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Topology optimization of reinforced concrete structure using composite truss-like model

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Abstract. Topology optimization of steel and concrete composite based on truss-like material model is studied in this paper. First, the initial design domain is filled with concrete, and the steel is distributed in it. The problem of topology optimization is to minimize the volume of steel material and solved by full stress method. Then the optimized steel and concrete composite truss-like continuum is obtained. Finally, the distribution of steel material is determined based on the optimized truss-like continuum. Several numerical results indicate the numerical instability and rough boundary are settled. And more details of manufacture and construction can be presented based on the truss-like material model. Hence, the truss-like material model of steel and concrete is efficient to establish the distribution of steel material in concrete.

Keywords: strut-and-tie; topology optimization; truss-like continuum; steel; concrete

1. Introduction

Topology optimization (Sigmund and Maute 2013) is effective to find optimal material distribution and establish reasonable initial structure configuration. The problem of topology optimization was solved using analytical method in Michell (1904). The homogenization method was proposed by Bendsoe and Kikuchi (1988). Zhou and Rozvany (1991) introduced solid isotropic material with penalization (SIMP) method. Level set method was studied in Osher and Sethian (1988). Eschenauer (1994) proposed the bubble method. Xie and Steven (1993) presented the evolutionary structural optimization (ESO). In recent years, structural topology optimization has also been emphasized in the theory, algorithm and engineering application (Eschenauer 2001, Rozvany 2001, Bendsoe 2003, Sigmund 2013, Deaton 2014). However, some numerical instabilities were found in previous studies, such as checkerboard phenomenon, mesh-dependency phenomenon and fuzzy boundaries. In addition, most researches just highlighted isotropic material.

In the field of civil engineering, steel and concrete are widely used. At present, the research of steel and concrete topology optimization method is mainly based on the theory of the strut and tie model (Kumar 1978, Rozvany 1996, Liang 2000, 2005, Liu and Qiao 2011, Valério 2013, Matteo Bruggi 2009, 2016). Strut and tie model is to calculate the stress of the structure itself. The tension zone is equivalent to the tie rod, and the compression zone is equivalent to the compression bar. Moreover, the joints between the compression bar and the tie rod are connected by a node, thus the structure is discretized into a system composed of struts, tie bars and joints.

At present, most topology optimization is based on structural optimization of isotropic material or single material. In order to make the topology optimization method more adaptable and reliable in practical engineering, it is necessary to study the structural topology optimization method of steel and concrete composite materials. Steel has superior tensile and compressive properties, while concrete has strong compressive and weak compressive properties. In the past, the truss-like model has been used for topology optimization based on single material (Zhou 2016, Qiao 2016, 2017). In this paper, the truss-like model is expanded and the method of the reinforced concrete composite material model is established. It is extremely useful to optimize the composite materials of steel and concrete, and the results of topology optimization are compared with the results of strut and tie model.

2. Composite material model of plane two phase orthogonal arrangement steel and concrete

2.1 Mechanical model

The truss-like topology optimization method is proposed by Zhou (2005, 2008, 2011), which is based on the single material model in previous study. Now it is expanded to two kinds of materials. It is assumed that there are two phase orthotropic steel materials at any point in the design domain. The direction of the principal axis of the steel material is the direction of the two groups of orthogonal steel. The t_1, t_2 are defined as the densities of the two steel materials. The elastic modulus of two-phase steel is Es. The

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stress and strain of two groups of steel materials are σ_1, σ_2 and ε_1, ε_2, respectively. The relationship of Stress-strain is,

$$\sigma_i = E_s t_i \varepsilon_i \text{ , } i=1,2 \tag{1}$$

The elastic matrix of steel material along the principal axis can be written as

$$\boldsymbol{D}^s(t_1,t_2,0) = E_s \cdot \text{diag}[t_1 \quad t_2 \quad 0] \tag{2}$$

