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Force density ratios of flexible borders to membrane in tension fabric structures

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Abstract. Architectural fabrics membranes have not only the structural performance but also act as an efficient cladding to cover large areas. Because of the direct relationship between form and force distribution in tension membrane structures, form-finding procedure is an important issue. Ideally, once the optimal form is found, a uniform pre-stressing is applied to the fabric which takes the form of a minimal surface. The force density method is one of the most efficient computational form-finding techniques to solve the initial equilibrium equations. In this method, the force density ratios of the borders to the membrane is the main parameter for shape-finding. In fact, the shape is evolved and improved with the help of the stress state that is combined with the desired boundary conditions. This paper is evaluated the optimum amount of this ratio considering the curvature of the flexible boarders for structural configurations, i.e., hypar and conic membranes. Results of this study can be used (in the absence of the guidelines) for the fast and optimal design of fabric structures.

Keywords: tension membranes; force density; curvature; saddle shaped; cone shaped; fabric

1. Introduction

The key mechanism to achieve the increased performance in tension membrane structures is the tensile force generation in a way to have smoother geometry. The tensioned fabric structures form a double curved surface satisfies the stress equilibrium of the surface and the boundary constraints. The detailed geometry of the hyperbolic paraboloid structures, which is necessary for the structural design and construction, must be obtained through the so-called “form-finding” procedure using physical or numerical models (Zhang 2010). Gaudi (1852-1926) was pioneer (Lahuerta 2003) in investigation on models of equilibrium for free hanging cables which forms a catenary curve carrying forces in tension (this was used for design and construction of many vaults such as Guell garden). If the aforementioned hanging cable is flipped, a perfect arc under pure compression could be shaped (Linkwitz 1999).

A significant part of the form-finding physical results, comes from Frie Otto’s work on soap film according to the research by Belgian physicist Josep Platea. He determined the analogue solution to minimization problems by dipping wire frame works into a bath of soap solution (Plateau 1873). Most of the recent applications in the form-finding is performed based on the numerical methods and the computer iterations. For example, Haber and Abel (1982a, b), and Veenendaal and Block (2012) tried to outline, compare and contract different numerical form-finding methods.

Tibert and Pellegrino (2011) provided an excellent review of form-finding methods for tensegrity structures. Huttner *et al.* (2017) presented a general framework to understand the basic principles of the form-finding process and their explanation of the very simple examples. The use of the finding a shape of a tension membrane that is in static equilibrium as an analogy with the search condition of minimal surfaces is explained. Xu *et al.* (2015a) presented a novel approach to construct minimal surface from a given boundary by quasi-harmonic Bezier approximation. Tang and Li (2017) developed an equivalent FDM as a shape-finding tool for cable-membrane structures. Shi *et al.* (2018) proposed a free-form optimization method to design cable-membrane structures for form-finding and stiffness maximization. This method is a kind of non-parametric optimization method based on the H1 gradient method. Lan *et al.* (2018) presented an effective form-finding method for form-fixed spatial network structures. They introduced the adaptive form-finding method along with the example of designing an ellipsoidal network dome with bar length variations being as small as possible. Moreover, the dynamic relaxation method is employed to explicitly integrate the node positions by applying residual forces.

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Shimoda and Yamane (2015) proposed a convenient numerical form-finding method to design the minimal surface, or the equally tensioned surface of membrane structures with specified arbitrary boundaries. They formulated the area minimization problems as a distributed-parameter shape optimization. In these techniques, it is assumed that the membrane is varied in the out-of-plane and/or the in-plane direction to the surface. Also, the shape sensitivity function for each problem is derived using the material derivative method. Alic and Persson (2016) implemented a method for form-finding with dynamic relaxation and non-uniform rational B-Splines based isogeometric membrane elements. Since the form-finding can be performed using coarse mesh, this procedure allows for rapid evaluation of the curved geometries. However, to minimize the bending strain energy a fine mesh is needed. Aish *et al.* (2015) described the theory of isotropic membrane stress under gravity load and introduced a particle method for its numerical simulation for the form-finding of shell structures. Ibrahim *et al.* (2018) proposed a form-findings method using nonlinear analysis of tensioned fabric structures in the form of Handkerchief surface. Popov *et al.* (2018) described a special 3D frame-grid template for efficient form-finding of the tensile structure. It is based on a frame object consisting of a set of spatial edges, connected with each other in vertices. The grid is hung on the frame object and is subject to optimal form-finding by the stretched grid method as close to the minimum surface as possible.

Force density method (FDM) which was first introduced in 1971 by Linkwitz and Schek (Linkwitz and Schek 1971), (Schek 1974) is a powerful tool for analytical form-finding of the self-stressed structures. This method was first proposed in context of cable nets, while the main idea was to convert the nonlinear static equilibrium equation at each node to a linear one and simplifying the computing process. Development of this method was based on the physical experiment on a free hanging membrane formed by Heinz Isler (Grundig *et al.* 2000, Fund 2008). The main feature of FDM lies on prescribing a force density coefficient (i.e., force-to-length ratio) for each element in the discretized model. Force density has been developed for form-finding of different structures such as Timber shells (Adriaenssens *et al.* 2014), tensigrity structures (Harichandran and Sreevalli 2016, Koohestani 2013, 2017), trusses (Ohsaki and Hayashi 2017), mesh reflectors (Li *et al.* 2017), cable nets (Aboul-Nasr and Mourad 2015), and membrane structures (Ye *et al.* 2012, Xu *et al.* 2015b).

