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Damage detection technique for irregular continuum structures using wavelet transform and fuzzy inference system optimized by particle swarm optimization

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Abstract. This paper presents a method for detecting damage in irregular 2D and 3D continuum structures based on combination of wavelet transform (WT) with fuzzy inference system (FIS) and particle swarm optimization (PSO). Many damage detection methods study regular structures. This method studies irregular structures and doesn't need response of healthy structures. First the damaged structure is analyzed with finite element methods, and damage response is obtained at the finite element points that have irregular distance, secondly the FIS, which is optimized by PSO is used to obtain responses at points, having equal distance by response at those points that previously obtained by the finite element methods. Then a 2D (for 2D continuum structures) or a 3D (for 3D continuum structures) matrix is performed by equal distance point response. Thirdly, by applying 2D or 3D wavelet transform on 2D or 3D matrix that previously obtained by FIS detail matrix coefficient of WT is obtained. It is shown that detail matrix coefficient can determine the damage zone of the structure by perturbation in the damaged area. In order to illustrate the capability of proposed method some examples are considered.

Keywords: discrete wavelet transform; damage detection; fuzzy inference system; particle swarm optimization

1. Introduction

Damage in a structural system may be caused by various parameters, such as corrosion, excessive loading, crack opening, wear and tear of some parts of structure, unpredictable environmental conditions and impact by a foreign object. The researcher's attention was on the structural damage detection in the recent decade. There are a lot of approaches in the field of damage identification, each of them has advantages and disadvantages, and WT is the most important approach among these approaches. The WT is a remedial method for precise signal analysis, which overcomes the problems exhibited by other techniques. Applying WT on response of damaged structures, produces acceptable results in the damage identification. Perturbation in the wavelet coefficients near a damage zone shows the presence of the damage. There are a lot of approaches that process the local changes in the structural parameters based on wavelets having emerged recently. Some of these are explained below.

Bajaba and Alnefaie (2005) proposed a new technique that couples the modal analysis and WT for detection of multiple damages in a cantilevered beam with single and multiple damages. Mallikarjuna Reddy and Swarnamani (2012) used mode shapes and strain energy data of the damaged plate to show the effectiveness of using spatial WT for detection and localization of small damages. Lotfollahi-Yaghin and Hesari (2008) used frequency analysis response of dam to identify crack in dam structure under wavelet analyzing. Magdalena (2011) used the first eight modes of a cantilever beam with damage in the form of a single notch of depth 20%, 10% and 5% of the beam height to present the results of experimental and numerical analyses of damage detection based on higher order modes. Bagheri and Kourehli (2013) proposed an effective method for the damage diagnosis of structures under seismic excitation via discrete WT based on changes in the seismic vibration responses. Balafas and Kiremidjian (2015) presented the development and validation of several novel data-driven damage sensitive features based on the continuous WT. Obrien *et al*. (2015) investigated a method for damage detection using a moving force identification algorithm. Yu *et al*. (2013) studied damage detection in a six-bay truss bridge model and used the fuzzy C-means clustering algorithm to categorize features for structural damage detection. Zhao (2012) identified the crack of the sprocket wheel by wavelet finite element method. Chen and Oyadiji (2017) identified damage property from the modal frequency curve via discrete WT. Hajizadeh *et al*. (2016) applied 2-D discrete wavelet to identify multi-cracks in plate structures by using static and dynamic responses. Hajizadeh *et al*. (2016) identified the damage type, damage existence and failure location in plate by wavelet and curvelet transform.

There has been no considerable research on irregular 2D and 3D continuum structures with the help of WT. In this

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study, the procedure of damage detection is performed based on using the distribution of coefficients of 2D and 3D discrete WT. The main aims of this study are:

1- How, combination of an optimized FIS, with a WT, can detect the damage of irregular structures.

2- Investigate the performance and accuracy of WT for damage detection of irregular plate and arch dams.

The arch dams are one of the important 3D continuum structures which are exposed to local failures over their useful life. By identifying the exact location of the failure, it can be repaired. This will increase the useful life of the structure and save on additional costs resulting from major repairs or the construction of a replacement structure.

2. An Overview of WT, FIS and PSO

2.1 WT

Wavelet analysis begins with the selection of a wavelet basic function among the available wavelets which are a function of location *x* (Kim and Melhem 2004, Chen *et al.* 2014). This basic wavelet function is called mother wavelet, $\psi(x)$. Then it would be delayed by m and transferred by n in space to form a set of basic functions $\psi_{m,n}(x)$ shown by Eq. (1).

$$\psi_{m,n}(x) = \left(\frac{1}{\sqrt{m}}\right)\psi\left(\frac{x-n}{m}\right) \tag{1}$$

The function is centralized in *n* with the spreading ratio of *m*. Continuous or discrete WT correlates wavelet function $f(x)$ with $\psi_{m,n}(x)$. Eq. (2) is continuous WTs which decompose a signal in the space domain into a two-dimensional function in the space-scale plane m, n. (Kim and Melhem 2004, Ovanesova and Suarez 2004).

$$C(m,n) = \left(\frac{1}{\sqrt{m}}\right)\int_{-\infty}^{\infty} f(x)\psi\left(\frac{x-n}{m}\right)dx = \int_{-\infty}^{\infty} f(x)\psi_{m,n}(x)\,dx \tag{2}$$

Where, $C\,(m,n)$ are wavelet coefficients and, *m*, *n* are real numbers and *m*≠0.

