Structural Engineering and Mechanics, *Vol. 67, No. 2 (2018) 165-173*
DOI: https://doi.org/10.12989/sem.2018.67.2.165

On transverse matrix cracking in composite laminates loaded in flexure under transient hygrothermal conditions

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(Received November 9, 2017, Revised April 20, 2018, Accepted April 22, 2018)

Abstract. A simple predicted model using a modified Shear-lag method was used to represent the moisture absorption effect on the stiffness degradation for $[0/90]_{2s}$ composite laminates with transverse cracks and under flexural loading. Good agreement is obtained by comparing the prediction model and experimental data published by Smith and Ogin (2000). The material properties of the composite are affected by the variation of temperature and moisture absorption. The transient and non-uniform moisture concentration distribution give rise to the transient elastic moduli of cracked composite laminates. The hygrothermal effect is taken into account to assess the changes in the normalised axial and flexural modulus due to transverse crack. The obtained results represent well the dependence of the stiffness properties degradation on the cracks density, moisture absorption and operational temperature. The composite laminate with transverse crack loaded in axial tension is more affected by the hygrothermal condition than the one under flexural loading. Through this theoretical study, we hope to contribute to the understanding of the moisture absorption on the composite materials with matrix cracking.

Keywords: absorption; bending; transverse cracks; hygrothermal effect; stiffness; Chamis model

1. Introduction

Compared with simple axial loading (Li and Hafeez 2009, Vingradov and Hashin 2010, Barbero and Cosso 2014, Hajikazemi and Sadr 2014, Huang *et al*. 2014, Katerelos *et al*. 2015), the development of matrix cracking damage in composite laminates loaded in flexure has received little attention in the literature. Earlier work on flexure concentrated on developing linear and nonlinear (large displacement) methodologies to analyse the stress state in undamaged composite beams and, in particular, plates under lateral loads and the implementation of phenomenological engineering failure criteria (Turvey 1980, Reddy and Pandey 1987, Reddy and Reddy 1992, Kam and Jan 1995, Reddy 1997). Comparison with the experimental data in these studies was limited.

Smith and Ogin (2000) proposed a one-dimensional model based on simple bending theory, which enables flexural stiffness as a function of crack density to be calculated and also gives predictions of the first ply failure based on fracture mechanics. The flexural stiffness as a function of crack density in one of the 90/90 plies of a $(0/90)_{2s}$ lay-up was shown to be in good agreement with the experimental data. Progressive cracking leads to a reduction in flexural modulus, although this is less than would be seen for tensile loading, due to the greater role of the surface 0° plies. Although the approach developed has been applied only to a relatively simple four-point bending geometry and the general methodology is applicable in more complex situations.

When a fibre-reinforced plastic composite material is exposed to a hygrothermal environment and mechanical loads, changes in material properties are expected (Tounsi *et al.* 2005, Amara *et al.* 2006, Benzair *et al.* 2006, Bouazza *et al.* 2007, Adda Bedia *et al.* 2008, Benkhedda *et al*. 2008, Rezoug *et al.* 2011, Amara *et al.* 2014, Khodjet-Kesba *et al.* 2015, Khodjet-Kesba *et al.* 2016). These changes in material properties are connected to an irreversible material degradation. Irreversible degradation in composite laminates due to moisture environment has been discussed by Wong and Broutman (1985), Jackson and Weitsman (1985). Usually one of the first observed damage modes in a laminated composite is matrix cracking. These cracks are in general not critical for final failure, but if they are connected to a surrounding moisture environment more rapid moisture absorption may be expected for the cracked laminate (Gillat and Broutman 1987, Collings and Stone 1987, Suri and Perreux 1995). The accelerated moisture absorption in a cracked material exposed to humid air is a result of the faster diffusion in air compared to the diffusion speed in the composite material (Loos and Springer 1981).

Faster moisture uptake may also develop a faster material degradation. This makes it important to know the moisture absorption behaviour in a cracked laminate. For an undamaged material, well-accepted moisture transportation models are available. The most common models for the transportation of moisture in undamaged polymeric composite materials are Fickian diffusion (Shen and

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http://www.techno-press.com/journals/sem&subpage=7
ISSN: 1225-4568 (Print), 1598-6217 (Online)

Springer 1981). If the material contains cracks that significantly affect the moisture uptake, then the original laws of Fickian are no longer valid for the whole laminate, but locally they still work. Moisture absorption models for cracked materials have been presented by Weitsman (1987), a link is made with continuum damage theories and thermodynamics, while Lundgren and Gudmundson (1998) use a micromechanical approach.

In the present paper, the transverse cracking problem in symmetrically cross-ply $[0/90]_{2s}$ laminate loaded in flexure is predicted by using the modified shear-lag model. The obtained results show a good agreement comparing with experimental data (Smith and Ogin 2000) without taking into account the moisture absorption and temperature effect. On the other hand, the cross-ply laminates are initially exposed to the hygrothermal ageing submitted to transient and non-uniform moisture absorption concentration distribution. The obtained results show the dependence of the axial and flexural stiffness degradation on the cracks density, moisture absorption and operational temperature.

2. Theoretical analysis

Transverse matrix cracking is a common damage mode in cross-ply laminates under flexure loading. It is assumed that the 90° ply has developed continuous intralaminar cracks in fibre direction which extend from edge to edge in the z direction. The cross-ply laminate is characterised by $2.t_{90}$ the width of 90° ply and the spacing between two cracks is $2.l_0$ (Fig. 1).