For textile structures, the continuous membrane is approximated by a discretized cable networks. This model is also applicable for woven fabrics with negligible shear stiffness (Grundig *et al.* 2000). In FDM, the geometry of a cable network structure is quantified by balancing the internal and external forces. The nonlinear version of FDM (NLFDM) increases the possibility of imposing extra criteria such as: relative distance among the nodes, the tensile level in the elements and/or their initial undeformed length (Malerba *et al.* 2012, Koohestani 2014). Miki and Kawaguchi (2010) applied the extended FDM on tension structures. Moreover, Tang *et al.* (2016) proposed an extended version of NLFDM in which a Newton-Raphson-based iterative algorithm is used to solve the nonlinear force density equations and optimize the initial geometrical shape.

Maurin and Motro developed similar technique which is called surface stress density method (SSDM) and takes into account the shear stresses as well (Maurin and Motro 1998, Singer 1995). Pauletti and Pimenta (2008) presented the extension of force density method with Argris concept of natural strain for the finite element analysis of membrane. This method preserves the linearity of original force density, while it can be applied on irregular finite element meshes. Lee and Han (2011) developed a modified formulation of the force density method with the help of finite element and replaced 3-node or 4-node membrane element with a linear line element. Xu *et al.* (2015b) proposed a modified NLFDM in which the stresses of membrane elements are transformed to the force-densities of cable nets using an equivalent method, and are used as initial conditions. Xiang *et al.* (2010) presented an improved nonlinear FDM considering 2D deformation of membrane surface because of changing their width during form finding.

In FDM, the value of force density can be determined by trial and error in a way that the final geometry and distributed stress meet both structural and architectural requirements. This process is time consuming and sometimes lead to invalid results. In fact, it is difficult to assess the internal force to length of the bars, especially for membrane structures with flexible boundaries. Ye *et al.* (2012) offered a modified FDM with the help of membrane stress and cable tension as initial conditions instead of their related force density values. In this work, first the curved surface is calculated by dynamic relaxation, and then the structure is discretized in to triangle elements and, then force density equations are imposed according to initial membrane stress and cable tension. Finally, convergence criteria tries to obtain uniform stress on the curved surface.
The last three studies have tried to solve the difficulties of defining force density amounts as initial parameters using membrane stress and cable forces in iterative cycles. In the following research it is tried to adjust the main influential parameter in force density form-finding method (i.e., force density ratio of boundary to membrane) in a way that in final self-stressed geometry the curvature of the boundary cables reach to their optimum value. For this purpose, first the force density method and its matrix formulation is briefly reviewed; then the importance of the boundary curvature in membrane structures, specially its relation with membrane stresses and cable forces, are explained. Finally, the force density ratios related to the optimum border curvature in two main membrane forms (i.e., saddle and conic shapes) are assessed using a set of parametric analyses.

2. Developing a linear form

The fundamental idea of FDM in the pin-jointed network structures assumes the state of equilibrium when the internal forces F and the external ones P are balanced as,

Fig. 1 (Linkwitz and Schek 197, Schek 1974)

$$\begin{aligned}
&\frac{F_1}{L_1}(x_1-x_5)+\frac{F_2}{L_2}(x_2-x_5)+\frac{F_3}{L_3}(x_3-x_5)+\frac{F_4}{L_4}(x_4-x_5)=P_x\\
&\frac{F_1}{L_1}(y_1-y_5)+\frac{F_2}{L_2}(y_2-y_5)+\frac{F_3}{L_3}(y_3-y_5)+\frac{F_4}{L_4}(y_4-y_5)=P_y\\
&\frac{F_1}{L_1}(z_1-z_5)+\frac{F_2}{L_2}(z_2-z_5)+\frac{F_3}{L_3}(z_3-z_5)+\frac{F_4}{L_4}(z_4-z_5)=P_z
\end{aligned} \tag{1}$$

where L_1, L_2 and L_3 are the nonlinear length functions of the coordinates. Using the force density parameters $\frac{F_1}{L_1}=q_1$, $\frac{F_2}{L_2}=q_2$, $\frac{F_3}{L_3}=q_3$, and $\frac{F_4}{L_4}=q_4$, the nonlinear nature of Eq. (1) can be altered into the following linear form

$$\begin{aligned}
&q_1(x_1-x_5)+q_2(x_2-x_5)+q_3(x_3-x_5)+q_4(x_4-x_5)=P_x\\
&q_1(y_1-y_5)+q_2(y_2-y_5)+q_3(y_3-y_5)+q_4(y_4-y_5)=P_y\\
&q_1(z_1-z_5)+q_2(z_2-z_5)+q_3(z_3-z_5)+q_4(z_4-z_5)=P_z
\end{aligned} \tag{2}$$

This method could be implemented in an efficient way by applying special matrix techniques to solve Eq. (2). First of all, the following vectors and matrices should be introduced (note that n and m present the nodes and branch numbers, respectively): $x_{sn_s\times 1}$, $y_{sn_s\times 1}$ and $z_{sn_s\times 1}$ are coordinates of the nodes. By numbering the fixed nodes after the free nodes, this vector is divided into a n×1 vector of <x, y, z> and a n_f×1 vector of <x_f, y_f, z_f> for the free and fixed nodes, respectively. $F_{m\times 1}$ is the tensile forces in the elements.

