Structural Engineering and Mechanics, *Vol. 67, No. 5 (2018) 517-525*
DOI: https://doi.org/10.12989/sem.2018.67.5.517

Analysis of boundary conditions effects on vibration of nanobeam in a polymeric matrix

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(Received April 12, 2017, Revised June 6, 2018, Accepted June 7, 2018)

Abstract. In this study, we investigate the vibration of single-walled carbon nanotubes embedded in a polymeric matrix using nonlocal elasticity theories with account arbitrary boundary conditions effects. A Winkler type elastic foundation is employed to model the interaction of nanobeam and the surrounding elastic medium. Influence of all parameters such as nonlocal small-scale effects, Winkler modulus parameter, vibration mode and aspect ratio of nanobeam on the vibration frequency are analyzed and discussed. The mechanical properties of carbon nanotubes and polymer matrix are treated and an analytical solution is derived using the governing equations of the nonlocal Euler-Bernoulli beam models. Solutions have been compared with those obtained in the literature and The results obtained show that the non-dimensional natural frequency is significantly affected by the small-scale coefficient, the vibrational mode number and the elastic medium.

Keywords: non-local; frequency; Winkler; boundary conditions; nanobeam

1. Introduction

Normally Carbon nanotube CNTs are cylindrical macromolecules composed of carbon atoms in a periodic hexagonal arrangement discovered by Iijima (1991), which have received tremendous attention from various branches of science. Varieties of experimental, theoretical, and computer simulation approaches indicate that carbon nanotubes (CNTs) possess superior electronic and mechanical properties (Dresselhaus and Avouris 2001, Bachtold *et al.* 2001), others studies have showed that they have good properties so they can be used for nanocomposites (Dai *et al.* 1996, Thostenson *et al.* 2001, Feldman and Aboudi 1997). In addition, CNTs are well known for their excellent rigidity, higher than that of steel and any other metal.

In recent years, carbon nanotubes and nanobeams hold a wide variety of potential applications, Sakhaee-Pour (2009) using beam element model for vibration analysis of single-walled carbon nanotubes. Bouazza *et al.* (2014) Employing the different gradient elasticity theories on buckling of multiwalled carbon nanotubes. In the same context, variety theoretical studies are used carbon nanotubes (CNTs) and graphene sheet (Hamidi *et al.* 2018, Bensattalah *et al.* 2016, Mokhtar *et al.* 2018, Karami *et al.* 2018e, Yazid *et al.* 2018, Bouazza *et al.* 2015).

Due to difficulties encountered in experimental methods to predict the responses of nanostructures under different loading conditions, the molecular dynamics (MD) simulations and the continuum mechanics methods are used. But the computational problem when using the (MD) is that the time steps involved in the (MD) simulations are limited by the vibration modes of the atoms to be of the order of femto-seconds (10-15 s) (Ranjbartoreh *et al.* 2007).

The continuum mechanics methods have been effectively used to study mechanical behaviors of not only single-walled carbon nanotubes (SWCNTs) (Ghorban *et al.* 2008, Mustapha and Zhong 2012, Boumia *et al.* 2014, Naceri *et al.* 2011) but also MWCNTs (Ghorban *et al.* 2011, Hajnayeb and Khadem 2015, Chemi *et al.* 2015, Rakrak *et al.* 2016, Chemi *et al.* 2018). Recently, the continuum mechanics approach has been widely and successfully used to study the responses of nanostructures, such as the static (Reddy and Pang 2008, Ahouel *et al.* 2016, Zemri *et al.* 2015, Karami *et al.* 2017a), the buckling (Xiaohu and Qiang 2007, Tounsi *et al.* 2016, Bellifa *et al.* 2017a, Larbi Chaht *et al.* 2015, Khetir *et al.* 2017), free vibration (Zidour *et al.* 2012, Saira *et al.* 2016, Bounouara *et al.* 2016, Mouffoki *et al.* 2017), wave propagation and forced vibration (Karami *et al.* 2017b, Wang and Yang 2005, Behrouz 2016, Moradi-Dastjerdi 2016, Ait Yahia *et al.* 2015, Behrouz 2018, Besseghier *et al.* 2017, Bouafia *et al.* 2017, Belkorissat *et al.* 2015).