The laminate is acted upon by a bending moment, M, which introduces a radius of curvature R or curvature k (k =1/R). The longitudinal stress in each ply is given as a function of the coordinate z by the following equation

$$\sigma_{xx}(z) = \frac{E(z)\cdot z}{R} \tag{1}$$

Integrating through the thickness of the laminate we derive a relationship between the bending moment and curvature as follows

$$\frac{M}{k} = \frac{wh^3}{12}\left(\frac{88E_0 + 40E_{90}}{128}\right) \tag{2}$$

where E_0 and E_{90} denote the moduli of the 0° and 90° plies in the x-direction.

Eq. (2) may be rewritten as

$$\frac{M}{k} = \frac{wh^3}{12}E^0_{flex} \tag{3}$$

Where E^0_{flex} is the initial flexural stiffness of the laminate derived using the simple bending theory, it's given by

$$E^0_{flex} = \left(\frac{88E_0 + 40E_{90}}{128}\right) \tag{4}$$

In the absence of a unified theory for the mechanical characterization of the composites materials with long fibres, many formulations were proposed. In the bibliography, we can quote, the rule of mixtures method, the contiguity method who is based on the fibres arrangement (Staab 1999, Maurice 2001), the semi-empirical method based of Halpin-Tsai (Haplin and Tsai 1968). In this paper, we used the rule of mixtures method applied to the composites with anisotropic fibres, which it's based on the fibres emplacement (Chamis 1983). Therefore, the elastic constants are obtained from the following equations

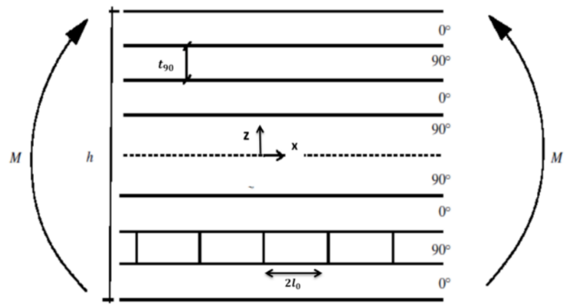

Fig. 1 Transverse cracked cross-ply laminate loaded in flexure

$$E_x = F_m E_m \cdot V_m + E_{fx}\cdot V_f \tag{5}$$

$$E_y = \left(1-\sqrt{V_f}\right)F_m E_m \frac{F_m E_m \sqrt{V_f}}{1-\sqrt{V_f}\left(1-\dfrac{F_m E_m}{E_{fy}}\right)} \tag{6}$$

$$G_{xy} = \left(1-\sqrt{V_f}\right)F_m G_m + \frac{F_m G_m \sqrt{V_f}}{1-\sqrt{V_f}\left(1-\dfrac{F_m G_m}{G_f}\right)} \tag{7}$$

$$\nu_{xy} = V_f\cdot\nu_f + V_m\cdot\nu_m \tag{8}$$

In the above equations, V_f and V_m are the fibre and matrix volume fractions and are related by

$$V_f + V_m = 1 \tag{9}$$

Where F_m is the matrix mechanical property retention ratio

E_f, G_f and $\underline{\nu_f}$ are the Young's modulus, shear modulus and Poisson's ratio, respectively, of the fibre and E_m, G_m and ν_m are the corresponding properties of the matrix.

2.1 The modified shear-lag model

To assign a stiffness value to any cracked 90° plies, we make use of a one-dimensional modified shear-lag analysis for a cross-ply laminate loaded in flexure. In doing this we are assuming that the basic morphology of the cracks generated in flexure is the same as that of the cracks resulting from tensile loading.

For that, when the loading is applied only in x-direction and the far field applied stress is defined by $\sigma_c=(1/2h)N_x$, where N_x is applied load. The following analysis will be performed assuming generalised plane strain condition

$$\varepsilon_y^0 = \varepsilon_y^{90} = \varepsilon_y = const \tag{10}$$

The symbol ($^{-}$) over stress and train components denotes volume average. They are calculated using the following expressions

- In the 0° layer

$$\overline{f^0} = \frac{1}{2l_0}\frac{1}{t_0}\int_{-l_0}^{+l_0}\int_{t_{90}}^{h} f^0 dx\,dz = \frac{1}{2\bar{l}_0}\frac{1}{\alpha}\int_{-\bar{l}_0}^{+\bar{l}_0}\int_{1}^{\bar{h}} f^0(\bar{x},\bar{z})d\bar{x}\,d\bar{z} \tag{11}$$

-In the 90° layer

$$\overline{f^{90}} = \frac{1}{2l_0}\frac{1}{t_{90}}\int_{-l_0}^{+l_0}\int_{0}^{t_{90}} f^{90} dx\,dz = \frac{1}{2\bar{l}_0}\frac{1}{\alpha}\int_{-\bar{l}_0}^{+\bar{l}_0}\int_{0}^{1} f^{90}(\bar{x},\bar{z})d\bar{x}\,d\bar{z} \tag{12}$$

