Structural Engineering and Mechanics, *Vol. 65, No. 4 (2018) 423-434*
DOI: https://doi.org/10.12989/sem.2018.65.4.423

Mechanical and thermal stability investigation of functionally graded plates resting on elastic foundations

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(Received May 25, 2017, Revised September 10, 2017, Accepted September 18, 2017)

Abstract. In present work, both the hyperbolic shear deformation theory and stress function concept are used to study the mechanical and thermal stability responses of functionally graded (FG) plates resting on elastic foundation. The accuracy of the proposed formulation is checked by comparing the computed results with those predicted by classical plate theory (CPT), first-order shear deformation theory (FSDT) and higher-order shear deformation theory (HSDT). Moreover, results demonstrate that the proposed formulation can achieve the same accuracy of the existing HSDTs which have more number of governing equations.

Keywords: functionally graded materials; mechanical-thermal buckling; plate; hyperbolic shear deformation theory; stress function

1. Introduction

Functionally graded materials (FGMs) are a novel type of composite materials. It has been widely employed in many engineering uses and environments, especially in large temperature gradients. The metal provides considerable roughness while the ceramic gives high temperature-resistant and high corrosion-resistant. Thus, they are extensively employed in aerospace engineering, aircraft structures, high-speed vehicle frames and so on. The concept of FGM was first presented by a group of material scientists in Japan in mid-1980s (Koizumi 1993, 1997). Since the main applications of FGMs have been in high temperature environments, most of the research on FGMs has been restricted to thermal stress analysis, thermal stability, fracture mechanics and optimization (Attia *et al.* 2015, Bakora and Tounsi 2015, Barati and Shahverdi 2016, Barka *et al.* 2016, Benferhat *et al.* 2016, Abdelhak *et al.* 2016, Bousahla *et al.* 2016, Chikh *et al.* 2016, El-Hassar *et al.* 2016, Fahsi *et al.* 2017, El-Haina *et al.* 2017).

A number of studies dealing with thermal stability of FG plates have been presented in the published literature. Javaheri and Eslami (2002a, b) studied stability analysis of FG plates under four types of thermal loads based on the CPT and the HSDT, respectively. Lanhe (2004) analytically investigated the thermal stability problem of a FG plate with moderately thickness and simply supported boundary conditions based on the FSDT. Matsunaga (2005) proposed a two-dimensional global HSDT for thermal stability of FG plates. He computed the critical stability temperatures of a simply supported FG plate subjected to uniformly and linearly distributed temperatures. Zhao *et al.* (2009) studied the stability behavior of FG plates under mechanical and thermal loads with arbitrary geometry, considering plates that contain square and circular holes at the center, is studied by employing the element-free kp-Ritz method. Bouazza *et al.* (2010) investigated the thermoelastic stability of FG plate using FSDT. Influences of varying plate characteristics, material composition and volume fraction of constituent materials on the critical temperature difference of FG plate with simply supported edges are also examined. Lee *et al.* (2010) have used element-free Ritz technique to investigate the post-buckling of FG plates under compressive and thermal loads. Tung and Duc (2010) developed a simple accurate analytical solution to study the buckling and post-buckling response of thin FG plates. By considering the initial imperfection for an FG plate, they proved that imperfect plates do not follow bifurcation-type buckling and commence to deflect by initiation of compression. They studied possible combinations of movable and immovable simply supported edges for each case of thermo-mechanical loading. Ahmed (2014) investigated the post-buckling behavior of FG sandwich beams by employing a consistent HSDT. Swaminathan and Naveenkumar (2014) proposed an analytical formulation for the stability analysis of simply supported FG sandwich plates based on two higher-order refined computational models. Additional works on buckling and post-buckling investigation of laminated composite and FG structures under thermomechanical load are presented in the literature by Panda and his co-workers (Kar and Panda 2015a, b, 2016a, b, Katariya and Panda 2016). Recently, Bouderba *et al.* (2016) discussed the thermal stability of FG sandwich

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ISSN: 1225-4568 (Print), 1598-6217 (Online)

plates using a simple FSDT.

The components of structures widely employed in aircraft, reusable space transportation vehicles and civil engineering are often supported by an elastic foundation. Therefore, it is necessary to consider the influences of elastic foundation for a better understanding of the stability behavior of plates. To describe the interactions of the plate and foundation we found various types of foundation models. The simplest model for the elastic foundation is Winkler or one-parameter model (Winkler 1867), which considers the foundation as a series of separated springs without coupling influences between each other. Pasternak (1954) improved this model by introducing a shear layer to Winkler model. Pasternak or two-parameter model is widely employed to describe the mechanical behavior of structure-foundation interactions. In spite of practical importance and increasing use of FG structures, investigation on stability of FG plates supported by elastic media is limited in number (Wang and Shen 2011, Duc and Tung 2011, Kiani *et al.* 2011, Naderi and Saidi 2011, Sobhy 2013, Yaghoobi and Torabi 2013, Yaghoobi and Yaghoobi 2013, Yaghoobi and Fereidoon 2014, Ait Amar Meziane *et al.* 2014, Belkorissat *et al.* 2015, Tounsi *et al.* 2016, Besseghier *et al.* 2017, Bellifa *et al.* 2017).

