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Thermo-mechanical vibration analysis of nonlocal flexoelectric/piezoelectric beams incorporating surface effects

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Abstract. This paper is concerned with thermo-mechanical vibration behavior of flexoelectric/piezoelectric nanobeams under uniform and linear temperature distributions. Flexoelectric/piezoelectric nanobeams have higher natural frequencies compared to conventional piezoelectric ones, especially at lower thicknesses. Both nonlocal and surface effects are considered in the analysis of flexoelectric/piezoelectric nanobeams for the first time. Hamilton's principle is employed to derive the governing equations and the related boundary conditions which are solved applying a Galerkin-based solution. Comparison study is also performed to verify the present formulation with those of previous data. Numerical results are presented to investigate the influences of the flexoelectricity, nonlocal parameter, surface elasticity, temperature rise, beam thickness and various boundary conditions on the vibration frequencies of thermally affected flexoelectric/piezoelectric nanobeam.

Keywords: vibration; flexoelectric/piezoelectric nanobeam; surface effect; nonlocal elasticity theory

1. Introduction

Recently, nanotechnology and nanostructures have gained an unbelievable role in the modern engineering and the rate of nanostructures' employment in various micro/nano electro-mechanical-systems (MEMS/NEMS) is rising with a high speed (Ebrahimi and Barati 2016a, b, c, d, e, Ebrahimi *et al.* 2016a, Ebrahimi and Dabbagh 2016, Ebrahimi and Hosseini 2016a, b). Therefore, such structures must be analyzed properly in different mechanical aspects and this process cannot be performed using classical continuum theory. The mechanical responses of nanosize structures are completely different from those in the macro scale and this is the main issue of developing size-dependent continuum theories.

The flexoelectricity is related to a particular electromechanical coupling phenomenon between polarization and strain gradients (Jiang *et al.* 2013). In fact, imposing a strain gradient to dielectrics can induce an electrical polarization by breaking the inversion symmetry. It is well known that the flexoelectricity provides an inherent size effect as the dimensions of nanostructured materials decrease. Contrary to flexoelectricity, the piezoelasticity cannot introduce such size effect for a wide range of dielectrics applied in NEMs. Also, having a large ratio of surface area to volume in nanomaterials, surface effects have been believed to involve the size-dependency of material properties. According to the surface elasticity theory developed by Gurtin and Murdoch (1975), the size-dependency of nanoscale structures due to the surface effects have been broadly researched by the modified continuum models from static and dynamic perspectives (Wang and Wang 2011, Hosseini *et al.* 2016).

Recently, a number of researches are performed to incorporate the surface effects in analysis of piezoelectric nanostructure. Yan and Jiang (2011) investigated surface effects on vibration and buckling of piezoelectric nanobeams with surface effects. Also, Yan and Jiang (2012) explored vibrational and stability behaviors of piezoelectric nanoplates considering surface effects and in-plane constraints. A Two-dimensional theory of surface piezoelasticity of plates is presented by Zhang *et al.* (2013). Also, Zhang *et al.* (2014a) researched wave propagation of piezoelectric nanoplates considering surface effects. Also, Zhang *et al.* (2014b) investigated the influence of surface piezoelasticity on the buckling behavior of piezoelectric nanofilms subjected to mechanical loadings. Recently, Li and Pan (2016) presented bending analysis of a sinusoidal piezoelectric nanoplate with surface effects. As a deficiency, the nonlocality of stress field is not considered in these papers.

Recently, modeling of nanostructures by using the nonlocal elastic field theory of Eringen (1972, 1983) has received wide importance. The prominence of nonlocal theory of elasticity has stimulated the researchers to investigate the behavior of the nanostructures much accurately (Ebrahimi and Barati 2016f, g, h, i, j, k, l, Ebrahimi and Barati 2017). This theory contains a nonlocal stress field parameter which introduces a stiffness-softening influence on the nanostructures. To include the nonlocal effects in analysis of piezoelectric nanostructures, Ke and Wang (2012) investigated thermal vibration of piezoelectric nanoscale beams according to the nonlocal theory. Wang and Wang (2012) researched the electromechanical coupling behavior of a piezoelectric nanowire incorporating both surface and nonlocal effects. Also, Liu *et al.* (2013)

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presented vibration analysis of piezoelectric nanoplates exposed to thermo-electro-mechanical loads based on the nonlocal theory. Thermal buckling and free vibration analysis of FG nanobeams subjected to temperature distribution have been exactly investigated by Ebrahimi and Salari (2015a, b, c) and Ebrahimi *et al.* (2015 a, b). Ebrahimi and Barati (2016o, p, q) investigated buckling behavior of smart piezoelectrically actuated higher-order size-dependent graded nanoscale beams and plates in thermal environment. Asemi *et al.* (2014) explored the Influence of initial stress on vibrational behavior of double-piezoelectric-nanoplate systems under different boundary conditions. Zang *et al.* (2014) investigated axial wave propagation of piezoelectric nanoplates considering surface and nonlocal effects. Liu *et al.* (2014) studied buckling and post-buckling behaviors of piezoelectric Timoshenko nanobeams under thermo-electro-mechanical loadings. Ke *et al.* (2015) reported vibration response of a nonlocal piezoelectric nanoplate considering various boundary conditions. Liu *et al.* (2015) presented large amplitude vibration of nonlocal piezoelectric nanoplates under electro-mechanical coupling. Asemi *et al.* (2015) researched the nanoscale mass detection using vibrating piezoelectric ultrathin films subjected to thermo-electro-mechanical loads. Ansari *et al.* (2016) presented thermo-electrical vibrational analysis of post-buckled piezoelectric nanosize beams according to the nonlocal elasticity theory. Ebrahimi and Barati (2016a) investigated dynamic behavior of non-homogenous piezoelectric nanobeams under magnetic field. Wang *et al.* (2016) investigated vibration response of piezoelectric circular nanoplates considering surface and nonlocal effects. Ebrahimi and Barati (2017a) presented buckling analysis of nonlocal third-order shear deformable piezoelectric nanobeams embedded in elastic medium. Liu *et al.* (2016) studied nonlinear vibration of piezoelectric nanoplates using nonlocal Mindlin plate theory.

