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Transient analysis of two dissimilar FGM layers with multiple interface cracks

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Abstract. The analytical solution of two functionally graded layers with Volterra type screw dislocation is investigated under anti-plane shear impact loading. The energy dissipation of FGM layers is modeled by viscous damping and the properties of the materials are assumed to change exponentially along the thickness of the layers. In this study, the rate of gradual change of shear moduli, mass density and damping constant are assumed to be same. At first, the stress fields in the interface of the FGM layers are derived by using a single dislocation. Then, by determining a distributed dislocation density on the crack surface and by using the Fourier and Laplace integral transforms, the problem are reduce to a system of singular integral equations with simple Cauchy kernel. The dynamic stress intensity factors are determined by numerical Laplace inversion and the distributed dislocation technique. Finally, various examples are provided to investigate the effects of the geometrical parameters, material properties, viscous damping and cracks configuration on the dynamic fracture behavior of the interacting cracks.

Keywords: two FGM layers; viscous damping; transient anti-plane loading; several interface cracks

1. Introduction

Predicting the behavior of cracked structures during impact and crash events has increased the importance of fracture mechanics considerably in engineering applications. Great efforts have been made to study the static fracture behavior of materials in recent years. Although most of the studies are related to static or quasi-static conditions the response of the cracks under dynamic loading is more important. Dynamic loading, are mainly categorized in two groups including harmonic loading and impact loading. Impact loads applied on the cracked structures lead to catastrophic failure of the system. Great progress has been made in the analysis of cracks in bodies made up of FG materials. The dynamic response of a crack in an FGM layer between two dissimilar half planes under anti-plane shear impact load was solved bay Babaei and Lukasiewicz (1998). They showed that the dynamic stress intensity factors depend on the crack length and the material properties. The transient dynamic stress intensity factor for an interface crack between two dissimilar half-infinite isotropic viscoelastic bodies under impact load was determined by Wei *et al*. (2000). They analyzed the dynamic SIF during a small time-interval and the effects of the viscoelastic material parameters on the dynamic SIF. Chen and Worswick (2000) studied the transient response of two cracks in a half space under anti-plane shear impact load. The effect of the geometry ratio and the depth of the crack on DSIFs were studied. Shul and Lee (2001) considered the dynamic response of the subsurface interface crack in multi-layered orthotropic half-space subjected to an anti-plane shear impact loading. The effects of the geometric constants and material properties were discussed. Shul and Lee (2002) considered the dynamic response of a subsurface eccentric crack in a functionally graded coating layer on the layered half-space under an anti-plane shear impact loading. The non-homogeneous parameter, geometric constants and the material properties were investigated on the normalized stress intensity factor. Zhang *et al.* (2003) solved the anti-plane transient problem of a finite crack in an infinite FGM and explored the effects of the material gradients of the FGM on the transient dynamic SIF and their dynamic overshoot corresponding with the static SIF. The transient in-plane problem was derived by Guo *et al.* (2005) for a coating-substrate structure with a cracked functionally graded interfacial layer subjected to an impact load. By using the integral transform techniques, the boundary value problem reduced to singular integral equations. The influences of the material nonhomogeneity constant and the geometric parameters on the DSIFs were discussed. Chen (2006) investigated the dynamic response of an electrically impermeable mode III crack in a transversely isotropic piezoelectric material subjected to the pure electric load. The stress intensity factor, the mechanical energy release rate and the total energy release rate were derived and expressed as a function of time for a given applied electric load. Yongdong *et al.* (2006) studied the anti-plane transient analysis for two functionally gradient half-planes with a weak/infinitesimal-discontinuous interface. The effects of the non-homogeneity parameters and the types of discontinuity were studied on the SIF mode-III. A periodic array of cracks in an infinite FGM under transient mechanical loading was investigated by Wang and Mai (2006). The effect of the material non-homogeneity on the crack tip intensity factors was discussed. Li *et al.* (2008) considered the dynamic SIF of two collinear mode-III cracks perpendicular to and on the two sides of a bi-FGM weak-discontinuous interface