A discrete type of the wavelet is often empowered by discretizing the dilation parameter m and the translation parameter n. The dilation and translation parameters are defined as $m = 2^r$ and $n = k2^r$ respectively, where *r* and *k* are set of integers. Discrete form of $\psi_{m,n}(x)$ is shown by Eq. (3).

$$\psi_{r,k}(x) = \psi(2^{-r}x - k) \tag{3}$$

Where $\psi_{r,k}(x)$ is discrete form of $\psi_{m,n}(x)$. This type of sampling from coordinates (*m*, *n*) is known as dyadic sampling, because the consecutive values of discrete scales are different by a factor of 2. (Kim and Melhem 2004).

Application of discrete scales can describe discrete WT which is shown by Eq. (4).

$$C_{r,k} = 2^{\left(\frac{-r}{2}\right)}\int_{-\infty}^{\infty} f(x)\psi(2^{-r}x-k)dx = \int_{-\infty}^{\infty} f(x)\psi_{r,k}(x)dx \tag{4}$$

Where, $C_{r,k}$ are wavelet coefficients in discrete WT form.

Resolution of signal is defined by the inverse scale $(1/m) = 2^{-r}$, where the integer r is related to the level. The smaller the level and scale, the higher the resolution would be available.

Note that the continues WT $C_{(m,n)}$ is possible for the small scales, $m < m_0$. In this case, complete information about $C_{(m,n)}$ for $m > m_0$ is required. To collect this information, it is necessary to produce another function $\emptyset\,(x)$, which returns to the scale function. Substituting $\psi\,(x)$ by $\emptyset\,(x)$ in Eq. (2), function $D(m_0\,,n)$ shown by Eq. (5) is obtained.

$$D(m_0\,,n) = \int_{-\infty}^{\infty} f(x)\frac{1}{\sqrt{m_0}}\emptyset\left(\frac{x-n}{m_0}\right)dx = \int_{-\infty}^{\infty} f(x)\emptyset_{m_0,n}(x)dx \tag{5}$$

The existence of function $\emptyset\,(x)$ for the numerical calculations of fast WT is very important. It must be noted that the dyadic scales are used for *m*, *n* and the reference level, *R*, must also be taken into account. By applying Eq. (4) to this case, a set of wavelet coefficients will be obtained. These set of wavelet coefficients is shown by Eq. (6). (Kim and Melhem 2004, Staszewski 1998).

$$cD_R(k) = \int_{-\infty}^{\infty} f(x)\psi_{R,k}(x)dx \tag{6}$$

The coefficients $cD_R(k)$ are known as the reference level *R* detail coefficients. By applying the dyadic scale and reference level *R*, Eq. (5) will tend to other set of coefficients which is shown by Eq. (7).

$$cA_R(k) = \int_{-\infty}^{\infty} f(x)\emptyset_{R,k}(x)dx \tag{7}$$

The coefficients $cA_R(k)$ are known as the approximation coefficients for reference level *R*.

The function $\sum_{k=-\infty}^{\infty} cD_R(k)\psi_{R,k}(x)$ is known as the detail function of reference level *R*.

$$D_r(x) = \sum_{k=-\infty}^{\infty} cD_r(k)\psi_{r,k}\,(x) \tag{8}$$

Now we have two sets of signals, but in damage detection, we are interested in the details of the signals. If $f(x)$is assumed to be a structural response (for example, a deformation curve), the signals $D_r(x)$ contain information necessary for determining damage in the structure (Kim and Melhem 2004, Ovanesova and Suarez 2004).

The wavelets in higher dimensions are obtained by the tensor product of one-dimensional wavelet, (Mallat 2008). In short, this concept is expressed in a two (R^2) and three-dimensional (R^3) state, in which R^2,R^3 is considered with coordinates (*x*, *y*) and (*x*, *y*, *z*) respectively.

