Structural Engineering and Mechanics, *Vol. 67, No. 4 (2018) 359-368*
DOI: https://doi.org/10.12989/sem.2018.67.4.359

Structural identification based on substructural technique and using generalized BPFs and GA

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(Received November 23, 2017, Revised May 21, 2018, Accepted June 12, 2018)

Abstract. In this paper, a method is presented to identify the physical and modal parameters of multistory shear building based on substructural technique using block pulse generalized operational matrix and genetic algorithm. The substructure approach divides a complete structure into several substructures in order to significantly reduce the number of unknown parameters for each substructure so that identification processes can be independently conducted on each substructure. Block pulse functions are set of orthogonal functions that have been used in recent years as useful tools in signal characterization. Assuming that the input-outputs data of the system are known, their original BP coefficients can be calculated using numerical method. By using generalized BP operational matrices, substructural dynamic vibration equations can be converted into algebraic equations and based on BP coefficient for each story can be estimated. A cost function can be defined for each story based on original and estimated BP coefficients and physical parameters such as mass, stiffness and damping can be obtained by minimizing cost functions with genetic algorithm. Then, the modal parameters can be computed based on physical parameters. This method does not require that all floors are equipped with sensor simultaneously. To prove the validity, numerical simulation of a shear building excited by two different normally distributed random signals is presented. To evaluate the noise effect, measurement random white noise is added to the noise-free structural responses. The results reveal the proposed method can be beneficial in structural identification with less computational expenses and high accuracy.

Keywords: system identification; substructural technique; block pulse operational matrix; genetic algorithm; noise effect

1. Introduction

Structural health monitoring (SHM) has been considered as a very important research field in civil engineering. The main parts of the SHM are damage detection, which are essential monitoring zones for structures after severe loading such as earthquakes (Garevski 2013).

System identification (SI) for dynamical parameters estimation of structures in recent years with the development of dynamic testing has become one of the useful methods for structural damage detection (Moaveni 2007). The basic idea is that modal parameters, such as frequencies, mode shapes, and modal damping, are functions of the physical properties of the structure (mass, stiffness, and damping) and damage changes the physical properties such as stiffness and it will cause changes in the modal properties and based on structural damage can be detected (Fan and Qiao 2011).

Therefore, accurate dynamical parameters estimation based on SI methods is a very important step in damage detection and also finite element model updating. Generally, SI is the process of determining relation or the parameters of a mathematical model from a physical system using a set of input-output information obtained from experiment. This technique in the various fields of engineering is known as inverse problem (Sirca Jr and Adeli 2012). In the case of structures, the input-outputs are excitation-responses of structure and the unknown parameters can be structural parameters such as mass, stiffness, damping and/or modal parameters such as natural frequencies, damping ratios and mode shapes.

There are many classical and non-classical methods for system identification in the time domain. Classical methods are mainly based on state space realization and eigen mode data analysis such as Eigen Realization Algorithm (Juang and Pappa 1985), Nature Excitation Technique (James *et al*. 1993), Generalized Realization Algorithm (De Callafon *et al*. 2008), Stochastic Subspace Identification (Overschee and Moor 1996).

Genetic Algorithm (GA) and Neural Network (NN) are now the two most commonly non-classical methods that used in structural parameters identification and damage detection problem. GA is a stochastic search technique based on natural selection and genetics, developed by Holland. GA is a robust tool for solving large and complicated optimization problems that can be used in the system identification. GA uses multiple points to search for the solution rather than a single points in the traditional gradient based optimization method and leads to the global optimum without converging to a local optimum or

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http://www.techno-press.com/journals/sem&subpage=7 ISSN: 1225-4568 (Print), 1598-6217 (Online)

diverging (Koh and Perry 2010, Marwala 2010, Monti *et al.* 2010, Wang 2009, Perry *et al.* 2006, Koh *et al.* 2000).

Shear structures are used to model tall buildings. A practical large structure usually requires a complex model that involves many degree-of-freedoms (DOFs) and unknown parameters. The main challenges are that data measurement and identification of an entire structure are not easy jobs and the difficulty and computation time required for convergence increase dramatically with the increase in the number DOFs. To overcome these problems, some researchers have been using the substructure method for large-scale structures. Substructure identification methods divide a large structure into many smaller substructures, each of which has far fewer DOFs and unknown parameters, and perform parameter identification for each substructure independently.

Since substructure identification methods highly decrease the size of unknown parameter search space for the optimization thus, convergence and ill-condition problems are reduced. Because only the structural responses related to the identified substructure are required in substructure identification, there is no need to monitor all DOFs simultaneously, which may greatly reduce the cost of structural health monitoring system. Several research works have proposed sub structuring methods for system identification and damage detection of large-scale structures. Koh *et al.* (1991) are considered to be the first to present the concept of substructure identification. They adapted the extended Kalman filter by adding a weighted global iteration algorithm to determine the stiffness matrix and the damping matrix of a substructure through solving the state and observation equations of the substructure. This approach needs acceleration, velocity and displacement responses of the substructures under consideration. This approach was further modified to use acceleration responses only with a non-classical approach of genetic algorithms was applied (Koh *et al*. 2003). Koh and Shankar (2003) proposed a substructure identification approach in frequency domain without the need of interface measurement. Tee *et al.* (2005) proposed a substructure identification method considering both first order and second order models. Xing and Mita (2012) proposed a substructure approach to divide a complete structure into several substructures in order to significantly reduce the number of unknown parameters for each substructure so that damage identification processes can be independently conducted on each substructure. Weng *et al.* (2012) proposed a substructural model updating method to obtain the independent substructural dynamic flexibility matrices under force and displacement compatibility conditions. The extracted substructural flexibility matrices are then used as references for updating the corresponding substructural models. Kuwabara *et al.* (2013) proposed a damage identification method for high-rise buildings which is devised to find the story shear and bending stiffness of a specific story from the floor accelerations just above and below the specific story. Zhang *et al.* (2014) presented a loop substructure identification method to estimate the parameters of any story in a shear structure using the cross power spectral densities (CPSD) of structural responses. Mei *et al.* (2015) proposed a substructural identification approach for shear structure based on changes in the first AR model coefficient matrix. Mei *et al.* (2016) proposed an improved substructural damage detection approach of shear structure based on auto regressive moving average with exogenous inputs (ARMAX) model to correct the former damage indicator. In this method the correction coefficient is defined as the normalized Kolmogorov-Smirnov (KS) test statistical distance between the two distinguished data sets of ARMAX model residual generalized from input-output data process for undamaged and damaged states.