The diag[.] is a diagonal matrix. The angle between the two phase steel material and the coordinate axis X and Y are α and $\alpha+\pi/2$ respectively, then the elastic matrix in the axis direction is,

$$\boldsymbol{D}^s(t_1,t_2,\alpha) = \boldsymbol{T}^T(\alpha)\boldsymbol{D}^s(t_1,t_2,0)\boldsymbol{T}(\alpha) \tag{3}$$

$\boldsymbol{T}(\alpha)$ is a strain coordinate transpose matrix,

$$\boldsymbol{T}(\alpha) = \begin{bmatrix} \cos^2\alpha & \sin^2\alpha & 0.5\sin 2\alpha \\ \sin^2\alpha & \cos^2\alpha & -0.5\sin 2\alpha \\ -\sin 2\alpha & \sin 2\alpha & \cos 2\alpha \end{bmatrix} \tag{4}$$

Take Eq. (4) into Eq. (3),

$$\boldsymbol{D}^s(t_1,t_2,\alpha) = E_s \sum_{b=1}^{2} t_b \sum_{r=1}^{5} s_{br} g_r(\alpha_b) \boldsymbol{A}_r \tag{5}$$

$$\boldsymbol{g}(\alpha) = [\cos 4\alpha \quad \sin 4\alpha \quad \cos 2\alpha \quad \sin 2\alpha \quad 1] \tag{6}$$

Constant matrix,

$$\boldsymbol{A}_1 = \frac{1}{8}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad \boldsymbol{A}_2 = \frac{1}{8}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}, \quad \boldsymbol{A}_3 = \frac{1}{2}\text{diag}[1 \quad -1 \quad 0]$$
$$\boldsymbol{A}_4 = \frac{1}{4}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \quad \boldsymbol{A}_5 = \frac{1}{8}\begin{bmatrix} 3 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{7}$$

$$\boldsymbol{s} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 & 1 \end{bmatrix} \tag{8}$$

Given the concrete filled with steel materials, the elastic matrix of concrete should be considered in the study of reinforced concrete composite optimization layout. Finally, the elastic matrix of the steel composite structure is obtained,

$$\boldsymbol{D} = \boldsymbol{D}^c + \boldsymbol{D}^s \tag{9}$$

$\boldsymbol{D}^c$, $\boldsymbol{D}^s$ are the elastic matrices of concrete and steel.

2.2 Finite element method, stiffness matrix

2.2.1 Elastic matrix at any position within the element

The design variables are the density $\boldsymbol{t_j}$ and the direction angle $\boldsymbol{a_j}$ of the steel material at the node position. By using the method of shape function interpolation, the elastic matrix of any point in the element is,

$$\boldsymbol{D}_e^s(\xi,\eta) = \sum_{j\in S_e} N_j(\xi,\eta)\boldsymbol{D}^s(t_{1j},t_{2j},\alpha_j) \tag{10}$$

The e is the unit number, $N_j(\xi,\eta)$ is the form function, Se is the node set of unit e. The Eq. (5) of elastic matrix of steel material is substituted into the Eq. (10), then,

$$\boldsymbol{D}_e^s(\xi,\eta) = E_s \sum_{j\in S_e} N_j(\xi,\eta) \sum_{b=1}^{2} t_{bj} \sum_{r=1}^{5} s_{br} g_r(\alpha_j) \boldsymbol{A}_r \tag{11}$$

2.2.2 The stiffness matrix of steel and concrete composite material

The steel concrete composite material is composed of two kinds of materials: steel and concrete. When the steel material is smaller in the design domain, and the influence of the superposition of the concrete and steel is ignored, the total stiffness matrix can be calculated by the following formula,

$$\boldsymbol{k}_e = \int_{V_e} \boldsymbol{B}^T(\boldsymbol{D}_e^c + \boldsymbol{D}_e^s)\boldsymbol{B}dV = \int_{V_e} \boldsymbol{B}^T\boldsymbol{D}_e^c\boldsymbol{B}dV + \int_{V_e} \boldsymbol{B}^T\boldsymbol{D}_e^s\boldsymbol{B}dV = \boldsymbol{k}_e^c + \boldsymbol{k}_e^s \tag{12}$$

