Structural Engineering and Mechanics, *Vol. 65, No. 5 (2018) 621-631*
DOI: https://doi.org/10.12989/sem.2018.65.5.621

Post-buckling analysis of shear-deformable composite beams using a novel simple two-unknown beam theory

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(Received October 15, 2017, Revised January 2, 2018, Accepted January 3, 2018)

Abstract. In this paper, an exact analytical solution is developed for the analysis of the post-buckling non-linear response of simply supported deformable symmetric composite beams. For this, a new theory of higher order shear deformation is used for the analysis of composite beams in post-buckling. Unlike any other shear deformation beam theories, the number of functions unknown in the present theory is only two as the Euler-Bernoulli beam theory, while three unknowns are needed in the case of the other beam theories. The theory presents a parabolic distribution of transverse shear stresses, which satisfies the nullity conditions on both sides of the beam without a shear correction factor. The shear effect has a significant contribution to buckling and post-buckling behaviour. The results of this analysis show that classical and first-order theories underestimate the amplitude of the buckling whereas all the theories considered in this study give results very close to the static response of post-buckling. The numerical results obtained with the novel theory are not only much more accurate than those obtained using the Euler-Bernoulli theory but are almost comparable to those obtained using higher order theories, Accuracy and effectiveness of the current theory.

Keywords: novel beam theory; composite beams; post-buckling; analytical modeling

1. Introduction

The use of composite structures has developed greatly in recent years in many fields such as aerospace, naval, automotive and constructions, for their superior performance and reliability (Ozturk, 2015, Draiche *et al.* 2016, Chikh *et al.* 2017). In order to predict accurately their structural responses; various beam theories with different approaches have been developed. A composite beam used as a structural element is often subjected to different types of compression which can cause buckling. Knowledge of critical and post-critical behaviour is then necessary in the dimensioning of these beams. However, composite materials are characterized by their anisotropic behavior. The behavior of composite structures has been identified since 1975 (Housner 1975, Jones 1998). However, the fact that the structures are composed of layers of superposed folds leads to specific modes of rupture. These are mainly delaminated and detachments due to the intrinsic weakness in transverse composite structures. Analysis of the stability of composite plates is much more difficult. Indeed, the critical load depends on other additional parameters such as fiber orientation, stacking sequence, geometric variables and boundary conditions (Hirano 1980, Fukunaga 1986, Miki *et al.* 1986, Yung *et al.* 1989). There are many theories that have been proposed to explain the shear deformation of moderately deep and highly anisotropic composite structures. The study of the behaviour of composite laminated plates is based on the theory of laminates. This theory uses the same hypotheses as the general theory of plates which are, in a scheme of first degrees, associated with the names of Reissner/Mindlin and Kirchoff-Love. Kirchoff-Love's theory is historically one of the first two-dimensional approaches to bending elastic plates. It is based on the preservation of normals, neglecting transverse shear. In the case of Thick plates, the contribution of transverse shears is not negligible and then the preservation hypothesis of normal is no longer applied. The Reissner/Mindlin theory allows transverse shearing to be taken into account. It is based on the kinematic assumption of the plane sections but introducing a shear correction factor (Bellifa *et al.* 2016). The higher order theory is best suited to thick plates; it is based on a nonlinear distribution of fields across the thickness. This model does not require correction factors. In this theory, the displacement fields of the components were

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http://www.techno-press.com/journals/sem&subpage=7 ISSN: 1225-4568 (Print), 1598-6217 (Online)