Block pulse functions (BPFs) have been extensively investigated and used as an elementary set of functions for signal characterizations in system identification. The BPFs set proved to be the most fundamental and it enjoyed prolific popularity in different applications. In comparison with other basal functions or polynomials, the BPFs can result more easily to solve concrete problems. Sannuti (1997) showed that the application of BPFs results in a great decrease of computational effort over Walsh function in the control system. Pacheco and Steffen (2002) with integration of motion equation of mechanical system and using specific properties of the BPFs obtained a simple algebraic equation which leads to the determination of the unknown parameters. Bouafoura *et al.* (2010) proposed an analytic method for the fractional state space realization of non-integer systems with BPFs. Yinggan *et al.* (2015) converted the fractional differential equation to an algebraic one through a generalized operational matrix of block pulse functions and identified the parameters of fractional-order systems. Ajorloo and Ghaffarzadeh (2017) proposed a method to identify the dynamical parameters of shear building based on continuous time state space estimation using BPFs and least squares algorithm.

The main goal of this research is the implementation of BPFs in identification of the dynamic parameters of the shear building and simplification of identification process using substructural technique and the special properties of block pulse functions. These identified parameters can be used in structural damage detection, vibration control, model updating and etc. Using generalized block pulse operational matrices, in the block pulse domain each substructure vibration equation separately can be integrated. This integration is performed twice on both sides of substructural equations and the simulated BP coefficients of response for each story can be achieved. Based on this simulated and original BP coefficients, a cost function can be defined for each story and structural parameters including mass, damping and stiffness can be obtained by minimizing cost function with genetic algorithm then modal parameters including natural frequencies, and mode shapes can be computed based on identified structural parameters. The validity of the proposed method is verified by numerical examples for structural system subjected to random signals also the effects of noise studied and finally identified parameters are compared with the original values.

2. Block pulse functions and generalized operational matrices

BPFs are the family of orthogonal functions used widely as a rudimentary set of functions for signal characterizations in system science because of their simple formulations, easy operations and exact approximation (Younespour and Ghaffarzadeh 2014, Maleknejad *et al.* 2011, Mohan and Kar 2013, Ghaffarzadeh and Younespour 2015, Babolian and Masouri 2008, Younespour and Ghaffarzadeh 2016). The basic view of using BPFs for system identification is simplification of problem by applying fundamental properties of the BPFs.

The BPFs are defined on a time interval [0, *T*) as (Jiang and Schaufelberger 1992)

$$\varphi_i(t) = \begin{cases} 1 & \frac{(i-1)T}{m} \leqslant t \leqslant \frac{iT}{m} \\ 0 & otherwise \end{cases} \tag{1}$$

Where φ_i is i-th element of the orthogonal basis and $i = 1, 2, \ \ldots \ , m$ with positive integer value for m. Also, consider $h = \frac{T}{m}$ and T is the time horizon.

BPFs possess disparate properties, the most salient characteristics are disjointness, orthogonality and completeness. The disjointness property can be easily proved from the definition of BPFs

$$\varphi_i(t)\varphi_j(t) = \begin{cases} \varphi_i(t) & i = j \\ 0 & i \neq j \end{cases} \qquad i, j = 1, 2, \ldots, m \tag{2}$$

The second property is orthogonally which can be represented as follows

$$\int_0^T \varphi_i(t)\varphi_j(t)\, dt = h\delta_{ij} \tag{3}$$

Where, δ_{ij} is Kronecker delta.

The last property is completeness. For every $f \epsilon L^2([0, T))$

Parseval's identity holds

$$\int_0^T f^2(t)dt = \sum_{i=1}^{\infty} f_i^2 \|\varphi_i(t)\|^2 \tag{4}$$

The orthogonality is the main property and based on arbitrary real bounded function *f*(*t*), that is square integrable in the interval *t* € [0,T), can be expanded into block pulse series as follow

$$f(t) \simeq \hat{f}_m(t) = \sum_{i=1}^{m} f_i \ \varphi_i(t) \tag{5}$$

Where f_i is block pulse coefficient corresponding to the ith BPF. This formulation can also be expressed in vector form as

$$f(t) \simeq \hat{f}_m(t) = F^T \Phi(t) \tag{6}$$

Where $, \boldsymbol{\Phi^T}(t) = [\varphi_1(t), \varphi_2(t) \ldots \ \varphi_m(t)]$ and $F^T = [f_1, f_2 \ \cdots \ f_m]$ is the block pulse coefficients vector. The criterion of this approximation is minimization of the mean square error between $f(t)$ and $\hat{f}_m(t)$ in a time interval $t \ \epsilon \ [0, \ T)$

$$\varepsilon = \frac{1}{T}\int_0^T \left(f(t) - \sum_{j=1}^{m} f_j \, \varphi_j(t) \right)^2 dt \tag{7}$$