Incorporation of flexoelectric effect in analysis of piezoelectric nanostructures is carried out by few researchers. Zhang *et al.* (2014) examined the flexoelectric effect on the electroelastic and vibration responses of piezoelectric nanoplates. Liang *et al.* (2014) showed the influences of surface and flexoelectricity on a piezoelectric nanobeam. Zhang and Jiang (2014) investigated bending behavior of piezoelectric nanoplates due to surface effects and flexoelectricity. Yang *et al.* (2015) examined electromechanical behavior of piezoelectric nanoplates with flexoelectricity under simply-supported boundary conditions. Liang *et al.* (2015) presented buckling and vibration behaviors of piezoelectric nanowires due to flexoelectricity. Liang *et al.* (2016) examined buckling and vibration of flexoelectric nanofilms under simply-supported boundary conditions. But, they did not consider the effects of surface piezoelasticity, nonlocality and other kinds of boundary conditions in their model. Most recently, surface and nonlocal effects on vibration behavior of flexoelectric nanobeams is examined by Ebrahimi and Barati (2017b). They reported that flexoelectricity effect is more prominent for slender beams. Thus, it is reliable to model a flexoelectric nanobeam by Euler-Bernoulli beam theory. They also showed that the mechanical behavior of piezoelectric nanobeams is significantly influenced by the presence of nonlocality. Therefore, there is a strong scientific need to investigate thermal vibration behavior of flexoelectric/piezoelectric nanobeams incorporating both surface piezoelasticity and nonlocal effects.

In this paper, thermo-electro-mechanical vibration behavior of size-dependent flexoelectric/ piezoelectric nanobeams is investigated based on nonlocal and surface elasticity theories. Flexoelectricity represents the coupling between strain gradients and electrical polarizations. The flexoelectric/piezoelectric nanobeam is exposed to uniform and linear temperature rises across the thickness. Nonlocal elasticity theory of Eringen is applied in analysis of flexoelectric/piezoelectric nanobeams for the first time. The residual surface stresses which are usually neglected in modeling of flexoelectric/piezoelectric nanobeams are incorporated into nonlocal elasticity to provide better understanding of the physic of problem. Applying a Galerkin-based solution which satisfies various boundary conditions the governing equations obtained from Hamilton's principle are solved. The reliability of present approach is verified by comparing obtained results with those provided in literature. Finally, the influences of nonlocal parameter, surface effect, plate geometrical parameters, thermal environments and boundary conditions on the vibration characteristics of flexoelectric/piezoelectric nanobeams are explored.

2. Nonlocal elasticity theory for the piezoelectric materials with flexoelectric effect

Suppose a nanobeam made of PZT-5H piezoelectric material, as shown in Fig. 1. According to the nonlocal elasticity model (Eringen 1972) which contains wide range interactions between points in a continuum solid, the stress state at a point inside a body is introduced as a function of the strains of all neighbor points. The influence of flexoelectricity due to the elastic polarization Pi induced by strain gradient, and the elastic stress created by electric field gradient, can be expressed by

$$\sigma_{ij}-(e_0\,a)^2\nabla^2\sigma_{ij}=C_{ijkl}\varepsilon_{kl}-e_{kij}E_k+f_{klij}\frac{\partial E_k}{\partial x_l}-C_{ijkl}\alpha_{kl}\Delta T \tag{1a}$$

$$P_i-(e_0\,a)^2\nabla^2P_i=\varepsilon_0\chi_{ij}E_j+e_{ikl}\varepsilon_{kl}+f_{ijkl}\frac{\partial\varepsilon_{kl}}{\partial x_j}-p_i\Delta T \tag{1b}$$

where σ_{ij}, ε_{ij}, E_k denote the stress, strain and electric field components, respectively; C_{ijkl}, e_{kij} and k_{ik} are elastic, piezoelectric and dielectric constant, respectively. Also, χ_{ij} is the relative dielectric susceptibility and f_{ijkl} is the flexoelectric coefficient. α_{kl}, ΔT and p_i are thermal expansion coefficient, temperature change and pyroelectric constant, respectively. Also, e_0a is nonlocal parameter which is introduced to describe the size-dependency of nanostructures. The effect of flexoelectricity is involved using the following expression of the electric enthalpy energy density as follows (Zhang *et al.* 2014)

$$H=-\frac{1}{2}a_{kl}E_kE_l+\frac{1}{2}c_{ijkl}\varepsilon_{ij}\varepsilon_{kl}-e_{kij}E_k\varepsilon_{ij}-\frac{1}{2}f_{klij}(E_k\frac{\partial\varepsilon_{ij}}{\partial x_l}-\varepsilon_{ij}\frac{\partial E_k}{\partial x_l}) \tag{2}$$

Finally, the constitutive relations incorporating nonlocal and flexoelectricity effects can be expressed by

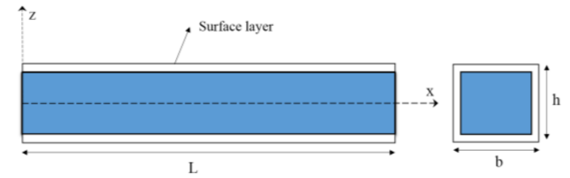


Fig. 1 Geometry and coordinates of flexoelectric/piezoelectric nanobeam

$$(1-(e_0\,a)^2\nabla^2)\sigma_{ij}=\frac{\partial H}{\partial \varepsilon_{ij}}=C_{ijkl}\varepsilon_{kl}-e_{kij}E_k+\frac{f_{klij}}{2}\frac{\partial E_k}{\partial x_l}-C_{ijkl}\alpha_{kl}\Delta T \quad (3a)$$

$$(1-(e_0\,a)^2\nabla^2)\tau_{ijl}=\frac{\partial H}{\partial(\partial \varepsilon_{ij}/\partial x_l)}=-f_{ijkl}E_k \quad (3b)$$

$$(1-(e_0\,a)^2\nabla^2)D_i=-\frac{\partial H}{\partial E_i}=a_{ij}E_j+e_{ikl}\varepsilon_{kl}+\frac{f_{ijkl}}{2}\frac{\partial \varepsilon_{kl}}{\partial x_j}-p_i\Delta T \quad (3c)$$

$$(1-(e_0\,a)^2\nabla^2)Q_{ij}=\frac{\partial H}{\partial(\partial E_i/\partial x_j)}=-\frac{f_{ijkl}}{2}\varepsilon_{kl} \quad (3d)$$

in which τ_{ijl} denotes the moment stress tensor due to the converse flexoelectric effect, D_i is the electric displacement vector and Q_{ij} denotes the electric quadrupole density due to flexoelectricity, respectively. The size-dependent phenomena in piezoelectric nanostructures due to flexoelectricity involved in Eq. (3) is reported in analysis of nanowires, nanobeams and nanoplates. Taking into account the surface effects, i.e., the residual surface stress, the surface elasticity, and the surface piezoelectricity, the surface internal energy density U_s can be defined by the surface strain and the surface polarization as (Zhang *et al.* 2014)