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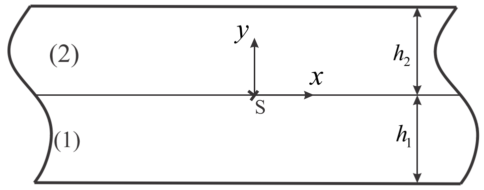

Fig. 1 Schematic view of the medium with screw dislocation

through studying the geometrical and physical parameters. Itou (2007) investigated the transient dynamic stress intensity factors around two rectangular cracks in a non-homogeneous interfacial layer sandwiched between two dissimilar elastic half-spaces. By using the Laplace and Fourier transforms, the problem reduced to the solution of a pair of dual integral equations. Also, a homogeneous linear elastic body containing multiple collinear cracks under anti-plane dynamic load was considered by Wu and Chen (2011). The dynamic stress intensity factor associated with the crack tips was calculated by a numerical inverse Laplace scheme. Vafa *et al.* (2015) used the Volterra-type screw dislocation technique and the Stehfest inversion method to simulate the dynamic response of the FG layers weakened by multiple cracks parallel to the boundary under anti-plane shear impact load.

According to the reviewed literature, there is not a promising investigation regarding the transient response of the several cracks in the interface of two FG layers. In this study, the mode-III fracture problem of multiple interface cracks between two FG layers under mechanical impacts is analyzed. The energy dissipation in the medium is showed by viscous damping. Using the distributed dislocation and integral transform techniques in conjunction with the Stehfest inversion method, the integral equations are derived in the form of Cauchy singular types, and are solved numerically for the dislocation density on the cracks faces. These solutions are employed to calculate the dynamic stress intensity factors for multiple interfacial cracks. Finally, the effects of the material properties, viscous damping, cracks length and cracks interaction on the dynamic stress intensity factors of the cracks are studied using different examples to demonstrate the advantage of this method.

2. Solution of dislocation

First, we consider two functionally graded layers, where the energy dissipation in the medium is modeled by viscous damping. The subscripts 1 and 2 correspond to the lower and upper FGM layers with thickness h_1 and h_2, respectively (Fig. 1). The single dislocation is situated at the origin of Cartesian coordinates along the interface of two layers.

The anti-plane displacement is assumed independent of z, as follows

$$u = 0, v = 0, w = w(x, y, t) \tag{1}$$

where $[u, v, w]$ denote the components of the displacement along x, y and z axis, respectively. The constitutive equations governing the anti-plane deformation of the FGM layers are given by

$$\tau_{zxi} = G_{zxi}(y)\frac{\partial w_i(x,y,t)}{\partial x} \quad \tau_{zyi} = G_{zyi}(y)\frac{\partial w_i(x,y,t)}{\partial y} \qquad i = 1,2 \tag{2}$$

where τ_{zx}, τ_{zy} are the anti-plane stress components and $G_{zx}(y)$, $G_{zy}(y)$ are the shear modules of elasticity of the FGM, respectively. In this case, the governing equations with structural energy dissipation, is written as

$$\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} = \rho(y)\frac{\partial^2 w}{\partial t^2} + \gamma(y)\frac{\partial w}{\partial t} \tag{3}$$

where $\rho(y)$ and $\gamma(y)$ are the mass density and the viscous damping coefficient per unit volume of the FGM, respectively. Substituting Eqs. (2) into Eq. (3) leads to

$$G_{zxi}(y)\frac{\partial^2 w_i}{\partial x^2} + G_{zyi}(y)\frac{\partial^2 w_i}{\partial y^2} + \frac{\partial G_{zyi}(y)}{\partial y}\frac{\partial w_i}{\partial y} = \rho_i(y)\frac{\partial^2 w_i}{\partial t^2} + \gamma_i(y)\frac{\partial w_i}{\partial t} \tag{4}$$