$\boldsymbol{D}_e^c$ and $\boldsymbol{D}_e^s$ are respectively the elastic matrix of concrete and steel material of unit e, $\boldsymbol{k}_e^c$ and $\boldsymbol{k}_e^s$ are respectively the elastic matrix of concrete and steel material of unit e. The Eq. (11) can calculate the stiffness matrix of steel material, then

$$\boldsymbol{k}_e^s = E_s \sum_{j\in S_e}\sum_{b=1}^{2} t_{bj} \sum_{r=1}^{5} g_r(\alpha_{bj}) \int_{V_e} N_j \boldsymbol{B}^T\boldsymbol{A}_r\boldsymbol{B}dV = \sum_{j\in S_e}\sum_{b=1}^{2} t_{bj} \sum_{r=1}^{5} s_{br} g_r(\alpha_j) \boldsymbol{H}_{ejr} \tag{13}$$

$\boldsymbol{H}_{ejr}$ is a constant matrix, which is independent of the design variables, and it is independent of the unit under the same design domain and rule cell grid. The Eq. is,

$$\boldsymbol{H}_{ejr} = E_s \int_{V_e} N_j \boldsymbol{B}^T\boldsymbol{A}_r\boldsymbol{B}dV \tag{14}$$

As a constant matrix, $\boldsymbol{A_r}$ can be calculated by the Eq. (7). $g_r(a_j)$ can be calculated by Eq. (6).

The whole stiffness of structural steel material can be calculated by the Eq. (13),

$$\boldsymbol{K}^s = \sum_e \boldsymbol{k}_e^s = \sum_e \sum_{j\in S_e}\sum_{b=1}^{2} t_{bj} \sum_{r=1}^{5} s_{br} g_r(\alpha_j) \boldsymbol{H}_{ejr} \tag{15}$$

By exchanging of the cumulative sequence of nodes and units, it can be obtained,

$$\boldsymbol{K}^s = \sum_e \boldsymbol{k}_e^s = \sum_{j=1}^{J}\sum_{e\in S_j}\sum_{b=1}^{2} t_{bj} \sum_{r=1}^{5} s_{br} g_r(\alpha_j) \boldsymbol{H}_{ejr} \tag{16}$$

S_j is a collection of cells connected to the node j.

Finally, the overall stiffness matrix of steel concrete composite structure is

$$\boldsymbol{K} = \boldsymbol{K}^c + \boldsymbol{K}^s \tag{17}$$

2.3 The calculation of principal stress and principal stress angle of steel and concrete composite materials

The structural displacement can be solved by following equation.

$$\boldsymbol{F} = \boldsymbol{K}\boldsymbol{U} \tag{18}$$

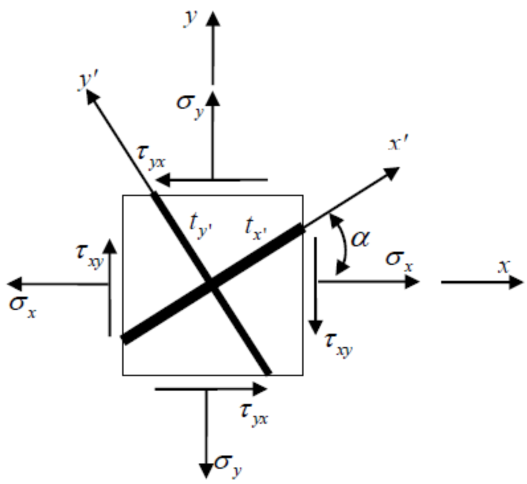


Fig. 2.1 Composite microelement of steel and concrete

Respectively the stress of the steel and concrete is,

$$\boldsymbol{\sigma}_{\mathrm{s}} = \boldsymbol{D}^{\mathrm{s}}\boldsymbol{\varepsilon} \tag{19}$$

$$\boldsymbol{\sigma}_{\mathrm{c}} = \boldsymbol{D}^{\mathrm{c}}\boldsymbol{\varepsilon} \tag{20}$$

In the initial layout, the direction of the principal stress of the steel and concrete material is not consistent. As shown in Fig. 2.1, in order to find the principal stress and the principal stress angle of the composite material, a micro element is analyzed. The axial direction of the steel material is x' and y', and the angle between the x axis and the y axis is α and $\alpha+\pi/2$ respectively, and the corresponding steel density is $t_{x'}$, $t_{y'}$.

