in Georgoussis (2017), are also shown in this Table. It may be of interest to compare the locations pointed by the inverted peaks in Figs. 5(a)-5(c) with the predicted data of Table 1. The predicted locations are somewhat lower than those shown in the aforementioned figures, an outcome that can be attributed to the effect of the higher modes of vibration, which are not accounted for in the analysis of section 3. However, the length difference of Eq. (22), as predicted from the data of Table 1 (i.e.,: $\Delta x = x^+ - x^-$), appears to be very close to that shown by the inverted peaks in Figs. 5(a) to 5(c). On the grounds of the data obtained from the numerical analysis of the case studies, the accuracy of the predicted results may be considered within the acceptable engineering limits and the proposed Eq. (26) may be used with confidence in the preliminary stage of structural designs.

7. Conclusions

The effect of mass eccentricities on the seismic torsional response of medium height multistory buildings was investigated in comparison to the response of the counterpart systems in which all floor masses are assumed to lie on the same vertical line. The influence of mass eccentricities on the torsional response of buildings is first studied on a theoretical basis, using the stiffness matrix methodology, with the assumption that the first mode of vibration is the dominant mode of response. It is demonstrated that for any heightwise mass eccentricity variation a structural configuration of minimum torsional response may be obtained when the locations of the lateral load resisting bents satisfy Eq. (26). In any structural configuration, composed by a given set of lateral load resisting bents, the procedure requires (i) the evaluation of the element frequencies, which are determined from the corresponding individual bents when they are assumed to carry, as planar frames, the mass of the complete structure and, (ii) the evaluation of the fundamental frequency of an uncoupled structure with fictitious floor masses. Both data may be easily determined by any commercial structural software, and, in practice, the procedure to determine a structural configuration of minimum torsional response is similar to that which evaluates the location of the Optimum Torsion Axis (OTA). The definition of the Optimum Torsion Axis applies to building systems where all floor masses are located on the same line (mass axis) and the main attribute of the OTA is that when its location coincides with the mass axis the seismic response is essentially translational. In practice the coincidence of the aforementioned axes is implemented by the suitable location (nominal location) of a certain element (key element). It is shown that when a spatial distribution of the floor masses is taken into account, as required by most recent building codes, the aforementioned translational response may be reserved by readjusting (shifting) the location of the key element. It was also demonstrated that by reversing the spatial distribution of floor masses, the required relocation of the key element is shifted to a symmetrical position with respect to its nominal location. Small shifts of the key element from its optimum location result in rather large torsional distortions, but these effects gradually become less significant as the key element moves further away from its optimum location. The accuracy of the proposed procedure is demonstrated in common 9-storey regular and irregular in elevation building structures, with a lateral load resisting system composed of flexural walls, coupled wall bents and moment resisting frames, in relation to the results provided by the structural analysis program SAP2000-V16. The proposed equation for assessing the location of the key element presents reasonable accuracy and may be used with confidence in structural applications.

Acknowledgments

This work has been (co-)financed by the Greek School of Pedagogical and Technological Education through the operational program “Research strengthening in ASPETE”- Project SSKK: “Torsional response of building structures under seismic excitations”.

References

Anagnostopoulos, S.A., Kyrkos, M.T. and Stathopoulos, K.G. (2015a), “Earthquake induced torsion in buildings: Critical review and state of the art", *Earthq. Struct.*, **8**(2), 305-377.

Anagnostopoulos, S.A., Kyrkos, M.T., Papalymperi, A. and Plevri, E. (2015b), "Should accidental eccentricity be eliminated from Eurocode 8?", *Earthq. Struct.*, **8**(2), 463-484.

ASCE. *Minimum Design Loads for Buildings and Other Structures*, *Standard ASCE/SEI 7-10*, American Society of Civil Engineers, Reston, VA: 2010.

Basu, D. and Jain, S.K. (2007), "Alternative method to locate center of rigidity in asymmetric buildings", *Earthq. Eng. Struct. Dyn.*, **36**, 965-973.

Basu, D., Constantinou, M.C. and Whittaker, A.S. (2014), "An equivalent accidental eccentricity to account for the effects of torsional ground motion on structures", *Eng. Struct.*, **69**, 1-11.