It is supposed that the material properties of FGM layers change exponentially along the y-axis. To make the analysis tractable, we use

$$[G_{zxi}(y), G_{zyi}(y), \rho_i(y), \gamma_i(y)] = [G_{zx0}, G_{zy0}, \rho_0, \gamma_0]e^{2\delta_i y} \tag{5}$$

where γ_0 is constant and $[G_{zx0}$, G_{zy0}, $\rho 0]$ are material properties at y=0. Also, δ_i, i=1,2 is the nonhomogeneity parameter of the FGM. By imposing Eq. (5) into Eq. (4), one may conclude that,

$$\frac{\partial^2 w_i}{\partial y^2} + 2\delta_i\frac{\partial w_i}{\partial y} + g^2\frac{\partial^2 w_i}{\partial x^2} = c^2\frac{\partial^2 w_i}{\partial t^2} + m\frac{\partial w_i}{\partial t} \tag{6}$$

where

$$c^2 = \frac{\rho_0}{G_{zy0}}, \qquad m = \frac{\gamma_0}{G_{zy0}} \tag{7}$$

The conditions representing the screw dislocation and the traction-free conditions on the outer boundary of the layers are expressed as

$$\begin{aligned} &w_2(x,0^+,t) - w_1(x,0^-,t) = b_{mz}(t)H(x) \\ &\tau_{zy2}(x,0^+,t) = \tau_{zy1}(x,0^-,t) \\ &\tau_{zy2}(x,h_2,t) = 0 \\ &\tau_{zy1}(x,-h_1,t) = 0 \\ &w_i(x,y,0) = \frac{\partial w_i}{\partial t}(x,y,0) = 0 \qquad i = 1,2 \end{aligned} \tag{8}$$

where $b_{mz}(t)$ stands for the dislocation Burgers vector and $H(.)$ is the Heaviside step function. Imposing the Laplace and complex Fourier transforms to Eq. (6) by assuming that the FGM layers are initially at rest, one may obtain the ordinary differential equations as follows

$$\frac{d^2\overline{w}_i^*(\zeta,y,s)}{dy^2}+2\delta_i\frac{d\overline{w}_i^*(\zeta,y,s)}{dy}-\beta^2\overline{w}_i^*(\zeta,y,s)=0 \qquad (9)$$

where

$$\beta=\sqrt{g^2\zeta^2+c^2s^2+ms} \qquad (10)$$

The general solutions of Eq. (9) for each layer, is derived as

$$\begin{aligned}&\overline{w}_1^*(\zeta,y,s)=A_1(\zeta,s)e^{(-\delta_1+\sqrt{\delta_1^2+\beta^2})y}+A_2(\zeta,s)e^{-(\delta_1+\sqrt{\delta_1^2+\beta^2})y} \qquad -h_1\le y\le 0\\&\overline{w}_2^*(\zeta,y,s)=A_3(\zeta,s)e^{(-\delta_2+\sqrt{\delta_2^2+\beta^2})y}+A_4(\zeta,s)e^{-(\delta_2+\sqrt{\delta_2^2+\beta^2})y} \qquad 0\le y\le h_2\end{aligned} \qquad (11)$$

The unknown coefficients in Eq. (11) are determined by applying the boundary conditions (8). The transformed out-of-plane displacement $w_1(x, y, t)$ in the Laplace and Fourier domain introduced as

$$\overline{w}_1^*(\zeta,y,s)=\frac{-b_{mz}(s)[\pi\delta(\zeta)-i/\zeta]\sinh(k_2h_2)e^{-\delta_1 y}[\delta_1\sinh(k_1(y+h_1))+k_1\cosh(k_1(y+h_1))]}{\sinh(k_1h_1)[-\delta_2\sinh(k_2h_2)+k_2\cosh(k_2h_2)]+\sinh(k_2h_2)[\delta_1\sinh(k_1h_1)+k_1\cosh(k_1h_1)]} \qquad (12)$$

where

$$\begin{aligned}k_1&=\sqrt{\delta_1^2+c^2s^2+ms}\\k_2&=\sqrt{\delta_2^2+c^2s^2+ms}\end{aligned} \qquad (13)$$