The problem of the choice of the models employed in the expansion of the different variables is important to adequately approximate the real response of a considered structure. In the past few decades, various shear deformation theories have been developed and implemented to investigation of plates. Moreover, increased employ of advanced materials in primary structures requires the development of precise mathematical model to accurately study the response of the structures. The researchers have paid much attention for modeling of the plates over the past few decades and a variety of plate models have been proposed. The CPT can only give reasonable results for thin plates since it neglects the influences of the transverse shear deformation and transverse normal stress. The FSDT (Reissner 1945, Mindlin 1951, Adda Bedia *et al.* 2015, Hadji *et al.* 2016, Bellifa *et al.* 2016) introduces the influence of transverse shear deformation, but this model needs a shear correction coefficient in order to respect zero transverse shear stress boundary conditions on the top and bottom. The various HSDTs were proposed to investigate the plates by Ambartsumian (1958), Levinson (1980), Touratier (1991), Soldatos (1992), Karama *et al.* (2003), Reddy (2004), Xiang *et al.* (2011), Akavci (2014), Mahi *et al.* (2015), Ait Yahia *et al.* (2015), Abdelbari *et al.* (2016), Houari *et al.* (2016). On the other hand, Shimpi (2002) has proposed a novel refined plate theory (RPT) which is simple to utilize. The RPT proposed by Shimpi is based on the supposition that the in-plane and vertical displacements consist of bending and shear components in which the bending components do not contribute toward shear forces and, likewise, the shear components do not contribute toward bending moments. Recently, various RPTs are proposed for composite structures with and without thickness stretching effect (Bouderba *et al.* 2013, Tounsi *et al.* 2013, Bessaim *et al.* 2013, Zidi *et al.* 2014, Bousahla *et al.* 2014, Hebali *et al.* 2014, Belabed *et al.* 2014, Fekrar *et al.* 2014, Bousahla *et al.* 2014, Bourada *et al.* 2015, Taibi *et al.* 2015, Hamidi *et al.* 2015, Bousahla *et al.* 2016, Bennoun *et al.* 2016, Beldjelili *et al.* 2016, Bounouara *et al.* 2016, Hebali *et al.* 2016, Bourada *et al.* 2016, Benbakhti *et al.* 2016, Boukhari *et al.* 2016, Draiche *et al.* 2016, Raminnea *et al.* 2016, Zidi *et al.* 2017, Khetir *et al.* 2017, Klouche *et al.* 2017, Chikh *et al.* 2017, Benchohra *et al.* 2017, Benahmed *et al.* 2017, Bouafia *et al.* 2017).

In the present work, the mechanical and thermal stability responses of FG plates are studied by employing a new HSDT with four unknowns in which instead of derivative terms in the displacement field, integral terms are utilized. Such kinematic, which can be further implemented in HSDTs, may require new mathematical strategies to analytically solve the proposed theory because of its novelty. Governing equations are obtained from the principle of minimum total potential energy. Analytical solutions for mechanical and thermal stability investigation of FG plates resting on elastic foundations are determined. Numerical results are presented to check the accuracy of the proposed theory. The present HSDT is reported for the first time and can be served as benchmark results for researchers to validate their models in the future.

2. Material properties of FG plate

In this work, material characteristics of a FG plate are assumed to vary according to a rule of mixtures as (Sugano 1990). Simple power law variation from pure metal at lower face ($z=-h/2$) to pure ceramic at the upper face ($z=+h/2$) in terms of the volume fractions of the constituents is considered (Praveen and Reddy 1998). The mechanical and thermal characteristics of FGMs are obtained from the volume fraction of the material constituents. We consider that the material characteristics such as the modulus of elasticity (E), the thermal conductivity (K), coefficient of thermal expansion (α) and Poisson's ratio (ν) can be obtained by (Yaghoobi and Torabi 2013, Yaghoobi and Fereidoon 2014, Meksi *et al.* 2015, Ait Atmane *et al.* 2015, Meradjah *et al.* 2015, Larbi Chaht *et al.* 2015, Laoufi *et al.* 2016, Ahouel *et al.* 2016, Meksi *et al.* 2017, Menasria *et al.* 2017, Mouffoki *et al.* 2017)

$$E(z)=E_M+\left(E_C-E_M\right)\left(\frac{2z+h}{2h}\right)^k \tag{1a}$$

$$K(z)=K_M+\left(K_C-K_M\right)\left(\frac{2z+h}{2h}\right)^k \tag{1b}$$

$$\alpha(z)=\alpha_M+\left(\alpha_C-\alpha_M\right)\left(\frac{2z+h}{2h}\right)^k,\quad \nu(z)=\nu=\text{constant} \tag{1c}$$

where k is the gradient index and subscripts M and C denote the metallic and ceramic components, respectively. The value of k equal to zero and infinity represents a fully ceramic and metal plate, respectively.