This yields to the determination of BP coefficients

$$f_i = \frac{1}{h}\int_{(i-1)h}^{ih} f(t)\, dt \tag{8}$$

A major attraction feature of BPFs is that the multiple integration of the real function in the BP domain is related to operational matrices. This idea is realized as generalized block pulse operational matrices by the Wang (1982) and can be expressed for *k*-times integration as follows

$$\int_0^t \ldots \int_0^t f(t)dt \ \ldots \ dt \cong F^T P_k \Phi(t) \tag{9}$$

Where P_k is the generalized operational matrix, which is given by

$$P_k = \frac{h^k}{(k+1)!} \begin{pmatrix} P_{k,1} & P_{k,2} & P_{k,3} & \ldots & \\ 0 & P_{k,1} & P_{k,2} & \ldots & \\ 0 \ 0 & P_{k,1} & \ldots & P_{k,m-2} & \\ \vdots \ \vdots & & \vdots \ \vdots & & \\ & 0 \ 0 \ 0 & \ldots & P_{k,1} & \end{pmatrix} \tag{10}$$

With

$$P_{k,j} = \begin{cases} 1 & for \ j = 1 \\ j^{k+1} - 2(j-1)^{k+1} + (j-2)^{k+1} & for \ j = 2, 3, \ldots, m \end{cases} \tag{11}$$

Since the BP operational rule of integration is much simpler than original function integration, the aim of simplification can be attained.

3. Substructural dynamic formulation of shear building and identification process

The equations of motion for multi degrees of freedom (MDOF) structure in generalized coordinate system can be written in matrix form as

$$M\ddot{X}(t) + C\dot{X}(t) + KX(t) = Bu(t) \tag{12}$$

Where M, C and K are $n \times n$ matrices of mass, damping and stiffness of a simulated structure, n is number of building stories respectively. $X(t), \dot{X}(t)$ and $\ddot{X}(t)$ are vectors of generalized displacement, velocity and acceleration, respectively and $u(t)$ is the $1 \times r$ input vector containing r external excitations acting on the system and B is the input matrix with dimension $n \times 1$. The mass matrix of shear structure is a diagonal matrix in which the mass of each story is sorted on its diagonal, as given in the following

$$M = \begin{bmatrix} m_1 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & m_n \end{bmatrix} \tag{13}$$

The structural stiffness matrix can be described based on the individual stiffness of each story k_i as follows (Note that the C matrix is the same configuration as K matrix)

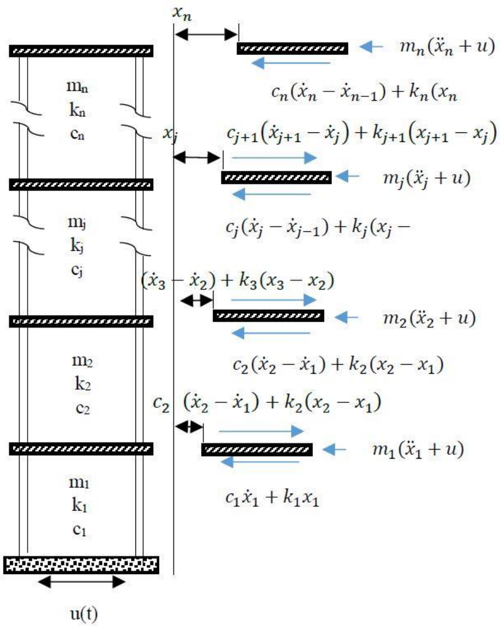


Fig. 1 Model and free-body diagram of shear structure

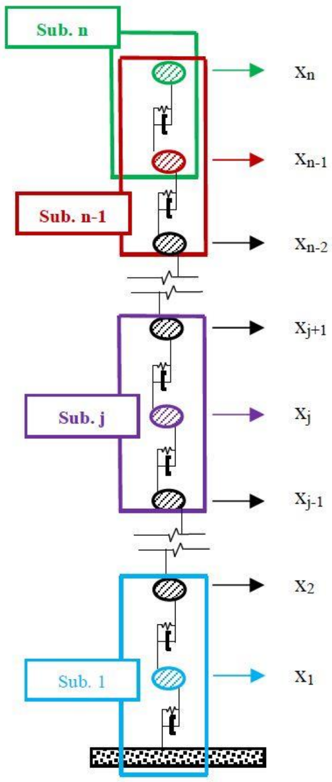


Fig. 2 Divided substructure model

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & \dots & 0 \\ -k_2 & k_2 + k_3 & -k_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ & & -k_{n-1} & k_{n-1} + k_n & -k_n \\ 0 & 0 & \dots & -k_n & k_n \end{bmatrix} \quad (14)$$

A sketch of a multi-story shear building under external excitation is shown in Fig. 1. The equation of motion for each degree of freedom can be written as

$$\begin{cases} m_1\ddot{x}_1 + c_1\dot{x}_1 - c_2(\dot{x}_2 - \dot{x}_1) + k_1x_1 - k_2(x_2 - x_1) = -m_1u(t) \\ \vdots \\ m_j\,\ddot{x}_j + c_j(\dot{x}_j - \dot{x}_{j-1}) - c_{j+1}(\dot{x}_{j+1} - \dot{x}_j) \\ +k_j(x_j - x_{j-1}) - k_{j+1}(x_{j+1} - x_j) = -m_j\,u(t) \\ \vdots \\ m_n\ddot{x}_n + c_n(\dot{x}_n - \dot{x}_{n-1}) + k_n(x_n - x_{n-1}) = -m_nu(t) \end{cases} \quad (15)$$

Where $x_i, \dot{x}_i, \ddot{x}_i$ are nodal displacement of the *i*-th story.