Each iteration is to arrange the steel material with the allowable stress of the steel material, in order to obtain the distribution density of steel along the x axis and the y axis, the stress of steel of x' and y' axes is orthogonally decomposed along X and Y axes,

$$\sigma_p^s t_x A' = \sigma_p^s t_{x'} A' \cos\alpha + \sigma_p^s t_{y'} A' \sin\alpha$$

$$\sigma_p^s t_y A' = \sigma_p^s t_{x'} A' \sin\alpha + \sigma_p^s t_{y'} A' \cos\alpha$$

Among them, the allowable stress of steel is σ_p^s, The cross section area of each element in the structure along the principal stress direction is A', further there,

$$t_x = t_{x'} \cos\alpha + t_{y'} \sin\alpha \tag{21}$$

$$t_y = t_{x'} \sin\alpha + t_{y'} \cos\alpha \tag{22}$$

Combined with the concrete stress, it can obtain the average stress of two kinds of materials on the axes. Based on the mean stress, the principal stress and principal stress angle of the composite material are calculated. Among them, σ_{sx}, σ_{sy} and γ_{sxy} are the three components of the steel stress $\boldsymbol{\sigma}_s$, respectively.

Let x, y axial cross-sectional area is respectively A_x and A_y. On the x side, on the same side, the proportion of steel and concrete area is respectively t_x and $1-t_x$. The x axial force of the micro element is,

$$F_x = \sigma_x A_x = \sigma_{cx}(1-t_x)A_x + \sigma_{sx} t_x A_x \tag{23}$$

Divide out A_x, the average axial stress of x axis can be obtained,

$$\sigma_x = \sigma_{cx}(1-t_x) + \sigma_{sx} t_x \tag{24}$$

In the same way,

$$\tau_{xy} = [\tau_{cxy}(1-t_x) + \tau_{sxy} t_x + \tau_{cxy}(1-t_y) + \tau_{sxy} t_y]/2 \tag{25}$$

$$\sigma_y = \sigma_{cy}(1-t_y) + \sigma_{sy} t_y \tag{26}$$

It can be found that the formulas for calculating the average principal stress and the principal stress angle of the planar micro element by material mechanics,

$$\sigma_{\min}^{\max} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}{}^2} \tag{27}$$

$$\alpha_j = \frac{1}{2}\arctan\left(\frac{2\tau_{xy}}{\sigma_x-\sigma_y}\right) \tag{28}$$

2.4 Derivation of steel material distribution standards

The full stress criterion method is used to deduce the material distribution criterion. For stress constrained minimum volume problem, a mathematical model is adopted,

$$\begin{array}{lll} \text{find} & t_{bj} \geq \underline{t}, \alpha_j & b=1,2 \\ \min & V & , \; j=1,2,\ldots,J \\ \text{s.t.} & \left|\sigma_{bj}^l\right| \leq \sigma_{\mathrm{p}} & l=1,2,\ldots,L \end{array} \tag{29}$$

Where, $\underline{t}$ is the lower limit of density, because there is no singular problem of stiffness matrix due to the arrangement of steel material under the concrete, the minimum can be 0. l and L are load case and the total number of working conditions respectively.

For steel and concrete composite materials, the distribution of steel materials with elastic state in the design domain is arranged as follows:

1) The strain in the elastic state of steel and concrete is synchronous, strain corresponding to allowable stress are to meet the ultimate strain of the elastic state.