3. New refined plate theory

3.1 Kinematics

In this work, the hyperbolic shear deformation plate theory is employed. The displacement field can then be expressed as (Akavci 2007, 2010)

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w(x,y)}{\partial x} + \Psi(z)\ \phi_x(x,y) \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w(x,y)}{\partial y} + \Psi(z)\ \phi_y(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{2a}$$

where the shape function is given by Akavci (2007, 2010) as

$$\Psi(z) = \frac{3}{2}\pi\, h \tanh\left(\frac{z}{h}\right) - \frac{3}{2} z\,\pi \left[\operatorname{sech}\left(\frac{1}{2}\right)\right]^2 \tag{2b}$$

where u_0, v_0 and w_0 are generalized displacement at the mid-plane of the plate in the x, y, and z directions, respectively; ϕ_x, ϕ_y are the slope rotations in the (x, z) and (y, z) planes, respectively; and h is the plate thickness.

The non-linear von Karman strain–displacement equations are as follows

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} + \Psi(z) \begin{Bmatrix} \eta_x \\ \eta_y \\ \eta_{xy} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \Psi'(z) \begin{Bmatrix} \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix}, \tag{3}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} u_{0,x} + (w_{0,x})^2/2 \\ v_{0,x} + (w_{0,y})^2/2 \\ u_{0,y} + v_{0,x} + w_{0,x} w_{0,y} \end{Bmatrix},$$

$$\begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} = \begin{Bmatrix} -w_{0,xx} \\ -w_{0,yy} \\ -2w_{0,xy} \end{Bmatrix}, \quad \begin{Bmatrix} \eta_x \\ \eta_y \\ \eta_{xy} \end{Bmatrix} = \begin{Bmatrix} \phi_{x,x} \\ \phi_{y,y} \\ \phi_{x,y} + \phi_{y,x} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} = \begin{Bmatrix} \phi_x \\ \phi_y \end{Bmatrix}, \tag{4}$$

3.2 Constitutive equations

The linear constitutive relations of a FG plate can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 & 0 & 0 \\ \nu & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1-\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1-\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x - \alpha\,\Delta T \\ \varepsilon_y - \alpha\,\Delta T \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{5}$$

where ΔT is temperature rise from stress free initial state or temperature difference between two surfaces of the FG plate.

By using the principle of minimum total potential energy, the expressions for the nonlinear equilibrium equations of the plate are obtained as

$$N_{x,x} + N_{xy,y} = 0 \tag{6a}$$

$$N_{xy,x} + N_{y,y} = 0 \tag{6b}$$

$$(M_{x,xx} + 2\,M_{xy,xy} + M_{y,yy}) + N_x w_{,xx} + 2\,N_{xy} w_{,xy} + N_y w_{,yy} - k_1 w + k_2 \nabla^2 w = 0 \tag{6c}$$

$$S_{x,x} + S_{xy,y} - Q_x = 0 \tag{6d}$$

$$S_{xy,x} + S_{y,y} - Q_y = 0 \tag{6e}$$

The force and moment resultants (N, Q, S and M) of the FG plate are obtained by

$$(N_i,\ M_i,\ S_i) = \int_{-h/2}^{h/2} \sigma_i\,(1, z, \Psi(z))\,dz, \quad (i = x, y, xy) \tag{7a}$$

$$Q_i = \int_{-h/2}^{h/2} \sigma_j\ \Psi'(z)\,dz, \qquad (i = x, y);\ \ (j = xz, yz) \tag{7b}$$

