Structural Engineering and Mechanics, *Vol. 67, No. 4 (2018) 417-425*
DOI: https://doi.org/10.12989/sem.2018.67.4.417

Bending analysis of bi-directional functionally graded Euler-Bernoulli nano-beams using integral form of Eringen's non-local elasticity theory

Mohammad Zamani Nejad[*1], Amin Hadi[2], Arash Omidvari[3] and Abbas Rastgoo[2]

[1]*Department of Mechanical Engineering, Yasouj University, P.O. Box: 75914-353, Yasouj, Iran*
[2]*School of Mechanical Engineering, College of Engineering, University of Tehran, Tehran, Iran*
[3]*Department of Mechanical Engineering, Shiraz University, Shiraz, Iran*

(Received November 2, 2017, Revised June 8, 2018, Accepted June 26, 2018)

Abstract. The main aim of this paper is to investigate the bending of Euler-Bernouilli nano-beams made of bi-directional functionally graded materials (BDFGMs) using Eringen's non-local elasticity theory in the integral form with compare the differential form. To the best of the researchers' knowledge, in the literature, there is no study carried out into integral form of Eringen's non-local elasticity theory for bending analysis of BDFGM Euler-Bernoulli nano-beams with arbitrary functions. Material properties of nano-beam are assumed to change along the thickness and length directions according to arbitrary function. The approximate analytical solutions to the bending analysis of the BDFG nano-beam are derived by using the Rayleigh-Ritz method. The differential form of Eringen's non-local elasticity theory reveals with increasing size effect parameter, the flexibility of the nano-beam decreases, that this is unreasonable. This problem has been resolved in the integral form of the Eringen's model. For all boundary conditions, it is clearly seen that the integral form of Eringen's model predicts the softening effect of the non-local parameter as expected. Finally, the effects of changes of some important parameters such as material length scale, BDFG index on the values of deflection of nano-beam are studied.

Keywords: bending; Euler-Bernoulli nano-beams; Bi-directional functionally graded material (BDFGM); integral form; non-local; Rayleigh-Ritz method

1. Introduction

Today, nanotechnology has become one of the most important research topics because of its extensive use in engineering, biology and other sciences. Perhaps the use of nanotechnology is not limited to engineering and other sciences such as medicine and chemistry, but the engineering science is the origin of nanoscience and nanostructural elements such as nano-beams, nanomembranes and nano-plates (Yane *et al.* 2015). Because Nanoscience is defined in very small sizes, the size effect is very significant and important (Gopalakrishnan and Narendar 2013, Hosseini *et al.* 2018). Size effect is what affects the properties of the material. Experimental and numerical results have shown that the size effects in the analysis of nanostructures cannot be neglected and classical continuum theories is not helpful (Nejad *et al.* 2016a). These new properties require new theories and equations for mechanical analysis of the nanostructures. Molecular dynamics simulation is a numerical method to study the mechanical properties of small size structures and size effects. Although, it is computationally expensive for large numbers of atoms (Daneshmehr *et al.* 2015, Gopalakrishnan and Narendar 2013). Undoubtedly, accurate theories for analyze nanostructures, reduce the waste of time and cost in experiments. The theory of non-local continuum mechanics initiated by Eringen is one of the size dependent theories that has been widely used to analyze the problems (Eringen 1972a, 1972b, 2002). Integral constitutive equation non-local theory describes that the stress tensor at any point depends on the strain tensor at this point and all other points in the domain. Integral constitutive equation non-local theory is difficult to manage. For this reason, a specific class of kernel functions introduced by Eringen (1983) for solving this barrier. By this a specific class of kernel functions, the non-local integral constitutive equation can be transformed in to a differential form. Many researchers have applied the differential form of non-local theory for bending, buckling and vibration analyses of nanostructures (Anjomshoa *et al.* 2014, Ansari *et al.* 2015, Ansari *et al.* 2015, Ansari *et al.* 2016, Ansari *et al.* 2016, Asemi *et al.* 2014, Babaei and Shahidi 2011, Barati and Shahverdi 2017, Barretta and Sciarra 2016, Barretta and Sciarra 2015, Barzoki *et al.* 2015, Behera and Chakraverty 2015, Challamel *et al.* 2016, Challamel *et al.* 2014, Daneshmehr *et al.* 2014, Ebrahimi and Barati 2016a, 2016b, 2017, Ebrahimi *et al.* 2016, Eltaher *et al.* 2016, Farajpour *et al.* 2016, Golmakani *et al.* 2012, Hadi *et al.* 2018a, 2018b, Kaghazian *et al.* 2017, Li and Hu 2017, Li *et al.* 2018, Zhu and Li 2017a, 2017b, 2017c, 2017d, Miandoab *et al.* 2014, Nejad *et al.* 2017a, Nejad and Hadi 2016a, Nejad and Hadi 2016b, Nejad *et al.* 2016a, Panyatong *et al.* 2016, Pour *et al.* 2015, Setoodeh and Rezaei 2017, Shahverdi and Barati 2017, Sobhy 2017, Tufekci *et al.* 2016, Wang *et al.* 2016, Xu *et al.* 2016, Yan *et al.* 2015, Yu *et al.* 2016, Zang *et al.*