By using the strains in the 0° layer (which is not damaged and, hence, strains are equal to laminate strains, $\varepsilon_x = \bar{\varepsilon}_x^0$, etc.) and assuming that the residual stresses are zero, the Young's modulus of the laminate with cracks may be defined by the following expression

$$E_x = \frac{\sigma_c}{\bar{\varepsilon}_x^0} \tag{13}$$

Note that the initial laminate modulus measured at the same load is $E_{x0}=\sigma_c/\varepsilon_{x0}$ and, hence

$$\frac{E_x}{E_{x0}} = \frac{\varepsilon_{x0}}{\bar{\varepsilon}_x^0} \tag{14}$$

Strain- stress equations are giving in the following form

a) In the 0° layer

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \varepsilon_z^0\end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13}\\ S_{12} & S_{22} & S_{23}\\ S_{13} & S_{23} & S_{33}\end{bmatrix}\begin{Bmatrix}\sigma_x^0\\ \sigma_y^0\\ \sigma_z^0\end{Bmatrix} \tag{15}$$

b) In the 90° layer

$$\begin{Bmatrix}\varepsilon_x^{90}\\ \varepsilon_y^{90}\\ \varepsilon_z^{90}\end{Bmatrix} = \begin{bmatrix} S_{22} & S_{12} & S_{23}\\ S_{12} & S_{11} & S_{12}\\ S_{23} & S_{12} & S_{22}\end{bmatrix}\begin{Bmatrix}\sigma_x^{90}\\ \sigma_y^{90}\\ \sigma_z^{90}\end{Bmatrix} \tag{16}$$

Where S_{ij} is the compliance matrix for $[0/90]_{2s}$ composite laminate.

By averaging Eqs. (15) and (16), we obtain averaged constitutive equations of the 90° and 0° layer. In averaged relationships we have $\bar{\sigma}_z^{90} = \bar{\sigma}_z^0 = 0$ which follows from the force equilibrium in z direction

$$\int_{-l_0}^{+l_0} \sigma_z^i dx = 0 \quad , i = 90, \beta \tag{17}$$

Averaged constitutive equations corresponding to in-plane normal stress and strain components are

$$\begin{Bmatrix}\overline{\varepsilon_x^0}\\ \varepsilon_y\end{Bmatrix} = \begin{bmatrix} S_{11} S_{12}\\ S_{12} S_{22}\end{bmatrix}\begin{Bmatrix}\overline{\sigma_x^0}\\ \overline{\sigma_y^0}\end{Bmatrix} \tag{18}$$

$$\begin{Bmatrix}\overline{\varepsilon_x^{90}}\\ \varepsilon_y\end{Bmatrix} = \begin{bmatrix} S_{22} S_{12}\\ S_{12} S_{11}\end{bmatrix}\begin{Bmatrix}\overline{\sigma_x^{90}}\\ \overline{\sigma_y^{90}}\end{Bmatrix} \tag{19}$$

Eqs. (18) and (19) are obtained from the 3D strain-stress relationships but because of Eq. (17) the result is similar as in classical thin-laminate theory (CLT). In fact, for an undamaged laminate, the averaged stresses and strains are equal to the laminate theory stresses and strains and Eqs. (18) and (19) are still applicable.

Force equilibrium equations for a damaged (or undamaged) laminate are

- In x-direction

$$N_x = \int_0^{t_{90}} \sigma_x^{90} dz + \int_{t_{90}}^{h} \sigma_x^0 dz = \sigma_c (t_{90} + t_0) \tag{20}$$

- Leading to

$$\overline{\sigma_x^{90}} t_{90} + \overline{\sigma_x^0} t_0 = \sigma_c (t_{90} + t_0) \tag{21}$$

- In y-direction

$$N_y = 0 \Rightarrow \int_0^{t_{90}} \sigma_y^{90} dz + \int_{t_{90}}^{h} \sigma_x^0 dz = 0 \tag{22}$$

-From which follows

$$\overline{\sigma_y^{90}} t_{90} + \overline{\sigma_y^0} t_0 = 0 \tag{23}$$

Eqs. (18), (19), (21) and (23) contain seven unknowns: four stress components and three strain components ($\bar{\varepsilon}_x^{90}$, $\bar{\varepsilon}_x^0$ and ε_y). The total number of equations is six. Hence, one of the unknowns may be considered as independent, and the rest of them can be expressed as linear functions of it. Choosing the stress $\bar{\sigma}_x^{90}$ as independent, and solving with respect to it the system of Eqs. (18), (19), (21) and (23) we obtain

$$\varepsilon_y = g_1 \overline{\sigma_x^{90}} + f_1 \sigma_c ; \qquad \overline{\varepsilon_x^{90}} = g_2 \overline{\sigma_x^{90}} + f_2 \sigma_c ; \qquad \overline{\varepsilon_x^0} = g_3 \overline{\sigma_x^{90}} + f_3 \sigma_c \tag{24}$$

Expressions for g_i, f_i, i=1,2,3 through laminate geometry and properties of constituents are given as follows

$$g_1 = t_{90}\frac{S_{12}S_{22} - S_{11}S_{12}}{S_{11}t_0 + S_{22}t_{90}} \quad ; f_1 = \frac{S_{11}S_{12}(t_0 + t_{90})}{S_{11}t_0 + S_{22}t_{90}} \tag{25}$$

$$g_2 = S_{22} - \frac{S_{12}(S_{12}t_0 + S_{12}t_{90})}{S_{11}t_0 + S_{22}t_{90}} \quad ; \quad f_2 = \frac{S_{12}S_{xy}^0(t_0 + t_{90})}{S_{11}t_0 + S_{22}t_{90}} \tag{26}$$

$$g_3 = \frac{t_{90}}{t_0}\left(S_{12}\frac{(S_{12}t_0 + S_{12}t_{90})}{S_{11}t_0 + S_{22}t_{90}}\right) - S_{11} ; \quad f_3 = \frac{t_0 + t_{90}}{t_0}\left(S_{11} - \frac{(S_{12})^2 t_{90}}{S_{11}t_0 + S_{22}t_{90}}\right) \tag{27}$$