Eq. (15) shows a complete structure can be divided into several substructures which have a considerably smaller number of degrees of freedom (DOFs), as shown in Fig. 2.

In the block pulse domain with two-times integration from 0 to t on both side of Eq. (15) and assuming zero initial conditions, the algebraic relations are obtained as follow

$$\begin{cases} m_1(X_1^{BP})^TP_0\Phi(t) + c_1(X_1^{BP})^TP_1\Phi(t) \\ -c_2\left((X_2^{BP})^TP_1\Phi(t) - (X_1^{BP})^TP_1\Phi(t)\right) + k_1(X_1^{BP})^TP_2\Phi(t) \\ -k_2((X_2^{BP})^TP_2\Phi(t) - (X_1^{BP})^TP_2\Phi(t)) = -m_1\,(U^{BP})^TP_2\Phi(t) \end{cases}$$

$$\vdots$$

$$\vdots$$

$$\begin{cases} m_j\,(X_j^{BP})^TP_0\Phi(t) + c_j\left((X_j^{BP})^TP_1\Phi(t) - (X_{j-1}^{BP})^TP_1\Phi(t)\right) \\ -c_{j+1}\left((X_{j+1}^{BP})^TP_1\Phi(t) - (X_j^{BP})^TP_1\Phi(t)\right) \\ +k_j\left((X_j^{BP})^TP_2\Phi(t) - (X_{j-1}^{BP})^TP_2\Phi(t)\right) \\ -k_{j+1}((X_{j+1}^{BP})^TP_2\Phi(t) - (X_j^{BP})^TP_2\Phi(t)) \\ = -m_j\,(U^{BP})^TP_2\Phi(t) \\ \vdots \\ m_n(X_n^{BP})^TP_0\Phi(t) + c_n\left((X_n^{BP})^TP_1\Phi(t) - (X_{n-1}^{BP})^TP_1\Phi(t)\right) \\ +k_n((X_n^{BP})^TP_2\Phi(t) - (X_{n-1}^{BP})^TP_2\Phi(t)) \\ = -m_n\,(U^{BP})^TP_2\Phi(t) \end{cases} \quad (16)$$

Where X_j^{BP} and U^{BP} are j-th output and input signals block pulse coefficients also P_1 and P_2 are corresponding with one and two times BP integration operational matrix respectively. Note that P_0 is equal to the identity matrix and is used just to keep up appearances.

The component of $\Phi(t)$ can be removed from both sides of Eq. (16) and this equation can be configured as follows

$$\begin{cases} (\hat{X}_1^{BP})^T = \dfrac{b_1(X_2^{BP})^T + d_1(U^{BP})^T}{a_1} \\ \vdots \\ (\hat{X}_j^{BP})^T = \dfrac{b_j(X_{j-1}^{BP})^T + z_j(X_{j+1}^{BP})^T + d_j(U^{BP})^T}{a_j} \\ \vdots \\ (\hat{X}_n^{BP})^T = \dfrac{b_n(X_{n-1}^{BP})^T + d_n(U^{BP})^T}{a_n} \end{cases} \quad (17)$$

Where

$$\begin{cases} a_1 = m_1P_0 + c_1P_1 + c_2P_1 + k_1P_2 + k_2P_2 \\ b_1 = c_2P_1 + k_2P_2 \quad , \quad d_1 = -m_1P_2 \\ \vdots \\ a_j = m_jP_0 + c_jP_1 + c_{j+1}P_1 + k_jP_2 + k_{j+1}P_2 \\ b_j = c_jP_1 + k_jP_2 \quad , \quad z_j = c_{j+1}P_1 + k_{j+1}P_2 \\ d_j = -m_jP_2 \\ \vdots \\ a_n = m_nP_0 + c_nP_1 + k_nP_2 \\ b_n = c_nP_1 + k_nP_2 \quad , \quad d_n = -m_nP_2 \end{cases} \tag{18}$$

The cost functions based on original responses BP coefficients obtained from Eq. (8) and estimated BP coefficients obtained from Eq. (17) can be defined as

$$\begin{cases} (\hat{m}_1, \hat{c}_1, \hat{k}_1, \hat{c}_2, \hat{k}_2) = min \sum \left(X_1^{BP} - \hat{X}_1^{BP}\right)^2 \\ \vdots \\ (\hat{m}_j, \hat{c}_j, \hat{k}_j, \hat{c}_{j+1}, \hat{k}_{j+1}) = min \sum \left(X_j^{BP} - \hat{X}_j^{BP}\right)^2 \\ \vdots \\ (\hat{m}_n, \hat{c}_n, \hat{k}_n) = min \sum \left(X_n^{BP} - \hat{X}_n^{BP}\right)^2 \end{cases} \tag{19}$$

By minimizing the cost functions, physical parameters of structure can be identified. To solve minimization problem, many conventional optimization techniques can be applied. In this research, genetic algorithm is selected from optimization toolbox of MATLAB software as minimization solver. In the following, the numerical simulation and setting of minimization algorithm will be discussed.

4. Numerical study

To demonstrate the feasibility and validity of proposed method a 4-story shear building model is considered and shown in Fig. 3. Divided substructures also shown in Fig. 4. At the beginning of identification process, it is assumed that the responses are known for all degrees of freedom and excitations data are available. To evaluate the generality of proposed identification method, two random signals with different sampling time $(\Delta t = h)$ are selected and cited in Table 1. This signals can also be an earthquake or artificial harmonic excitation.