∗Corresponding author, Ph.D.
E-mail: m_zamani@yu.ac.ir ; m.zamani.n@gmail.com

ISSN: 1225-4568 (Print), 1598-6217 (Online)

2014, Zenkour and Sobhy 2013, Zhang *et al.* 2016).

Unlike its popularity, differential form of Eringen's non-local model propels to some contradictions that have been illustrated recently for the cantilever beams by indicating the differences between the integral and the differential forms of the Eringen's non-local model, which shows the importance and necessity of applying the original integral model (Tuna and Kirca 2016). In addition, for all boundary and loading cases, different from the differential form, it is clearly seen that the Eringen's model in the integral form predicts the softening effect of the non-local parameter as expected. Fernández-Sáez *et al.* (2016) show that, in general, the differential form of the Ernigen's model is different with the Eringen's model in the integral form. Also they offer a general method to solve the problem precisely in the integral form. By considering the integral formulation, the inconsistency that appears when solving the bending analysis of cantilever beam with the differential form of the Eringen's non-local model is solved, which is one of the main benefits of their work. Abdollahi and Boroomand (2013) present low-residual approximate solutions for non-local 1D and 2D elasticity problems defined according to Eringen's integral model. The results of this research are particularly helpful for the validation and convergence studies when numerical methods are to be used for investigate of material properties and solution of the non-local elasticity problems. It is known that the strain-driven nonlocal integral law proposed by Eringen in 1983 is ill-posed for continuous structural elastic problems on bounded domains. This occurrence is due to conflicting constitutive boundary conditions and equilibrium requirements (Apuzzo *et al*. 2017, Barretta *et al*. 2017, Romano and Barretta 2016, 2017a, 2017b, Romano *et al*. 2017, Romano *et al*. 2017). When a discrete interpolation is adopted for numerical computations, ill-posedness is bypassed since the equilibrium requirements are by far less stringent.

Functionally graded materials (FGMs) are advanced composite materials in which mechanical properties change continuously from one side to the other (Ozturk and Gulgec 2011). Functional grading of material can be applied to attain a variety of purposes, such as relief of residual stresses, resolving interface problems, reducing stresses during lifetime of the structure, improvement of stability and dynamic behavior, preventing fracture and fatigue (Birman 2014, Kahrobaiyan *et al.* 2012). A number of papers considering various aspects of FGM have been published in recent years (Asghari *et al*. 2011, Ben-Oumrane *et al.* 2009, Dehghan *et al.* 2016, Fatehi and Nejad 2014, Ghannad *et al.* 2009, Ghannad and Nejad 2010, Ghannad and Nejad 2013, Ghannad *et al.* 2012, Ghannad *et al.* 2013, Jabbari *et al.* 2015, Jabbari *et al.* 2016a, Jabbari *et al.* 2016b, Kashkoli and Nejad 2015, Kashkoli *et al.* 2017, Nejad *et al*. 2016b, Jabbari and Nejad 2018, Kashkoli *et al.* 2018, Mazarei *et al.* 2016, Nejad *et al.* 2017, Nejad and Rahimi 2009, Nejad *et al.* 2009, Nejad and Rahimi 2010, Gharibi *et al*. 2017, Afshin *et al.* 2017, Nejad and Kashkoli 2014, Nejad *et al.* 2014a, Nejad *et al.* 2014b, Nejad and Fatehi 2015, Nejad *et al.* 2015a, Nejad *et al.* 2015b, Nejad *et al.* 2015c, Nejad *et al.* 2017b, Petrova and Schmauder 2012, Radman *et al.* 2014, Şimşek and Reddy 2013, Xue and Pan 2013, Ziegler and Kraft 2014). It should be mentioned that most of the above researches are related to FGMs with material properties changing in one direction only. However, there are practical cases which require appropriate grading of macroscopic mechanical properties in two or three directions. Therefore, it is more important to develop novel FGMs with mechanical properties changing in two or three directions for example to endure a more general temperature field (Lü *et al.* 2008).