Variety theoretical methods has been used based on the continuum mechanics sash as, (FSDT) (Bouderba *et al.* 2016, Bellifa *et al.* 2016, Al-Basyouni *et al.* 2015, Youcef *et al.* 2018), (HSDT) (Bousahla *et al.* 2016, El-Haina *et al.* 2017, Bellifa *et al.* 2017b, Menasria *et al.* 2017, Zidi *et al.* 2014, Beldjelili *et al.* 2016, Bouderba *et al.* 2013, Boukhari *et al.* 2016, Belabed *et al.* 2014, Chikh *et al.* 2017, Fourn *et al.* 2018, Mahi *et al.* 2015, Houari *et al.* 2016, Zidi *et al.* 2017, Tounsi *et al.* 2013, Tounsi *et al.* 2013, Zine *et al.*

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ISSN: 1225-4568 (Print), 1598-6217 (Online)

2018). On the other hand, recent research takes into account the effect of normal stress (stretching effect). This novelty is studies by many recent work, (Bourada *et al.* 2015, Hebali *et al.* 2014, Bennoun *et al.* 2016, Bousahla *et al.* 2014, Draiche *et al.* 2016, Belabed *et al.* 2018, Bouhadra *et al.* 2018, Hamidi *et al.* 2018, Abualnour *et al.* 2018, Younsi *et al.* 2018, Benchohra *et al.* 2018).

At nanoscale, the classical continuum theories are deemed to fail, because the length dimensions at this scale are often sufficiently small such that call the applicability of classical continuum theories into the question. At macroscopic scale, the mechanical characteristics of structures are often significantly different from their behavior at nanoscale. Consequently, many non-local theories that consider the scale effect have been proposed such as micro-polar theory (Eringen 1967) and the nonlocal theory of elasticity (Eringen 1972), these theories take into account the influence of the screen introducing the intrinsic scale length in the constituent relations. Among the theories mentioned previously, non-local elasticity theory developed by Eringen (1983) when the stress state at a reference point is considered as a function of strain states of all points in the body. Then Peddieson *et al.* (2003) the first who applied the nonlocal theory in continuous nano technology, static deformations of the beam are obtained by using nonlocal simplified model of the beams based on the non-local elastic theory (Eringen 1983). Bending and shearing responses for dynamic analysis of single-layer graphene sheets under moving load (Shahsavari and Janghorban 2017) porous functionally graded nanoplates and sandwich piezoelectric nanoplates with functionally graded core are studied based on the non-local elastic theory by (Karami *et al.* 2018a, Karami *et al.* 2018b). Shahsavari *et al*. (2017) analyzed dynamic characteristics of viscoelastic nanoplates under moving load using the non-local elastic theory. Karami *et al.* (2018f) used nonlocal strain gradient for anisotropic spherical nanoparticles.

In theoretical studies, boundary conditions are usually used to evaluate constants of integration when you are performing an indefinite integral. Abdelaziz *et al.* (2017) utilized various boundary conditions for bending, buckling and free vibration of FGM sandwich plates. A simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions is studied by (Ait Amar Meziane *et al.* 2014). Post-buckling analysis of shear-deformable composite beams using a novel simple two-unknown beam theory has been studied by (Kaci *et al.* 2018). Wang *et al.* (2006) studied the buckling of micro- and nano-rods/tubes based on nonlocal Timoshenko beam theory under various boundary conditions. In recent years, mechanics, electronics and engineering study's and applications shows the need to study the carbon nanotube under boundary conditions. It is in this scientific context that this work was realized for free vibration of nano beam under various boundary conditions. In the past fifty years, linear and nonlinear problems which appeared in physical, chemistry, mechanics, engineering applications and various of scientific areas are modelled and they are investigated by using so many approximating methods. Some of these numerical methods are Differential Transformation Method (DTM), Homotopy Perturbation Method (HPM) (He 2005), Adomian Decomposition Method (ADM) (Wazwaz 2002), Differential quadrature method. (Hasan Rahimi Pour *et al.* 2015), Variational Iteration Method (VIM) (Ganji *et al.* 2008), (GDQ) (Pradhan, and Phadikar 2009) and Homotopy Analysis Method (HAM) (Liao 2004). Many authors studied linear and nonlinear models to compute approximate solutions and their convergences with Differential Transformation Method (DTM) (Abdel-Halim Hassan 2002). To model the interaction of structure and the surrounding elastic medium A Winkler type elastic foundation is widely employed, (Karami *et al.* 2018c, Shahsavari *et al.* 2018a, Karami *et al.* 2018d). (Shahsavari *et al.* 2018b) used Winkler/Pasternak/Kerr foundation for studied a free vibration of FG plates with porosities.