chosen in different shapes in order to satisfy the zero-shear stress on the upper and lower fibers. The well-known higher-order plate theories with five unknown functions are as follows: the third-order beam theory (TBT) (Levinson 1980, Murthy 1981, Reddy 1984), the sinusoidal beam/plate theory (SBT) (Touratier 1991, Hamidi *et al.* 2015), the hyperbolic beam/plate theory (HBT) (Soldatos 1992, Saidi *et al.* 2016, Zidi *et al.* 2017) and the exponential theory (EBT) (Karama *et al.* 2003). Recently, new shear deformation theories are developed with few unknowns (Bessaim *et al.* 2013, Bouderba *et al.* 2013, Tounsi *et al.* 2013, Ait Amar Meziane *et al.* 2014, Fekrar *et al.* 2014, Akavci 2014, Bousahla *et al.* 2014, Zidi *et al.* 2014, Hebali *et al.* 2014, Belabed *et al.* 2014, Attia *et al.* 2015, Ait Yahia *et al.* 2015, Larbi Chaht *et al.* 2015, Al-Basyouni *et al.* 2015, Taibi *et al.* 2015, Zemri *et al.* 2015, Belkorissat *et al.* 2015, Bourada *et al.* 2015, Mahi *et al.* 2015, Ahouel *et al*. 2016, Kar and Panda 2016, 2017, Bennoun *et al.* 2016, Houari *et al.* 2016, Beldjelili *et al.* 2016, Barati *et al.* 2016, Boukhari *et al.* 2016, Bounouara *et al.* 2016, Besseghier *et al.* 2017, Mouffoki *et al.* 2017, Bellifa *et al.* 2017a, Fahsi *et al.* 2017, Hirwani *et al.* 2017, Benadouda *et al.* 2017, Bouafia *et al.* 2017, Yazid *et al.* 2018, Meksi *et al.* 2018, Youcef *et al.* 2018, Attia *et al.* 2018, Benchohra *et al.* 2018, Abualnour *et al.* 2018). The aforementioned theories are applied at the fold level, which requires many unknowns for multi-layered plates and is often the calculations are tedious to get accurate results. Recently, Bouderba *et al.* (2016) proposed a new simple theory of first-order shear deformation is developed and validated for a variety of numerical examples of the thermal buckling response of an FGM plate with different boundary conditions with only four unknowns, unlike to the first theory of conventional first-order shear deformation and has strong similarities with classical plate theory in aspects such as motion equations, and constraint expressions. The study of the stability of the plates was also of interest to Bakora and Tounsi (2015) investigated the thermo-mechanical post-buckling of an FGP plate on an elastic foundation. Panda and Singh (2009) studied thermal post-buckling behavior of laminated composite cylindrical/hyperboloid shallow shell panel using nonlinear finite element method. Panda and Singh (2010) analyzed thermal post-buckling analysis of a laminated composite spherical shell panel embedded with shape memory alloy fibres using non-linear finite element method. Panda and Singh (2013a) presented nonlinear finite element analysis of thermal post-buckling vibration of laminated composite shell panel embedded with SMA fibre. Panda and Singh (2013b) examined post-buckling behavior of laminated composite doubly curved panel embedded with SMA fibers subjected to thermal environment. Panda and Singh (2013c) studied thermal postbuckling behavior of laminated composite spherical shell panel using NFEM. Daneshmehra *et al.* (2013) presented a post-buckling analysis of functionally graded beams according to different shear deformation theories. Swaminathan and Naveenkumar (2014) presented higher order refined computational models for the stability analysis of functionally graded plates. Akbaş (2015a) investigated the wave propagation of a functionally graded beam in thermal environments. Akbaş (2015b) examined the post-buckling analysis of axially functionally graded three-dimensional beams. Katariya and Panda (2016) studied thermal buckling and vibration response of laminated composite curved shell panel. Barka *et al.* (2016) studied thermal post-buckling behavior of imperfect temperature-dependent sandwich functionally graded plates resting on Pasternak elastic foundation. Abdelaziz *et al.* (2017) presented an efficient hyperbolic shear deformation theory for bending, buckling and free vibration of FG sandwich plates with various boundary conditions. Katariya *et al.* (2017) predicted nonlinear eigenfrequency of laminated curved sandwich structure using higher-order equivalent single-layer theory. Kar *et al.* (2017) examined the influence of different temperature load on thermal post-buckling response of functionally graded shallow curved shell panels. The instability of buckling is related to the geometry of the structure and its loading (Ahmed 2014). Baseri *et al.* (2016) used an analytical solution for buckling of embedded laminated plates based on higher order shear deformation plate theory. Bousahla *et al.* (2016) studied the thermal stability of plates with functionally graded coefficient of thermal expansion. Sekkal *et al.* (2017a, b) presented both 2D and quasi 3D HSDT for buckling and vibration of FG plate. Menasria *et al.* (2017) presented a new and simple HSDT for thermal stability analysis of FG sandwich plates. Aldousari (2017) presented a bending analysis of different material distributions of functionally graded beam. Bellifa *et al.* (2017b) proposed a nonlocal zeroth-order shear deformation theory for nonlinear postbuckling of nanobeams. Khetir *et al.* (2017) investigated the thermal buckling behavior of embedded nanosize FG plates using a new nonlocal trigonometric shear deformation theory. El-Haina *et al.* (2017) presented a simple analytical approach for thermal buckling of thick functionally graded sandwich plates. Rahmani *et al.* (2017) employed various nonlocal higher-order shear deformation beam theories to study the bending and buckling behavior of FG beam. A slender structure loaded in compression in its plane or along its axis, as opposed to transverse loading or bending, enters a state of unstable equilibrium from of a certain level of loading. In other words, beyond this level of loading, an Infinitesimal disturbance modifies the deformation mode of the structure (Timoshenko and Gere 1963). The buckling therefore corresponds to a change of equilibrium branch, Branch fundamental to a secondary branch. After buckling (in the postbuckling phase), the structure deforms in the directions transverse to the axis or to the loading plane. Part of the deformation energy in the plane, or along the axis, is thus transformed into bending deformation energy and of transverse shear. The new state of equilibrium may itself be stable or unstable in the sense that a disturbance of the imposed effort leads to an undetermined increase of the deflection. In post-buckling, the secondary branch et is theoretically unstable (horizontal) (Stolz 2003), but in practice the loading may increase slightly with deflection of geometric non-linearity.

In this paper an efficient and simple refined theory with only two unknown functions is developed to study and examine the importance of shear deformation on the post-buckling response of composite beams. The ends of the beam are assumed to be axially blocked, and consequently the geometrical non-linearity due to the stretching of the mean plane becomes significant. The constitutive equations for composite beams were obtained using the Von-Karman theory for large deflections (Kar *et al.* 2015, Kar and Panda

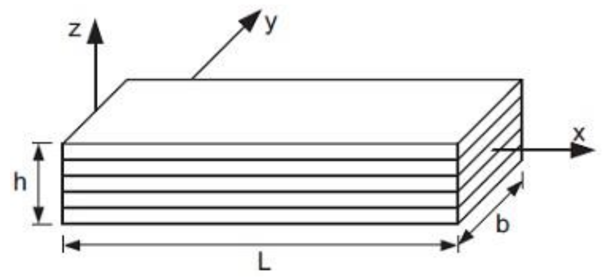

Fig. 1 Geometry of a laminated composite beam

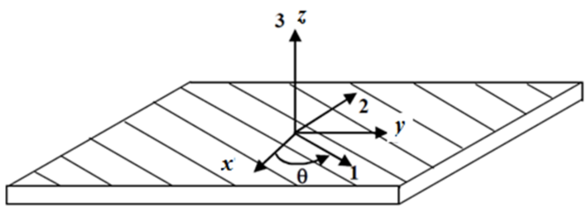

Fig. 2 Representation of global and individual axis acting on a single lamina

Table 1 The shape function $f(z)$ describing the shear deformation according to different beam theories

Beam theory	Function $f(z)$
Reddy (1984)	$f(z)=z\left[1-\dfrac{4z^2}{3h^2}\right]$
Touratier (1991)	$f(z)=\dfrac{h}{\pi}\sin\left(\dfrac{\pi z}{h}\right)$
Soldatos (1992)	$f(z)=h\sinh\left(\dfrac{z}{h}\right)-z\cosh\left(\dfrac{1}{2}\right)$
Karama *et al.* (2003)	$f(z)=ze^{-2(z/h)^2}$

2015). The governing equations composed of two partial nonlinear differential equations in terms of axial and lateral displacements in the mean plane. For the static problem, the model is manipulated to become explicitly independent of the axial displacement, and consequently a bending model is obtained. Assuming a symmetrically supported laminated beam, the displacement field is postulated to satisfy the boundary conditions. The post-buckling static amplitude is obtained by solving the nonlinear equations while the critical buckling load is obtained by solving their linear counterparts. The results showing the variation of the post-buckling amplitude with the applied axial load are presented. Increases in static post-buckling intervention such as shear deformation have been shown to become more important.