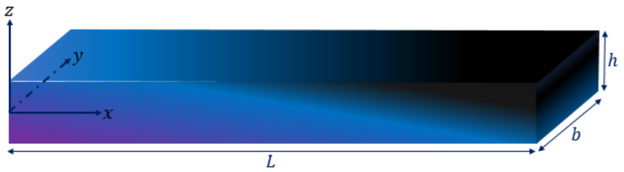

Fig. 1 Geometry of the bi-directional functionally graded Euler-Bernoulli nano-beam

Nejad *et al.* (2016) presented buckling analysis of arbitrary bi-directional functionally graded Euler-Bernoulli nano-beams based on non-local elasticity theory in the differential form. In other studies, Nejad and Hadi (2016a) presented free vibration analysis and bending analysis (Nejad and Hadi 2016b) of arbitrary bi-directional functionally graded Euler-Bernoulli nano-beams based on non-local elasticity theory in the differential form.

In this article, bending analysis of BDFG nano-beam using Eringen's non-local theory in the integral form with compared the differential form is investigated. The effects of changes of some important parameters such as material length scale, FG index on the bending analysis are studied.

2. Analysis

Consider a nano-beam of length L, width b, and thickness h made of bi-directional functionally graded materials (Fig. 1). Cartesian coordinates (x,y,z) are considered.

The modulus of elasticity E and density ρ are assumed to vary as arbitrary functions in both axial and thickness directions, as indicated below

$$E = f(x)\,g(z) \tag{1}$$

where $f(x)$ and $g(z)$ are arbitrary functions.

Based on the Euler-Bernoulli beam theory, the axial displacement u and the transverse displacement of any point of the beam, w, are given by

$$\begin{cases} u = -z\dfrac{dw}{dx} \\ w = w(x) \end{cases} \tag{2}$$

By assuming the small deformations, the only nonzero strain of the Euler-Bernoulli beam theory is

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = -z\frac{d^2 w}{dx^2} \tag{3}$$

According to the non-local elasticity theory, in contrast to classical elasticity, the stress tensor at an arbitrary point x

in the domain of material depends not only on the strain tensor at x but also on strain tensor at all other points in the domain. Both atomistic simulation results and experimental observations on phonon dispersion confirm the accuracy of this observation. According to this theory, a stress-strain relationship for a homogeneous elastic solid is expressed as (Eringen 1983)

$$\sigma_{ij}^{nl} = \int_0^L \alpha\left(\left|x' - x\right|, \tau\right)\sigma_{ij}^{l}\, dx' \tag{4}$$