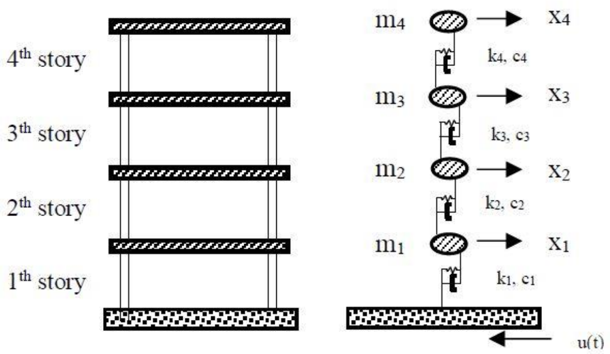


Fig. 3 Four-story shear building

Table 1 Properties of normally distributed random signals

Excitation	Total time(sec)	Number of data (m)	Sampling time (Δt=h)	mean	standard deviation
1	10	500	0.02	0	0.2
2	10	2000	0.005	0	0.1

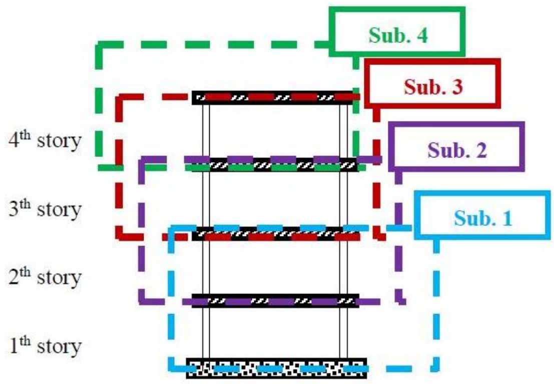


Fig. 4 Structure division

Table 2 Mass, stiffness and damping values of the original structure

Story	Mass (KN s^{2}m^{-1})	Stiffness (KN m^{-1})	Damping (KN sm^{-1})
1	5.50	1200.00	7.00
2	5.00	1000.00	6.00
3	4.50	800.00	5.00
4	4.00	600.00	4.00

The physical properties of the original structure consist of mass, damping and stiffness are listed in Table 2 and the responses of the structure assessed from the ordinary linear dynamic analysis via finite element method.

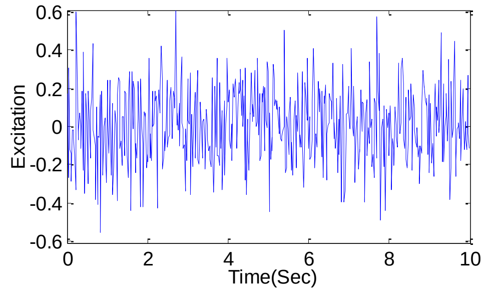
In order to use operation rules of block pulse functions, BP coefficients of continuous signal should be calculated. In the result, BP coefficients should be evaluated from Eq. (8) but in using this formula, evaluation is conceivable only when the analytical expressions of continuous signals are known or when measurements between the sampling instants are accessible.

In the lack of any information about the variations of signal, BP coefficients of the continuous signal must be approximated by numerical methods. The simplest approximation of BP coefficients is the average value of the signal at two end point connected subinterval. This approximation is shown below

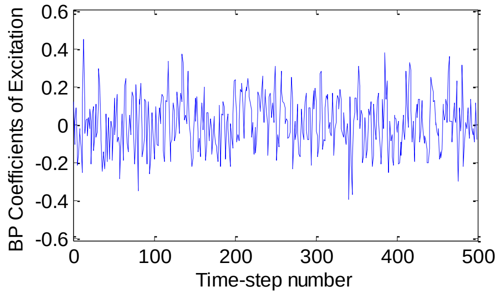
$$U_k^{BP} = \frac{1}{h}\int_{(k-1)h}^{kh} u(t)\,dt \;\cong \frac{1}{2}(\bar{u}_{k-1} + \bar{u}_k) \tag{20}$$

Where $\bar{u}_{k-1}$ and $\bar{u}_k$ are the sampled values of the continuous signal $u(t)$ at the time instants $t = (k-1)h$ and $t = kh$ respectively. Figs. 5-6 Show external excitations and their BP coefficients. In the following, BP coefficients for original responses are calculated according to Eq. (20) and based on this input-output BP coefficients data the process of identification is done.

To evaluate the accuracy of Eq. (17) in the BP coefficients estimation, all parameters of the equation, including mass, damping and stiffness set with the original

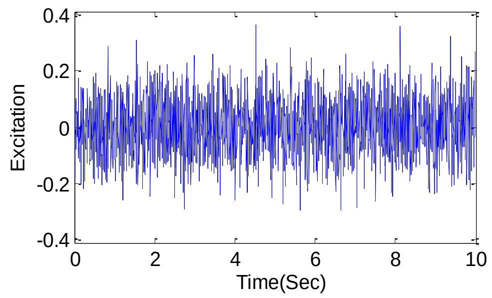


(a) Excitation 1

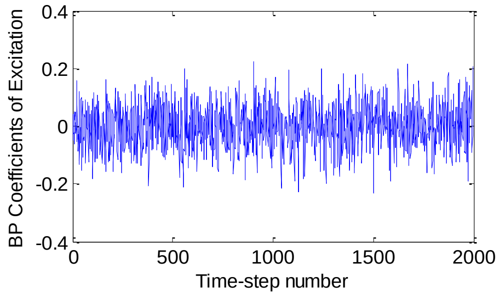


(b) BP coefficients of excitation 1

Fig. 5 External excitations time history



(a) Excitation 2



(b) BP coefficients of excitation 2

Fig. 6 External excitations time history

values according to Table 2 and the obtained coefficients are compared with the Eq. (20). The sum of squared error (SSE) and the root mean square error (RMSE) are used as the evaluation criterions and the results are listed in Table 3.