2. Theoretical formulation

2.1 Kinematics of the present beam model

A laminated composite beam with rectangular section $(b\times h)$ and length L is subjected to a compressive axial load $\overline{N}$ as shown in Fig. 1.

For the development of the present shear deformation theory, the following displacement field is assumed

$$\begin{aligned} u(x,z)&=u_0(x)-z\frac{\partial w_0(x)}{\partial x}-\beta\, f(z)\frac{\partial^3 w_0(x)}{\partial x^3} \\ w(x,z)&=w_0(x) \end{aligned} \tag{1}$$

where u_0 and w_0 are the two unknown functions of mid-plane displacements of beam, respectively. $f(z)$ represents the shape functions determining the distribution of the transverse shear strains and stresses along the thickness. Table 1 presents the function $f(z)$ according to different beam theories. For the transverse shear stress behaviour, it is very important that the first derivative of the transverse shear stress function must give a parabolic response in the thickness direction of the laminate and satisfy the boundary conditions.

2.2 The strain field

The non-linear von Karman strain–displacement equations are as follows

$$\varepsilon_x=\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \tag{2a}$$

$$\varepsilon_{xz}=\frac{1}{2}\left[\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right] \tag{2b}$$

On the basis of the displacement field given in Eq. (1), Eq. (2) becomes

$$\varepsilon_x=\varepsilon_x^0+zk_x+\beta\, f(z)\eta_x \tag{3a}$$

$$\gamma_{xz}=\beta\, g(z)\gamma_{xz}^0 \tag{3b}$$

where

$$\begin{gathered} \varepsilon_x^0=\frac{\partial u_0}{\partial x}+\frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2, k_x=-\frac{\partial^2 w_0}{\partial x^2}\quad ,\eta_x=-\frac{\partial^4 w_0}{\partial x^4} \\ \gamma_{xz}^0=-\frac{\partial^3 w_0}{\partial x^3} \end{gathered} \tag{4}$$

and

$$g(z)=f'(z) \tag{5}$$

2.3 Constitutive relations

The stresses acting on a lamina (ply of the laminate) are three-dimensional state stresses, in general. However, the thickness of a lamina is very thin. Hence, a generalized state of plane stress is assumed. That means, no interlaminar stresses are included in this analysis. Considering x-y-z as the global coordinate and 1-2-3 as the principal material coordinate (Fig. 2), the stress-strain relationship in 1-2-3 coordinate can be written as equation

(6).

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}^{(k)} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & C_{55} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xy} \end{Bmatrix}^{(k)} \tag{6}$$

where ($\sigma_x, \sigma_y, \tau_{xy}, \tau_{yz}, \tau_{xz}$) are the constraints and ($\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{yz}, \gamma_{xz}$) are the strains relative to the coordinates (1-2-3) of the layer and the matrix are stiffness coefficients of the k-th layer in the axis reference system (1-2-3).
where components of the $[C]$ matrix depend only on the material properties of the lamina.

$$C_{11} = \frac{E_1}{1-\upsilon_{12}\upsilon_{21}};\ C_{22} = \frac{E_2}{1-\upsilon_{12}\upsilon_{21}};$$
$$C_{12} = \frac{\upsilon_{12} E_2}{1-\upsilon_{12}\upsilon_{21}} = \frac{\upsilon_{21} E_1}{1-\upsilon_{12}\upsilon_{21}};\ C_{66} = G_{12};\ C_{44} = G_{23} \tag{7}$$
$$C_{55} = G_{13}$$

Then if the principal axes of the lamina are rotated to coincide with the global x and y axes, the transformation of stiffness matrix $[Q]^{(k)}$ is expressed in equation

$$[Q]^{(k)} = [T_1]^{-1}[C]^{(k)}[T_2] \tag{8a}$$

where

$$[T_1] = \begin{bmatrix} m^2 & n^2 & 2mn & 0 & 0 \\ n^2 & m^2 & -2mn & 0 & 0 \\ -mn & mn & m^2-n^2 & 0 & 0 \\ 0 & 0 & 0 & m & -n \\ 0 & 0 & 0 & n & m \end{bmatrix} \tag{8b}$$

$$[T_2] = \begin{bmatrix} m^2 & n^2 & mn & 0 & 0 \\ n^2 & m^2 & -mn & 0 & 0 \\ -2mn & 2mn & m^2-n^2 & 0 & 0 \\ 0 & 0 & 0 & m & -n \\ 0 & 0 & 0 & n & m \end{bmatrix} \tag{8c}$$

where $m = \cos\theta$ and $n = \sin\theta$

In global coordinates (x, y, z), the stress-strain relationship for the k-th layer, is written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}^{(k)} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} & 0 & 0 \\ Q_{12} & Q_{22} & Q_{26} & 0 & 0 \\ Q_{16} & Q_{26} & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & Q_{45} \\ 0 & 0 & 0 & Q_{45} & Q_{55} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^{(k)} \tag{9}$$