$$U_s=\Gamma_{\alpha\beta}\varepsilon^s_{\alpha\beta}-\frac{1}{2}a^s_{\gamma\kappa}E^s_\gamma E^s_\kappa+\frac{1}{2}c^s_{\alpha\beta\gamma\kappa}\varepsilon^s_{\alpha\beta}\varepsilon^s_{\gamma\kappa}-e^s_{\kappa\alpha\beta}E^s_\kappa\varepsilon^s_{\alpha\beta} \quad (4)$$

in which $\Gamma_{\alpha\beta}$ denotes the surface residual stress tensor, $a^s_{\gamma\kappa}$ and $c^s_{\alpha\beta\gamma\kappa}$ denote the surface permittivity and surface elastic constants. Also, $e^s_{\kappa\alpha\beta}$ and E^s_κ are the surface piezoelectric tensor and surface electric field. Finally, the nonlocal surface constitutive relations can be written as

$$(1-(e_0\,a)^2\nabla^2)\sigma^s_{\alpha\beta}=\frac{\partial U_s}{\partial \varepsilon_{\alpha\beta}}=\Gamma_{\alpha\beta}+c^s_{\alpha\beta\gamma\kappa}\varepsilon^s_{\gamma\kappa}-e^s_{\kappa\alpha\beta}E^s_\kappa \quad (5a)$$

$$(1-(e_0\,a)^2\nabla^2)D^s_\gamma=-\frac{\partial U_s}{\partial E^s_\gamma}=a^s_{\gamma\kappa}E^s_\kappa+e^s_{\gamma\alpha\beta}\varepsilon^s_{\alpha\beta} \quad (5b)$$

where $\sigma^s_{\alpha\beta}$ and D^s_γ are the surface Cauchy stress and surface electric displacement.

3. Theoretical formulation

Here, the classical beam theory is employed for modeling of a piezoelectric nanobeam with surface, nonlocal and flexoelectric effects. The displacement field at any point of the nanobeam can be written as

$$u_1(x,y,z)=u-z\frac{\partial w}{\partial x} \quad (6a)$$

$$u_3(x,y,z)=w \quad (6b)$$

where u is displacement of the mid-surface and w is the bending displacement. Nonzero strains and strain-gradients of the present beam model are expressed as

$$\varepsilon_{xx}=\frac{\partial u_1}{\partial x}=\frac{\partial u}{\partial x}-z\frac{\partial^2 w}{\partial x^2},\quad \eta_{xxz}=\frac{\partial \varepsilon_{xx}}{\partial z}=-\frac{\partial^2 w}{\partial x^2}. \quad (7)$$

Through extended Hamilton's principle, the governing equations can be derived as follows

$$\int_0^t \delta(\Pi_S-\Pi_K+\Pi_W)\,dt=0 \quad (8)$$

where $\prod_s$ and $\prod_W$ are strain energy and external forces work, respectively and $\prod_K$ is kinetic energy. The strain energy can be written as

$$\delta\Pi_S=\int_V(\sigma_{xx}\delta\varepsilon_{xx}+\tau_{xxz}\delta\eta_{xxz})\,dV+\int_S(\sigma^s_{xx}\delta\varepsilon_{xx})\,dS \quad (9a)$$

Substituting Eq. (7) into Eq. (9a) yields

$$\delta\Pi_S=\int_0^L[(N_{xx}+N^s_{xx})\frac{\partial\delta u}{\partial x}-(M_{xx}+M^s_{xx})\frac{\partial^2\delta w}{\partial x^2}+P_{xxz}\frac{\partial^2\delta w}{\partial x^2}]dx \quad (9b)$$

in which the variables introduced in arriving at the last expression are defined as follows

$$\begin{aligned}(N_{xx},M_{xx})&=\int_A(1,z)\sigma_{xx}\,dA,\\ P_{xxz}&=\int_A\tau_{xxz}\,dA.\end{aligned} \quad (9c)$$

The work done by applied forces can be written in the form

$$\delta\Pi_W=\int_0^L(-N^T\frac{\partial w}{\partial x}\frac{\partial\delta w}{\partial x})dx \quad (10)$$

where N^T is thermal load. The first variational of the virtual kinetic energy of present beam model can be written in the form as

$$\delta\Pi_K=\int_0^L I_0(\frac{\partial u}{\partial t}\frac{\partial\delta u}{\partial t}+\frac{\partial w}{\partial t}\frac{\partial\delta w}{\partial t})-I_1(\frac{\partial u}{\partial t}\frac{\partial\delta w}{\partial x\partial t}+\frac{\partial w}{\partial x\partial t}\frac{\partial\delta u}{\partial t})+I_2(\frac{\partial w}{\partial x\partial t}\frac{\partial\delta w}{\partial x\partial t}+\frac{\partial w}{\partial y\partial t}\frac{\partial\delta w}{\partial y\partial t})]dx \quad (11a)$$

in which the mass inertias are defined as

$$(I_0,I_1,I_2)=b\int_{-h/2}^{h/2}(1,z,z^2)\rho dz=b\{\rho h,0,\frac{\rho h^3}{12}\} \quad (11b)$$

The following equations are obtained by inserting Eqs. (9b), (10) and (11) in Eq. (8) when the coefficients of δu, δw are equal to zero

$$\frac{\partial(N_{xx}+N^s_{xx})}{\partial x}=I_0\frac{\partial^2 u}{\partial t^2} \quad (12a)$$

$$\frac{\partial^2(M_{xx}+M_{xx}^s)}{\partial x^2}+\frac{\partial^2 P_{xxz}}{\partial x^2}+(2b\sigma_0-bN^T)\nabla^2 w=I_0\frac{\partial^2 w}{\partial t^2}-I_2\nabla^2(\frac{\partial^2 w}{\partial t^2}) \quad (12b)$$

and the associated boundary conditions

$$u=0,\ or\ \ (N_{xx}+N_{xx}^s)n_x=0 \quad (13a)$$

$$w=0,\ or\ \ n_x(\frac{\partial(M_{xx}+M_{xx}^s)}{\partial x}+\frac{\partial P_{xxz}}{\partial x}-N^T\frac{\partial w}{\partial x})=0 \quad (13b)$$

$$\frac{\partial w}{\partial x}=0,\ or\ \ (M_{xx}+M_{xx}^s)n_x=0 \quad (13c)$$

For a piezoelectric nanobeam with the flexoelectric effect, the nonlocal constitutive relations for the bulk may be written as

$$\sigma_{xx}-(e_0a)^2\nabla^2\sigma_{xx}=c_{11}\varepsilon_{xx}+e_{31}\frac{\partial\varphi}{\partial z}-\frac{f_{31}}{2}\frac{\partial^2\varphi}{\partial z^2}-c_{11}\alpha_1\Delta T \quad (14)$$

$$\tau_{xxz}-(e_0a)^2\nabla^2\tau_{xxz}=+\frac{f_{31}}{2}\frac{\partial\varphi}{\partial z} \quad (15)$$

$$D_z-(e_0a)^2\nabla^2 D_z=e_{31}\varepsilon_{xx}-k_{33}\frac{\partial\varphi}{\partial z}+\frac{f_{31}}{2}\eta_{xxz}+p_3\Delta T \quad (16)$$

$$Q_{zz}-(e_0a)^2\nabla^2 Q_{zz}=-\frac{f_{31}}{2}\varepsilon_{xx} \quad (17)$$

where φ is the electrostatic potential and $E_z=-\frac{\partial\varphi}{\partial z}$. Also, the nonlocal constitutive relations for the surface layer can be expressed by

$$\sigma_{xx}^s-(e_0a)^2\nabla^2\sigma_{xx}^s=\sigma_{xx}^0+c_{11}^s\varepsilon_{xx}+e_{31}^s\frac{\partial\varphi}{\partial z} \quad (18)$$