α is the non-local modulus or kernel function, which contains the small scale effects incorporating into constitutive equations the non-local effects at the reference point x produced by local strain at the source x'. This function depends on two variables $|x-x'|$ and α, as can be seen from the above equation. $|x-x'|$ represents the distance in Euclidean form. $\tau = e_0 a$ is a material constant that depends on internal and external characteristic length (such as the lattice spacing and wavelength). The parameter e_0 is vital for the validity of non-local models. This parameter is determined by matching the dispersion curves based on atomistic models. σ^{nl} is the non-local stress tensor at the reference point and σ^{i} is the classical stress tensor at local point. In addition, the classical stress tensor is defined in the following way (Reddy and Mahaffey 2013)

$$\sigma^{l} = C : \varepsilon \tag{5}$$

Here C is the fourth order elasticity tensor and ':' denotes the double dot product. In this study, kernel function is taken as (Eringen 1983)

$$\alpha = \frac{1}{2\tau} e^{-\frac{|x-x'|}{\tau}} \tag{6}$$

According to the non-local theory in integral form, the strain energy density of an isotropic linear elastic material with volume Ω experiencing an infinitesimal displacement is defined as

$$U = \frac{1}{2}\int_{\Omega}\left(\sigma_x^{nl}\varepsilon_x\right)dv \tag{7}$$

Therefore

$$U = \frac{1}{2}\int_{\Omega}\left(\left[\int_0^L \frac{1}{2\tau} e^{-\frac{|x-x'|}{\tau}} f\left(x'\right) g\left(z\right)\left(-z\,\frac{d^2 w}{dx'^2}\right)dx'\right]\left(-z\,\frac{d^2 w}{dx^2}\right)\right)dv = \frac{I_2}{4\tau}\int_0^L\left(\left[\int_0^L e^{-\frac{|x-x'|}{\tau}} f\left(x'\right)\left(\frac{d^2 w}{dx'^2}\right)dx'\right]\left(\frac{d^2 w}{dx^2}\right)\right)dx \tag{8}$$

here

$$I_2 = \int_A z^2 g\left(z\right) dA \tag{9}$$

The work done by the external applied force, q, is expressed as

$$V = \int_0^L q w\, dx \tag{10}$$

Now, for achieving the governing equations, the total potential energy can be defined as

$$R = U - V \tag{11}$$

By substitution of Eqs. (8) and (10) into the Eq. (11), the total potential energy is expressed as

$$R = \frac{I_2}{4\tau}\int_0^L\left(\left[\int_0^L e^{-\frac{|x-x'|}{\tau}} f\left(x'\right)\left(\frac{d^2 w}{dx'^2}\right)dx'\right]\left(\frac{d^2 w}{dx^2}\right)\right)dx - \int_0^L q w\, dx \tag{12}$$

For convenience, the following nondimensionalizations are used

$$\bar{w} = \frac{w}{w_0}, \quad \bar{x} = \frac{x}{L}, \quad \bar{q} = \frac{q}{q_0}, \quad \bar{\tau} = \frac{\tau}{L} \tag{13}$$

with $w_0 = q_0 L^4 / I_2$ and q_0 a characteristic value of the transverse load. The non-dimensional total potential energy expression can be obtained as

$$\bar{R} = \frac{1}{4\bar{\tau}}\int_0^1\left(\left[\int_0^1 e^{-\frac{|\bar{x}-\bar{x}'|}{\bar{\tau}}} f\left(\bar{x}'\right)\left(\frac{d^2 \bar{w}}{d\bar{x}'^2}\right)d\bar{x}'\right]\left(\frac{d^2 \bar{w}}{d\bar{x}^2}\right)\right)d\bar{x} - \int_0^1 \bar{q}\bar{w}\, d\bar{x} \tag{14}$$

The exact solution for the nano-beam is difficult to obtain since it is a nonlinear system with nonlinear material properties. Thus, such a problem is often solved by the approximate analytical solution. The Rayleigh-Ritz method is a feasible method. An approximate solution $\bar{w}\left(\bar{x}\right)$ can be constructed in the form of a linear combination of admissible trial functions

$$\bar{w}\left(\bar{x}\right) = \sum_{i=1}^{N} a_i \phi_i\left(\bar{x}\right) \tag{15}$$

where a_i are coefficients to be determined and $\phi_i\,(x)$ are the trail functions. By substitution of Eq. (15) into the Eq. (14), the total potential energy is expressed as

$$\bar{R} = \frac{1}{4\bar{\tau}}\int_0^1\left(\left[\int_0^1 e^{-\frac{|\bar{x}-\bar{x}'|}{\bar{\tau}}} f\left(\bar{x}'\right)\sum_{i=1}^{N} a_i\,\frac{d^2\phi_i\left(\bar{x}'\right)}{d\bar{x}'^2}d\bar{x}'\right]\sum_{i=1}^{N} a_i\,\frac{d^2\phi_i\left(\bar{x}\right)}{d\bar{x}^2}\right)d\bar{x} - \int_0^1 \bar{q}\left(\sum_{i=1}^{N} a_i\phi_i\left(\bar{x}\right)\right)d\bar{x} \tag{16}$$