where

$$\begin{aligned}
Q_{11}^{(k)} &= m^4 C_{11} + 2m^2n^2(C_{12} + 2C_{66}) + n^4 C_{22} \\
Q_{12}^{(k)} &= n^2m^2(C_{11} + C_{22} - 4C_{66}) + (n^4 + m^4)C_{12} \\
Q_{16}^{(k)} &= nm\left[m^2(C_{11} - C_{12} - 2C_{66}) + n^2(C_{12} - C_{22} + 2C_{66})\right] \\
Q_{22}^{(k)} &= n^4 C_{11} + 2m^2n^2(C_{12} + 2C_{66}) + m^4 C_{22} \\
Q_{26}^{(k)} &= nm\left[n^2(C_{11} - C_{12} - 2C_{66}) + m^2(C_{12} - C_{22} + 2C_{66})\right] \\
Q_{66}^{(k)} &= n^2m^2(C_{11} - 2C_{12} + C_{22}) + (n^2 - m^2)^2 C_{66} \\
Q_{44}^{(k)} &= m^2 C_{44} + n^2 C_{55} \\
Q_{45}^{(k)} &= mn(C_{55} - C_{44}) \\
Q_{55}^{(k)} &= n^2 C_{44} + m^2 C_{55}
\end{aligned} \tag{10}$$

where $[Q]^{(k)}$ is elastic stiffness matrix in the global coordinates.

The stress-strain relationship of the kth-layer (Eqs. (9, 10)) for beam is written in the simplified form as

$$\sigma_x^{(k)} = Q_{11}^{(k)}\left[\varepsilon_x^0 + zk_x + \beta\, f(z)\eta_x\right] \tag{11a}$$

$$\tau_{xz}^{(k)} = Q_{55}\beta\, g(z)\gamma_{xz}^0 \tag{11b}$$

2.4 Governing equations

The governing equations can be obtained using the principle of virtual work. The principle can be stated in the following form

$$\int_{-h/2}^{h/2}\int_{\Omega}(\sigma_x \delta\,\varepsilon_x + \tau_{xz}\delta\,\gamma_{xz})d\Omega dz - \int_{\Omega}\overline{N}\frac{\partial^2 \delta w}{\partial x^2}d\Omega = 0 \tag{12}$$

where Ω is the top surface.

Substituting Eqs. (3), (4) and (5) into Eq. (12) and integrating through the thickness of the plate, Eq. (12) can be rewritten as

$$\begin{aligned}
&\int_{\Omega}\left[N_x\delta\,\varepsilon_x^0 + M_x\delta\,k_x + \beta(S_x\delta\,\eta_x + Q_{xz}\delta\,\gamma_{xz}^0)\right]d\Omega \\
&-\int_{\Omega}\left(\overline{N}\frac{\partial^2\delta w}{\partial x^2}\right)d\Omega = 0
\end{aligned} \tag{13}$$

In which the stress resultants N_x, M_x, S_x and Q_x are defined by

$$\{N_x; M_x; S_x\} = \sum_{k=1}^{n} b\int_{h_k}^{h_{k+1}}(1, z, \beta\, f(z))\sigma_x^{(k)}dz \tag{14a}$$

$$Q_x = \sum_{k=1}^{n} b\int_{h_k}^{h_{k+1}}\tau_{xz}^{(k)}\,\beta\, g(z)dz \tag{14b}$$

where h_k and h_{k+1} are the top and bottom z-coordinates of the k[th]-layer.

The governing equations can be obtained from Eq. (13) by integrating the displacement gradients by parts and setting the coefficients δu_0 and δw_0 zero separately. Thus, one can obtain the equilibrium equations associated with the present shear deformation theory.

$$\delta u_0: \frac{\partial N_x}{\partial x} = 0 \tag{15a}$$

$$\delta w_0: \frac{\partial}{\partial x}\left(N_x \frac{\partial w}{\partial x}\right) + \frac{\partial^2 M_x}{\partial x^2} + \beta \frac{\partial^4 S_x}{\partial x^4} - \beta \frac{\partial^3 Q_{xz}}{\partial x^3} - \bar{N} \frac{\partial^2 w}{\partial x^2} = 0 \tag{15b}$$

2.5 Governing equations in terms of displacements

By substituting Eq. (4) into Eq. (8) and the subsequent results into Eq. (14), the stress resultants can be written as below

$$\begin{Bmatrix} N_x \\ M_x \\ S_x \end{Bmatrix} = \begin{bmatrix} A_{11} & B_{11} & \beta B_{11}^s \\ B_{11} & D_{11} & \beta D_{11}^s \\ \beta B_{11}^s & \beta D_{11}^s & \beta^2 H_{11}^s \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ k_x \\ \eta_x \end{Bmatrix} \tag{16a}$$

$$Q_{xz} = \beta^2 A_{55}^s \gamma_{xz}^0 \tag{16b}$$

where

$$\{A_{11} \quad B_{11} \quad D_{11} \quad B_{11}^s \quad D_{11}^s \quad H_{11}^s\} = \sum_{k=1}^{n} \int_{h_k}^{h_{k+1}} Q_{11}^{(k)} \left(1, z, z^2, f(z), z\, f(z), f^2(z)\right) b dz \tag{17a}$$

$$A_{55}^s = \sum_{k=1}^{n} \int_{h_k}^{h_{k+1}} Q_{55} [g(z)]^2 b dz, \tag{17b}$$

By substituting Eq. (16) into Eq. (15), the governing equations can be written in terms of generalized displacements (u_0 and w_0) as

$$\delta u_0: A_{11} \frac{\partial}{\partial x}\left(\frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right) - B_{11} \frac{\partial^3 w_0}{\partial x^3} - \beta B_{11}^s \frac{\partial^5 w_0}{\partial x^5} = 0, \tag{18a}$$

$$\delta w_0: A_{11} \frac{\partial}{\partial x}\left(\left(\frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right) \frac{\partial w}{\partial x}\right) + B_{11} \frac{\partial^3 u_0}{\partial x^3} - D_{11} \frac{\partial^4 w_0}{\partial x^4} + \beta \left[B_{11}^s \frac{\partial^5 u_0}{\partial x^5} - 2 D_{11}^s \frac{\partial^6 w_0}{\partial x^6}\right] - \beta^2 \left[H_{11}^s \frac{\partial^8 w_0}{\partial x^8} - A_{55}^s \frac{\partial^6 w_0}{\partial x^6}\right] - \bar{N} \frac{\partial^2 w}{\partial x^2} = 0 \tag{18b}$$