Assume that $\emptyset$ and ψ are the scale and mother wavelet function, the two-dimensional separable functions are shown by Eqs. (9)-(10).

$$\phi_{rkst}(x,y) = \phi_{rk}(x)\phi_{\mathrm{st}}(y) \tag{9}$$

$$\psi_{rkst}(x,y) = \psi_{rk}(x)\psi_{st}(y) \quad (10)$$

Where r, k, s, t are integers and ϕ_{rk}, ψ_{rk}, ϕ_{st}, ψ_{st} are shown by Eqs. (11)-(14)

$$\phi_{rk}(x) = 2^{\frac{r}{2}}\phi(2^r x - k) \quad (11)$$

$$\psi_{rk}(x) = 2^{\frac{r}{2}}\psi(2^r x - k) \quad (12)$$

$$\phi_{st}(y) = 2^{\frac{s}{2}}\phi(2^s y - t) \quad (13)$$

$$\psi_{st}(y) = 2^{\frac{s}{2}}\psi(2^s y - t) \quad (14)$$

These bases are orthogonal. In addition, each ψ_{rk} is perpendicular to all ϕ_{rk} and each ψ_{st} is perpendicular to all ϕ_{st}, indices r and s vary between 0 and upper limit (integer). The indices k and t correspond to the transferred components, which depend on arbitrary amplitude.

In three-dimensional separable case, 3D wavelet functions are shown by Eqs. (15)-(16). (Kaarna *et al* 2008, Mallat 2008).

$$\phi_{rkstpq}(x,y,z) = \phi_{rk}(x)\phi_{\mathrm{st}}(y)\phi_{\mathrm{pq}}(z) \quad (15)$$

$$\psi_{rkst\mathrm{pq}}(x,y,z) = \psi_{rk}(x)\psi_{\mathrm{st}}(y)\psi_{\mathrm{pq}}(z) \quad (16)$$

Where r, k, s, t, p, q are integers, $\psi_{pq}(z)$ is one-dimensional orthonormal discrete mother wavelet which its equation is shown by Eq. (18) and $\phi_{pq}(z)$ is scale function and its equation shown by Eq. (17).

$$\phi_{pq}(z) = 2^{\frac{p}{2}}\phi(2^p z - q) \quad (17)$$

$$\psi_{pq}(z) = 2^{\frac{p}{2}}\psi(2^p z - q) \quad (18)$$

2.2 FIS

Concept of fuzzy set was introduced for the first time by Lotfizadeh (1965) who is among the leading developers of fuzzy logic as a substitute for the Aristotelian logic. In the Aristotelian logic, each proposition or phrase might be true or false, which is attributed to 1 or 0, respectively. However, in the fuzzy logic, a proposition might have a value between 0 and 1, the proposition might be not completely true or false, but with the degree of trueness and falseness. For more information on features of the fuzzy sets, see references, (Sivanandam *et al.* 2007, Nguyen and Walker 2005).

In this paper, fuzzy is used as an approximator. This article only focused on FIS in forming the appropriate structure of two and three-dimensional matrices. The corresponding structural analysis is used for two and three-dimensional WTs.

2.3 PSO algorithm

PSO algorithm was first introduced by Eberhart and Kennedy (1995). The algorithm is inspired by the lives of birds, living in groups and meeting their requirements such as searching for foods in flocks. For more information on features of the PSO algorithm, see references: (Li *et al.* 2007, Perez and Behdinan 2007, Shi and Eberhart 1998).

In this article, the PSO algorithm is used to optimize FIS.

3. Optimal fuzzy system design with PSO intelligent algorithm

A fuzzy system contains name, type, rules, combining methods of conditional proposition (and method) and conditional proposition (or method), input, output, aggregation method and defuzzification method.

In general, there are two types of fuzzy system, including Mamdani and Sugeno. (Mamdani and Assilian 1975, Sugeno 1985). The first two parts of the fuzzy inference process, fuzzifying the inputs and applying the fuzzy operator, are exactly the same. The main difference between Mamdani and Sugeno is that the Sugeno output membership functions are either linear or constant. Since Sugeno is more compressed and computationally efficient compared to the Mamdani, therefore, in creating the fuzzy models, it can be benefited from optimization techniques. In this paper, Sugeno type is used in FIS.

Training data included the input data and output data. Each input data contains a few variables, for instance, the three-variable input vector X is $X^T = \{X_1\ X_2\ X_3\}$, and output Y is $Y = \{Y_1\}$, and corresponding to each input data X, will be an output data Y. The relationship between input vectors and outputs is expressed by the rules.

Conditional proposition (and method), describes how to combine inputs and output based on their membership functions in each rule. Aggregation combines rules. Conditional proposition (or method) describes how to combine rules and defuzzification extracts the numeric value of output from aggregation.

Each input data variable and each output data contain three sections, including the name such as X_1, X_2, X_3, range of operation, and membership function. Membership function is a function that specifies the value of the attributes of the variables in the desired fuzzy set. There are many membership functions, some of which included Gaussian, triangular, trapezoidal, sigmoid functions.

The structure of a fuzzy set with three inputs and one output shown in Fig. 1.