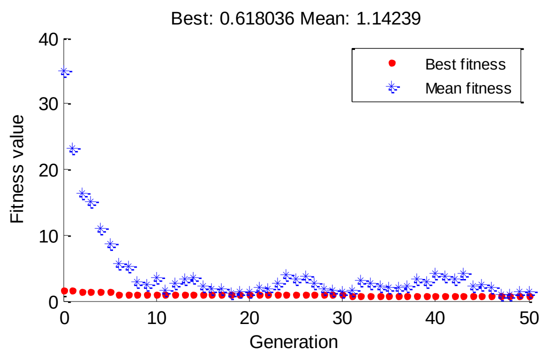
As the results show, error values are acceptable and with fewer sampling time $(\Delta t = h)$ RMSE values are reduced and the accuracy of the estimation increases. The reason is that, according to Eq. (8) the better BP approximation can be obtained if a smaller width of block pulse $(\Delta t = h)$ is chosen. Therefore, small rate of h makes more accurate approximation of the BP operational matrix P_k according to the Eq. (10) and this increases the accuracy of estimation.

Table 3 Error values in estimation of BP coefficients of responses based on original parameters

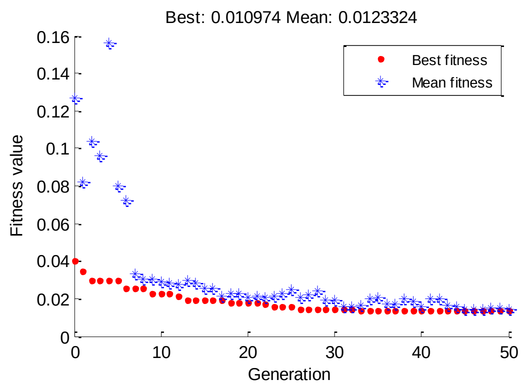
story	Excitation 1		Excitation 2	
	SSE	RMSE	SSE	RMSE
First	0.0776	0.0125	0.0109	0.0023
Second	0.1092	0.0148	0.0152	0.0028
Third	0.1313	0.0162	0.0178	0.0030
Fourth	0.6124	0.0350	0.1653	0.0091

Table 4 Parameters and operators of GA

Parameters	Values
Maximal generation	50
Population size	30
Fitness scaling function	Rank
Selection function	Stochastic uniform
Crossover function	Arithmetic
Mutation function	Constraint dependent
Search Ranges	$\begin{cases} m_i\ [1,10] \\ k_i\ [500,1500] \\ c_i\ [1,10] \end{cases}$



(a) Excitation 1-the fourth story



(b) Excitation 2-the first story

Fig. 7 Fitness values of the costs

Now the cost function according to Eq. (19) can be formed. As previously mentioned, Genetic algorithm has been used from MATLAB optimization toolbox to minimize the fitness (cost) functions. Table 4 shows settings and assigned values for the parameters of the algorithm. The general parameters and operators of GA which is suitable for the optimization problem, are considered and adapted by inspection to give better performance. Although

Table 5 Identified mass, stiffness and damping values

Parameter	Story	Excitation 1	Error (%)
Mass ($kN\ s^2m^{-1}$)	m_1	5.481	0.34
	m_2	4.958	0.84
	m_3	4.476	0.53
	m_4	3.964	0.90
Stiffness ($kN\ m^{-1}$)	k_1	1196.596	0.28
	k_2	995.216	0.49
	k_3	796.637	0.42
	k_4	594.971	0.84
Damping ($kN\ s\ m^{-1}$)	c_1	6.975	0.36
	c_2	5.965	0.58
	c_3	4.972	0.56
	c_4	3.949	1.27

Table 6 Identified mass, stiffness and damping values

Parameter	Story	Excitation 2	Error (%)
Mass ($kN\ s^2m^{-1}$)	m_1	5.492	0.14
	m_2	4.995	0.10
	m_3	4.497	0.067
	m_4	3.991	0.22
Stiffness ($kN\ m^{-1}$)	k_1	1198.253	0.14
	k_2	998.665	0.13
	k_3	798.450	0.19
	k_4	597.282	0.45
Damping ($kN\ s\ m^{-1}$)	c_1	6.984	0.23
	c_2	5.997	0.05
	c_3	4.993	0.14
	c_4	4.007	0.17

the sensitivity analysis could be performed but it was not taken into account for the parameters GA.

After minimization process, identified parameters including mass, damping and stiffness and the percent of relative errors are summarized in Table 5 and Table 6. It is worth noting that the error for each parameter is calculated as

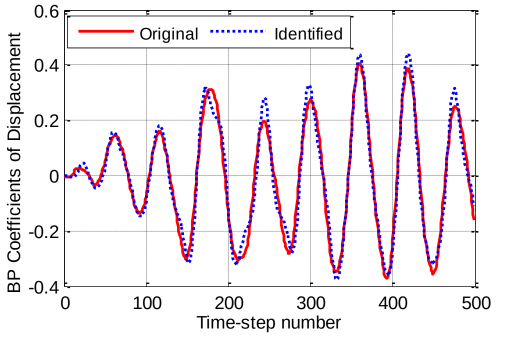
$$Error = \frac{Original\ parameter - Identified\ parameter}{Original\ parameter} \tag{21}$$

Also in Figs. 7-8 the best and mean values of fitness functions are shown in each generation and estimated BP coefficients of responses are compared with the theoretical (original) values, respectively.

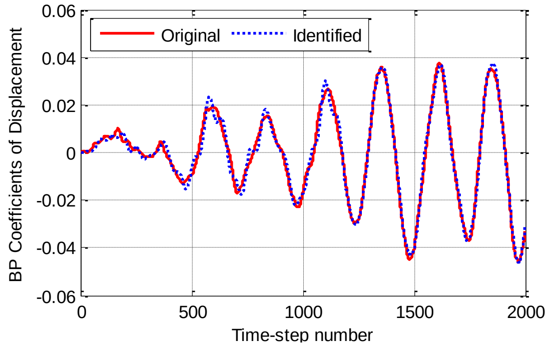
The results indicate that the identified physical parameters obtained from BPFs and GA have excellent consistency with those of the finite element model. As the results show, excitation 2 error is lower than that of excitation 1 and the proposed method has high accuracy in identifying structural parameters based on noise free data.