One notes that Eq. (18a) may be solved for the axial displacement u_0, and hence it can be eliminated from the other two equations. This will lead to a flexural model that is given in terms of only the displacements unknowns w_0. It is worth noting that this is applicable regardless of the symmetry property of the structural laminate. Integrating Eq. (18a) with respect to the spatial coordinate x yields

$$\delta u_0: A_{11}\left(\frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right) - B_{11} \frac{\partial^2 w_0}{\partial x^2} - \beta B_{11}^s \frac{\partial^4 w_0}{\partial x^4} = c_1 \tag{19}$$

where c_1 is a constant that represents the induced axial tension force due to midplane stretching as it will be shown. Integrating Eq. (19) once more, we obtain

$$u_0(x) = -\int_0^x \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 d\xi + \frac{B_{11}}{A_{11}} \frac{\partial w_0}{\partial x} + \beta \frac{B_{11}^s}{A_{11}} \frac{\partial^3 w_0}{\partial x^3} + \frac{c_1}{A_{11}} x + c_2 \tag{20}$$

For the midplane stretching to be significant, the beam ends must be restrained (Nayfeh and Mook 1979). The boundary conditions for the axial displacement are assumed as follows

$$u_0(0) = u_0(L) = 0 \tag{21}$$

The constants c_1 and c_2 are now given by

$$c_1 = \frac{A_{11}}{2L} \int_0^L \left(\frac{\partial w_0}{\partial x}\right)^2 dx - \frac{B_{11}}{L}\left[\frac{\partial w_0(L)}{\partial x} - \frac{\partial w_0(0)}{\partial x}\right] - \beta \frac{B_{11}^s}{L}\left[\frac{\partial^3 w_0(L)}{\partial x^3} - \frac{\partial^3 w_0(0)}{\partial x^3}\right] \tag{22a}$$

$$c_2 = -\frac{1}{A_{11}}\left[B_{11} \frac{\partial w_0(0)}{\partial x} + \beta B_{11}^s \frac{\partial^3 w_0(0)}{\partial x^3}\right] \tag{22b}$$

If B_{11} and B_{11}^s vanish, as in the case of a symmetric laminate, or $\partial w_0/\partial x$ and $\partial^3 w_0/\partial x^3$ vanish at the beam ends, as in the case of clamped–clamped beams, one can easily note that the constant c_1 yields the well-known tension force due to midplane stretching. Now, Eq. (19) can be rewritten as follows

$$\frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 = \frac{1}{2L} \int_0^L \left(\frac{\partial w_0}{\partial x}\right)^2 dx + \frac{B_{11}}{A_{11}} \frac{\partial^2 w_0}{\partial x^2} + \beta \frac{B_{11}^s}{A_{11}} \frac{\partial^4 w_0}{\partial x^4} - \frac{B_{11}}{A_{11} L}\left[\frac{\partial w_0(L)}{\partial x} - \frac{\partial w_0(0)}{\partial x}\right] - \beta \frac{B_{11}^s}{A_{11} L}\left[\frac{\partial^3 w_0(L)}{\partial x^3} - \frac{\partial^3 w_0(0)}{\partial x^3}\right] \tag{23}$$

Eq. (18a) and its first derivative can be expressed as follows

$$\frac{\partial}{\partial x}\left(\frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right) = \frac{B_{11}}{A_{11}} \frac{\partial^3 w_0}{\partial x^3} + \beta \frac{B_{11}^s}{A_{11}} \frac{\partial^5 w_0}{\partial x^5} \tag{24}$$

and

$$\frac{\partial^3 u_0}{\partial x^3} = -\frac{\partial^4 w_0}{\partial x^4} + \frac{B_{11}}{A_{11}}\frac{\partial^4 w_0}{\partial x^4} + \beta\frac{B_{11}^s}{A_{11}}\frac{\partial^6 w_0}{\partial x^6} \tag{25}$$

$$\frac{\partial^5 u_0}{\partial x^5} = -\frac{\partial^6 w_0}{\partial x^6} + \frac{B_{11}}{A_{11}}\frac{\partial^6 w_0}{\partial x^6} + \beta\frac{B_{11}^s}{A_{11}}\frac{\partial^8 w_0}{\partial x^8} \tag{26}$$

Substituting Eqs. (23)-(26) into Eq. (18b), we obtain

$$\begin{aligned} &\beta^2\left(\frac{{B_{11}^s}^2}{A_{11}} - H_{11}^s\right)\frac{\partial^8 w_0}{\partial x^8} \\ &+\beta\left[2\left(\frac{B_{11}B_{11}^s}{A_{11}} - D_{11}^s\right) + \beta A_{55}^s\right]\frac{\partial^6 w_0}{\partial x^6} \\ &+\left(\frac{B_{11}^2}{A_{11}} - D_{11}\right)\frac{\partial^4 w_0}{\partial x^4} \\ &-\left(\overline{N} + \chi - \frac{A_{11}}{2L}\int_0^L\left(\frac{\partial w_0}{\partial x}\right)^2 dx\right)\frac{\partial^2 w}{\partial x^2} = 0 \end{aligned} \tag{27}$$

where χ is a constant defined by

$$\begin{aligned} \chi = \frac{1}{L}\Bigg\{&B_{11}\left[\frac{\partial w_0(L)}{\partial x} - \frac{\partial w_0(0)}{\partial x}\right] \\ &+\beta B_{11}^s\left[\frac{\partial^3 w_0(L)}{\partial x^3} - \frac{\partial^3 w_0(0)}{\partial x^3}\right]\Bigg\} \end{aligned} \tag{28}$$