In this study, the governing equations and boundary conditions for the free vibration of a nonlocal Euler-Bernoulli beam have been extracted via the theory of nonlocal continuum elasticity. The mathematical derivations and numerical investigations are presented and performed while the emphasis is placed on investigating the impact of different parameters such as nonlocal small-scale effects, Winkler modulus parameter, and vibration mode. Comparisons of present approach with the results from the existing literature are provided and the good agreement between the results of the proposed method and those available in literature validated the presented approach.

2. Nonlocal Euler-Bernoulli elastic beam models

The theory of nonlocal continuum elasticity proposed by Eringen (1983) assumed that the stress at a reference point is considered to be a functional of the strain field at every point in the body. In the limit when the effects of strains at points other than x are neglected, one obtains local or classical theory of elasticity. For homogeneous and isotropic elastic solids, the constitutive equation of non-local elasticity can be given by Eringen (1972). Non-local stress tensor (t) at point (x') is defined by

$$\begin{aligned} &\sigma_{ij,j} = 0 \\ &\sigma_{ij}(x) = \int K(|x-x'|,\ \tau) C_{ijkl}\varepsilon_{kl}(x')dV(x'), \quad \forall x \in V \\ &\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \end{aligned} \tag{1}$$

Where (C_{ijkl}) is the classical, macroscopic stress tensor at point x', σ_{ij} and ε_{ij} are stress and strain tensors respectively. $K(|x-x'|,\ \tau)$ is the kernel function and ($\tau = e0a/l$) is a material constant that depends on internal and external characteristic length (such as the lattice spacing and wavelength), where (e0) is a constant appropriate to each material, a is an internal characteristic length, e.g., length of (C-C) bond, lattice parameter, granular distance, and (l) is an external characteristic length.

Non-local constitutive relations for present nano-beams can be approximated to a one-dimensional form as

$$\sigma_x - (e0a)^2 \frac{\partial^2 \sigma_x}{\partial x^2} = E\,\varepsilon_x \tag{2}$$

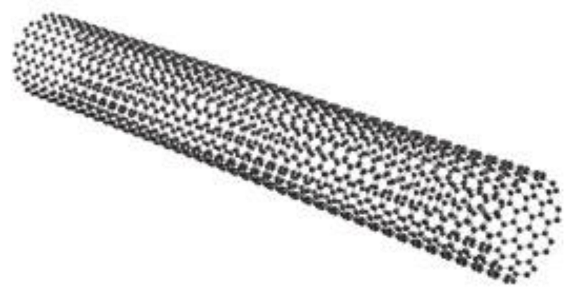

Fig. 1 The illustration of carbon nanotube

Where (E) is the Young's modulus, and the scale coefficient (e0a) in the modelling will lead to small-scale effect on the response of structures at nano-size. Assume that the displacement of the beam along the z axis is w(x,t) in terms of spatial coordinate x and time variable t (Fig. 1).

For transverse vibration of nanotube, the equilibrium conditions of the Euler-Bernoulli beam can be written as (Rakrak *et al*. 2016)

$$\frac{\partial V}{\partial x} = \rho A \frac{\partial^2 w}{\partial t^2} - P(x) \tag{3a}$$

$$V = \frac{\partial M}{\partial x} \tag{3a}$$

where V and M are resultant shear force and bending moment of the beam, ρ is the mass density, A is the area of the cross-section of the beam, w is the transverse displacement of the microtubules, $P(x)$ is the inter action pressure per unit axial length between the nanotube and the surrounding elastic medium, and t is the time variable.