The connectivity matrix of whole nodes (fixed and free) $C_{s\ m\times n_s}$ is assumed as

$$C_s=\begin{cases}+1 & \textit{for starting nodes}\\ -1 & \textit{for ending nodes}\\ 0 & \textit{other branches}\end{cases} \tag{3}$$

and the differences between the couples of coordinates can be written as

$$\begin{cases}u=C_s x_s=Cx+C_f x_f\\ v=C_s y_s=Cy+C_f y_f\\ w=C_s z_s=Cz+C_f z_f\end{cases} \tag{4}$$

where C and C_f are branch node matrix of free and fixed nodes, respectively. By introducing U= diag(u), V= diag(v), W= diag(w), Eq. (2) can be expressed as

$$\begin{cases}C^T U L^{-1} F=P_x\\ C^T V L^{-1} F=P_y\\ C^T W L^{-1} F=P_z\end{cases} \xrightarrow{q=L^{-1}F} \begin{cases}C^T U q=P_x\\ C^T V q=P_y\\ C^T W q=P_z\end{cases} \tag{5}$$

Further, assuming Q=diag (q), the diagonal matrix of force density becomes $Uq=Qu$, $Vq=Qv$ and $Wq=Qw$. This diagonal matrix of force density, Q, includes force density amounts of all bars in the network system, as well as borders and interior elements.

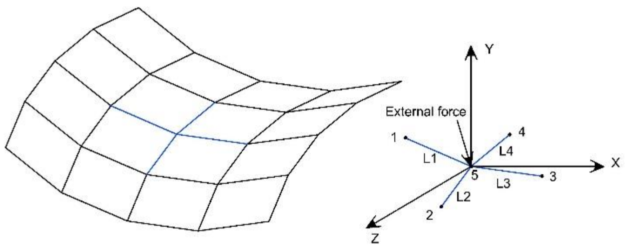


Fig. 1 Saddle shape network

Substituting these diagonal equations into the equation of equilibrium using differences of coordinates yields to

$$\begin{cases}C^T QCx+C^T QC_f x_f=P_x\\ C^T QCy+C^T QC_f y_f=P_y\\ C^T QCz+C^T QC_f z_f=P_z\end{cases} \xrightarrow{D=C^T QC and D_f=C^T QC_f} \begin{cases}x=D^{-1}(P_x-D_f x_f)\\ y=D^{-1}(P_y-D_f y_f)\\ z=D^{-1}(P_z-D_f z_f)\end{cases} \tag{6}$$

This final equation allows to find a unique equilibrium configuration of the system. This method has many advantages. First of all, the only input parameters are the force densities of all branches in the net, the coordinates of any fixed nodes and the connectivity of the structure, which simply defines all branches with their two connecting points. Second, the output is a system of linear equations which can be solved though a single operation without iteration. Furthermore, the equations for this initial shape do not contain anything related to the materials of the structure, which would only be considered if other geometrical constraints on the structure are required. Last but not least, aside from the fixed nodes, this method eliminates the need to specify any initial coordinates.

Based on the above discussion, the force density method is one of the best analytical methods to find the initial equilibrium problem of tension membrane structures with lowest possible amount of predefined parameters. However, definition of the amount of force density for borders and membrane play an important role. The most fundamental question is: “how to choose the initial force densities?” In this research, a Matlab (2016) code is developed to quantify the impact of the force densities on the initial shape of equilibrium. The optimum value then is discussed based on the flexible border curvatures of a simple hypars consist of two high and two low points and cone shapes with five fixed nodes.

3. Importance of borders curvatures

In pre-stressed membrane structures, the borders of the surfaces are playing an important role in the initial shape of the equilibrium which consequently affects the load-carrying capacity of the whole anticlastic membrane. These borders might be materialized by flexible edge cables or rigid elements such as beams. Additionally, during the installation of fabric patches between two anchoring points, curvature of the flexible borders is influencing mainly the relationship between the magnitude of tensile forces in the boundary cables and the adjacent interior membranes.