For symmetric laminates, the stiffness's B_{11} and B_{11}^s vanishes. A simply supported beam has the following boundary conditions

$$w_0 = M_x = S_x = 0 \quad at \quad x = 0, L \tag{29}$$

In view of Eq. (16), the stress resultants M_x and S_x are given by

$$M_x = -D_{11}\frac{\partial^2 w_0}{\partial x^2} - \beta D_{11}^s\frac{\partial^4 w_0}{\partial x^4} \tag{30a}$$

$$S_x = -\beta D_{11}^s\frac{\partial^2 w_0}{\partial x^2} - \beta^2 H_{11}^s\frac{\partial^4 w_0}{\partial x^4} \tag{30b}$$

These two equations can be solved for $\partial^2 w_0/\partial x^2$ and $\partial^4 w_0/\partial x^4$ at the boundaries and obtain

$$\left[\left(D_{11}^s\right)^2 - H_{11}^s D_{11}\right]\frac{\partial^2 w_0(\xi)}{\partial x^2} = 0 \tag{31}$$

$$\left[\left(D_{11}^s\right)^2 - H_{11}^s D_{11}\right]\frac{\partial^4 w_0(\xi)}{\partial x^4} = 0, with \quad \xi = 0, L \tag{32}$$

Since D_{11}^s, H_{11}^s and D_{11} do not vanish, the boundary conditions in terms of the displacements can be expressed as follows

$$\frac{\partial^2 w_0}{\partial x^2} = 0 \text{ and } \frac{\partial^4 w_0}{\partial x^4} = 0 \text{ at } \quad x = 0, L \tag{33}$$

The first buckling mode was proofed to be the only stable equilibrium position (Nayfeh *et al*. 2008). For simply supported boundary conditions outlined above, the following displacement field is assumed

$$w(x) = a\sin\pi\frac{x}{L} \tag{34}$$

where a is unknown to be determined. Substituting Eq. (34) into Eq. (27), yields three solutions: the first is the trivial solution, $a = 0$, that corresponds to the equilibrium position in the prebuckling state and the other two solutions, $a \neq 0$, correspond to the stable equilibrium positions in the postbuckling state. As it is well-known, the prebuckling equilibrium position becomes unstable beyond the state of buckling. The postbuckling response can be obtained as follows

$$a = \pm\frac{2}{\pi\sqrt{A_{11}}}\sqrt{L^2\overline{N} - D_{11}\pi^2 + \frac{\left(2\beta D_{11}^s - \beta^2 A_{55}^s\right)\pi^4 L^2 - \beta^2 H_{11}^s\pi^6}{L^4}} \tag{35}$$

We note that the buckling amplitude a, corresponds to the maximum buckling level that occurs at the midspan of the beam where $x = L/2$.

On the other hand, the critical buckling load, $\overline{N}_{cr}$, can be obtained by solving the linear counterpart of Eq. (27). The result is

$$\overline{N}_{cr} = \frac{\pi^2}{L^2}\left(D_{11} + \frac{\beta^2 H_{11}^s\pi^4}{L^4} + \frac{\pi^2}{L^2}\left(\beta^2 A_{55}^s - 2\beta D_{11}^s\right)\right) \tag{36}$$

and

$$\beta = \frac{L^2\left(A_{11}D_{11}^s - B_{11}B_{11}^s\right)}{A_{11}H_{11}^s\pi^2 - \left(B_{11}^s\right)^2\pi^2 + A_{11}A_{11}^s L^2} \tag{37}$$

For convenience, the following non-dimensional critical buckling load, $\overline{P}_{cr}$, is used

$$\overline{P}_{cr} = \frac{L^2}{bh^3 E_2}\overline{N}_{cr} \tag{38}$$

3. Results and discussion

In this section, the present theory with only two functions unknown has been verified by comparing the obtained results and to investigate the responses of composite beams for postbuckling problems with those available in the literature. We consider a (0°/90°/0°) cross-ply symmetrically laminated simply supported beam with the following properties (Aydogdu 2009):

✓ Material I: $E_1/E_2 = 10, G_{12} = G_{13} = 0.6E_2$, $G_{23} = 0.5E_2, \nu_{12} = \nu_{13} = 0.25$

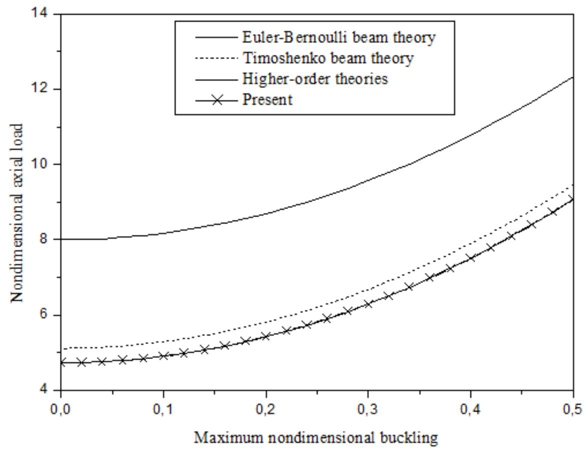

Fig. 3 Variation of the maximum buckling with the applied axial load for L/h = 5 and E_1/E_2 = 10

Table 2 Non-dimensional first critical buckling load using different beam theories

$\frac{E_1}{E_2}$	Beam theory	L/h				
		5	10	20	50	100
10	Euler-Bernoulli	8.001	8.001	8.001	8.001	8.001
	Timoshenko	5.113	7.011	7.728	7.956	7.989
	Reddy	4.727	6.814	7.666	7.945	7.987
	Karama *et al.*	4.734	6.815	7.666	7.945	7.987
	Soldatos	4.727	6.813	7.666	7.945	7.987
	Touratier	4.734	6.815	7.666	7.945	7.987
	Present	4.727	6.813	7.666	7.945	7.987
40	Euler-Bernoulli	31.760	31.760	31.760	31.760	31.760
	Timoshenko	9.797	20.353	27.857	31.064	31.583
	Reddy	8.613	18.832	27.086	30.906	31.542
	Karama *et al.*	8.699	18.862	27.092	30.906	31.542
	Soldatos	8.644	18.834	27.083	30.905	31.542
	Touratier	8.699	18.862	27.092	30.906	31.542
	Present	8.644	18.834	27.083	30.905	31.542