In addition the pressure per unit axial length, acting on the outermost tube due to the surrounding elastic medium, can be described by a Winkler type model (Dihaj *et al*. 2018).

$$P(x) = -K_{win} w(x,t) \tag{4}$$

The elasto-dynamics differential equation that governs the mechanical vibration of the nanotube SWCNT based on the nonlocal Euler-Bernoulli beam theory is

$$\begin{aligned}&(EI)\frac{\partial^4 w(x,t)}{\partial x^4} + \left(-e0a^2 k_{win}\right)\frac{\partial^2 w(x,t)}{\partial x^2} + k_{win} w(x,t) \\ &- e0a^2 \rho A \frac{\partial^4 w(x,t)}{\partial t^2 \partial x^2} + \rho A \frac{\partial^2 w(x,t)}{\partial t^2} = 0\end{aligned} \tag{5}$$

Eq. (5) describes the normal modes shapes of geometrically nonlinear vibrations of o beam resting on elastic foundation. The natural frequencies functions are determined also. This two elements, and determine the normal mode shape. For the integration of Eq. (5), let us assume the solution is in the form of a sinusoidal variation of w with circular frequency ω

$$w(x,t) = W(x) e^{i\omega t} \tag{6}$$

Substituting Eq. (6) into Eq. (5), equations of motion is expressed as follows

$$\begin{aligned}&(EI)\frac{\partial^4 W}{\partial x^4} + \left(e0a^2 \rho A \omega^2 - e0a^2 k_{win}\right)\frac{\partial^2 W}{\partial x^2} \\ &+ \left(k_{win} - \rho A \omega^2\right) k_{win} W = 0\end{aligned} \tag{7}$$

3. Non-dimensionalization

The following non-dimensional variables are introduced in the present analysis to simplify the equations and to make comparisons in the studies possible. The non-dimensional parameters for the Euler-beam on the Winkler foundation are defined as

$$\xi = \frac{x}{L}, \quad \overline{W} = \frac{W}{L}, \quad \mu^2 = \frac{k_{win} L^4}{EI}, \quad \Omega = \omega . L^2 \sqrt{\frac{\rho A}{EI}}, \quad \alpha = \frac{e0a}{L}$$

Using these parameters, the non-dimensional form of Eq. (7) can be written as

$$\frac{d^4 \overline{W}}{d\xi^4} + (\alpha^2 \Omega^2 - \alpha^2 \mu^2) . \frac{d^2 \overline{W}}{d\xi^2} + (\mu^2 - \Omega^2)\overline{W} = 0 \tag{8}$$

and it concludes that the free dynamic response is harmonic and dependent of the frequency ω. The first equality of Eq. (8) is written

$$\overline{W}^{IV}(x) + \gamma^2 . \overline{W}''(x) - \phi^4 . \overline{W}(x) = 0 \tag{9}$$

Where

$$\gamma^2 = (\alpha^2 \Omega^2 - \alpha^2 \mu^2); \ \phi^4 = (\mu^2 - \Omega^2) \tag{10}$$

4. The homogeneous solution of differential equation

Eq. (9) is a linear equation, homogeneous, with constant coefficients, which has the particular solutions by the form of $e^{\lambda x}$. The characteristic equation is

$$\lambda^4 + \gamma^2 . \lambda^2 - \phi^4 = 0 \tag{11}$$

The frequency parameter λ is given by the lower real root, i.e.,

$$\lambda = \sqrt{\frac{\sqrt{\gamma^4 + 4\phi^4} - \gamma^2}{2}} \tag{12}$$

Thus, the general solution of Eq. (9) is

$$\begin{aligned}\overline{W}(\xi) &= A_1 \cos(\lambda . \xi) + A_2 \sin(\lambda . \xi) \\ &+ A_3 \cosh(\lambda . \xi) + A_4 \sinh(\lambda . \xi)\end{aligned} \tag{13}$$

and the constants A_1, A_2, A_3 and A_4 will be determined from the boundary conditions of the problem.