By using the inverse complex Fourier transforms, Eqs. (12) may be written as

$$\begin{aligned}\overline{w}_1(x,y,s)=&\frac{-b_{mz}(s)e^{-\delta_1 y}}{2}\times\\&\frac{\sinh(k_2h_2)[\delta_1\sinh(k_1(y+h_1))+k_1\cosh(k_1(y+h_1))]}{\sinh(k_1h_1)[-\delta_2\sinh(k_2h_2)+k_2\cosh(k_2h_2)]+\sinh(k_2h_2)[\delta_1\sinh(k_1h_1)+k_1\cosh(k_1h_1)]}\\&\frac{-b_{mz}(s)e^{-\delta_1 y}}{\pi}\times\\&\int_0^\infty\frac{\sinh(k_2h_2)[\delta_1\sinh(k_1(y+h_1))+k_1\cosh(k_1(y+h_1))]\sin(\zeta x)d\zeta}{\zeta\left(\sinh(k_1h_1)[-\delta_2\sinh(k_2h_2)+k_2\cosh(k_2h_2)]+\sinh(k_2h_2)[\delta_1\sinh(k_1h_1)+k_1\cosh(k_1h_1)]\right)}\end{aligned} \qquad (14)$$

The component of the stress in the Laplace domain is obtained from the constitutive equations and Eq. (14) as follows

$$\begin{aligned}\overline{\tau}_{zy}(x,y,s)=&\frac{G_{zy0}b_{mz}(s)e^{\delta_1 y}}{\pi}\times\\&\int_0^\infty\frac{(\delta_1^2-k_1^2)\sinh(k_2h_2)\sinh(k_1(y+h_1))\sin(\zeta x)d\zeta}{\zeta\left(\sinh(k_1h_1)[-\delta_2\sinh(k_2h_2)+k_2\cosh(k_2h_2)]+\sinh(k_2h_2)[\delta_1\sinh(k_1h_1)+k_1\cosh(k_1h_1)]\right)}\end{aligned} \qquad (15)$$

In order to clearly describe the singular behavior of the stress component, the asymptotic behavior of the integrands in Eq. (15) must be considered. Hence, Eq. (15) may be recast to more suitable forms as below

$$\begin{aligned}\overline{\tau}_{zy}(x,y,s)=&\frac{G_{zy0}b_{mz}(s)e^{\delta_1 y}}{\pi}\times\\&\left[\int_0^\infty\{(\frac{(\delta_1^2-k_1^2)\sinh(k_2h_2)\sinh(k_1(y+h_1))}{Q}+0.5e^{g\zeta y})\sin(\zeta x)d\zeta\}-0.5\frac{gx}{x^2+(gy)^2}\right]\end{aligned} \qquad (16)$$

where

$$Q=\zeta\left(\sinh(k_1h_1)[-\delta_2\sinh(k_2h_2)+k_2\cosh(k_2h_2)]+\sinh(k_2h_2)[\delta_1\sinh(k_1h_1)+k_1\cosh(k_1h_1)]\right) \qquad (17)$$

Ignoring the details of the derivations, the stress component is obtained as Cauchy singularity at the dislocation position, which is a well-known characteristic of the stress fields caused by Volterra-type dislocations.

3. Two FGM layers weakened by multiple interface cracks

In this section, the fundamental concept of the distributed dislocation technique and the calculation of the numerical inversion Laplace transform are introduced. The distributed dislocation technique has been used by several investigators for the analyses of cracked bodies under mechanical loading (Weertman 2015). The dislocation method in previous section is extended to deal with two FGM layers weakened by N interface cracks. The configuration of the cracks is expressed in a parametric form as

$$\begin{aligned}&x_i(q)=x_i+l_iq\\&y_i(q)=y_i \qquad i=1,2,\ldots,N \qquad -1\le q\le 1\end{aligned} \qquad (18)$$

where (x_i, y_i) are the coordinates of the center of the cracks and l_i is the half-length of the cracks. By employing the superposition principle, the components of the stress on a given crack surface are obtained. The system of singular integral equations on the face of i-th crack is written in the following form employed in the numerical procedure.