✓ Material II: $E_1/E_2=40, G_{12}=G_{13}=0.6E_2,$ $G_{23}=0.5E_2, \nu_{12}=\nu_{13}=0.25$

In order to validate and demonstrate the accuracy and efficiency of the new shear deformation theory developed in the present study, with only two unknown functions, we determine the nondimensional first critical buckling load for laminated beams with different length-to-height ratios and compare the results with those available in the literature.

Table 2 shows the critical buckling loads given by Eq. (38), for laminates (0°/ 90°/0°) with different length-to-height ratios L/h = 5, 10, 20, 50, 100 using different theories of beams. Current solutions are validated against those derived from higher order theories (TBT, SBT, HBT, EBT) as well as the Euler-Bernoulli theory and that of Timoshenko. As can be seen from the results found in this table, the shear deformation has a significant effect on the critical buckling loads. The results given by the new theory of deformations of higher order with two variables are in excellent agreements with the solutions of the different models are observed. Note also for the new theory of shear deformation with two unknown functions that the effect of deformation by shear is greater for the composites with high orthotropy, that is to say a high ratio. These results are in good agreement with those of Soldatos (Soldatos 1992). We can also observe that the theories of Levinson (1980), Murthy (1981) and Reddy and Soldatos as well as the new theory of shear deformation with two unknown functions give results more precise than the other theories of shear deformation. After having verified the accuracy and precision of this new theory of high-order in the calculation of the critical buckling load, we will show that this new theory is also precise in the contribution of the resulting postbuckling response which is the subject of this study.

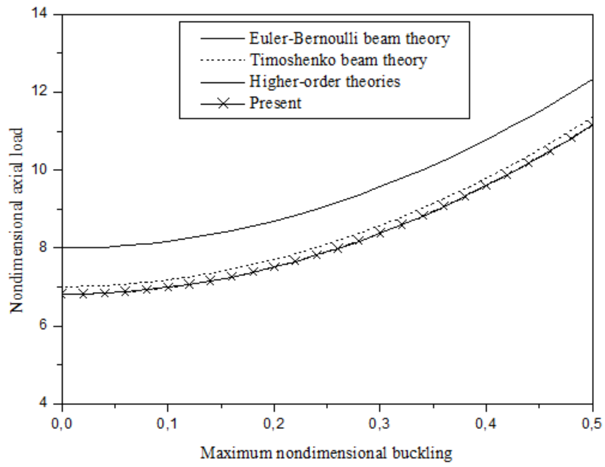

Fig. 4 Variation of the maximum buckling with the applied axial load for L/h = 10 and E_1/E_2 = 10

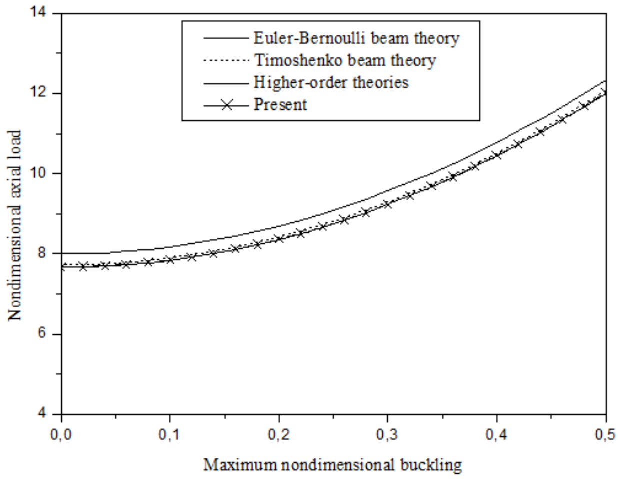

Fig. 5 Variation of the maximum buckling with the applied axial load for L/h = 20 and E_1/E_2 = 10

As the first example, (0°/90°/0°) composite beams with material I and $E_1/E_2=10$ is considered. Their mid-span postbuckling amplitude with the applied axial load with 5 ratios of length to-depth, L/h=5, 10, 20, 50 are given in Figs. 3-6. Which studied the variation of the buckling amplitude $\bar{a}$ with the applied axial load $\overline{P}$.

As can be noted from the figures, the length-to-

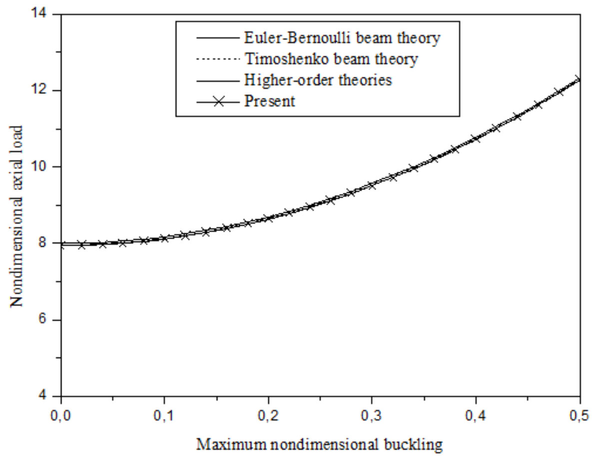

Fig. 6 Variation of the maximum buckling with the applied axial load for L/h = 50 and $E_1/E_2 = 10$