5. The non-dimensionalized boundary conditions

Table 1 The associated nondimensionalized boundary conditions

Simply supported ends	$W'' = \frac{\partial^2 W}{\partial x^2} = 0$	at ξ=0, 1
Clamped - Simply supported	$W = W' = \frac{\partial W}{\partial x} = 0$	at ξ = 0
	$W = \frac{\partial^2 W}{\partial x^2} = 0$	at ξ =1
Clamped ends	$W' = \frac{\partial w}{\partial x} = 0$	at ξ = 0, 1
Cantilever	$W' = \frac{\partial W}{\partial x} = 0$	at ξ =0
	$\frac{\partial^2 W}{\partial x^2} = \frac{\partial^3 W}{\partial x^3} = 0$	at ξ =1

The associated non-dimensionalized boundary conditions handled in this paper are given in Table 1.

By substituting Eq. (13) into the boundary conditions, the eigenvalue problem may be expressed as

$$[S]\times\{A\}=\{0\} \tag{14}$$

where $\{A\}=\{A_1, A_2, A_3, A_4\}$ and the matrix [S] is given below for nanotubes with various end conditions

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ -\lambda^2 & 0 & \lambda^2 & 0 \\ \cos(\lambda) & \sin(\lambda) & \cosh(\lambda) & \sinh(\lambda) \\ -\lambda^2\cos(\lambda) & \lambda^2\sin(\lambda) & \lambda^2\cosh(\lambda) & \lambda^2\sinh(\lambda) \end{bmatrix} \tag{15a}$$

For Simply supported beam

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & \lambda & 0 & \lambda \\ \cos(\lambda) & \sin(\lambda) & \cosh(\lambda) & \sinh(\lambda) \\ -\lambda^2\cos(\lambda) & \lambda^2\sin(\lambda) & \lambda^2\cosh(\lambda) & \lambda^2\sinh(\lambda) \end{bmatrix} \tag{15b}$$

For Clamped-Simply supported

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & \lambda & 0 & \lambda \\ \cos(\lambda) & \sin(\lambda) & \cosh(\lambda) & \sinh(\lambda) \\ -\lambda\sin(\lambda) & \lambda\cos(\lambda) & \lambda\sinh(\lambda) & \lambda\cosh(\lambda) \end{bmatrix} \tag{15c}$$

For Clamped ends

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ \cos(\lambda) & \sin(\lambda) & \cosh(\lambda) & \sinh(\lambda) \\ -\lambda\sin(\lambda) & \lambda\cos(\lambda) & \lambda\sinh(\lambda) & \lambda\cosh(\lambda) \end{bmatrix} \tag{15d}$$

For Cantilevered beam

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & \lambda & 0 & \lambda \\ -\lambda^2\cos(\lambda) & -\lambda^2\sin(\lambda) & \lambda^2\cosh(\lambda) & \lambda^2\sinh(\lambda) \\ -\lambda(\lambda^2+\gamma^2)\sin(\lambda) & \lambda(\lambda^2+\gamma^2)\cos(\lambda) & \lambda(\lambda^2+\gamma^2)\sinh(\lambda) & \lambda(\lambda^2+\gamma^2)\cosh(\lambda) \end{bmatrix} \tag{15e}$$

For a nontrivial solution, the determinant of the matrix [S] must vanish. This yields the following characteristic equations

$\sin(\lambda)=0$ For Simply supported beam

$\tan(\lambda)=\tanh(\lambda)$ For Clamped-Simply supported

$\cos(\lambda).\cosh(\lambda)-1=0$ For Clamped ends

$\cos(\lambda).\cosh(\lambda)+1=0$ For Cantilevered beam

From Eq. ($\sin(\lambda)=0$) results the natural frequencies values

$$\lambda = j\pi \ , \ j = 1,2,3,..... \tag{16}$$

From Eq. (12), with the absolute value of $|\lambda|$, results

$$\lambda^2 = \frac{\sqrt{\gamma^4 + 4\phi^4} + \gamma^2}{2} = j^2\pi^2 \tag{17}$$

To obtain natural frequencies values ω_j, also ϕ_j^4, will proceed further

$$\omega_j^2 = \left(-\frac{\mu^2 + j^4\pi^4 + j^2\pi^2(\alpha^2\mu^2)}{(1+j^2\pi^2\alpha^2)}\right)\cdot\left(\sqrt{\frac{EI}{\rho AL^4}}\right) \tag{18}$$

6. Results and discussion

In the present study the impact of all parameters such as nonlocal small-scale effects, Winkler modulus parameter, vibration mode for four types of boundary conditions e.g., simply supported, clamped-simply, clamped ends and Cantilever beam on first, second and third frequencies of the SWCNTs have been studied. As a validation example, the first three natural frequencies of nonlocal Euler-Bernoulli beam with various boundary conditions are studied and compared with Wang *et al.* (2007).