4.1 Noise effect

In real structure, measured responses are always corrupted by noise components from environmental factors. To simulate the proposed method with practice, the original responses of structure are contaminated by white noise effects. Therefore, a random noise of standard normal distribution with zero mean and a specified standard deviation is created and added to

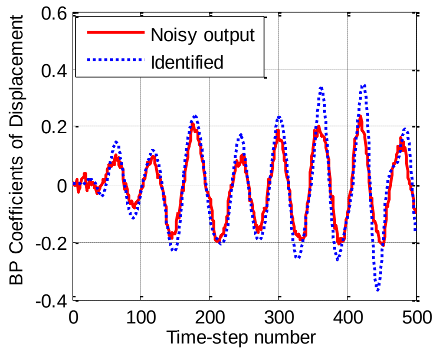


(a) Excitation 1-the fourth story

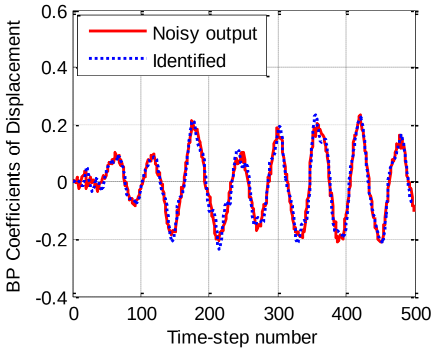


(b) Excitation 2-the first story

Fig. 8 Original and estimated BP coefficients of responses



(a) The first generation



(b) The final generation

Fig. 9 Minimization process of BP coefficient to estimate the physical parameters of the second story based on excitation1 and 10% noise polluted output data

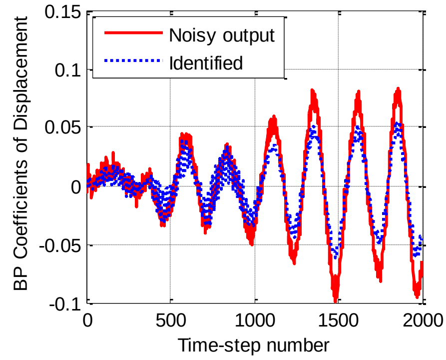
theoretical displacement responses. The noise level is considered as a ratio of the standard deviations (SD) between noise and a simulated response (Khanmirza *et al*. 2011). For

Table 7 Identified mass, stiffness and damping values of the four-story shear building under excitation 1 and noisy output

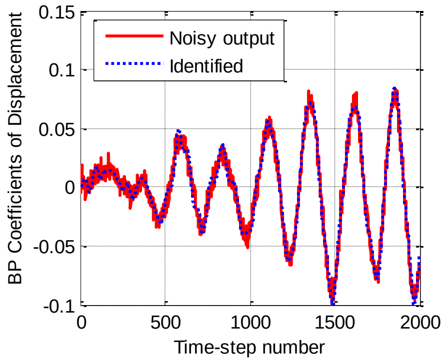
Parameters		Identified parameters			
		5%	Error (%)	10%	Error (%)
Mass ($kN\ s^2m^{-1}$)	m_1	5.178	5.85	4.975	9354
	m_2	4.553	8.94	4.311	13.78
	m_3	4.026	10.53	3.896	13.42
	m_4	3.501	12.47	3.287	17.82
Stiffness ($kN\ m^{-1}$)	k_1	1128.519	5.95	1090.562	9.12
	k_2	892.724	10.73	872.699	12.73
	k_3	725.680	9.29	677.135	15.36
	k_4	543.421	9.43	520.547	13.24
Damping ($kN\ s\ m^{-1}$)	c_1	6.346	9.34	5.884	15.94
	c_2	5.308	11.53	4.862	18.96
	c_2	4.241	15.18	3.975	20.50
	c_4	3.319	17.02	4.979	24.47

Table 8 Identified mass, stiffness and damping values of the four-story shear building under excitation 2 and noisy output

Parameters		Identified parameters			
		5%	Error (%)	10%	Error (%)
Mass ($kN\ s^2m^{-1}$)	m_1	5.363	2.49	5.261	4.35
	m_2	4.921	1.58	4.683	6.34
	m_3	4.445	1.22	4.042	10.18
	m_4	3.761	5.98	3.679	8.02
Stiffness ($kN\ m^{-1}$)	k_1	1175.458	2.04	1144.085	4.66
	k_2	985.413	1.46	907.615	9.24
	k_3	765.032	4.37	733.129	8.36
	k_4	577.980	3.67	534.063	10.99
Damping ($kN\ s\ m^{-1}$)	c_1	6.599	5.73	6.422	8.25
	c_2	5.729	4.52	5.543	7.62
	c_2	4.513	9.74	4.347	13.05
	c_4	4.225	5.62	3.385	15.37



(a) The first generation



(b) The final generation

Fig 10 Minimization process of BP coefficient to estimate the physical parameters of the second story based on excitation 2 and 10% noise polluted output data