Among all the blocks of this structure, the input and output, and among the various parts of input and output, the membership function is focused. Each membership function, depending on the type of function, has parameters that are working as function regulator. For instance, the Gaussian membership function parameters are the mean and standard deviation of the Gaussian function. The optimization techniques can make the membership parameters more efficient, so that the fuzzy system becomes able to simulate the data in the best possible manner.

Using PSO optimization algorithm, the parameters are regulated in such a way that the fuzzy inference system provides the best overlap from the response space. The optimization algorithm of membership function parameters of the fuzzy inference system using PSO method is as follows:

1. Obtaining the training data.

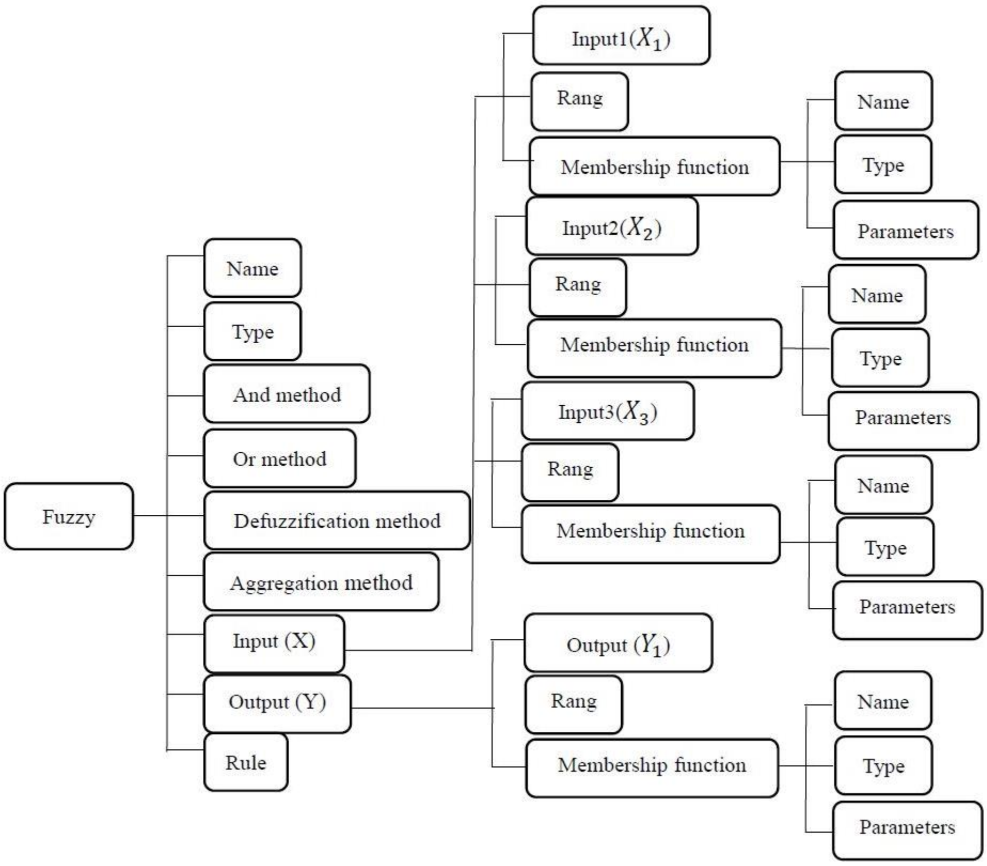

Fig. 1 The structure of a fuzzy set with three inputs and one output

2. Creating a basic fuzzy system.
3. Regulating the membership function parameters of the basic fuzzy system using modeling error function (minimizing the mean square error) by the PSO intelligent optimization algorithm.
4. Obtaining the fuzzy system with best values for membership function parameters.

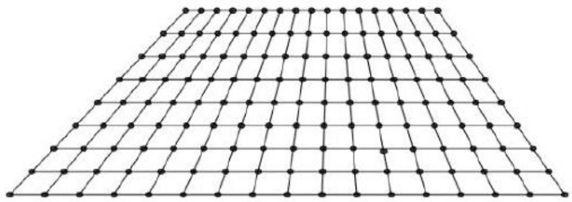

Fig. 2 Geometrically irregular two-dimensional plate

4. Wavelet damage detection procedure

Response of structure is an important data, which is required to find damage in a structure. In theoretical problem response of structure is obtained by finite element methods. In this study, linear elastic analysis is performed. In practical problem response of structure is obtained by sensors, which are installed in some points of structure. In irregular structure, damage detection by wavelet is formed in four stages mentioned bellow.

1. Obtaining the response of structure.
2. Regulating the response of structure.
3. Applying wavelet transform on regulated response.
4. Finding perturbation in detail matrix.