2) Under compression, the compressive stress is not greater than the allowable compressive stress of concrete, and is assumed by concrete; otherwise, the steel and concrete can bear together.

3) The tension under tensile stress is not greater than the allowable tensile stress of concrete tension, from concrete to bear; otherwise, can be made of steel and concrete to bear.

The allowable stress of concrete is $\sigma_{\mathrm{py}}^{\mathrm{c}}$, and the allowable stress of tension is $\sigma_{\mathrm{pt}}^{\mathrm{c}}$. The allowable stress of steel under tension and compression is $\sigma_{\mathrm{p}}^{\mathrm{s}}$. The density of

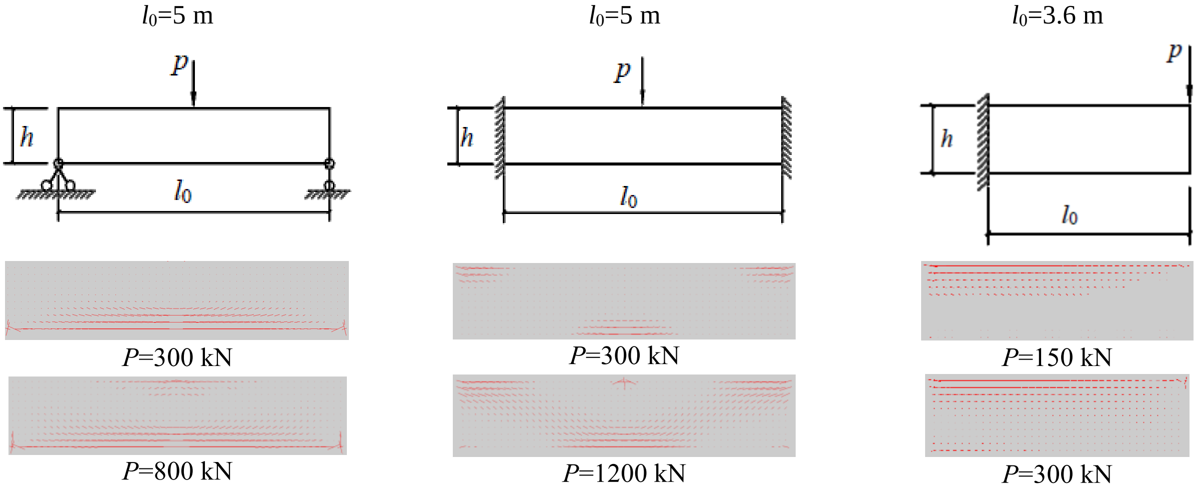
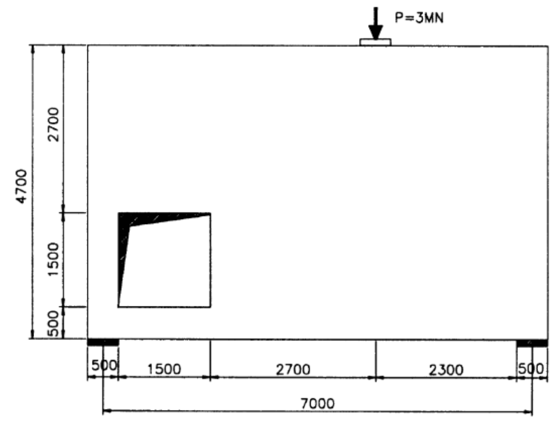
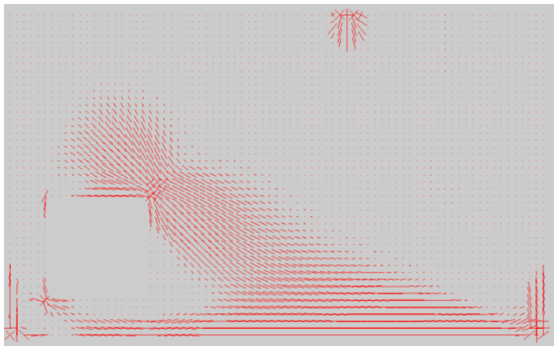