Curvature of the edge cable and the tension forces in

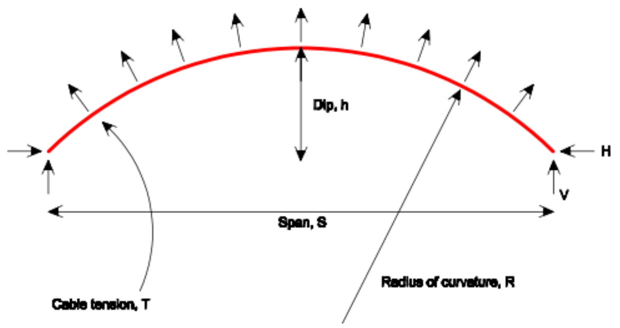


Fig. 2 Curvature of edge cable and reaction forces

Table 1 Mean final forces distributed after from finding by changing force densities amount

Initial shape of equilibrium	Mode	Force density of border elements (N/m)	Force density of inner elements (N/m)	Ratio	Mean value of final distributed force (N/m)
Hypar	a	1	1	1	0.1534
	b	10	10	1	1.5339
	c	5	1	5	0.3364
	d	5	10	0.5	1.1026
	e	2	1	2	0.2119
	f	2	10	0.2	0.6685
Conic	a	1	1	1	0.1738
	b	10	10	1	1.7826
	c	5	1	5	0.2896
	d	5	10	0.5	1.5059
	e	2	1	2	0.2156
	f	2	10	0.2	1.2606

cables are related by Young-Laplace equation for a structure spanning in one direction (Borgart 2010): Tension = Uniform applied load, $w\times$ radius of curvature, R, Fig. 2. Using the simplifying assumption that the weight of the edge cable is negligible compared to the applied load and hence the cable forms a circular arc, the cable tension, T, can be written in terms of the horizontal, H, and vertical, V, reaction forces

$$T = \sqrt{H^2 + V^2} : H = \frac{ws^2}{8h} ; V = \frac{ws}{2} \tag{7}$$

where s is span and h is dip. By replacing the reaction forces (i.e., H and V) in the cable tension formula, the following relationship is acquired

$$h = \frac{ws^2}{4\sqrt{4T^2 - w^2 s^2}} \tag{8}$$

According to the above formulation, the sag of the cable in a given span is related to the cable tension and the uniform applied loads. Many designers use Young-Laplace equation to find the appropriate form by adjusting the tension forces and membrane pre-stress to achieve the given boundary curvatures in a way to minimize the reaction forces that consequently results smaller cross sections in supporting structures (Forster *et al.* 2004). However, as it was already mentioned in the force density method, the predefined parameters are force densities of both membrane and boundary elements. This means that Young-Laplace equation are not applicable and a new approach is required in order to find the appropriate functional form and adjusting the border curvatures.

In the previous studies about FDM, the corresponding relationship between force density of cable and membranes, and also the final balanced shape is somehow hard to assess because the length of branches is changed during form-finding and it is impossible to define them precisely at first with the help of intended membrane stress and cable forces. Consequently, defining force density amount need some trials to reach the desired stress distribution on the surface and intended curvature in border elements. In the following section, the influential parameters, especially force density amounts, on border curvatures of equilibriuted geometry will be assessed.

4. Impact of the force density choices on the net configuration

As discussed earlier, the force density of membrane and edge elements are one of the input parameters in FDM. For this reason, first the impact of the force densities is assessed using the developed Matlab (2016) code for two main geometry of membrane structures. They include: 1) a simple square saddle shape with 400 nodes (four fixed nodes) and 760 elements and 2) a cone-shape with 401 nodes (five fixed nodes) and 800 elements. Six different combinations of the force density values in boarder and inner elements are studied as illustrated in Table 1.

As seen in Figs. 3 and 4, the initial shape of the equilibrium and the border curvatures are only dependent on the ratio of the force densities and not their magnitude. Modes (a) and (b) (in both shapes) have different force density values, while the ratio between the border and the internal elements is constant. Subsequently, they both generate the same geometry. Additionally, the larger the force density of the edge cables relative to that of the net, the straighter cables stand (see mode c). On the other side, when the force-densities of the net are increased, the net tends to contract itself and thus, the higher curvature of the boundary cable can be observed (mode f) (Linkwitz 1999). This means that for a given net, its initial equilibrium shape depends only upon the ratio of the force density of the border to the interior elements and not their actual values. These actual values affect the amount of the final pre-stressing which is distributed on the whole surface and consequently on the stiffness of the net based on Table 1 (Linkwitz 1999, Grundig *et al.* 2000). According to Fig. 5, the curvature of the border remains constant by changing the height of the corners and the mid-mast in hypar and conic shape, respectively, while the aforementioned ratio is identical.