All of these are used in regular structure except stage 2. Stage 2 is an important subject in damage detection of irregular structure by wavelet, which is proposed in this paper, and discussed in this section.

Before proceeding to the subject, it is necessary to define regularity in 2D and 3D continuum structures.

In this study, a 2D or 3D continuum structure is defined as a 2D or 3D regular continuum structure if the structural meshes have an equal distance in two or three directions, otherwise it would be an irregular continuum structure.

A geometrically irregular 2D plate, is shown in Fig. 2. The elements of plate are configured according to the following mesh.

As can be observed, the sizes of the elements are not the same. Consequently, the element nodes that the structural responses are obtained at them have different distances. In order to apply the 2D wavelet transform, these intervals must be equal. The question is, how to overcome this problem?

The plate shown in Fig. 2 is supposed to be positioned inside a regular plate (ABCD) and this regular plate is divided into regular vertical and horizontal distances, until the following mesh plate shown in Fig. 3 would be obtained. This mesh would provide the expected regular two-dimensional matrix for 2D discrete wavelet transform.

Here are two things that need to be considered.

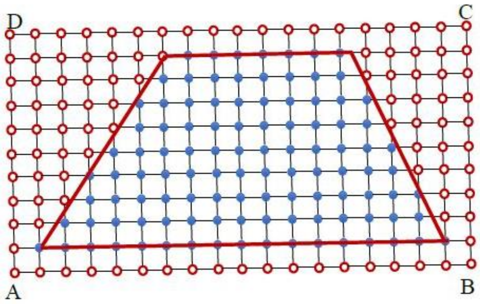

Fig. 3 The irregular plate placed inside a hypothetical regular rectangle

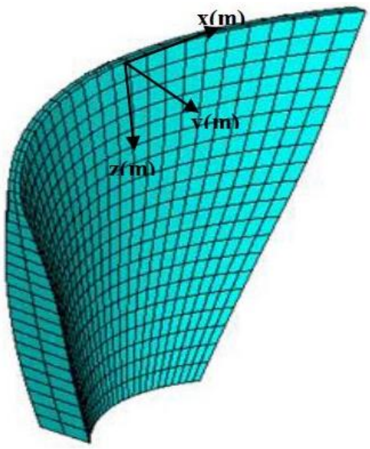

Fig. 4 Geometrically irregular three-dimensional

1- There are some points in the meshing that are located outside the studied structure which they did not actually exist (empty circle with red color).

2- There are some points (solid circle with blue color) on the studied structure that they are not the points, which their responses are obtained from finite elements analysis (the mesh in Fig. 2). FIS, which optimized by PSO algorithm, is used to find the structural responses at these points.

It is recommended to add some points near or outside the structure' boundary with zero value to the mesh input points responses to have more precise results for fuzzy system. The results of optimized FIS are acceptable if the correlation coefficient between real data (which is obtained by finite element analysis) and approximation data (which is obtained by optimized FIS) reached the maximum. The ordering of input data into the FIS program varies so much to achieve this goal.

The mathematical formula for computing correlation coefficient between two variables (x, y) is shown by Eq. (19)

$$f(x.y) = \left(\frac{n\sum xy - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2}\sqrt{n(\sum y^2) - (\sum y)^2}}\right) \tag{19}$$

where, f is correlation coefficient and n is number of pair of data, x is real data, y is approximation data. If x and y have a strong positive linear correlation, correlation coefficient is close to number +1. A correlation coefficient value of exactly +1 indicates a perfect positive fit.

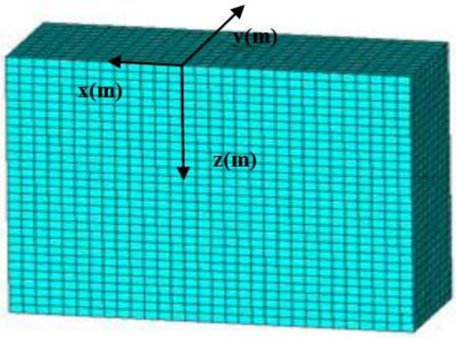

Fig. 5 The regular cubic with equal intervals in three directions

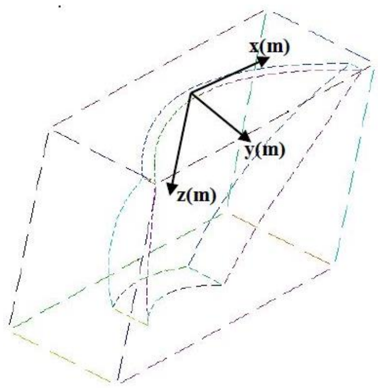

Fig. 6 The irregular dam placed inside a cubic

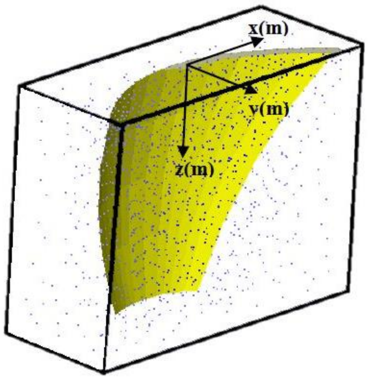

Fig. 7 Some point which add near or outside the dam's boundary

The presented dam is a geometrically irregular three-dimensional dam, which its elements are configured according to the mesh, shown in Fig. 4.