In order to obtain an expression for average stress $\bar{\sigma}_x^{90}$ in the repeatable unit, we consider the axial stress perturbation caused by the presence of two cracks. Without losing generality the axial stress distribution may be written in the following form

$$\sigma_x^{90} = \sigma_{x0}^{90} - \sigma_{x0}^{90} f_1(\bar{x}, \bar{z}) \tag{28}$$

$$\sigma_x^0 = \sigma_{x0}^0 - \sigma_{x0}^{90} f_2(\bar{x}, \bar{z}) \tag{29}$$

Where σ_{x0}^{90} is the CLT stress in 90° layer and σ_{x0}^0 is CLT stress in the 0° layer (laminate theory routine),

$-\sigma_{x0}^{90} f_1(\bar{x},\bar{z})$ and $\sigma_{x0}^{90} f_2(\bar{x},\bar{z})$ are stress perturbation caused by the presence of a crack. Normalising factors in the form of far field stresses in perturbation functions are used for convenience. Averaging Eqs. (28) and (29) using the integral force equilibrium in the x-direction (Eq. (20)), we obtain

$$\overline{\sigma_x^{90}} = \sigma_{x0}^{90} - \sigma_{x0}^{90} \frac{1}{2\bar{l}_0} R(\bar{l}_0) \tag{30}$$

$$\overline{\sigma_x^{\beta}} = \sigma_{x0}^{\beta} - \sigma_{x0}^{90} \frac{1}{2\alpha \bar{l}_0} R(\bar{l}_0) \tag{31}$$

In the following function

$$R(\bar{l}_0) = \int_{-\bar{l}_0}^{\bar{l}_0} \int_0^1 f_1(\bar{x},\bar{z})\, d\bar{z}\, d\bar{x} \tag{32}$$

$R(\bar{l}_0)$ is called the stress perturbation function. It's related to axial stress perturbation in the 90° layer and depends on the crack density.

The average stress $\bar{\sigma}_x^{90}$ involved in Eq. (24) is now expressed through the stress perturbation function (Eq. (32)). Conditions used to obtain expressions (Eq. (24)) are the same as used in CLT. Hence, substituting $\bar{\sigma}_x^{90} = \sigma_{x0}^{90}$ where σ_{x0}^{90} is the x-axis stress in the 90° layer according to CLT, we obtain CLT solution: $\bar{\varepsilon}_x^{90} = \varepsilon_{x0}^{90} = \varepsilon_{x0}$,$\bar{\varepsilon}_x^{0} = \varepsilon_{x0}^{0} = \varepsilon_{x0}$ and $\varepsilon_y = \varepsilon_{y0}$.

Substituting Eq. (30), which contains two terms, in Eq. (24) the result has two terms. The first term according to the discussion above is equal to CLT strain but the second one is a new term related to the stress perturbation function $R(\bar{l}_0)$

$$\varepsilon_y = \varepsilon_{y0} - \sigma_{x0}^{90} g_1 \frac{1}{2\bar{l}_0} R(\bar{l}_0) \tag{33}$$

$$\overline{\varepsilon_x^{90}} = \varepsilon_{x0} - \sigma_{x0}^{90} g_2 \frac{1}{2\bar{l}_0} R(\bar{l}_0) \tag{34}$$

$$\overline{\varepsilon_x^{0}} = \varepsilon_{x0} - \sigma_{x0}^{90} g_3 \frac{1}{2\bar{l}_0} R(\bar{l}_0) \tag{35}$$

The stress σ_{x0}^{90} in the 90° layer of an undamaged laminate under mechanical loading may be calculated using CLT

$$\sigma_{x0}^{90} = Q_{22}\varepsilon_{x0}(1-\nu_{12}\nu_{xy}^{0}) \tag{36}$$

Here, υ_{xy}^0 is the Poisson's ratio of the undamaged laminate.

We use definition Eq. (13) and substitute Eq. (35) in this relationship, finally the modified expression of the longitudinal Young's modulus of the $[0/90]_{2s}$ composite laminate due to transverse cracks

$$\frac{E_x}{E_{x0}} = \frac{1}{1+\dfrac{E_{90}t_{90}}{E_0 t_0}\dfrac{(1-\upsilon_{12}\upsilon_{xy}^0)}{(1-\upsilon_{12}\upsilon_{21})}\dfrac{1}{2a}R(\bar{l}_0)\left(1+\upsilon_{xy}^0\dfrac{(s_{12}t_{90}+s_{12}t_0)}{(s_{22}t_{90}+s_{11}t_\beta)}\right)} \tag{37}$$

The model developed by Berthelot *et al.* (1996) is used. This latter is modified by introducing the stress perturbation function

$$R(\bar{l}_0) = \int_{-a}^{+a} \frac{\cosh(\xi \bar{x})}{\cosh(\xi a)} d\bar{x} = \frac{2}{\xi}\tanh(\xi \bar{l}_0) \tag{38}$$

Where, ξ is the shear-lag parameter

$$\xi^2 = \bar{G}\frac{t_{90}(t_{90}E_{90}+t_0E_0)}{t_0E_0E_{90}} \tag{39}$$

Where $\bar{\rho} = \dfrac{1}{2\cdot \bar{l}_0}$; $\left(\bar{l}_0 = \dfrac{l_0}{t_{90}}\right)$ is a normalised crack density for composite laminate with matrix cracks.