Again, according to Table 2, by increasing the relative

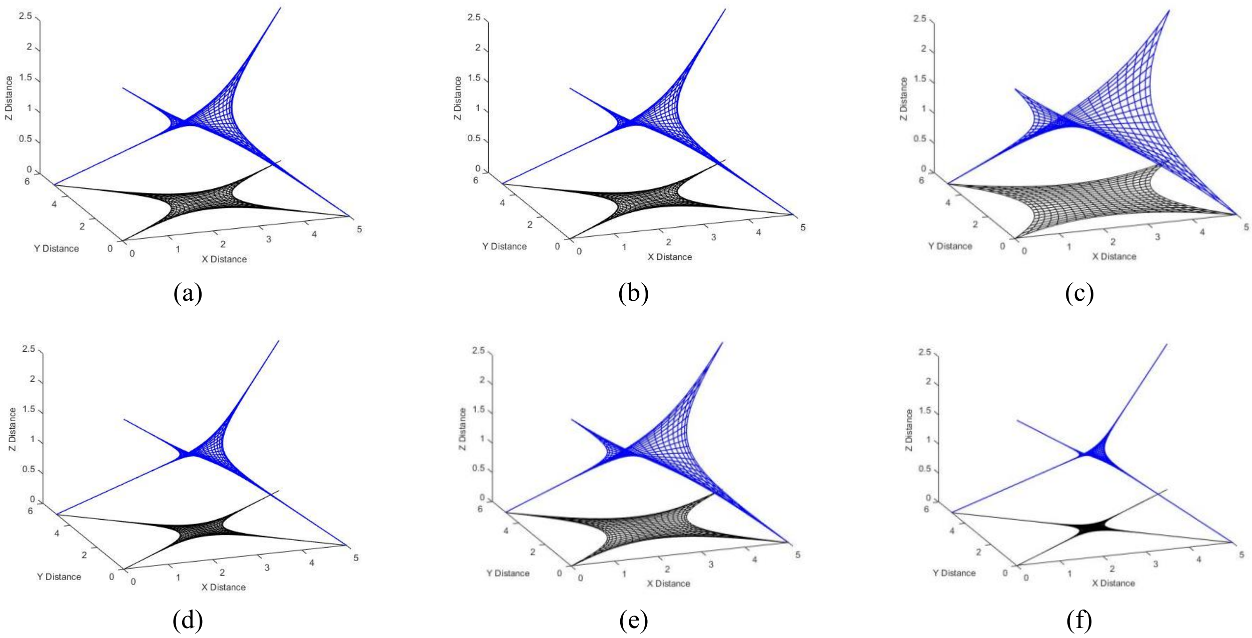


Fig. 3 Equilibrium configuration of conic shape network made of 400 nodes (five fixed) and 800 elements

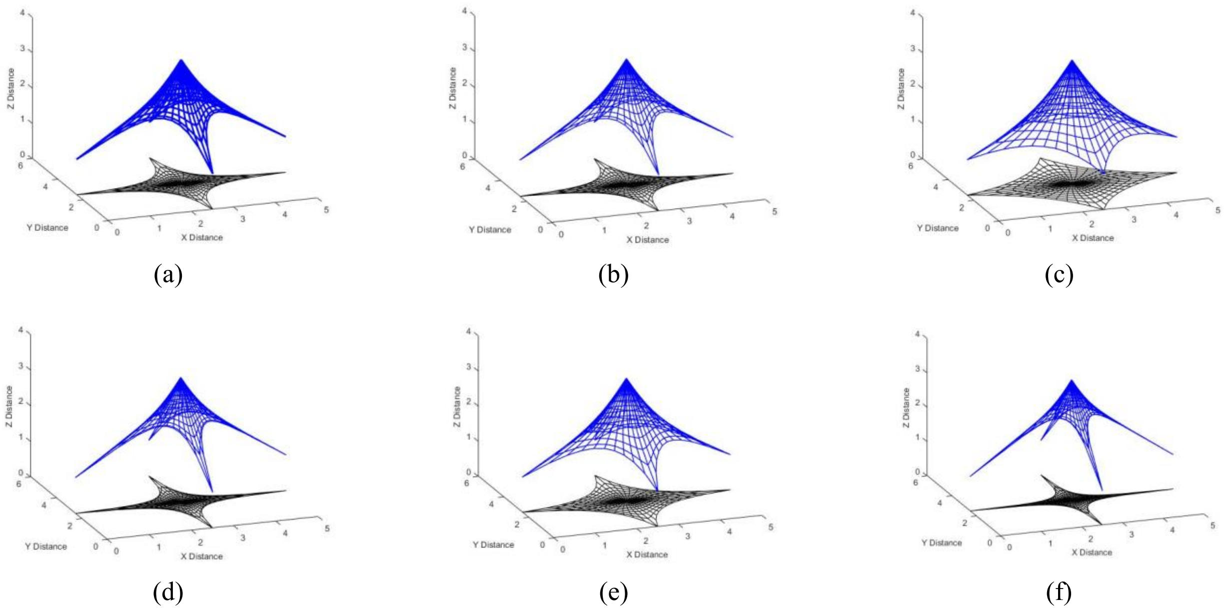


Fig. 4 Equilibrium configuration of square saddle shape network made of 400 nodes (four fixed) and 760 elements

height of the fixed nodes in both saddle and conic shapes, both the double curvatures of anticlastic surface increase while the curvature of the borders are identical. Ascending trend of the main curvatures leads to the increase in final pre-stressing distribution and consequently stiffness of the surfaces.

Number of the nodes can change the geometry and also curvature of the borders. As it is shown in Fig. 6 by increasing the node numbers, the curvature increases as well. In fact, when the number of the nodes in the cable borders goes up, more perpendicular tensile elements contribute in formation of the edge cable curvatures during the form-finding process.