Under the open circuit condition, the electric displacement on the surface is zero. Therefore, one can obtain the electric field an electric field gradient as

$$E_z=-(\frac{e_{31}}{k_{33}}\frac{\partial u}{\partial x})+(z\frac{e_{31}}{k_{33}}+\frac{f_{31}}{k_{33}})(\frac{\partial^2 w}{\partial x^2}) \quad (19)$$

Finally, the electric field gradient can be written as

$$E_{z,z}=\frac{e_{31}}{k_{33}}\frac{\partial^2 w}{\partial x^2} \quad (20)$$

Using Eqs. (14) and (15), the nonlocal constitutive relations for the bulk and surface can be expressed by the following form

$$\sigma_{xx}-(e_0a)^2\nabla^2\sigma_{xx}=(c_{11}+\frac{e_{31}^2}{k_{33}})\frac{\partial u}{\partial x}-(c_{11}+\frac{e_{31}^2}{k_{33}})z\frac{\partial^2 w}{\partial x^2}-(\frac{e_{31}f_{31}}{2k_{33}})\frac{\partial^2 w}{\partial x^2}-c_{11}\alpha_1\Delta T \quad (21)$$

$$\tau_{xxz}-(e_0a)^2\nabla^2\tau_{xxz}=(\frac{e_{31}f_{31}}{2k_{33}})\frac{\partial u}{\partial x}-(\frac{e_{31}f_{31}}{2k_{33}})z\frac{\partial^2 w}{\partial x^2}-(\frac{f_{31}^2}{2k_{33}})\frac{\partial^2 w}{\partial x^2} \quad (22)$$

$$\sigma_{xx}^s-(e_0a)^2\nabla^2\sigma_{xx}^s=\sigma_{xx}^0+(c_{11}^s+\frac{e_{31}^s e_{31}}{k_{33}})\frac{\partial u}{\partial x}-(c_{11}^s+\frac{e_{31}^s e_{31}}{k_{33}})z\frac{\partial^2 w}{\partial x^2}-(\frac{e_{31}^s f_{31}}{k_{33}}\frac{\partial^2 w}{\partial x^2}) \quad (23)$$

Therefore, by integrating Eqs. (21)-(23) over the beam's cross-section area, the force and moment stress resultants can be rewritten in the following form

$$N_{xx}-(e_0a)^2\nabla^2 N_{xx}=A_{11}\frac{\partial u}{\partial x}-B_{11}\frac{\partial^2 w}{\partial x^2}-N_{xx}^T \quad (24)$$

$$M_{xx}-(e_0a)^2\nabla^2 M_{xx}=-C_{11}\frac{\partial^2 w}{\partial x^2} \quad (25)$$

$$P_{xxz}-(e_0a)^2\nabla^2 P_{xxz}=B_{11}\frac{\partial u}{\partial x}-D_{11}\frac{\partial^2 w}{\partial x^2} \quad (26)$$

and the cross-sectional rigidities are defined as

$$\begin{aligned}&A_{11}=(c_{11}+\frac{e_{31}^2}{k_{33}})bh,\quad B_{11}=(\frac{e_{31}f_{31}}{2k_{33}})bh,\\&C_{11}=(c_{11}+\frac{e_{31}^2}{k_{33}})b\frac{h^3}{12},\quad D_{11}=(\frac{f_{31}^2}{2k_{33}})bh,\end{aligned} \quad (27)$$

And the force and moment stress resultants due to surface piezoelasticity may be expressed as

$$N_{xx}^s-(e_0a)^2\nabla^2 N_{xx}^s=A_{11}^s\frac{\partial u}{\partial x}-B_{11}^s\frac{\partial^2 w}{\partial x^2} \quad (28)$$

$$M_{xx}^s-(e_0a)^2\nabla^2 M_{xx}^s=F_{11}^s\frac{\partial u}{\partial x}-C_{11}^s\frac{\partial^2 w}{\partial x^2} \quad (29)$$

in which

$$\begin{aligned}&A_{11}^s=2(c_{11}^s+\frac{e_{31}e_{31}^s}{k_{33}})h,\quad B_{11}^s=(c_{11}^s+\frac{e_{31}e_{31}^s}{k_{33}})\frac{bh^2}{2}+2(\frac{e_{31}^s f_{31}}{k_{33}})h,\\&F_{11}^s=(c_{11}^s+\frac{e_{31}e_{31}^s}{k_{33}})\frac{bh^2}{2},\quad C_{11}^s=(c_{11}^s+\frac{e_{31}e_{31}^s}{k_{33}})\frac{h^3}{6}+(\frac{e_{31}^s f_{31}}{k_{33}})\frac{bh^2}{2},\end{aligned} \quad (30)$$

The nonlocal governing equations of a piezoelectric nanobeam with surface and flexoelectric effects in terms of the displacement can be derived by substituting Eqs. (24)-(29), into Eq. (12) as follows

$$(A_{11}+A_{11}^s)\frac{\partial^2 u}{\partial x^2}-(B_{11}+B_{11}^s)\frac{\partial^3 w}{\partial x^3}-I_0\frac{\partial^2 u}{\partial t^2}+(e_0a)^2\nabla^2(+I_0\frac{\partial^2 u}{\partial t^2})=0 \quad (31)$$

$$\begin{aligned}&(B_{11}+F_{11}^s)\frac{\partial^3 u}{\partial x^3}-(C_{11}+C_{11}^s+D_{11})\frac{\partial^4 w}{\partial x^4}+2b\sigma_0(\frac{\partial^2 w}{\partial x^2})\\&-(e_0a)^2 2b\sigma_0(\frac{\partial^2}{\partial x^2})(\frac{\partial^2 w}{\partial x^2})-bN^T(\frac{\partial^2 w}{\partial x^2})+(e_0a)^2 bN^T(\frac{\partial^2}{\partial x^2})(\frac{\partial^2 w}{\partial x^2})\\&-I_0\frac{\partial^2 w}{\partial t^2}+I_2\nabla^2(\frac{\partial^2 w}{\partial t^2})+(e_0a)^2\nabla^2(+I_0\frac{\partial^2 w}{\partial t^2}-I_2\nabla^2(\frac{\partial^2 w}{\partial t^2}))=0\end{aligned} \quad (32)$$