The coefficient $\bar{G}$ depends on used assumptions about the longitudinal displacement and shears stress distribution:

a) In the case of the assumption of a parabolic variation of longitudinal displacement in both 0° and 90° layers, the coefficient $\bar{G}$ is done by

$$\bar{G} = \frac{3G}{t_{90}} \tag{40}$$

The shear modulus G of the elementary cell

$$G = \frac{G_{xz}^{90}}{1-3\dfrac{G_{xz}^{90}}{G_{xz}^{0}}\dfrac{f(t_{90})}{t_{90}f'(t_{90})}} \tag{41}$$

By replacing the function $f(z) = z^2 - 2(t_0+t_{90})z + \frac{2}{3}t_0^2 + 2t_0t_{90} + t_{90}^2$ in the (41), the shear modulus for parabolic assumption will be in the form (Berthelot *et al.* 1996)

$$G = \frac{G_{xz}^{90}}{1+\alpha\dfrac{G_{xz}^{90}}{G_{xz}^{0}}} \tag{42}$$

b) In the case when the variation of the longitudinal displacement is supposed progressive in 0^0 -layer

We use the function $f(z) = \dfrac{\sinh \alpha\eta_t}{\alpha\eta_t} - \cosh\eta_t\left(1+\alpha-\dfrac{z}{t_{90}}\right)$ in the (41), the shear modulus for progressive shear assumption will be in the form (Berthelot *et al.* 1996)

$$G = \frac{G_{xz}^{90}}{1+3\alpha\dfrac{\alpha\eta_t(\tanh\alpha\eta_t)^{-1}-1}{\alpha\eta_t^2}\dfrac{G_{xz}^{90}}{G_{xz}^{0}}} \tag{43}$$

2.2 Flexural stiffness as a function of crack density

When the 90° layer towards the tensile face of the laminate undergoes matrix cracking, the effective modulus of the ply is reduced from E_{90} to E_{90}^*. The value of E_{90}^* can be estimated using a modified shear-lag analysis of a representative cross-ply laminate of $[0/90]_s$ lay-up. If we write the reduced modulus as

$$E_{x0} = \frac{t_0 E_0 + t_{90} E_{90}^*}{h} \tag{44}$$

Then, equating (37) and (44) and rearranging give

$$E_{90}^* = \frac{E_{90}\left(1 - \frac{\tanh(\xi \bar{l}_0)}{\xi \bar{l}_0}\right)}{\left(1 + \frac{t_{90} E_{90}}{t_0 E_0} \frac{\tanh(\xi \bar{l}_0)}{\xi \bar{l}_0}\right)} \tag{45}$$

The reduced flexural stiffness of the laminate can now be derived following the method detailed in Smith and Ogin (1999). We note first that, as a result of the reduced stiffness of the 90° ply, the neutral axis of the laminate will move towards the compression face of the beam. If we assume that the distance moved, δ_{NA}, is sufficiently small that the neutral axis remains within the central 90/90 layer, then it can be shown that δ_{NA} is given by

$$\delta_{NA} = \frac{5(E_{90} - E_{90}^*) t_{90}}{8E_0 + 2E_{90}^* + 6E_{90}} \tag{46}$$

The reduced flexural stiffness can then be found as (Smith and Ogin 2000)

$$\frac{E_{flex}}{E_{flex}^0} = \frac{2(88E_0 + 21E_{90} + 19E_{90}^*) - \left(\frac{75(E_{90} - E_{90}^*)^2}{8E_0 + 2E_{90}^* + 6E_{90}}\right)}{2(88E_0 + 40E_{90})} \tag{47}$$

Which may be approximated at large crack spacing as

$$\frac{E_{flex}}{E_{flex}^0} = 1 - \frac{19E_{90}}{88E_0 + 40E_{90}} \left(1 + \frac{t_{90} E_{90}}{t_0 E_0}\right) \frac{2}{\xi} \left(\frac{1}{2\rho}\right) \tag{48}$$

Table 1 Material properties of GFRP laminate used in calculations (Smith and Ogin 2000)

Properties Material	E_L (GPa)	E_T (GPa)	G_{LT} (GPa)	$G_{TT'}$ (GPa)	υ_{LT}	V_f
GFRP	37	9.5	4	3.345	0.28	0.52

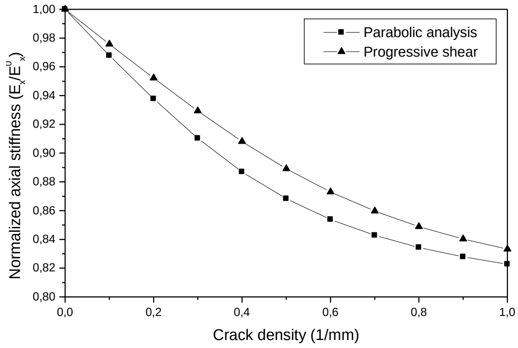

Fig. 2 The normalised axial stiffness of $[0/90]_{2s}$ GFRP laminate loaded in tension as a function of crack density

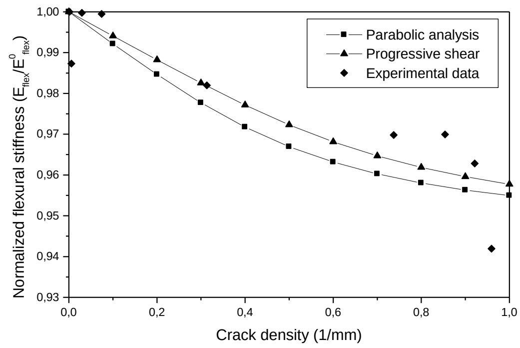

Fig. 3 The normalised flexural stiffness of $[0/90]_{2s}$ GFRP laminate loaded in flexure as a function of crack density

3. Results and discussion

A computer code based on the preceding equations was written to compute the stiffness properties degradation.