The responses are obtained at finite element points (mesh in Fig. 4), As can be observed, the sizes of the elements are not the same. Consequently, the element nodes have different distances. In order to apply the 3D discrete WT, these intervals must be equal as shown in Fig. 5. The question is how to overcome this problem?

The dam shown in Fig. 4 is supposed to be positioned inside a regular cubic Fig. 6 and this regular cubic is divided into regular distances to provide the expected regular three- dimensional matrix for 3D discrete WT. It is recommended to add some point Fig. 7 near or outside the

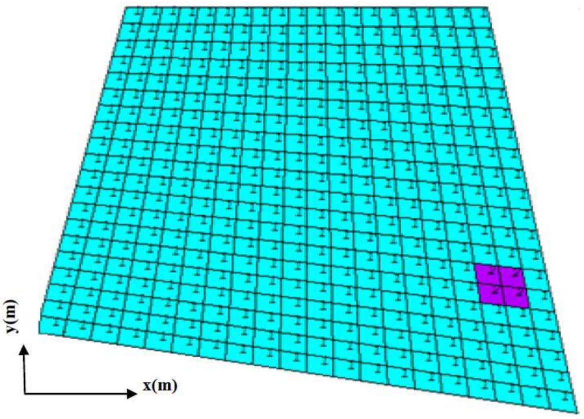

Fig. 8 Irregular plate which the damaged area shown by number (2) and purple color

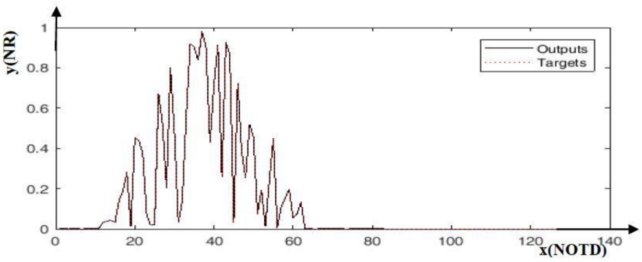

Fig. 9 Graph of real data and approximation data (which is obtained by optimized FIS)

dam’s boundary with zero value to the mesh input points responses to have more precise results for FIS. There are some points on the studied dam that they are not the points, which their responses are obtained from finite elements analysis. FIS, which optimized by PSO algorithm, is used to find the structural responses at these points.

5. Numerical example

In order to show the capabilities of the proposed method for identifying structural damage, three illustrative test examples are considered. The first example is an irregular plate with one damaged zone, the second one is a hole plate with one damaged zone and the third example is a dam with one damaged zone.

5.1 Plate with one damaged zone

The presented irregular plate shown in Fig. 8 is considered with the height of 6 m, width of 5 m at the top, width about of 6 m at the bottom, thickness of 0.10 m, and elasticity module of $3\,E10\ N/M^2$. In the area shown by number (2), elasticity modulus of the plate was reduced by 50%. It has fixed support in all borders and a uniform load ($50000\,N/M^2$). The mentioned plate was modeled, loaded, and analyzed by finite elements method. The displacement was considered as the response. The responses were provided as the input to the FIS which optimized by PSO algorithm. Conformity of real data and approximation data is shown in Fig. 9, in this figure *x* axis shows number of

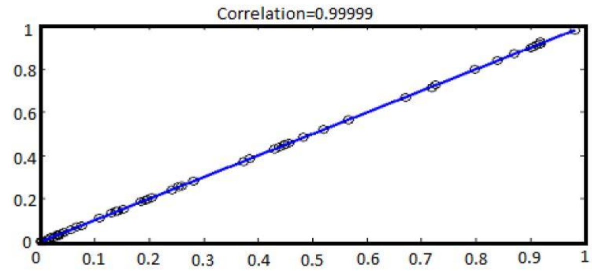

Fig. 10 Correlation coefficient (Eq. (19)) of real data and approximation data

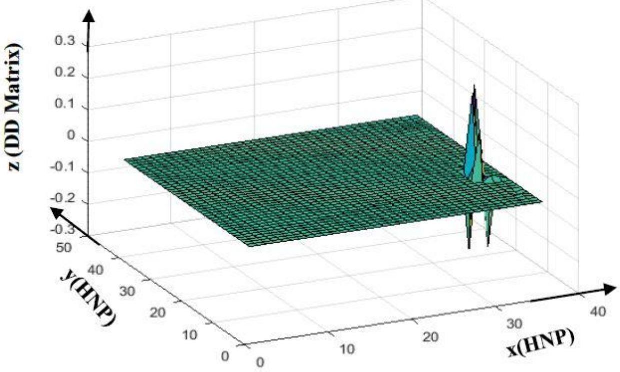

Fig. 11 Perturbation in damaged area (detail matrix)