Fig. 3.1 Several common beam

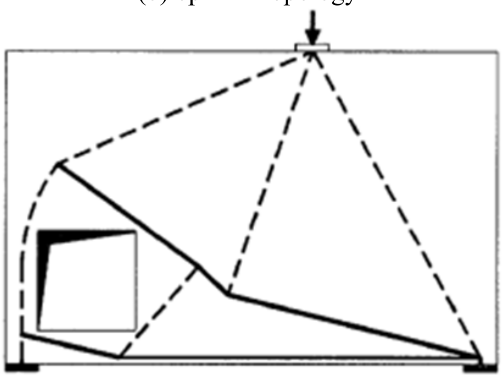


(a) structure facade(mm)



(b) optimal topology

(c) optimal topology



(d) optimal strut-and-tie

Fig. 3.2 Deep beam with large hole

steel is $\boldsymbol{t} = [t_x\ t_y]$. The mean principal stress $\boldsymbol{\sigma} = \begin{bmatrix}\sigma_x & \sigma_y\end{bmatrix}$ can be obtained by formula (11). The orthogonal section area of each element in the structure is $\boldsymbol{A} = [A_x\quad A_y]$ in the direction of the average principal stress, then

$$\boldsymbol{A}|\boldsymbol{\sigma}| \le \boldsymbol{A}(1-\boldsymbol{t})\sigma_{\mathrm{p}}^{\mathrm{c}} + \boldsymbol{A}\boldsymbol{t}\sigma_{\mathrm{p}}^{\mathrm{s}}$$

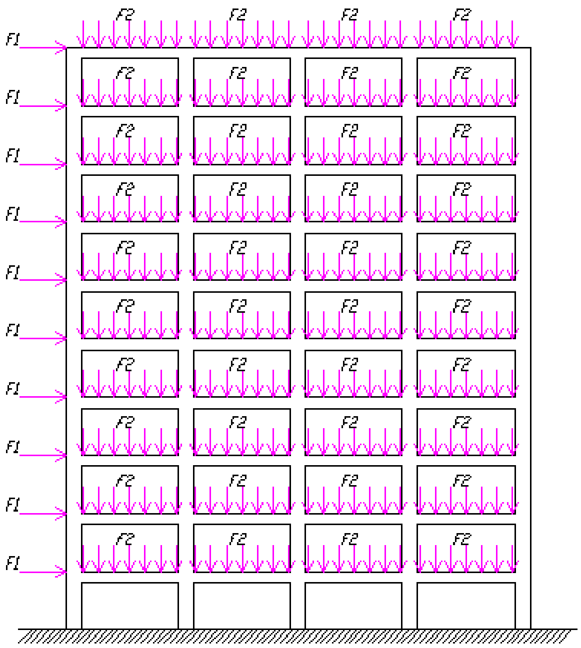
$$\text{Under compression, } \sigma_{\mathrm{p}}^{\mathrm{c}} = \sigma_{py}^{\mathrm{c}}, \tag{30}$$

$$\text{Under tension, } \sigma_{\mathrm{p}}^{\mathrm{c}} = \sigma_{pt}^{\mathrm{c}},$$

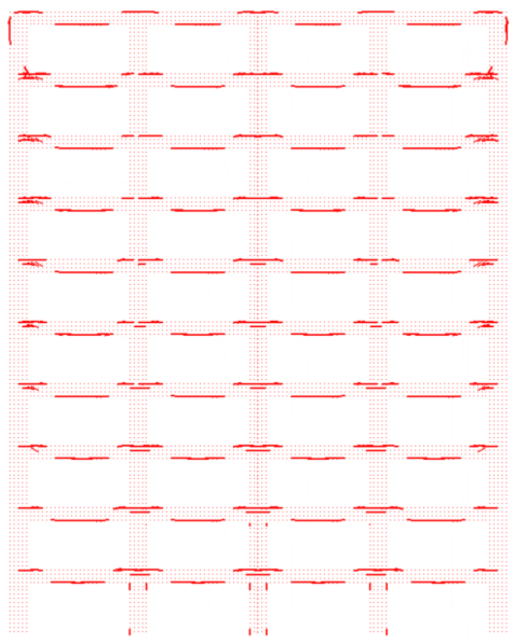
Further, the formula for calculating the density of steel materials can be obtained,