3.1 Comparison of predictions with experimental data

In this section, we will validate the results of the present programme without taking into account the hygrothermal effect on the material properties. The results are compared with experimental data for GFRP laminate (Smith and Ogin 2000). The material properties of the chosen composite, as well as their geometrical characteristics, are summarised in Table 1.

The degradation of the axial and flexural stiffness with respect to the transverse crack density, for $[0/90]_{2s}$ GFRP laminate loaded in axial tension and flexure respectively are shown in Figs. 2 and 3. Fig. 3 exhibits the prediction by the modified shear-lag model and the experimental data (Smith and Ogin 2000). It can be seen that good agreement is obtained between the predicted model and the experimental data (Smith and Ogin 2000). The largest measured flexural stiffness reductions were generally in the range 2-4% (Fig. 3). These reductions are less than would be seen in similar lay-ups loaded in tension, where the axial stiffness reduction is 15-20% (Fig. 2). The smaller reduction in flexure is a consequence of only 25% of the available 90° plies cracking under load, as well as the greater role of the 0° plies in determining the flexural (as opposed to tensile) modulus of the laminates.

3.2 Influence of hygrothermal conditions on the reduced stiffness modulus

The study here has been focussed on the axial and flexural stiffness properties reduction due to transverse cracking in $[0/90]_{2s}$ composite laminate when this latter is initially exposed to different environmental conditions. The model which will enable us to introduce ageing and to see its development on the fibre and matrix scales is Chamis model (1983). Since the effect of temperature and moisture is dominant in matrix material, the hygrothermal degradation of the fibre composite material property is estimated by degrading the matrix property only. The matrix mechanical property retention ratio is expressed as

$$F_m = \left(\frac{T_{gw} - T_{opr}}{T_{go} - T_{rm}}\right)^{1/2} \tag{49}$$

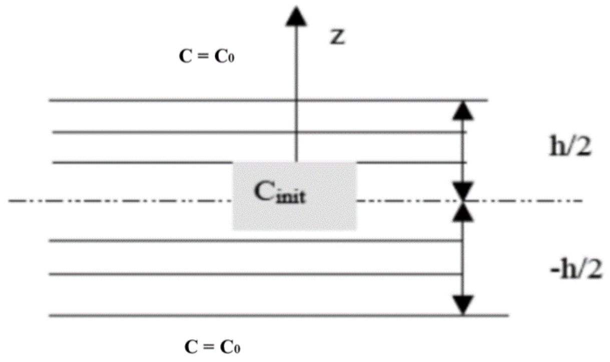

Fig. 4 The one-dimensional problem of moisture diffusion in plates

Where T_{gw} and T_{go} are the glass transition temperature for wet and reference dry conditions, T_{opr} is the operating temperature and T_{rm} is the room temperature. The glass transition temperature for wet material is determined as (Chamis 1983)

$$T_{gw} = \left(0.005C^2 - 0.1C + 1\right) T_{go} \tag{50}$$

Where C is the weight percent of moisture in the matrix material.

Let us consider a laminated plate of thickness h made of polymer matrix composite, submitted on it two sides to the different dry environment. If the laminate edges are moisture insulated and the free surfaces x = 0 and x = h are exposed to a constant moisture environmental change C_0 for t = 0, then a one-dimensional moisture diffusion through the thickness is obtained. Provided that the laminate has a uniform moisture concentration C_I for t=0. The moisture concentration inside the plate is described by Fick equation (Shen and Springer 1981, Tounsi *et al*. 2005, Benkhedda *et al*. 2008) with diffusivity D_z.

$$\frac{\partial C}{\partial t} = D_z \frac{\partial^2 C}{\partial z^2} \tag{51}$$

With

$$C_I = C_{init} \quad \text{at} \quad t = 0 \quad \text{for} \quad 0 < z < h \tag{52}$$

$$C = C_0 \text{ at } t = 0 \text{ for } z = 0 \text{ and } z = h \tag{53}$$

The solution is well known (Crank 1975) and can be written as

$$C(z,\tau) = C_0 + (C_I - C_0)\sum_{n=1}^{\infty} \frac{4}{(2n+1)\pi} e^{-(2n+1)^2\pi^2\tau} \sin\left(\frac{(2n+1)\pi z}{h}\right) \; ; \; \begin{cases} 0 \le z \le h \\ \tau \ge 0 \end{cases} \tag{54}$$

Where, the closure time

$$\tau = \frac{D_z t}{h^2} \tag{55}$$

If the composite laminate also contains matrix cracks that are in connection with the surrounding moisture environment, a faster moisture uptake is expected. This change in moisture uptake, for a cracked cross-ply laminate with matrix cracks in the 90° plies, is here estimated with the model given by Lundgren and Gudmundson (1998). For the case when the moisture environment instantly reaches the crack surfaces in the interior of the laminate the model becomes

$$C(z,\tau) = C_0 + (C_I - C_0)e^{-\mu\tau}\sum_{n=1}^{\infty} \frac{4}{(2n+1)\pi} e^{-(2n+1)^2\pi^2\tau} \sin\left(\frac{(2n+1)\pi z}{h}\right) \tag{56}$$

μ is the moisture transfer coefficient which is related to the crack density, ρ, according to Lundgren and Gudmundson (1998) as

$$\mu = N^2 \cdot a \cdot \rho^b \tag{57}$$

a and b are constants and N is the total number of plies. From Lundgren and Gudmundson (1998) the values of a=5.96 and b=1.43 are obtained.