4. Solution procedure

Here, an analytical solution of the governing equations for vibration of a flexoelectric nanobeam having simply-supported (S) and clamped (C) boundary conditions is presented which they are given as:

- Simply-supported (S)

$$w = N_{xx} = M_{xx} = 0 \quad \text{at x=0,L} \tag{33}$$

- Clamped (C)

$$w = \frac{\partial w}{\partial x} = 0 \quad \text{at x=0,L} \tag{34}$$

To satisfy above-mentioned boundary conditions, the displacement quantities are presented in the following form

$$u = \sum_{n=1}^{\infty} U_m \frac{\partial X_m(x)}{\partial x} e^{i\omega_n t} \tag{35}$$

$$w = \sum_{m=1}^{\infty} W_m X_m(x) e^{i\omega_n t} \tag{36}$$

where (U_m, W_m) are the unknown coefficients. Inserting Eqs. (35) and (36) into Eqs. (31) and (32) respectively, leads to

$$\left\{ \begin{pmatrix} k_{1,1} & k_{1,2} \\ k_{2,1} & k_{2,2} \end{pmatrix} - \bar{\omega}_n^{\,2} \begin{pmatrix} m_{1,1} & m_{1,2} \\ m_{2,1} & m_{2,2} \end{pmatrix} \right\} = 0 \tag{37}$$

where

$k_{1,1} = (A_{11} + A_{11}^s)\kappa_{12}, \quad k_{1,2} = (B_{11} + F_{11}^s)\kappa_{13},$

$k_{2,1} = -(B_{11} + B_{11}^s)\kappa_{12},$

$k_{2,2} = -(C_{11} + C_{11}^s + D_{11})\kappa_{13} - b(N^T - 2\sigma_0)(\kappa_{11}) + (e_0 a)^2 b(N^T - 2\sigma_0)(\kappa_{13}),$

$m_{1,1} = +I_0\kappa_6 - (e_0 a)^2 I_0 \kappa_{12}\,, \quad m_{1,2} = 0, \quad m_{2,1} = 0,$

$m_{2,2} = +I_0\kappa_1 - I_2(\kappa_{11}) - (e_0 a)^2 I_0(\kappa_{11}) + (e_0 a)^2 I_2(\kappa_{13}),$

in which

$$\begin{aligned} \kappa_1 &= \int_0^L (X_m X_m)dx \\ \kappa_6 &= \int_0^L (X'_m X'_m)dx \\ \kappa_{11} &= \int_0^L (X''_m X_m)dx \\ \kappa_{12} &= \int_0^L (X'''_m X'_m)dx \\ \kappa_{13} &= \int_0^L (X''''_m X_m)dx \end{aligned} \tag{38}$$

By finding determinant of the coefficients of above matrix and setting it to zero, we can find natural frequencies. The function X_m for different boundary conditions is defined by

S-S:

$$X_m(x) = \sin(\lambda_m x), \qquad \lambda_m = \frac{m\pi}{L} \tag{39}$$

C-C:

$$X_m(x) = \sin(\lambda_m x) - \sinh(\lambda_m x) - \xi_m(\cos(\lambda_m x) - \cosh(\lambda_m x))$$

$$\xi_m = \frac{\sin(\lambda_m x) - \sinh(\lambda_m x)}{\cos(\lambda_m x) - \cosh(\lambda_m x)} \tag{40}$$

$$\lambda_1 = 4.730, \lambda_2 = 7.853, \lambda_3 = 10.996, \lambda_4 = 14.137, \quad \lambda_{m\geq 5} = \frac{(m+0.5)\pi}{L}$$

C-S:

$$X_m(x) = \sin(\lambda_m x) - \sinh(\lambda_m x) - \xi_m(\cos(\lambda_m x) - \cosh(\lambda_m x))$$

$$\xi_m = \frac{\sin(\lambda_m x) + \sinh(\lambda_m x)}{\cos(\lambda_m x) + \cosh(\lambda_m x)} \tag{41}$$

$$\lambda_1 = 3.927, \lambda_2 = 7.069, \lambda_3 = 10.210, \lambda_4 = 13.352, \quad \lambda_{m\geq 5} = \frac{(m+0.25)\pi}{L}$$

5. Types of thermal loadings

5.1 Uniform temperature rise

Assume the case that the temperature of the nanoplate uniformly raised through-the-thickness as

$$T(z) = \Delta T. \tag{42}$$

Therefore, the pre-buckling force N^T is

$$N^T = c_{11}\alpha_1 h \Delta T \tag{43}$$

5.2 Linear temperature rise

Now let us consider the temperature rise varies linearly across the nanoplate thickness as

$$T(z) = T_0 + \Delta T\left(\frac{z}{h} + \frac{1}{2}\right) \tag{44}$$

The pre-buckling force N^T is

$$N^T = \frac{1}{2} c_{11} \alpha_1 h \Delta T \tag{45}$$

6. Numerical results and discussions

In this section, results are provided to investigate the thermal vibration behavior of flexoelectric/piezoelectric nanobeams employing nonlocal elasticity theory incorporating surface effect. Various types of boundary conditions are considered in this analysis (C-C, C-S and S-S). In the present paper it is assumed that the flexoelectric/piezoelectric nanobeam is made of PZT-5H where the elastic properties are considered as c_{11}=102 Gpa, c_{12}=31 Gpa, c_{66}=35.5 Gpa and the piezoelectric and dielectric coefficients are assumed as e_{31}=17.05 C/m^2 and k_{33}=1.76×10^{-8} C/(Vm). The flexoelectric coefficient is also considered as f_{31}=10^{-7} (Yang *et al.* 2015). The surface elastic and piezoelectric constants for PZT-5H can be considered as: c_{11}^s=102 N/m, c_{12}^s=3.3 N/m, c_{66}^s=2.13 N/m and e_{31}^s =-3×10^{-8} C/m.

Comparison is performed with those of a piezoelectric nanobeam presented by Yan and Jiang (2011). To this end, effect of nonlocality, flexoelectricity and thermal loading are omitted. In Fig. 2 the frequency ratio (ω/ω^0) is presented as a function of nanobeam thickness. Also, ω^0 is the natural frequency of piezoelectric nanobeam without surface effect.