$$\boldsymbol{t} = \begin{bmatrix} t_x & t_y \end{bmatrix} \ge \begin{bmatrix} \dfrac{|\sigma_x| - \sigma_{\mathrm{c}}}{\sigma_{\mathrm{p}}^{\mathrm{s}} - \sigma_{\mathrm{c}}} & \dfrac{|\sigma_y| - \sigma_{\mathrm{c}}}{\sigma_{\mathrm{p}}^{\mathrm{s}} - \sigma_{\mathrm{c}}} \end{bmatrix} \tag{31}$$

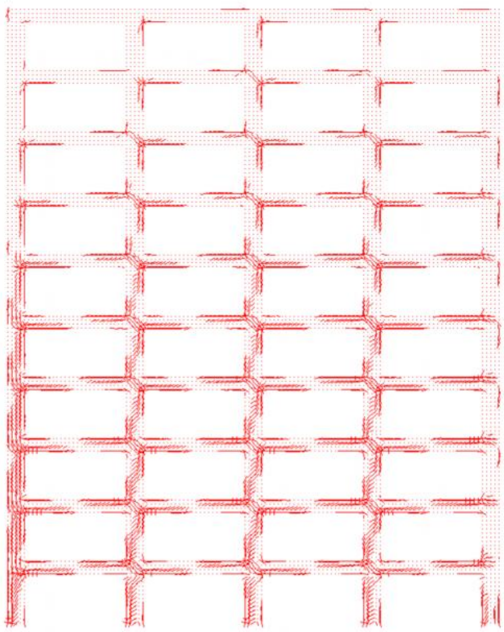
The arrangement direction of the steel material is the same as that of the principal stress.



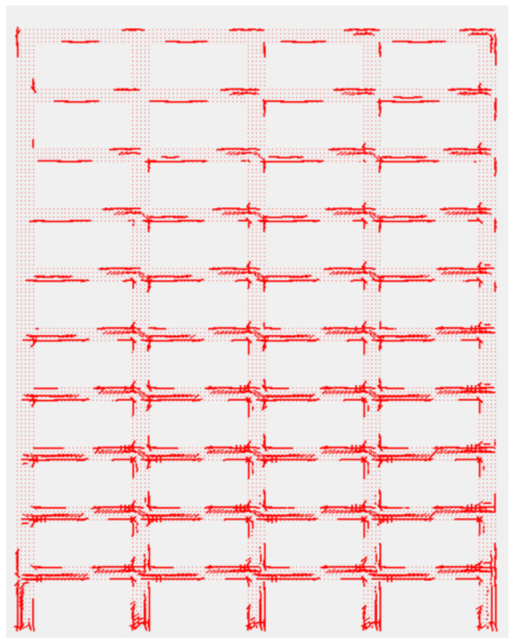
(a) structure facade



(b) $F1$=0 KN, $F2$=75 KN/m, optimal topology



(c) $F1$=300 KN, $F2$=0 KN, optimal topology



(d) $F1$=300 KN, $F2$=75 KN/m, optimal topology

Fig. 3.3 10-storey frame structure

3. Numerical examples

3.1 Several beams

The beam section is $b\times h$ = 250×1000 mm. Concrete material parameters: concrete strength grade is C30, elastic modulus E_c = 7.15 GPa, Poisson's ratio P_r = 0.2, allowable tensile stress σ_{pt}^{c} = 1.43 Mpa, allowable tensile stress σ_{py}^{c} = 14.3MPa. Steel material parameters: the allowable values of tensile and compressive strength are σ_{p}^{s} = 360 MPa, elastic modulus, poisson's ratio P_{sr} = 0.2658.