Very often, the laminated composite plate is symmetrical about the central plane (Fig. 4), and for the formulae of concentrations are the most convenient if we take the surfaces at $z = \pm\frac{h}{2}$. A few modification are considered for Eq. (57), when z is replaced by $z \pm \frac{h}{2}$ and

$$C_I = C_{init} \quad \text{at} \quad t = 0 \quad \text{for} \quad -\frac{h}{2} < z < \frac{h}{2} \tag{58}$$

$$C = C_0 \quad \text{at} \quad t = 0 \quad \text{for} \quad z = \frac{h}{2} \text{ and } z = -\frac{h}{2} \tag{59}$$

The presented model is applied to $[0/90]_{2s}$ glass/epoxy reported by Talerja 1986, which the mechanical properties of fibre and matrix of the lamina are listed in Table 2. The Single ply thickness=0.203 mm, $\sqrt{\tau} = 0.3$ and Fibre volume fraction V_f=0.45.

Table 2 Fibre and matrix characteristics of glass/epoxy material (Talerja 1986)

	E (Gpa)	G_{12} (Gpa)	υ_{12}
Fibre	84	33.6	0.27
Matrix	3.2	1.26	0.27

3.2.1 Analysis of relative axial and flexural stiffness reduction

The first step is to compute the on-axis free expansions E_x, E_y, G_{xy} and υ_{xy}. These expansions are computed at each point z of the thickness. Finally, the normalised axial and flexural stiffness degradation in $[0/90]_{2s}$ composite laminate as of crack density are evaluated compared to the initial axial and flexural stiffness of the same uncracked laminate and for the same environmental case. We note that this initial stiffness of the uncracked laminate is a function of temperature and moisture distribution. Consequently, Eqs. (37) and (47) become

$$\frac{E_{x(i)}}{E_{x0(i)}} = \frac{1}{1 + \frac{E_{90(i)}t_{90}}{E_{0(i)}t_0}\frac{\left(1-\upsilon_{12}\upsilon_{xy}^0\right)}{\left(1-\upsilon_{12}\upsilon_{21}\right)}\frac{1}{2a}R_{(i)}\left(\bar{l}_0\right)\left(1+\upsilon_{xy}^0\frac{\left(s_{12}t_{90}+s_{12}t_0\right)}{\left(s_{22}t_{90}+s_{11}t_0\right)}\right)} \tag{60}$$

$$\frac{E_{flex(i)}}{E^0_{flex(i)}} = \frac{2(88E_{0(i)} + 21E_{90(i)} + 19E^*_{90(i)}) - \left(\frac{75(E_{90(i)} - E^*_{90(i)})^2}{8E_{0(i)} + 2E^*_{90(i)} + 6E_{90(i)}}\right)}{2(88E_{0(i)} + 40E_{90(i)})} \tag{61}$$

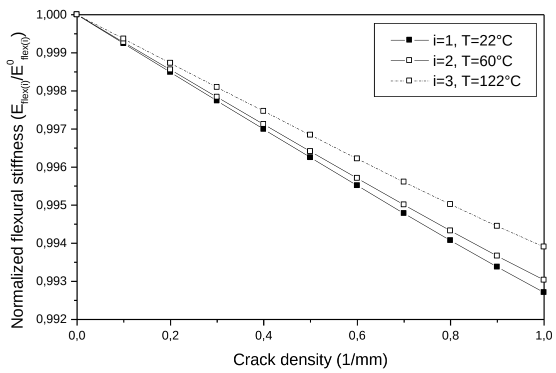

Fig. 5 Relative flexural stiffness reduction as a function of crack density for a $[0/90]_{2s}$ glass/epoxy laminate

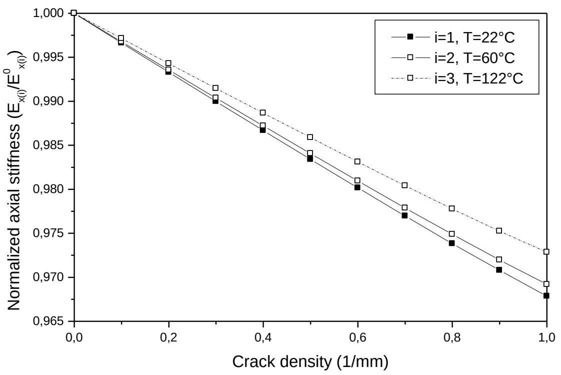

Fig. 6 Relative axial stiffness reduction as a function of crack density for a $[0/90]_{2s}$ glass/epoxy laminate

The index (i) represents the considered case of the environmental conditions

The normalised flexural and axial stiffness degradation are represented in cross-ply $[0/90]_{2s}$ cracked laminate exposed to environmental conditions with a parabolic variation of longitudinal displacement in both 0° and 90° layers (transverse cracks are in 90° layers). Moisture absorption in glass-fibre/epoxy laminate with transverse cracks has been selected to represent the effect of high temperature.