The material properties used in the present study are the mass density ρ=2300 kg/m^3, the poisson ratio υ=0.19, the Young's modulus E=5.5 TPa (Yao and Han 2006).

In the Table 2 compares the first three non-dimensional frequency of nonlocal nanobeam for four kinds of boundary conditions and L/d=10 obtained by the present method with the results of Wang *et al.* (2007). It can be seen in Table 6 the good agreement of the proposed method of solution with various small scale parameters.

Winkler modulus parameter has been considered here. The detailed of First three non-dimensional frequency $\sqrt{\Omega}$ for four kinds of boundary conditions with and without elastic medium using nonlocal Euler-Bernoulli beam model are listed in Table 3. The ratio of the length to the diameter, (L/d), is 10 and the scale coefficients, (α=0, 0.5, 1).

The results show the dependence of the frequency on the mode number and the elastic medium. It is noted that the frequency increases when elastic medium is neglected, this increasing is attributed to the stiffness of the elastic medium. With higher values of mode number the rate of increase of frequency reduces, and becomes more significant with the higher of small-scale parameter. This is interpreted as the small-scale effect makes the CNTs more flexible as CNT being assumed as atoms linked by springs, the external elastic medium "grips" the SWCNTs and forces

Table 2 First three non-dimensional frequency $\sqrt{\Omega}$ of nonlocal Euler-Bernoulli beam for Simply supported, Clamped–simply, Clamped ends and Cantilever beam with (kwin=0, L/d=10)

	Mode 1		Mode 2		Mode 3	
A	Wang *et al.* (2007)	Present	Wang *et al.* (2007)	Present	Wang *et al.* (2007)	Present
Simply supported beam						
0	3.1416	3.141593	6.2832	6.283185	9.4248	9.424777
0,1	3.0685	3.068531	5.7817	5.781668	8.0400	8.039987
0,3	2.6800	2.679996	4.3013	4.301343	5.4422	5.442246
0,5	2.3022	2.302231	3.4604	3.460401	4.2941	4.294061
0,7	2.0212	2.021245	2.9585	2.958479	3.6485	3.648549
Clamped–simply supported beam						
0	3.9266	3.926602	7.0686	7.068583	10.2102	10.210174
0,1	3.8209	3.820892	6.4649	6.464884	8.6517	8.651699
0,3	3.2828	3.282839	4.7668	4.766755	5.8371	5.837546
0,5	2.7899	2.789928	3.8325	3.832499	4.6105	4.611530
0,7	2.4364	2.436436	3.2776	3.277570	3.9201	3.921486
Clamped beam						
0	4,7300	4.730041	7,8532	7.853205	10,9956	10.995606
0,1	4,5945	4.594457	7,1402	7.140250	9,2583	9.258343
0,3	3,9184	3.918368	5,1963	5.196310	6,2317	6.236756
0,5	3,3153	3.315323	4,1561	4.156066	4,9328	4.948517
0,7	2,8893	2.889340	3,5462	3.546228	4,1996	4.223876
Cantilever beam						
0	1.8751	1.875104	4.6941	4.694091	7.8548	7.854758
0,1	1.8792	1.879171	4.5475	4.547483	7.1459	7.145895
0,3	1.9154	1.915370	3.7665	3.766536	5.2988	5.298544
0,5	2.0219	2.021921	2.9433	2.943266	-	-
0,7	-	-	-	-	-	-

it to be stiffer. In additional, it is clearly that the frequency increases when the vibrational mode number increases.