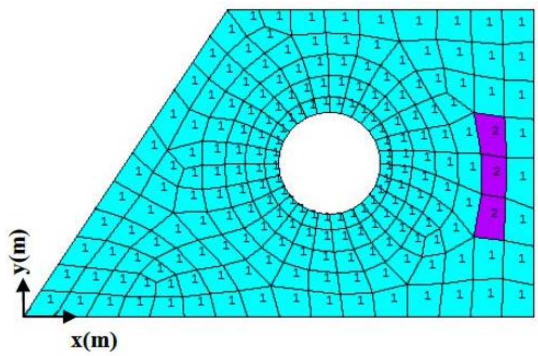

Fig. 12 Irregular hole plate which the damaged area is shown by number (2) and purple color

outputs and targets data (NOTD) and *y* axis shows normalized response (NR) of these data. Correlation coefficient (Eq. (19)) between real data and approximation data is shown in Fig. 10.

In the next step, using the FIS which optimized by PSO algorithm, the original 2D matrix representing plate response was formed. This matrix was also analyzed using the 2D wavelet transform (mother wavelet is shown by Eq. (10)) for the displacement responses, and in the detailed matrix, a jump in the damaged area is shown in Fig. 11. In this figure, the *x* and *y* axes indicate the half number of points (HNP) that the optimized FIS calculates the response matrix at those points and in those directions.

5.2 Plate with one hole and one damaged zone

The presented irregular plate shown in Fig. 12 is considered with a hole with the height of 6 m, width of 5 m

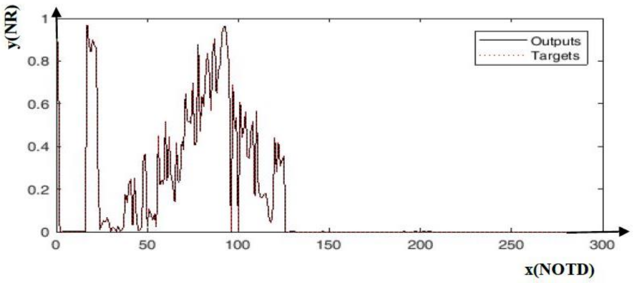

Fig. 13 Graph of real data and approximation data (which is obtained by optimized FIS)

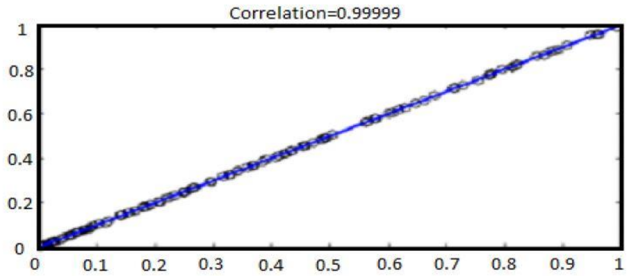

Fig. 14 Correlation coefficient (Eq. (19)) of real data and approximation data (which is obtained by optimized FIS)

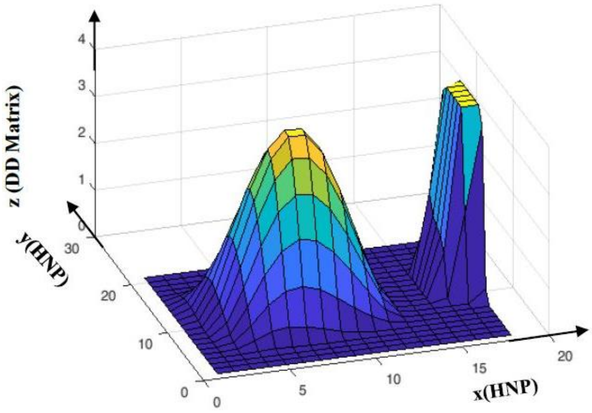

Fig. 15 Perturbation in damaged zone (detail matrix)

at the top, width of 8m at the bottom, thickness of 0.10m, and elasticity module of $3\,E10\ N/M^2$. In the area shown by number (2), elasticity module of the plate was reduced by 50%.

It has fixed support in all borders and a uniform load ($50000\,N/M^2$). All of the process mentioned in previous example is done in this plate and the simulate results are shown in Figs. 13-15.