Furthermore, when the node numbers and the ratio of the force densities are identical, the size of the saddle and the cone shape structures does not have any effect on the border curvatures. This can be seen from plan view of Fig. 7 in which the border lines are parallel.

According to Fig. 8(a) by increasing the node numbers from 25 (5×5) to 121 (11×11), 441 (21×21) and finally 961 (31×31), the border curvatures of a hypar shape increases at different force density ratio of boundary elements to net, and from 2,601 (51×51) it remains nearly constant. This means that the discrete cable network systems which are

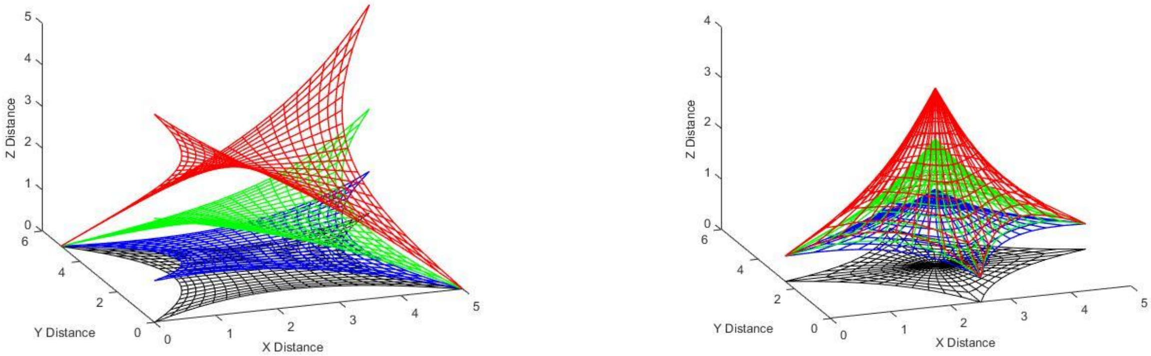
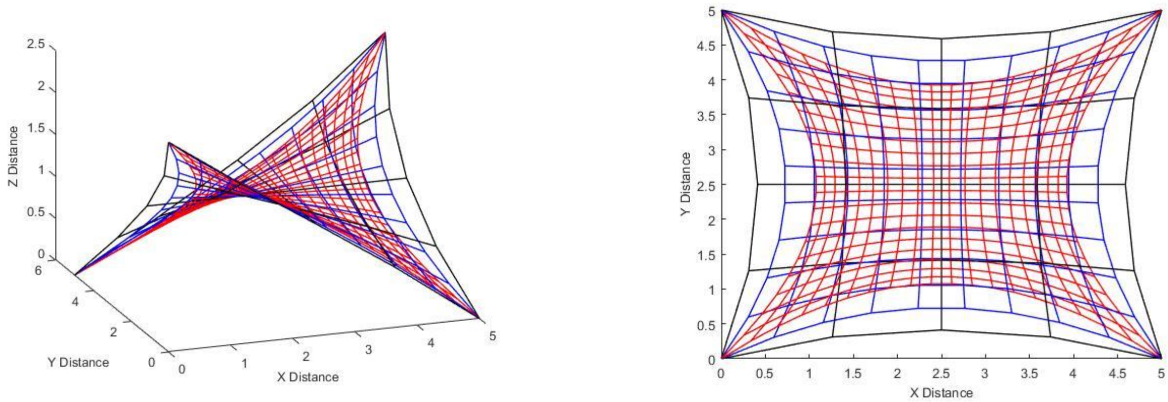
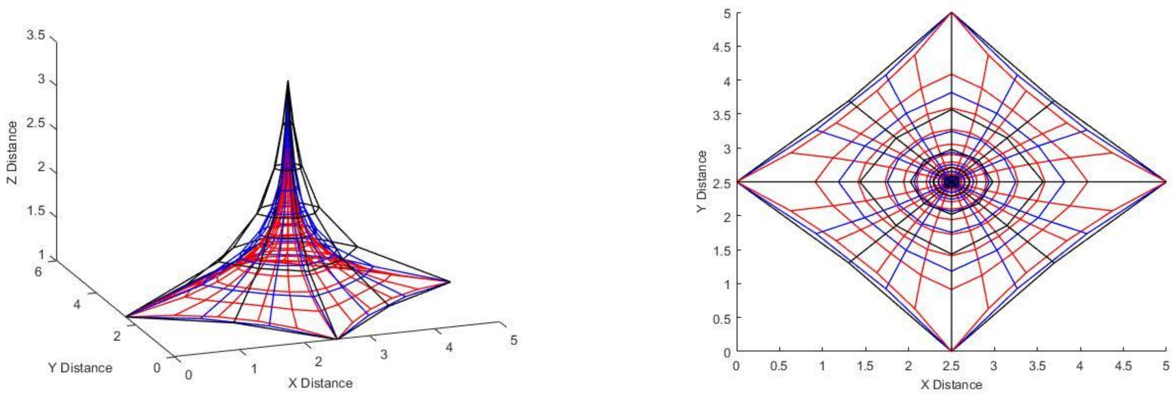