The red line distribution represents the distribution of the steel material, and the direction is the direction of the steel material layout.

3.2 Deep beam with large hole

The red line distribution represents the distribution of the steel material, and the direction is the direction of the steel material layout.(a), (c) and (d) are form Liang (2000).

3.3 10-storey frame structure

A frame is taken as a design domain. The total width is 24 m, the total height is 30 m, and the height of the layer is 3 m. Column cross section: 400×800 mm, beam cross section: 400×600 mm. Each beam is distributed vertical load 75 KN/m, each layer of horizontal load 300 KN. The rest of the parameters are the same as 3.1. The red line distribution represents the distribution of the steel material, and the direction is the direction of the steel material layout.

4. Discussions

Comparison of figures in Fig. 3.2, some conclusions can be presented as follows. Other examples can be seen in the same situation.

Most results of the strut-and-tie model using topology optimization are characterized by mesh dependence, checkerboard phenomenon, and fuzzy boundary, like Fig. 3.2(c). These problems are solved by the topology optimization method of truss-like model in Fig. 3.2(b).

Under the strut-and-tie model, the steel structure is modified by removing the unnecessary compressive material (concrete), and the steel structure is formed only in the tensile zone, like Fig. 3.2(c). Thus, the overall stiffness of the original structure is changed, and the actual internal force distribution is affected. The concrete is not changed in the original structure, but only the steel material is arranged along the principal stress beyond the allowable stress of concrete in Fig. 3.2(b).

The strut-and-tie model is confined to concrete and affects the bearing capacity, but truss-like material model takes precedence over concrete when it is pressed, the bearing capacity is increased by placing steel material when the concrete exceeds the compressive capacity of concrete.

It is the same arrangement of steel materials during tension. But the strut-and-tie model directly aggregates the tie bars, which is different from the actual steel concrete structure. Truss-like material model method is to disperse the steel material in the concrete and keep the original structure.

Fig. 3.3 presents the results of the current research on the 10-storey frame structure, which clearly shows the layout of the steel bars of the beams and columns. In the same way, the current research can be applied to frame-shear wall, shear wall and other structural systems composed of reinforced concrete materials.

5. Conclusions

In this paper, the truss-like topology optimization model of the single material is extended, and the topology optimization of reinforced concrete structure using composite truss-like material model is established. The results are compared with those of the similar field in the past. Several conclusions can be drawn:

- Most of the optimized results by strut-and tie model have mesh dependence, checkerboard phenomenon, boundary blur and so on. The topology optimization method based on truss-like material model can solve these problems very well.
- The strut-and-tie model of topology optimization just takes into account the difference between the allowable stress and strain of the tension and compression, or only the difference of the elastic modulus is taken into account. The topology optimization of two dissimilar materials combinations is not fundamentally considered in previous study. Two kinds of anisotropic materials of steel and concrete are used in this paper.
- The strut-and-tie model removes the unnecessary compressive concrete material, and the tensile zone only has the steel material, which affects the overall stiffness and actual internal force distribution of the original structure. This paper is to disperse the steel material in the concrete and keep the original structure, and take precedence over concrete when it is pressed. It can also solve the problem of reinforcing bar arrangement under the structure system.

At the same time, for the method described in this paper, there are the following expectations:

- The final result of this paper is the distribution of the steel material, which is the preliminary arrangement. However, the distribution of the steel material is mainly horizontal and vertical arrangement in the actual engineering, which can be further studied.
- The method of superposition of steel and concrete in the derivation is considered. The concrete is full of the design domain, and the concrete stiffness of the space occupied by the steel material should be deducted accurately. When the steel material is relatively small, it has little effect on the results. But there is a big error when the steel material is large (such as Steel Reinforced Concrete), which should be corrected in the future.
- Based on elastic plane analysis, elasto-plastic analysis and space analysis are still to be studied.
- The application of topology optimization to the structure system can be further deepened.

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