The displacement forms in a Rayleigh-Ritz procedure must be continuous and satisfy all geometric constraints. Spatial functions that satisfy these conditions are called trail functions. This is illustrated in the following examples.

1- Simply-simply (S-S) supported nano-beam

Geometric constraints

$$\phi_i\Big|_{x=0,L} = 0 \tag{17}$$

Trail functions for S-S supported nan-obeam is as

$$\phi_i\left(x\right) = \mathrm{Sin}\left(i\pi\bar{x}\right)\ , \ i = 1,2,3,4,\ldots \tag{18}$$

2- Simply-clamped (S-C) supported nano-beam
Geometric constraints

$$\begin{cases} \phi_i \Big|_{x=0,L} = 0 \\ \dfrac{d\phi_i}{dx}\Big|_{x=0} = 0 \end{cases} \tag{19}$$

Trail functions for C-S supported nano-beam is as

$$\phi_i(x) = (\bar{x})^i (1-\bar{x}) \quad , \quad i = 2,3,4,\ldots \tag{20}$$

The system is in static equilibrium when the first-order derivative of the total potential energy with respect to the coefficient a_i equals zero, then the following expression should be satisfied

$$\frac{\partial \bar{R}}{\partial a_i} = 0 \quad , \quad i = 1,2,3,4,\ldots \tag{21}$$

The non-dimensional governing equation expression can be obtained as

$$\frac{1}{4\bar{\tau}}\frac{\partial}{\partial a_i}\int_0^1\left(\left[\int_0^1 e^{-\frac{|\bar{x}-\bar{x}'|}{\bar{\tau}}} f(\bar{x}')\sum_{i=1}^{N} a_i \frac{d^2\phi_i(\bar{x}')}{d\bar{x}'^2}d\bar{x}'\right]\right.$$
$$\left.\sum_{i=1}^{N} a_i \frac{d^2\phi_i(\bar{x})}{d\bar{x}^2}\right)d\bar{x} - \frac{\partial}{\partial a_i}\int_0^1 \bar{q}\left(\sum_{i=1}^{N} a_i\phi_i(\bar{x})\right)d\bar{x} = 0 \tag{22}$$

Therefore, deflection of nano-beam and corresponding coefficient a_i are calculated with simultaneously solving the system of Eq. (22). All integrals are calculated by Gaussian quadrature method. An integral over [0,1] must be changed into an integral over [−1,1] before applying the Gaussian quadrature rule. This change of interval can be done in the following way.

$$\bar{x} = \frac{1}{2}(1+y) \tag{23}$$

From Eq. (22), the non-dimensional governing equation is defined as follows

$$\frac{1}{\bar{\tau}}\frac{\partial}{\partial a_i}\int_{-1}^1\left(\left[\int_{-1}^1 e^{-\frac{0.5|y-y'|}{\bar{\tau}}} f(y')\sum_{i=1}^{N} a_i \frac{d^2\phi_i(y')}{dy'^2}dy'\right]\right.$$
$$\left.\left(\sum_{i=1}^{N} a_i \frac{d^2\phi_i(y)}{dy^2}\right)dy - \frac{\partial}{\partial a_i}\int_{-1}^1 \bar{q}\left(\sum_{i=1}^{N} a_i\phi_i(y)\right)dy = 0 \tag{24}$$

Solving the system of equations gives the deflection $(\bar{w})$ of the BDFGM Euler-Bernoulli nano-beams based on Eringen's non-local theory in the integral form.