Bosco, M., Ferrara, G.A.F., Ghersi, A., Marino, E.M. and Rossi, P.P. (2015), "Seismic assessment of existing r.c. framed structures with in-plan irregularity by nonlinear static methods", *Earthq. Struct.*, **8**(2), 401-422.

Bosco, M., Ghersi, A., Marino, E.M. and Rossi, P.P. (2017), "Generalized corrective eccentricities for nonlinear static analysis of buildings with framed or braced structure", *Bullet. Earthq. Eng.*

Cheung, V.W.T. and Tso, W.K. (1986), "Eccentricity in irregular multi-story building", *Can. J. Civil Eng.*, **13**, 46-52.

Chopra, A.K. (2008), *Dynamics of Structures*, 3rd Edition, Prentice Hall, New Jersey, U.S.A.

De-La-Colina, J., Gonzalez, J.V. and Perez, C.A.G. (2013), "Experiments to study the effects of foundation rotation on the seismic building torsional response of a reinforced concrete space frame", *Eng. Struct.*, **56**, 1154-1163.

De-La-Llera, J.C. and Chopra, A.K. (1994a), "Accidental torsion in buildings due to stiffness uncertainties", *Earthq. Eng. Struct. Dyn.*, **23**, 117-136.

De-La-Llera, J.C. and Chopra, A.K. (1994b), "Using accidental eccentricity in code specified static and dynamic analyses of buildings", *Earthq. Eng. Struct. Dyn.*, **23**, 947-967.

De-La-Llera, J.C. and Chopra, A.K. (1995), "Estimation of accidental torsion effects for seismic design of buildings", *J. Struct. Eng.*, **121**(1), 102-114.

EAK-2000 *Greek Aseismic Code* (2000), *Greek Ministry of Environment*, City Planning and Public Works, Greece.

EC-8/2004 Eurocode 8 (2004), *Design Provisions for Earthquake Resistance of Structures*, European Standard EN/1998.

Georgoussis, G.K. (2010), "Modal rigidity center: Its use for assessing elastic torsion in asymmetric buildings", *Earthq. Struct.*, **1**(2), 163-175.

Georgoussis, G.K. (2014), "Modified seismic analysis of multistory asymmetric elastic buildings and suggestions for minimizing the rotational response", *Earthq. Struct.*, **7**(1), 39-52.

Georgoussis, G.K. (2015), "Minimizing the torsional response of inelastic multistory buildings with simple eccentricity", *Can. Civil Eng.*, **42**(11), 966-969.

Georgoussis, G.K. (2016), "An approach for minimum rotational response of medium-rise asymmetric structures under seismic excitations", *Adv. Struct. Eng.*, **19**(3), 420-436.

Georgoussis, G.K. (2017), "Locating optimum torsion axis in asymmetric buildings subjected to seismic excitation", *Proceedings ICE*.

Makarios, T. and Anastassiadis, K. (1998a), "Real and fictitious elastic axis of multi-storey buildings: Theory", *Struct. Des. Tall Build.*, **7**(1), 33-45.

Makarios, T. and Anastassiadis, K. (1998b), "Real and fictitious elastic axis of multi-storey buildings: Applications", *Struct. Des. Tall Build.*, **7**(1), 57-71.

Marino, E.M. and Rossi, P.P. (2004), "Exact evaluation of the location of the optimum torsion axis", *Struct. Design Tall Spec. Build.*, **13**, 277-290.

Newmark, N.M. and Rosenblueth, E. (1971), *Fundamentals of Earthquake Engineering*, Prentice-Hall, New Jersey, U.S.A.

Sheikhabadi, M.R.F. (2014), "Simplified relations for the application of rotational components to seismic design codes", *Eng. Struct.*, **59**, 141-152.

Stathopoulos, K.G. and Anagnostopoulos, S.A. (2010), "Accidental design eccentricity: Is it important for the inelastic response of buildings to strong earthquakes?", *Soil Dyn. Earthq. Eng.*, **30**(9), 782-797.

Wolff, E.D., Ipek, C., Constantinou, M.C. and Morillas, L. (2014), "Torsional response of seismic isolated structures revisited", *Eng. Struct.*, **59**, 462-468.

CC