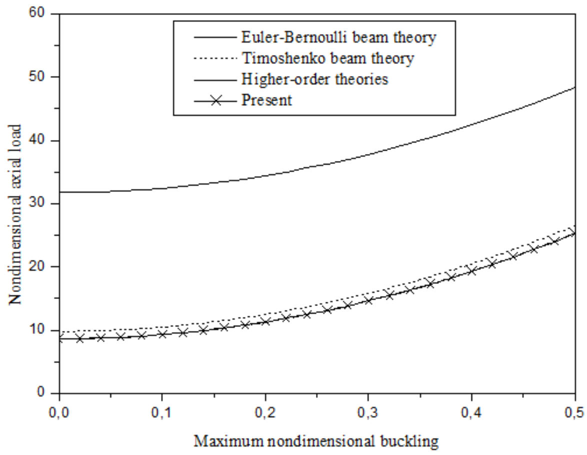

Fig. 7 Variation of the maximum buckling with the applied axial load for L/h = 5 and E1/E2 = 40

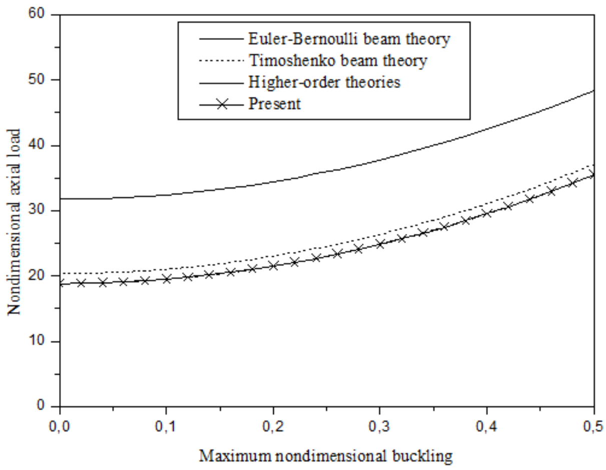

Fig. 8 Variation of the maximum buckling with the applied axial load for L/h = 10 and $E_1/E_2 = 40$

thickness ratio is a crucial parameter in the analysis of post-buckling of composite beams. It can be noted that, every time that the report length on height increases the different theories giving identical results. For a ratio of length to-depth *L*/*h*=5 the choice of the theory is important.

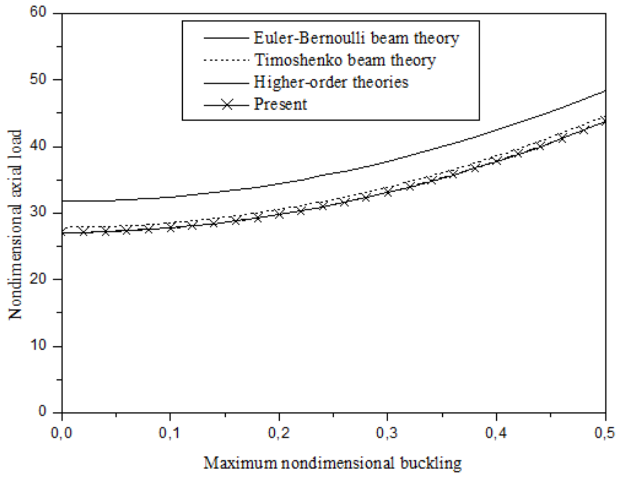

Fig. 9 Variation of the maximum buckling with the applied axial load for L/h = 20 and $E_1/E_2 = 40$

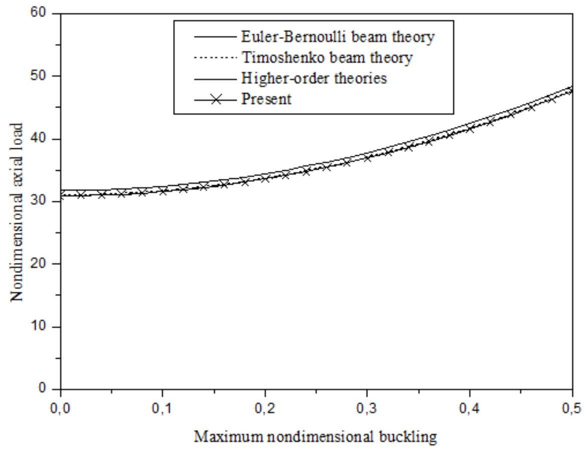

Fig. 10 Variation of the maximum buckling with the applied axial load for L/h = 50 and $E_1/E_2 = 40$

As shown in Table 2 when determining the critical buckling load, the new theory of higher order shear deformation at two unknown functions is in good agreement with the results found with the other five-variable theories, these figures also show that the solutions are also identical in post-buckling without forgetting the effect of the deformation theory. The results found by the new theory of higher order shear deformation to two unknown functions in post-buckling are very close. We also note that the theory of first-order shear deformation always underestimates the amplitude of buckling in relation to higher order theories. For a length / height ratio of 50, the shear deformation effect can be neglected.

It should be noted that the Figs. 7-10 present the variation of the amplitude of buckling with the axial load for material II where $E_1/E_2 = 40$. These figures clearly show that the responses are similar to those found using material I, however, the latter are much more important because the material II at high orthotropy, i.e. a high ratio. The new theory of higher order shear deformation at two unknown functions is in good agreement with the results found with the other five-variable theories, however the post-buckling response found using the Euler-Bernoulli theory is under Estimated in relation to higher order theories. FIG. 8 shows that for a slender girder all theories are combined in this case so that the shear deformation is neglected, the length-thickness ratio must be greater than 50.

4. Conclusions

An efficient and simple theory of refined plates has been successfully developed for post-buckling analysis of composite beams. The composite beams are subjected to an axial compression load and their non-linear geometry is introduced into the deformation-strain equations based on the Von-Karman assumptions. The principle of virtual works as well as current theory and the first and third order shear deformation theories are used to predict the amplitude of the composite beams. It can be concluded that the present theory is not only accurate but also effective in predicting the post-buckling response of the composite beam compared to other shear deformation beam theories such as Timoshenko and higher order theories (TBT, SBT, HBT, EBT).

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