3. Results and discussion

In this section, the bending analysis of BDFG Euler-Bernoulli nano-beams based on non-local elasticity theory in the integral form is investigated by numerical results. In order to verify the validity and reliability of the present work, when $f(x)$ is neglected, a comparison of the dimensionless deflection of beams at $\bar{x}$=0.5 versus $\bar{\tau}$ with

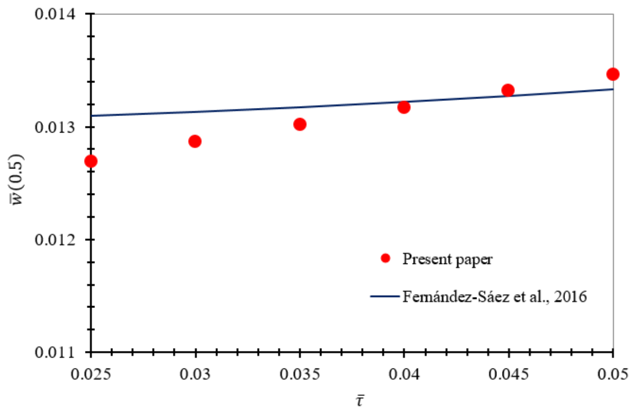

Fig. 2 Displacement of the section at $\bar{x}$=0.5 for a S-S supported beam submitted to a uniform distributed load, versus non-local parameter $\bar{\tau}$ for β=0

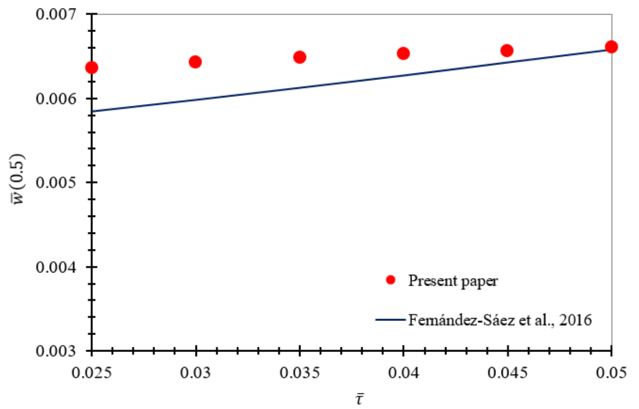

Fig. 3 Displacement of the section at $\bar{x}$=0.5 for a C-S supported beam submitted to a uniform distributed load, versus non-local parameter $\bar{\tau}$ for β=0

various boundary conditions (S-S supported and S-C supported) at two ends is made with Fernández-Sáez *et al.* (Fernández-Sáez *et al.* 2016), as shown in Figs. 2 and 3. It can be seen that there is an excellent agreement between the results obtained in this paper and those reported in (Fernández-Sáez *et al.* 2016).

It is proposed that the modulus of elasticity and density of the nano-beam material vary in the x and z directions, as follows

$$E(x,z) = e^{\frac{\beta}{L}x}\left[E_c\left(\frac{2z+h}{2h}\right)^n + E_m\left(1-\left(\frac{2z+h}{2h}\right)^n\right)\right] \tag{25}$$

Figs. 4 and 5 illustrate the variation of the dimensionless modulus of elasticity through the thickness and length of the beam for various values of n and β. The value of n equal to zero represents a fully ceramic beam, whereas infinite n indicates a fully metallic beam. The variation of the combination of ceramic and metal is linear for n=1. According to Fig. 5, in the same position (0<x/L<1), it is observed that for higher values of β, the stiffness increases.