The results are in an excellent agreement with those of Yan and Jiang (2011) for a simply-supported nanobeam. Also, for better presentation of the results the following dimensionless quantity is adopted

$$\bar{\varpi} = \omega \frac{L^2}{h} \sqrt{\frac{\rho}{c_{11}}}, \quad \mu = \frac{(e_0 a)}{L} \tag{46}$$

Fig. 3 determines the surface and flexoelectricity effects on the variation of natural frequencies of piezoelectric nanobeams with respect to thickness for S-S boundary conditions at μ=0.1. In this figure, NL refers to nonlocal piezoelectric nanobeam without surface and flexoelectric effects. NL-Flexoelectric refers to a nonlocal flexoelectric nanobeam without surface effect. Also, NL-SE denote a nonlocal piezoelectric nanobeam without flexoelectric effect. It is observable from this figure that neglecting the surface effect leads to lower natural frequencies. In fact, inclusion of surface effect enhances the stiffness of flexoelectric nanobeams and natural frequencies increase. It is found that flexoelectricity effect leads to higher natural frequencies, especially at smaller values of nanobeam thickness. Therefore, the maximum natural frequencies are observed for NL-SE-Flexoelectric nanobeam, while nonlocal (NL) piezoelectric nanobeam has the minimum buckling load. For the nonlocal (NL) piezoelectric nanobeams, natural frequencies are not dependent on the value of nanobeam thickness. But, when the flexoelectric effect is involved, natural frequencies reduce as the value of

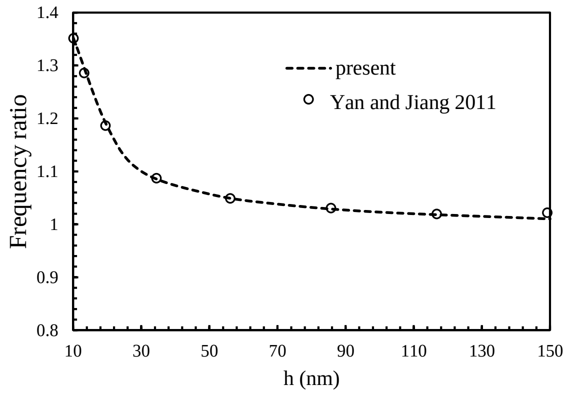


Fig. 2 Comparison of frequency ratio of S-S piezoelectric nanobeams (L/h=20)

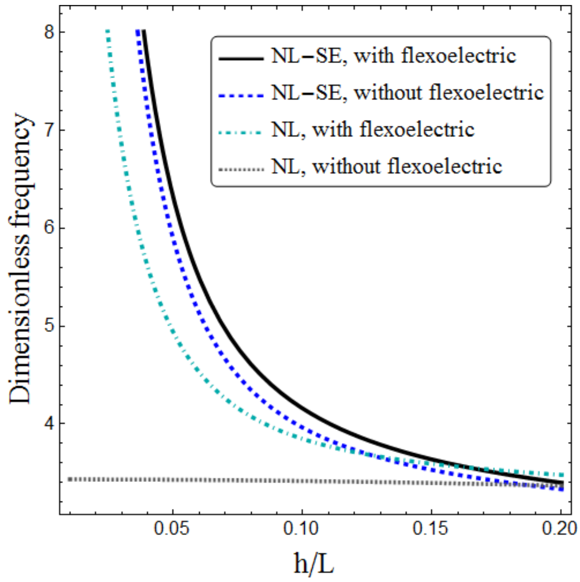
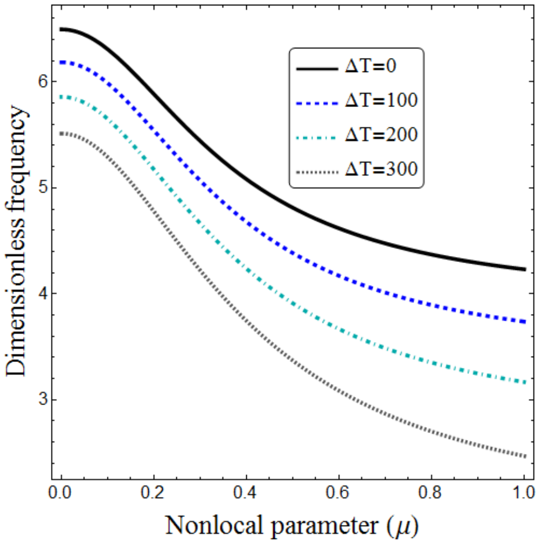


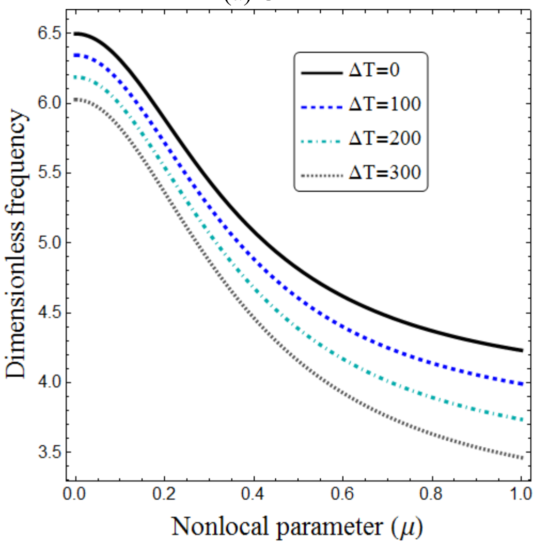
Fig. 3 Surface and flexoelectricity effects on vibration frequency of nonlocal S-S piezoelectric nanobeams with respect to thickness (μ=0.1)

thickness rises. So, flexoelectricity has an important size effect on vibration behavior of piezoelectric nanobeams. It can be concluded that surface and flexoelectric effects are important at lower thicknesses. In other words, effects surface elasticity and flexoelectricity are negligible at large thicknesses.