The Figs. 2-4 show the dependence of the frequency on the small scale and vibrational mode number of (SWCNTs) embedded in an elastic medium. The frequency serves as an index to assess quantitatively the scale effect on CNT vibration solution. It is observed from Figs. 2-4 that the frequency ratios are peak for local Euler-Bernoulli beam model if the scale effect between the individual carbon atoms in CNTs is neglected. However for larger values of α, this dependence becomes very largest. However, the small scale effect makes the beam more flexible. In additional, the ranges of the frequency for kinds of boundary conditions are quite different. For Clamped-simply Fig. 4, the range is the smallest for Clamped ends Fig. 2, but the range is the largest for simply supported Fig. 3. The reason for this difference is attributed to the boundary conditions effects. Furthermore, it is clearly that as the vibrational mode number increases, the frequency decreases. This significance in higher modes is attributed to the influence of small wavelength for higher modes. For smaller

Table 3 The effect of Winkler modulus parameter on First three non-dimensional frequency $\sqrt{\Omega}$ of nonlocal Euler-Bernoulli beam for four kinds of boundary conditions and (L/d=10)

	Without elastic medium			With elastic medium		
α	mode 1	mode 2	mode 3	mode 1	mode 2	mode 3
Simply supported beam						
0	3.141593	6.283185	9.424778	3.102895	6.278431	9.423370
0.5	2.302231	3.460401	4.294057	2.198958	3.431617	4.279104
1	1.730201	2.491001	3.061400	1.435812	2.411026	3.019494
Clamped-simply supported beam						
0	3.926602	7.068583	10.210176	3.907000	7.065245	10.209068
0.5	2.789928	3.832503	4.609050	2.734029	3.811403	4.598745
1	2.078038	2.760644	3.289952	1.932109	2.702868	3.257411
Clamped beam						
0	4.730041	7.853205	10.995598	4.718871	7.850772	10.994718
0.5	3.315323	4.156077	4.925093	3.282515	4.139579	4.929401
1	2.461007	2.982564	3.528925	2.377862	2.937074	3.494937
Cantilever beam						
0	1.875104	4.694091	7.854758	1.663131	4.682661	7.852326
0.5	2.021921	2.943266	4.745964	1.861268	2.895940	-
1	3.430133	5.158199	-	4.316503	5.259422	-

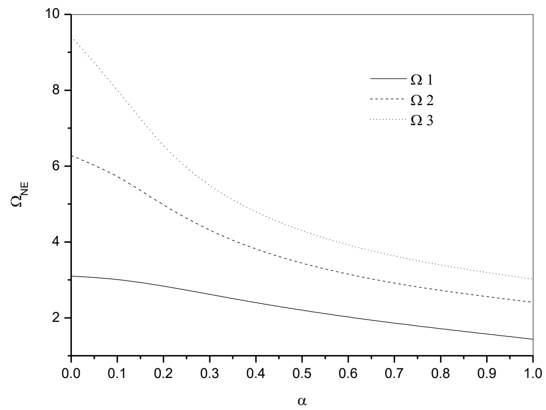

Fig. 2 small scale effect on different frequency modes for simply supported beam and (kwin=0.1, L/d=10)

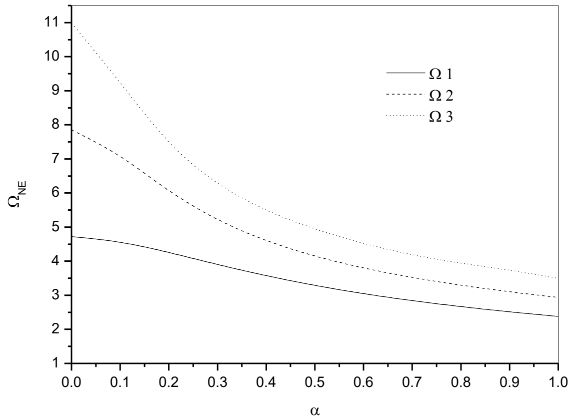

Fig. 3 small scale effect on different frequency modes for Clamped beam and (kwin=0.1, L/d=10)

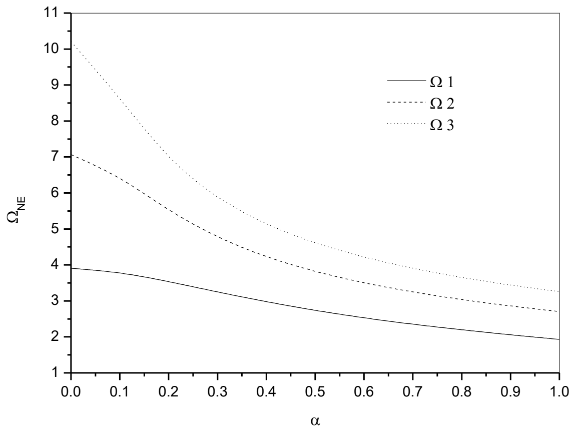

Fig. 4 small scale effect on different frequency modes for Clamped simply supported beam and (kwin=0.1, L/d=10)