$$\overline{\sigma}_{yz_i}(x_i(q),y_i(q),s)=\sum_{j=1}^{N}\int_{-1}^{1}\overline{K}_{ij}(q,p,s)B_{mzj}(p,s)\,l_j\,dp \qquad (19)$$

where $B_{mzj}(p, s)$ is the Laplace transform of the dislocation density on the face of j-th crack. Form Eq. (16), the kernel of integral Eq. (19) is given by

$$\begin{aligned}\overline{K}_{ij}(q,p,s)=&\frac{G_{zy0}e^{\delta_1(y_i-y_j)}}{\pi}\times\\&\left[\int_0^\infty\{(\frac{(\delta_1^2-k_1^2)\sinh(k_2h_2)\sinh(k_1(y_i-y_j+h_1))}{Q}+0.5e^{g\zeta(y_i-y_j)})\sin(\zeta(x_i-x_j+l_iq-l_jq))d\zeta\}\right.\\&\left.-0.5\frac{g(x_i-x_j+l_iq-l_jq)}{(x_i-x_j+l_iq-l_jq)^2+(g(y_i-y_j))^2}\right]\end{aligned} \qquad (20)$$

The crack opening displacement across the j-th crack is represented using the definition of dislocation as

$$\overline{w}_j^-(q,s)-\overline{w}_j^+(q,s)=\int_{-1}^{s}l_jB_{mzj}(p,s)dp \qquad (21)$$

The displacement fields must be a single-valued parameter; and so, the following closure conditions for embedded cracks is employed

$$l_j\int_{-1}^{1}B_{mzj}(p,s)dp=0 \qquad (22)$$

The numerical inversion of Laplace transform is carried out via Stehfest's method (2007). We introduce a time-dependent function $f(t)$ as

$$f(t)\approx\frac{ln2}{t}\sum_{n=1}^{M}\upsilon_nF(\frac{ln2}{t}n) \qquad (23)$$

where, $F(.)$ is the Laplace transform of $f(t)$, M is a chosen even number, and the coefficients

$$\upsilon_n=(-1)^{\frac{M}{2}+n}\sum_{k=[0.5(n+1)]}^{\min(\frac{M}{2},n)}\frac{k^{\frac{M}{2}}(2k)!}{(\frac{M}{2}-k)!k!(k-1)!(n-k)!(2k-n)!} \qquad (24)$$

and [.] signifies the integral part of the quantity. From Eq. (23), the calculation of $f(t)$ at a fixed time t depends on the computation of $F(s)$ at M points $s=\frac{ln2}{t}n,\ n\in\{1,2,\ldots,M\}$.

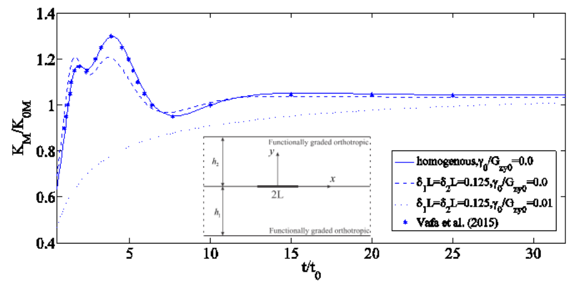

Fig. 3 The variations of the normalized stress intensity factor of an interface crack versus t/t_0

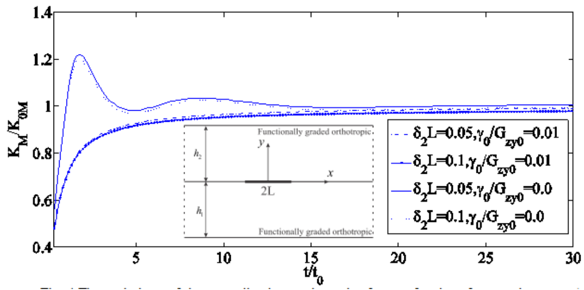

Fig. 4 The variations of the normalized stress intensity factor of an interface crack versus t/t_0