Influences of uniform and linear temperature changes (ΔT) on natural frequencies of flexoelectric nanobeam with surface effect for different nonlocal parameters is presented in Figs. 4 and 5, respectively for S-S and C-C boundary conditions. It is found that presence of temperature field has a significant effect on the vibration behavior flexoelectric nanobeam. In fact, temperature rise makes the flexoelectric nanobeam more flexible and vibration frequencies reduce at a constant nonlocal parameter. Moreover, the vibration frequencies of thermally affected flexoelectric nanobeam depend on the value of nonlocal parameter. It is observed that increasing the value of nonlocal parameter leads to reduction in dimensionless vibration frequencies of flexoelectric nanobeam at every magnitude of temperature

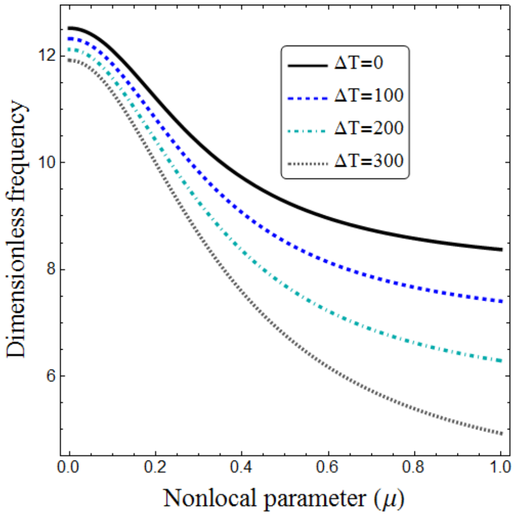


(a) UTR

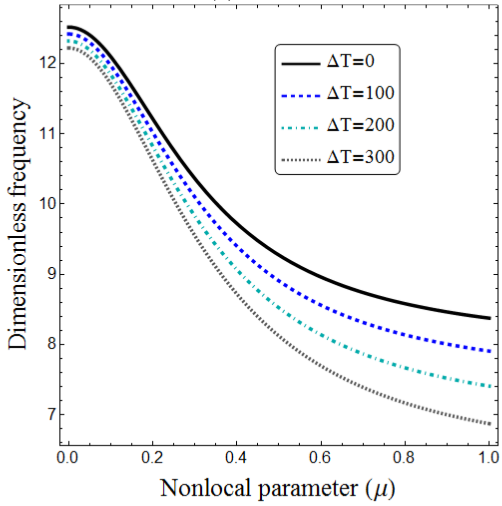


(b) LTR

Fig. 4 Temperature change effects on vibration frequency of nonlocal S-S piezoelectric nanobeams with respect to nonlocal parameter (L/h=20)



(a) UTR



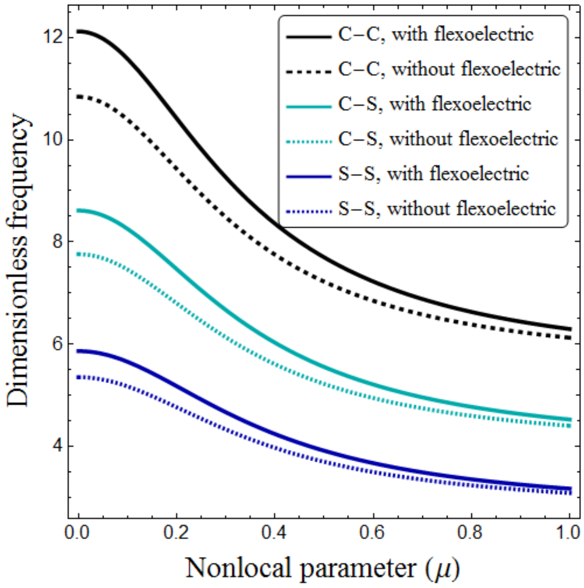
(b) LTR

Fig. 5 Temperature change effects on vibration frequency of nonlocal C-C piezoelectric nanobeams with respect to nonlocal parameter (L/h=20)

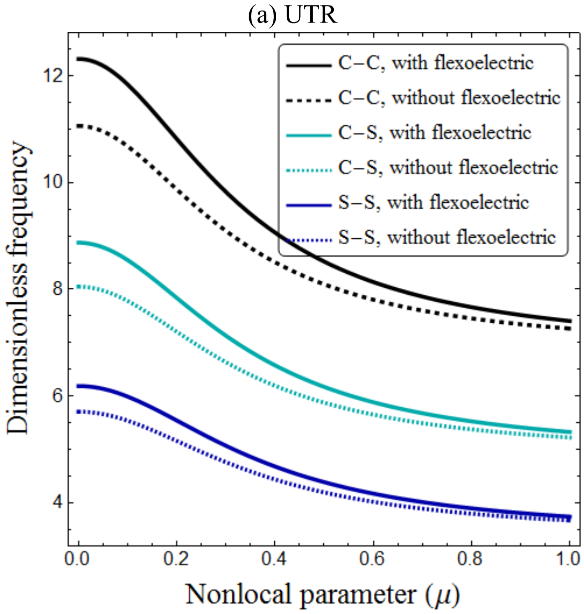
change. This is due to stiffness reduction of flexoelectric nanobeam by considering the nonlocal stress field parameter.

Examination of flexoelectric and nonlocal effects on vibration behavior of flexoelectric nanobeams under S-S, C-S and C-C boundary conditions when L/h=20 and ΔT=200 K is presented in Fig. 6. It is observable from this figure that neglecting the flexoelectric effect leads to lower natural frequencies at a fixed nonlocal parameter. It is also found that the nonlocal flexoelectric nanobeam has lower natural frequencies compared with local flexoelectric nanobeam (μ=0 nm^2), regardless of the type of boundary conditions. So, inclusion of nonlocal stress field parameter reduces the natural frequencies of a flexoelectric nanobeam. Such observation is neglected in all previous analyzes on flexoelectric nanobeams. So, by ignoring the effect of nonlocality in analysis of flexoelectric nanobeams, the obtained results are overestimated. Hence, it can be concluded that the vibration behavior of flexoelectric nanobeams is sensitive to the nonlocal parameter. The maximum and minimum natural frequencies of flexoelectric nanobeam are obtained for C-C and S-S boundary conditions. In fact, stronger supports at ends make the flexoelectric nanobeam stiffer and natural frequencies rise. It is known that nanobeam under uniform temperature rise (UTR) is more flexible than linear temperature rise (LTR). So, LTR gives larger frequencies for a piezoelectric nanobeam with and without flexoelectric effect.

Figs. 7 and 8 present the variation of vibration frequency of nonlocal S-S and C-C piezoelectric nanobeams under uniform and linear thermal loadings with respect to temperature rise for various nonlocal parameters at L/h=20. Increasing temperature leads to reduction in stiffness of flexoelectric nanobeam and natural frequencies reduce. In fact, as the temperature rise, vibration frequency reduces until it becomes close to zero at a critical point. After this critical point, increase of temperature yields larger frequencies. The most important observation from