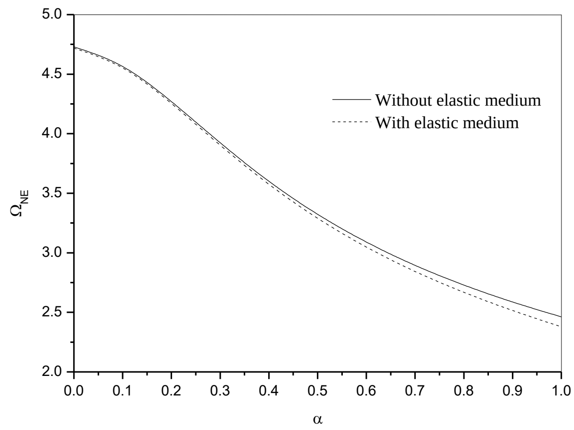

Fig. 5 The effect of elastic medium on first non-dimensional frequency of a short-SWCNT for clamped ends with different parameter α and L/d=10

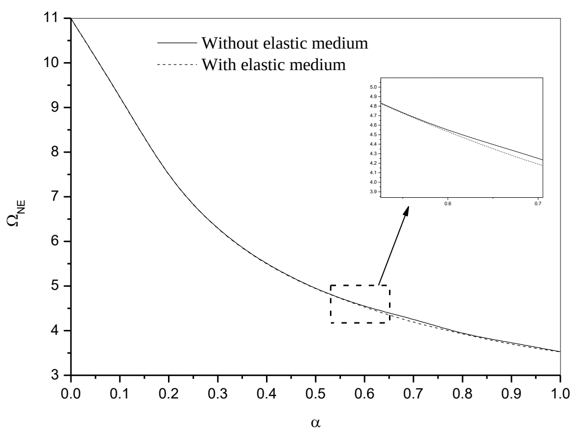

Fig. 6 The effect of elastic medium on third non-dimensional frequency of a short-SWCNT for clamped ends with different parameter *α* and L/d=10

wavelengths, interactions between atoms are increasing and this leads to an increasing in the small scale effect.

The effect of elastic medium on first and third non-dimensional frequency of a short-SWCNT for clamped ends with different parameter α is shown in Figs. 5-6 with the aspect ratio is (L/d=10). It can be seen that the difference between the frequency with and without elastic medium it is very weak for small values of α and for the higher values this difference become clearly. In additional, the range of the frequency ratios without elastic medium is the smallest for frequency with elastic medium because the elastic medium grips the CNT and forces it to be stiffer.

7. Conclusions

In this paper, we provide the analysis analytical of non-dimensional frequency of (SWCNTs) embedded in an elastic medium, based on non-local Euler-Bernoulli beam theory. The model analytical we mainly applied in this study to predict the vibration of a short SWCNT. Theoretical formulations include the small scale effect, the mode number, the aspect ratio and the elastic medium for kinds of boundary conditions. A very good agreement was observed when the comparisons are made with the studies in literature.

According to the study, the results showed the dependence of the non-dimensional frequency on the small-scale coefficients, Aspect Ratio, mode number and Winkler modulus parameter. However, it is observed that the small scale effect makes the beam more flexible and the ranges of the frequency ratios for kinds of boundary conditions are quite different. The reason for this difference is attributed to the boundary conditions effects. Furthermore it is clearly that as an increase on the vibrational mode number leads to the largest dependence. This dependence in higher modes is attributed to the influence of small wavelength. In this study, the non-dimensional frequency is more affected by elastic medium. The reason of this more affected is attributed to the rigidity of elastic medium when the elastic medium grips the CNT and forces it to be stiffer.

Acknowledgments

This research was supported by the Algerian National Thematic Agency of Research in Science and Technology (ATRST) and university of Sidi Bel Abbes (UDL SBA) in Algeria.

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