Fig. 5 Variation of changing in height difference of fixed nodes with curvature of borders in both hypar and conic shapes, force density of borders=5 and force density of inner nets=1



(a) Saddle shape (force density of border=5 N/m and force density of membrane=1 N/m, Black: 25 nodes, blue: 100 nodes and red: 400 nodes)



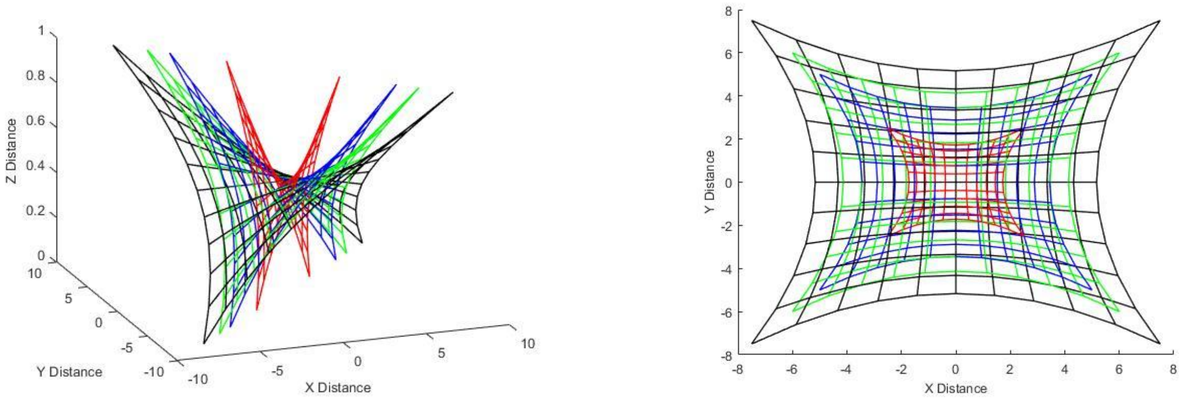
(b) cone shape (force density of border=5 N/m and force density of membrane=1 N/m, Black: 41 nodes (5 circle × 8 radial lines), blue: 121 nodes (10 circle ×12 radial lines), and red: 421 nodes (20 circle × 24 radial lines)

Fig. 6 Changes in initial equilibrium geometry by changing the node numbers

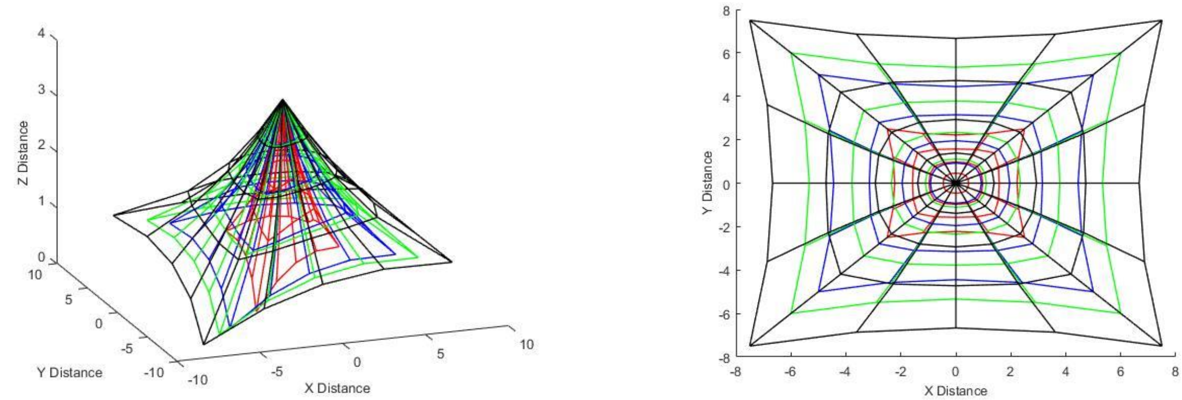
assumed for force density method needs a dense mesh to model a realized membrane structures. It should be mention that the range of changes in boundary curvature become wider by increasing force density ratio of the border to membrane. Note that the cone shape has similar situation, Fig. 8(b). This means, by increasing the number of nodes, the curvatures are increased until they reach to an identical value.

5. Optimum force density ratio

As it was found in the previous section, the only parameter affecting the curvature is the force density ratio of border to the membrane with a maximum node numbers.