Table 9 The modal characteristics

	Original			
	First mode	Second mode	Third mode	Fourth mode
Frequencies	5.219	13.134	19.894	25.012
Modal shapes	$\begin{Bmatrix}1.00\\2.05\\3.01\\3.68\end{Bmatrix}$	$\begin{Bmatrix}1.00\\1.25\\0.22\\-1.44\end{Bmatrix}$	$\begin{Bmatrix}1.00\\0.02\\-1.25\\0.77\end{Bmatrix}$	$\begin{Bmatrix}1.00\\-1.24\\0.81\\-0.25\end{Bmatrix}$

example, a noise level of %8 Considers that the SD of the measurement noise is %8 that of the displacement responses. For this point, white noise is added with

$$\{\bar{x}\}_t = \{x\}_t + E_p N_{noise}\, \sigma[\{x\}_t] \tag{22}$$

Where E_p demonstrates the percentage noise level, N_{noise} indicates standard distribution vector with zero mean and unit standard deviation and $\sigma[\{q\}_t]$ indicates the standard deviation of the computed displacement response. To assess the effect of noise, the noises equal to 5% and 10% of the variance of the N/S ratio are randomly generated and added to the noise-free structural responses and minimization process is applied. Then, the cost functions according to Eq. (19) are formed and the minimization processes are done until BP coefficients obtained from the Eq. (17) converging to noise polluted original coefficients. After minimization process, identified parameters including mass, damping and stiffness and the relative errors are provided in Table 7 and Table 8. Also Figs. 9-10 show minimization process for both excitation based on 10% noise polluted output data.

As the results show in Tables 7 and 8, the relative errors in mass, stiffness and damping identification process in low-level noise (less than 5%) are insignificant. As expected in the case of 10% noise level, the relative errors appear remarkable compared to the case of 5% and the maximum relative error is reached about 25%.

After identifying the physical parameters of the structure, the mass and stiffness matrices can be formed according to Eqs. (13)-(14) and based on the natural frequencies and mode shapes of the structure are determined by solving eigenvalue problem as follows

$$\left|[K] - \omega_i^2[M]\right| \begin{Bmatrix}\emptyset_{1i}\\ \vdots \\ \emptyset_{ni}\end{Bmatrix} = 0 \tag{23}$$

Where ω and $\{\emptyset\}$ are the natural frequencies and modal shapes, respectively. Table 9 shows the original values of these modal parameters for structural model.

To compare the agreement between the identified and the original modal shapes, the modal assurance criterion

Table 10 Identified frequency values based on noise-free data

Excitation 1	First mode	Second mode	Third mode	Fourth mode
Frequencies	5.228	13.145	19.889	25.032
Error (%)	0.17	0.08	0.02	0.08
Mac	1.000	1.000	1.000	1.000
Excitation 2				
Frequencies	5.218	13.125	19.877	25.003
Error (%)	0.02	0.07	0.08	0.03
Mac	1.000	1.000	1.000	1.000

Table 11 Identified frequency values based on noisy data

Excitation 1	5% Noise level			
	First mode	Second mode	Third mode	Fourth mode
Frequencies	5.287	13.222	19.916	24.836
Error (%)	1.303	0.670	0.110	0.704
MAC	1.000	0.999	1.000	0.998
Excitation 2				
Frequencies	5.245	13.149	19.821	24.923
Error (%)	0.50	0.11	0.37	0.36
MAC	1.000	0.999	1.000	0.999

(MAC) index is employed (pastor *et al.* 2012)

$$MAC = (\{\emptyset_{io}\}, \{\emptyset_{il}\}) = \frac{|\{\emptyset_{io}\}^T\{\emptyset_{il}\}|^2}{\{\emptyset_{io}\}^T\{\emptyset_{io}\}\{\emptyset_{il}\}^T\{\emptyset_{il}\}} \quad (24)$$

Where $\{\emptyset_{io}\}$ and $\{\emptyset_{il}\}$ are the *i*-th original and *i*-th identified mode shape, respectively. The value of MAC varies between 0 and 1. When this quantity is equal to 1, the two vectors $\{\emptyset_{io}\}$ and $\{\emptyset_{il}\}$ display exactly the similar mode shape and when the two mode shapes are orthogonal the MAC value is zero. Tables 10, 11 and 12 show identified frequencies and relative errors and MAC values for each mode. As the MAC criterion is to provide degree of consistency between estimated and original modal vectors, the high value of MAC near 1 indicate high degree of consistency. The results show in Table 12 the relative errors in identified frequencies based on 10% noise level are less than 2% in the general.

5. Conclusions

In this paper, a method based on substructural technique is proposed to identification of multistory shear building structures from their dynamic responses and excitation data. In this method simultaneously identification structural parameters contains mass, damping and stiffness can be done without any prior knowledge of structural dynamic parameters. By employing special properties of generalized block pulse operational matrices, BP coefficients of response for each degree of freedom were calculated. Using the original and estimated BP coefficients the cost functions were defined for each story and structural parameters obtained by minimizing cost functions with genetic algorithm. Finally natural frequencies was computed based on identified structural parameters. The feasibility and accuracy of the presented method have been verified by numerical studies on 4-story shear building using two different normally distributed random excitation with different sampling time. Noise free and noisy responses data are used as inputs of identification. The results of investigation indicated good consistency with those of the finite element models and high accuracy for shear building dynamical parameters identification based on noise-free data. The relative errors in low-level noise (less than 5%) were insignificant. when measurement noise level reaching up to 10% of the variance of the noise-to-signal ratio, the relative errors in identified frequencies become less than 2% in the general.

Table 12 Identified frequency values based on noisy data

Excitation 1	10% Noise level			
	First mode	Second mode	Third mode	Fourth mode
Frequencies	5.316	13.274	19.893	24.894
Error (%)	1.86	1.07	0.00	0.47
MAC	0.999	0.999	1.000	0.999
Excitation 2				
Frequencies	5.240	13.002	19.770	24.732
Error (%)	0.40	1.005	0. 62	1.12
MAC	0.999	0.999	1.000	0.998

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