5.3 Dam with one damaged zone

The presented dam shown in Fig. 16 has the height of 140 m, width of 190 m at the top, and width of 90 m at the bottom. In the zone shown darker (purple color), elasticity module of the dam was reduced by 50%. The above-mentioned dam was modeled, loaded (including weight plus hydro static load), and analyzed by a finite element software: the principal stress (S1) was considered as the response. The response was provided as the input to the FIS

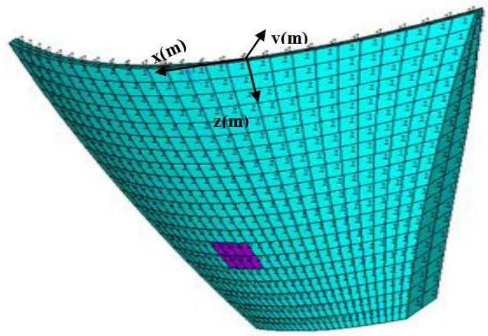

Fig. 16 The dam which in the zone shown darker (illustrated by purple color), elasticity module of the dam was reduced by 50%

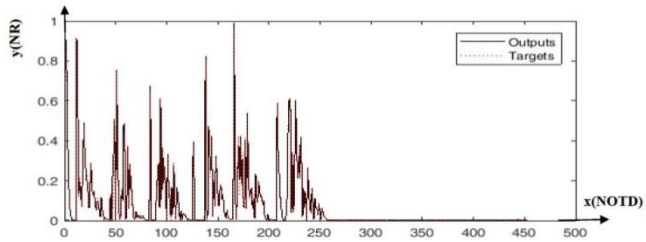

Fig. 17 Graph of real data and approximation data (which is obtained by optimized FIS)

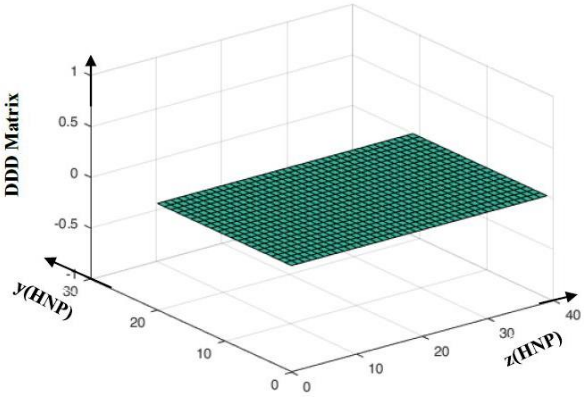

Fig. 18 detail Matrix (Out of the damaged region)

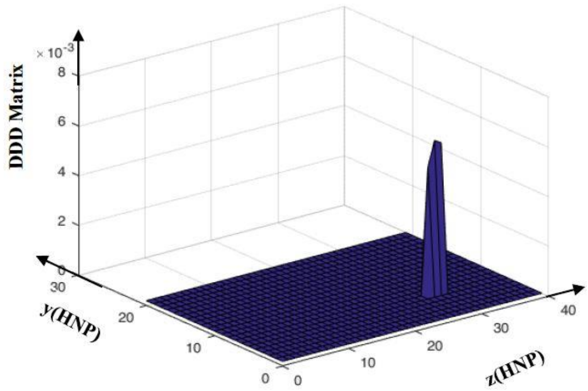

Fig. 19 detail Matrix (middle of damaged region)

which optimized by PSO algorithm then the FIS which optimized by PSO algorithm perfume the process until the correlation coefficient (Eq. (19)) of real data and approximation data (which is obtained by optimized FIS) reached the maximum (close to number +1). Conformity of real data and approximation data is shown in Fig. 17.

Then, using the FIS which optimized by PSO algorithm, the original 3D matrix representing dam response was formed. This matrix was also analyzed using the 3D discrete WT (mother wavelet is shown by Eq. (16)) for principle stress responses. It is shown Figs. 18-19 that detail matrix can be specified the damaged zone of dam by perturbation in this area. In these figures, the *y* and *z* axes indicate the half number of points (HNP) that the optimized FIS calculates the response matrix at those points and in those directions.

6. Conclusions

In this paper, a new method for irregular two and three-dimensional structural damage detection is based on a combination of WT and FIS, which optimized by PSO is proposed. Based on the numerical results the main conclusions are as follows:

1. Structural responses matrix of irregular 2D or 3D continuum structure can be obtained by transferring the irregular structure inside a regular imaginary plate or cube.
2. The FIS, which optimized by PSO is suitable to estimate the structural responses of the irregular points to a regular domain. This ability can be empowered by adding a number of points near or outside the structure's boundary with zero value to the mesh input points. This capability can prepare suitable two and three-dimensional structural response matrix (because the responses of a continuum structure should be evaluated at equal distance) for applying WT.
3. The details matrix which is obtained by applying 2D or 3D WT to structural response matrix can specify the damaged zones of 2D or 3D continuum structures by perturbation in these areas.

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