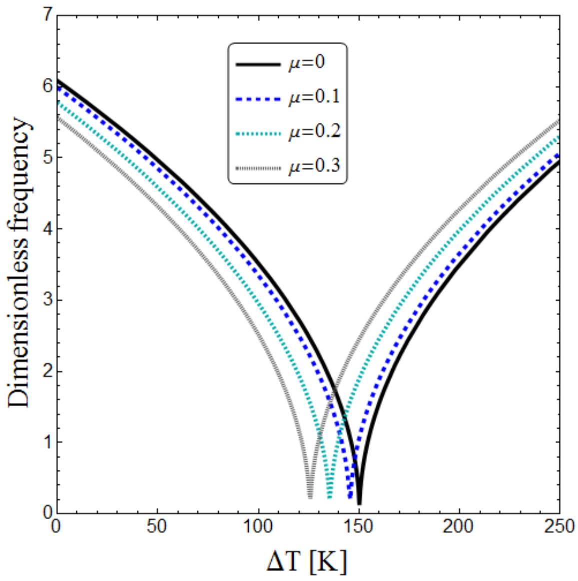


(a) UTR

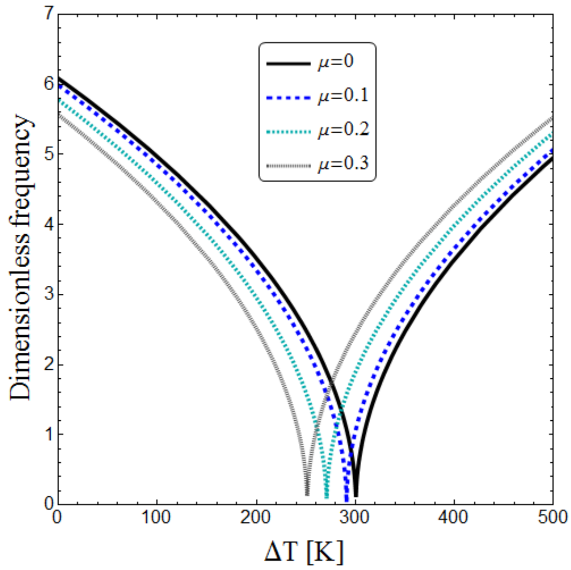


(b) LTR

Fig. 6 Flexoelectric and nonlocal effects on vibration frequency of nonlocal piezoelectric nanobeams for various boundary conditions (L/h=20, ΔT=200 K)



(a) UTR



(b) LTR

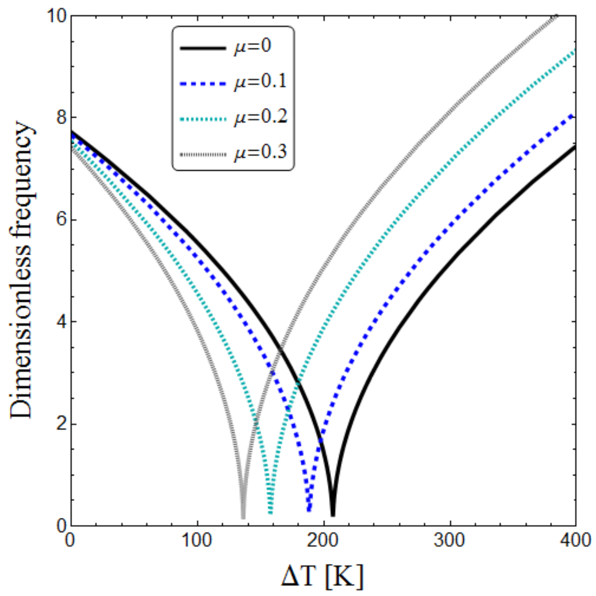
Fig. 7 Variation of vibration frequency of nonlocal S-S piezoelectric nanobeams under uniform and linear thermal loading with respect to temperature rise for various nonlocal parameters (L/h=20)

this figure is that nonlocal flexoelectric nanobeam gives larger critical temperatures than local flexoelectric nanobeam. In fact, inclusion of nonlocal effect diminishes the stiffness of flexoelectric nanobeams and critical temperatures reduce.

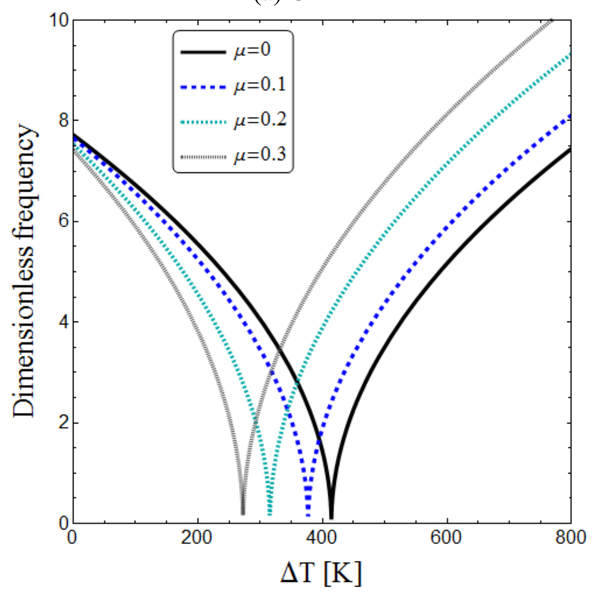
In fact, as the value of nonlocal parameter increases, the critical temperature is transferred to the left. So, it is showed that the nonlocal effect which is neglected in all previous papers on flexoelectric nanobeams affects significantly the vibration frequencies. In fact, by ignoring nonlocal effect the critical temperatures of flexoelectric nanobeams is overestimated. Also, it is clear that a flexoelectric nanobeam under linear temperature rise (LTR) has larger critical temperature than a flexoelectric nanobeam under uniform temperature rise (UTR).

7. Conclusions

In this research, thermal vibration characteristics of a flexoelectric/piezoelectric nanobeam under uniform and linear thermal loadings are investigated based on nonlocal elasticity theory considering surface effects. This non-classical nanobeam model contains flexoelectric effect to capture coupling of strain gradients and electrical polarizations. Moreover, the nonlocal elasticity theory is employed to study the nonlocal and long-range interactions between the particles. The present model can degenerate into the classical model if the nonlocal parameter, flexoelectric and surface effects are omitted. Hamilton's principle is employed to derive the governing equations and the related boundary conditions which are solved applying a



(a) UTR



(b) LTR

Fig. 8 Variation of vibration frequency of nonlocal C-S piezoelectric nanobeams under uniform and linear thermal loadings with respect to temperature rise for various nonlocal parameters (L/h=20)

Galerkin-based solution. From the results analyzed above, it is found that inclusion of nonlocal parameter leads to lower vibration frequencies by reducing the bending stiffness of flexoelectric/piezoelectric nanobeam. Besides, the non-dimensional vibration frequencies are found to be decreased by increasing the thickness value, however effect of flexoelectricity on vibration frequencies is more prominent at lower thicknesses. Increase of temperature reduces the stiffness of flexoelectric/piezoelectric nanobeam and leads to lower vibration frequencies. However, a flexoelectric/piezoelectric nanobeam under linear temperature rise has larger vibration frequencies compared with a flexoelectric nanobeam under uniform temperature rise.

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