In Figs. 5 and 6, the relative flexural and axial stiffness are plotted as a function of crack density with different values of operating temperatures. It can be observed that the flexural and axial modulus reduces monotonically with an increase of crack density and also with a decrease in temperature. The degradation of axial stiffness represents more reduction with a decrease of temperature from 122°C to 22°C, instead of less than 0.15% of reduction of flexural stiffness in the same condition. This leads that, the composite laminate under flexural load is less affected by the hygrothermal condition than the one under tension loading.

3.2.2 Analysis of total axial and flexural stiffness reduction

In this section, the total reduction of axial and flexural stiffness modulus is determined compared to the axial and flexural modulus of the uncracked laminate when this latter is exposed initially to the environmental condition of case

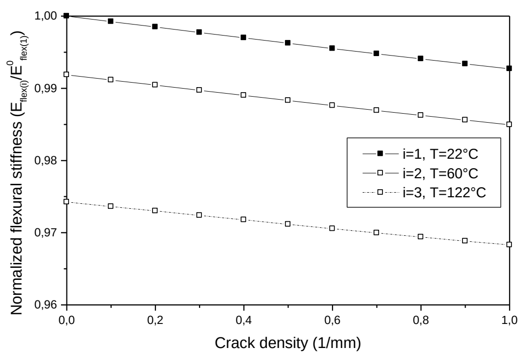

Fig. 7 Total flexural stiffness reduction as a function of crack density for a $[0/90]_{2s}$ glass/epoxy laminate

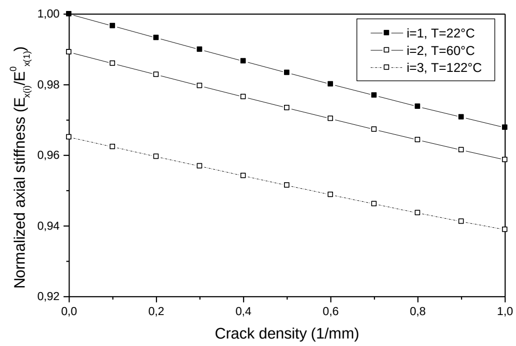

Fig. 8 Total axial stiffness reduction as a function of crack density for a $[0/90]_{2s}$ glass/epoxy laminate

1. Consequently, this total reduction of axial and flexural stiffness takes into account the reduction due to the crack density and to the variation of moisture and temperature. Eqs. (37) and (47) become

$$\frac{E_{x(i)}}{E_{x0(i)}} = \frac{(t_0 E_{0(i)} + t_{90} E_{90(i)})}{\left(1 + \frac{E_{90(i)} t_{90}}{E_{0(i)} t_0} \frac{\left(1 - \upsilon_{12}\upsilon^0_{xy}\right)}{\left(1 - \upsilon_{12}\upsilon_{21}\right)} \frac{1}{2a} R_{(i)}\left(\bar{l}_0\right)\left(1 + \upsilon^0_{xy} \frac{\left(s_{12} t_{90} + s_{12} t_0\right)}{\left(s_{22} t_{90} + s_{11} t_0\right)}\right)\right)(t_0 E_{0(1)} + t_{90} E_{90(1)})} \tag{62}$$

$$\frac{E_{flex(i)}}{E^0_{flex(i)}} = \frac{\left(2(88E_{0(i)} + 21E_{90(i)} + 19E^*_{90(i)}) - \left(\frac{75(E_{90(i)} - E^*_{90(i)})^2}{8E_{0(i)} + 2E^*_{90(i)} + 6E_{90(i)}}\right)\right)\left(88E_{0(i)} + 40E_{90(i)}\right)}{2\left(88E_{0(i)} + 40E_{90(i)}\right)\left(88E_{0(1)} + 40E_{90(1)}\right)} \tag{63}$$

In Figs. 7 and 8, the total flexural and axial stiffness respectively are plotted as a function of crack density with different values of temperatures. When we analyse the hygrothemal effect of cracked laminate under different temperature compared to uncracked laminate with the standard condition (case 1: T=22°C and HR=0%), we note that the big degradation of stiffness is at zero crack density. It can be observed that the total flexural and axial modulus reduces monotonically with an increase of moisture absorption, operating temperature and also increase of crack density. This leads that, the hygrothermal effect has more impact on the axial and flexural stiffness degradation for uncracked composite laminates than one with the transverse crack.

4. Conclusions

The normalised flexural stiffness reduction was predicted using a modified shear-lag model for the cross-ply $[0/90]_{2s}$ composite laminates with transverse cracks and under flexure load. A good agreement is obtained by comparing the prediction models and experimental data. The material properties are considered to be dependent on the temperature and moisture absorption, which are given explicitly in terms of the fibre and matrix properties and fibre volume ratio. The modified shear lag analysis has been extended to study the stiffness properties degradation of high-temperature polymer composite material. On the basis of the present results, the following conclusions can be drawn:

• The axial and flexural stiffness degradation of the $[0/90]_{2s}$ composite laminates largely depend on the increase of crack density.

• The increases of the temperature and moisture absorption have a small impact on the relative axial and flexural modulus reduction.

• The total axial and flexural modulus reduces monotonically with an increase of temperature and moisture absorption.

• The composite laminate with transverse crack and under axial load is more affected by the hygrothermal condition than the one under flexural loading.

• The hygrothermal effect has more impact on the axial and flexural stiffness degradation for uncracked composite laminates than one with the transverse crack.

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