Fig. 6 shows dimensionless displacement for a cantilever beam at $\bar{x}$=1, under a linear distributed load $\bar{q} = \bar{x}$, versus non-local parameter $\bar{\tau}$ for β=2. In this

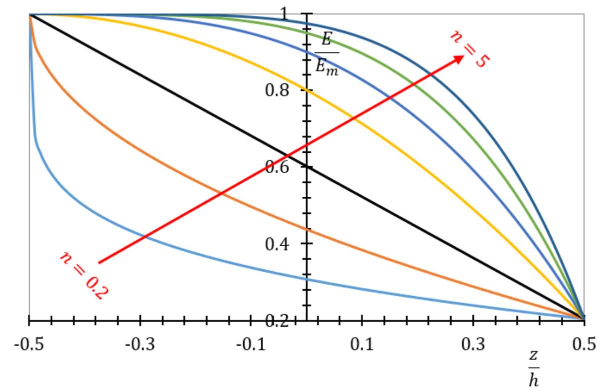

Fig. 4 Distribution of dimensionless modulus of elasticity versus dimensionless *z/h* at *x*=0 for *β*=0

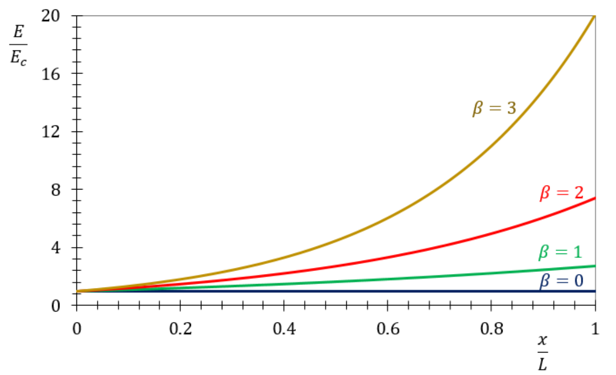

Fig. 5 Distribution of dimensionless modulus of elasticity versus *x/L* at *z*=−*h*/2 for *n*=0

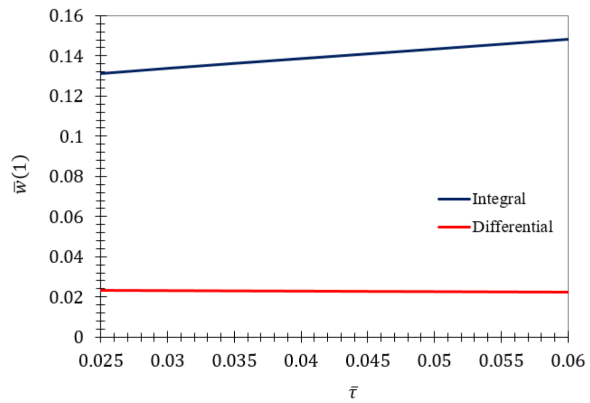

Fig. 6 Displacement of the section at $\bar{x}$=1 for a cantilever beam under a linear distributed load $\bar{q}=\bar{x}$, versus non-local parameter $\bar{\tau}$ for *β*=2

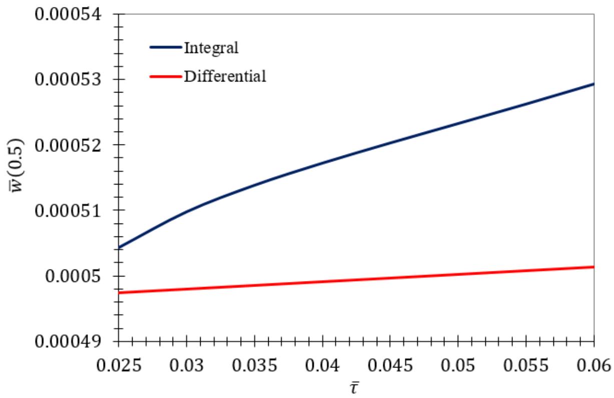

Fig. 7 Displacement of the section at $\bar{x}$=0.5 for a S-C supported beam submitted to a distributed load $\bar{q}=\bar{x}(1-\bar{x})$, versus non-local parameter $\bar{\tau}$ for *β*=2