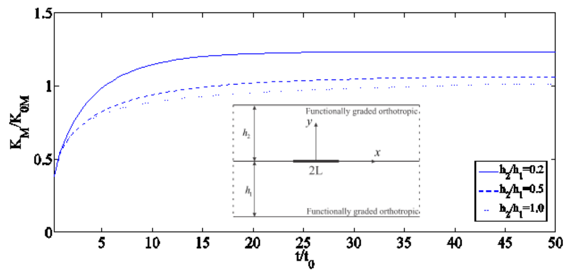

Fig. 5 The variations of the normalized stress intensity factor of an interface crack versus t/t_0

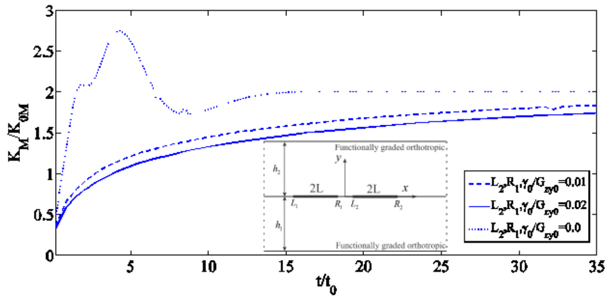

Fig. 6 The variations of the normalized stress intensity factor for two interface cracks versus t/t_0

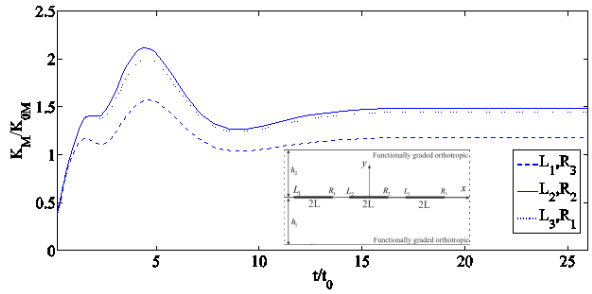

Fig. 7 The variations of the normalized stress intensity factor for three equal length cracks versus t/t_0

Applying the procedure to Eqs. (19) and (22) results in

$$\bar{\sigma}_{yz_i}(x_i(q), y_i(q), \frac{\ln 2}{t}n) = \sum_{j=1}^{N}\int_{-1}^{1} \bar{K}_{ij}(q,p,\frac{\ln 2}{t}n)B_{mzj}(p,\frac{\ln 2}{t}n)\, l_j\, dp$$
$$l_j\int_{-1}^{1} B_{mzj}(p,\frac{\ln 2}{t}n)dp = 0 \qquad i\in\{1,2,...,N\}, \quad n\in\{1,2,...,M\} \qquad (25)$$

The stress fields at a tip of an embedded crack behave like $1/\sqrt{r}$, where r is the distance from the crack tip. Consequently, the dislocation densities are taken as

$$B_{mzj}(p,\frac{\ln 2}{t}n) = \frac{G_{mzj}(p,\frac{\ln 2}{t}n)}{\sqrt{1-p^2}}, \quad -1\le t\le 1, \ , \ j\in\{1,2,...,N\} \qquad (26)$$

Substituting Eq. (26) into Eq. (25) and applying the numerical technique expanded by Erdogan *et al.* (1973) for the solution of singular integral equations, the resultant equations are solved. By using Eq. (23), the inverse Laplace transforms of the solution yields

$$g_{mzj}(q,t) = \frac{\ln 2}{t}\sum_{n=1}^{M}\upsilon_n G_{mzi}(q,\frac{\ln 2}{t}n), \qquad -1\le q\le 1, \ i\in\{1,2,...,N\} \qquad (27)$$

The stress intensity factors for the embedded cracks in FG materials is represented by

$$(K_{III}^{m})_{Li} = \frac{gG_{zy}(y_{Li})}{2}\left[[(x_i'(-1)]^2+[y_i'(-1)]^2\right]^{\frac{1}{4}} g_{mzi}(-1,t)$$
$$(K_{III}^{m})_{Ri} = -\frac{gG_{zy}(y_{Ri})}{2}\left[[(x_i'(1)]^2+[y_i'(1)]^2\right]^{\frac{1}{4}} g_{mzi}(1,t) - \qquad (28)$$

where L and R designate the left and right tips of the crack, respectively. Eqs. (27) are substituted into (28) to determine the stress intensity factors. The details of the derivation of stress intensity factors in Eq. (28) were presented by Bagheri *et al.* (2015).