(a) Saddle shape (121 nodes (four fixed), force density of border=5 N/m and force density of membrane=1 N/m, curvature of borders=0.155)



(b) cone shape (65 nodes (five fixed), force density of border=5 N/m and force density of membrane=1 N/m, curvature of borders=0.055)

Fig. 7 Same edge curvature in structures with different sizes

Table 2 Mean final forces distributed after from finding on hypar and conic shapes by changing the relative heights of fixed nodes, force density of borders = 5 N/m and force density of inner nets = 1 N/m

Initial shape of equilibrium	Height differences of fixed nodes (m)	Mean value of final distributed force (N/m)
Hypar	0	0.3141
	1.0	0.3178
	2.5	0.3364
	5.0	0.3945
Conic	0	0.2314
	1.5	0.2345
	2.5	0.2560
	3.5	0.2896

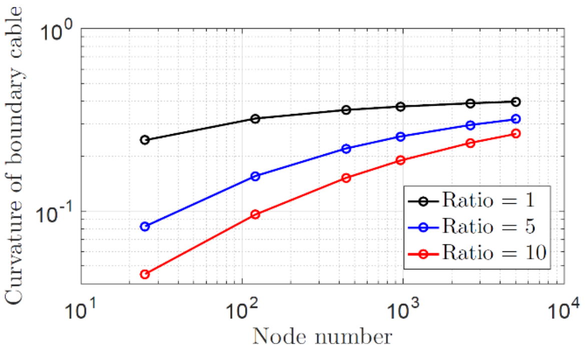
Note that the border curvature does not change meaningfully after it reaches to the optimum node number. In such a condition, the discrete model could act as a membrane surface. Bridgens and Brichall (2012) showed that sag-to-span ratio of the flexible borders between 0.1 to 0.15 could cover surrounded area efficiently while it leads to lower cable forces. Lower cable forces correspond to the smaller cable diameter and subsequently delicate end fittings connections and supporting members.

In the following, force density ratio related to the proposed optimum border curvature is assessed in both saddle and cone shape structures with four and five fixed nodes, respectively. Changes of force density ratio in both square hypar and conic shapes that leads to different sag-to-span ratio of flexible borders are depicted in Fig. 9. The optimum number of nodes are obtained to be 2,601 and 641 for saddle and conic shapes, respectively.

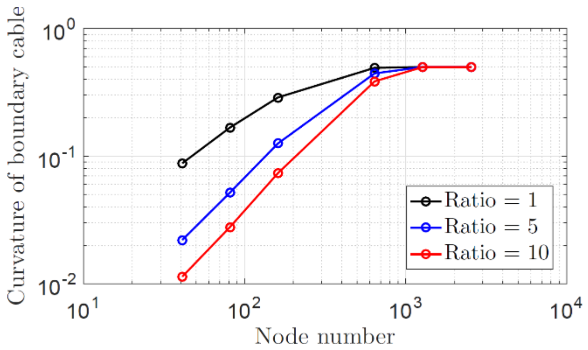
Fig. 9 shows that the relationship between the force density ratios and curvature of the flexible border is not linear and for dip-to-span ratios less than 0.05 the cable force densities increases dramatically. The optimum suggested value of curvature is between 0.1 to 0.15 correspond to the proportion of force densities among 25 to 45 and 62 to 105 for hypar and conic anticlastic surfaces, respectively.

6. Conclusions

Lightweight tensile structures are used worldwide for



(a) square hypar shape



(b) conic shape

Fig. 8 Changing of boundary curvature by node numbers

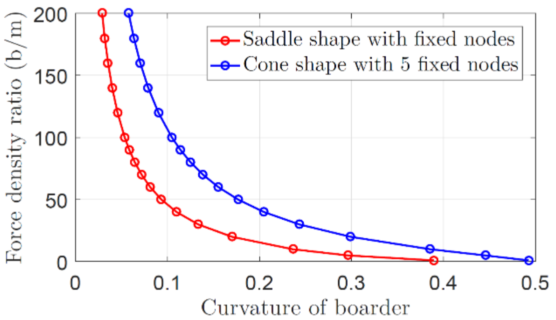


Fig. 9 Variation of curvature of borders with different ratios of force densities

high profile, large-scale structures, as well as for many smaller applications to efficiently provide shelter from rain, sun and wind. Lack of design codes or guidance, coupled with complex material and structural behavior, can limit the application of fabric structures or prevent their full utilization.

A profusion of graphical design tools for tensile structures enable membrane forms to be explored and generated with ease. However, it is important to temper this creative ease and freedom with a clear understanding of the limits to efficient and functional forms. Efficiency in lightweight structures does not just mean being able to specify a lighter grade of fabric, hence giving a small reduction in the overall project cost. Much more important is to minimize the reaction forces that consequently results more elegant connection details and supporting structures. All of these delicate details and supporting elements are in accordance with the light weight nature of tension membrane structures.

In this study two main shape of anticlastic surfaces (i.e., hypar and conic forms), have been studied with varying geometric parameters and ratios of force densities. The aim is to establish a rule of thumb for the safe and efficient initial equilibrium form considering the optimum border curvatures. For both shapes, the relationship between the force density values and the border curvature is highly nonlinear, and for dip-to-span ratios less than 0.05 the cable force densities increase dramatically. This leads to larger cable sizes, and more importantly larger connection details that are at odds with the lightweight, minimalist aesthetic. The relation between forces in the edge cable and the interior of the net influences significantly the curvature of the edge-line. Therefore, according to the optimal curvature of borders, the proportion of the force density of the borders to membrane between 25 to 45 for square saddle with 4 fixed nodes and 62 to 105 for conic shape with 5 fixed nodes including various dimensions and corner heights, are proposed.

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