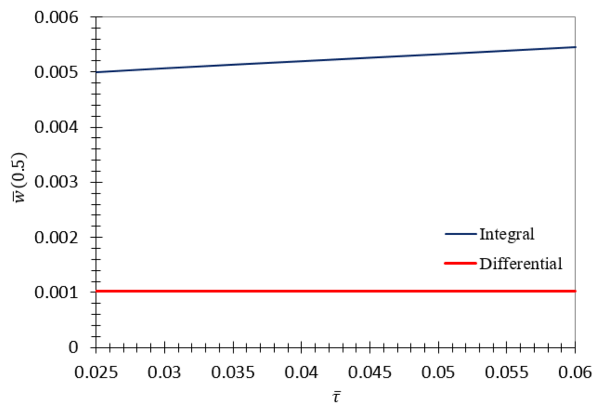

Fig. 8 Displacement of the section at $\bar{x}$=0.5 for a S-S supported beam submitted to a distributed load $\bar{q}=\bar{x}(1-\bar{x})$, versus non-local parameter $\bar{\tau}$ for *β*=2

figure results obtained by means of the integral form with those obtained with the differential form of the Eringen's model have been compared. This figure demonstrates that in the differential form, by increasing the non-local parameter, the displacement is decreased, in other word with increasing size effect, the flexibility of the nano-beam decreases, that this result is unreasonable. This problem has been resolved in the integral form of the Eringen's model. In the integral form, by increasing the non-local parameter, the nano-beam becomes more flexible.

Fig. 7 illustrates dimensionless displacement for a S-C supported beam at $\bar{x}$=0.5 under a parabolic distributed load $\bar{q}=\bar{x}(1-\bar{x})$, versus non-local parameter $\bar{\tau}$ for *β*=2. Unlike Fig. 6, this figure demonstrates that in the differential form, by increasing the non-local parameter, the nano-beam becomes more flexible that it is a correct result. But, the increasing of flexibility in the integral form is more than the differential form of the Eringen's model, and we can conclude that the integral form has better result than the differential form.

Fig. 8 represents dimensionless displacement for a S-S supported beam at $\bar{x}$=0.5 under a parabolic distributed load $\bar{q}=\bar{x}(1-\bar{x})$, versus non-local parameter $\bar{\tau}$ for *β*=2. As well as Fig. 6, this figure demonstrates that in the differential form, by increasing size effect, the flexibility of the nano-beam decreases, that this result is irrational. This problem has been resolved in the integral form of the Eringen's model. In the integral form, by increasing the non-local parameter, the nano-beam becomes more flexible.

Fig. 9 illustrates the deflection profile of S-S supported BDFG nano-beam under a uniform distributed load, against the $\bar{x}$ for various values of *β*. This figure shows that with increases in the value of *β*, the deflection of nano-beam decreases.

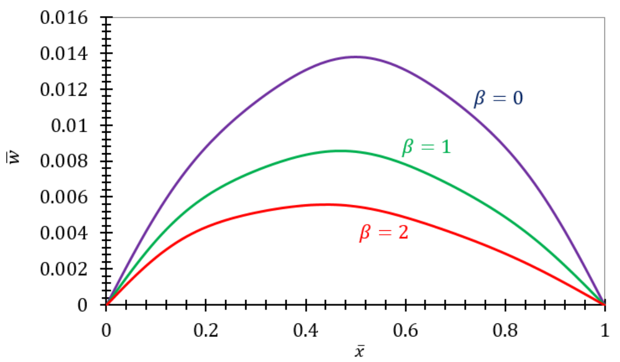

Fig. 9 Deflection of a S-S supported BDFG nano-beam under a uniform distributed load versus β for various values of β for $\bar{\tau} = 0.06$

4. Conclusions

One of the important results of this paper is to show the difference between Eringen's non-local theory in the integral form and the differential form. Numerical results of this paper show a conflict. For a cantilever beam, under a linear distributed load in the differential form, by increasing the non-local parameter, the displacement is decreased, that this result is unreasonable. This problem has been resolved in the integral form of the Eringen's model. In the integral form, by increasing the non-local parameter, the nano-beam becomes more flexible. All of results illustrate deflection of nano-beam in the integral form bigger than the differential form. Also in small nan-local parameter, the difference between Eringen's non-local theory in the integral form and the differential form is low. Moreover, an increase in the material inhomogeneity constant β leads to lower deflection of BDFGM Euler-Bernoulli nano-beams. The presented results also show that the material inhomogeneity constant has a significant influence on the mechanical behaviors of the bi-directional functionally graded Euler-Bernoulli nano-beams.

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