4. Numerical solutions

The preceding formulation provides the analysis of several cracks situated at the interface of two FG layers subjected to the anti-plane loading. The thickness of the layers are assumed as h_1=h_2=0.05(m), and the number of points for the inversion of Laplace transform by employing Stehfest's method is M=10, and the deviser of the normalizing time is t_0=Lc, where L is the half-crack length and 1/c is the shear wave velocity at the interface.

To display the validity of the current paper, the cracks with length 2l under uniform shear traction τ_0 are considered. The quantity of interest in the fracture problems is the dimensionless stress intensity factors (SIF), i.e., K_M/K_0. In this study, the dynamic SIFs are normalized by $K_{0M}=\tau_0\sqrt{l}$ for constant loading.

Figs. 2 and 3 display the effect of non-homogenous parameters and damping coefficient on the non-dimensional SIFs versus t/t_0. These examples show the effect of dimensionless time on the dynamic stress intensity factor in a homogenous layer and also in the FG layer. It is interesting to note that the normalized stress intensity factor K_M/K_0 increases quickly at first with time to reach its maximum value, and then gets approximately stable. Finally, the dynamic stress intensity factor approaches the corresponding static value.

In comparison with FG layers, the SIF peak of the homogenous layer occurs in shorter time. The SIFs attenuate as the damping effects enhance. The SIF of a crack in homogenous layer is in good agreement with that presented by Vafa *et al*. (2015) for a crack in strip. As shown in Figs. 2 and 3, by growing the crack length, the dynamic stress intensity factor increases too.

Furthermore, a central crack with $L/h_1=L/h_2=0.2$ is considered in an interface of two FG layers with $\delta_1=5$ under uniform traction $\tau_0 H(t)$. The effects of the damping coefficients and the non-homogeneous parameters of the upper FG layer on the dynamic stress intensity factors are plotted in Fig. 4. According to the results, by increasing the damping coefficients and FG constant parameters of the upper layer, the stress intensity factors decrease.

The effect of the dimensionless time on the behavior of the crack tips is plotted in Fig. 5 for three thickness ratios h_2/h_1. The material properties of layers are $\delta_1=5$ and $\delta_2=10$. The crack with length $2L/h_1=1.0$ is situated at the interface of the layers. The parameter K_M/K_{0M} decreases significantly with increasing of the thickness ratio h_2/h_1. One may conclude that for these patterns, the parameter h_2/h_1 has a great influence on the dynamic stress intensity factor.

In Fig. 6, two FGM layer weakened by two interface cracks is considered under anti-plane uniform traction $\tau_0 H(t)$. The effect of the damping coefficients on the dynamic stress intensity factors is illustrated (Fig. 6). The peak value of the normalized SIFs increases with decreasing of the damping coefficients.

In the last example, three equal-length cracks with length $2_L/h_1=2L/h_2=1.0$ are studied. The variations of the dynamic stress intensity factors of the adjacent crack tips (i.e., L_2 and R_2) are too large, whereas for other crack tips (i.e., L_1 and R_3) are not significant.

5. Conclusions

Two FGM layers weakened by several interface cracks are studied under anti-plane shear impact loading. Using the Fourier and Laplace transform methods, the associated boundary value problem is reduced to a system of singular integral equations for the Volterra dislocation density. The results are verified by considering a single crack in a strip. The examples of the multiple interfacial cracks show that the dynamic stress intensity factor at the crack tips increases by growing the crack length, decreases by growing the damping coefficient, and decreases by increasing the thickness ratio. The results are in excellent agreement with the analytical solutions obtained in Vafa *et al.* (2015).

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