Substitution of Eqs. (3) and (5) into Eq. (7) yields the constitutive relations as

$$\begin{aligned} (N_x,\ M_x, S_x) = \frac{1}{1-\nu^2} [&(E_1, E_2, E_3)(\varepsilon_x^0 + \nu\,\varepsilon_y^0) + (E_2, E_4, E_5)(k_x + \nu\,k_y) \\ &+ (E_3, E_5, E_7)(\eta_x + \nu\,\eta_y) - (1+\nu)(\Phi_1, \Phi_2, \Phi_3)] \end{aligned} \tag{8a}$$

$$\begin{aligned} (N_y,\ M_y,\ S_y) = \frac{1}{1-\nu^2} [&(E_1, E_2, E_3)(\varepsilon_y^o + \nu\,\varepsilon_x^0) + (E_2, E_4, E_5)(k_y + \nu\,k_x) \\ &+ (E_3, E_5, E_7)(\eta_y + \nu\,\eta_x) - (1+\nu)(\Phi_1, \Phi_2, \Phi_3)] \end{aligned} \tag{8b}$$

$$\begin{aligned} (N_{xy},\ M_{xy},\ S_{xy}) = \frac{1}{2(1+\nu)} [&(E_1, E_2, E_3)\gamma_{xy}^0 + (E_2, E_4, E_5)k_{xy} \\ &+ (E_3, E_5, E_7)\eta_{xy}] \end{aligned} \tag{8c}$$

$$(Q_x, Q_y) = \frac{1}{2(1+\nu)} E_8 \left(\gamma_{xz}^0, \gamma_{yz}^0\right) \tag{8d}$$

where

$$(E_1, E_4, E_5, E_7) = \int_{-h/2}^{h/2} \left(1, z^2, z\Psi(z), \Psi(z)^2\right) E(z)dz, \quad (E_2, E_3) = \int_{-h/2}^{h/2} (z, \Psi(z))\, E(z)dz = (0,0), \quad E_8 = \int_{-h/2}^{h/2} \left(\Psi'(z)\right)^2 E(z)\,dz \tag{9a}$$

$$(\Phi_1, \Phi_2, \Phi_3) = \int_{-h/2}^{h/2} (1, z, \Psi) E(z)\,\alpha(z)\,\Delta T(z)\,dz \tag{9b}$$

The last three equations of Eq. (6) may be rewritten into two equations in terms of variables w_0 and $\phi_{x,x}+\phi_{y,y}$ by substituting Eqs. (4) and (8) into Eqs. (6c)-(6e). Subsequently, elimination of the variable $\phi_{x,x}+\phi_{y,y}$ from two the resulting equations lead to the following system of equilibrium equations

$$\begin{aligned} &N_{x,x} + N_{xy,y} = 0 \\ &N_{xy,x} + N_{y,y} = 0 \\ &(D_1 D_3 - D_2^2)\nabla^6 w - D_1 D_4\ \nabla^4 w - D_3 \nabla^2 (N_x w_{,xx} + 2N_{xy} w_{,xy} + N_y w_{,yy} - k_1 w + k_2 \nabla^2 w) \\ &+ D_4\,(N_x w_{,xx} + 2N_{xy} w_{,xy} + N_y w_{,yy} - k_1 w + k_2 \nabla^2 w) = 0 \end{aligned} \tag{10}$$

where

$$D_1 = \frac{E_4}{\left(1-\nu^2\right)}, \quad D_2 = \frac{E_5}{\left(1-\nu^2\right)}, \quad D_3 = \frac{E_7}{\left(1-\nu^2\right)}, \quad D_4 = \frac{E_8}{2(1+\nu)}. \tag{11}$$

For a FG plate, Eq. (10) are modified into form as

$$\begin{aligned} &\left(D_2^2 - D_1 D_3\right)\nabla^6 w + D_1 D_4 \nabla^4 w + D_3 \nabla^2 \left[f_{,yy}\left(w_{,xx}\right) - 2 f_{,xy}\left(w_{,xy}\right) + f_{,xx}\left(w_{,yy}\right)\right] \\ &- D_4 \left[f_{,yy}\left(w_{,xx}\right) - 2 f_{,xy}\left(w_{,xy}\right) + f_{,xx}\left(w_{,yy}\right)\right] = 0 \end{aligned} \tag{12}$$

where $f(x, y)$ is stress function defined by

$$N_x = f_{,yy}\ ,\ N_y = f_{,xx}\ ,\ N_{xy} = -f_{,xy} \tag{13}$$

The geometrical compatibility equation for a FG plate is expressed as

$$\varepsilon^0_{x,yy} + \varepsilon^0_{y,xx} - \gamma^0_{xy,xy} = w^2_{,xy} - w_{,xx}w_{,yy} \tag{14}$$

From the constitutive relations (8) and Eq. (13) one can write

$$\begin{aligned}(\varepsilon^0_x, \varepsilon^0_y) &= \frac{1}{E_1}[(f_{,yy}, f_{,xx}) - \nu(f_{,xx}, f_{,yy}) + \Phi_1(1,1)] \\ \gamma^0_{xy} &= -\frac{1}{E_1}[\ 2(1+\nu) f_{,xy}\ ]\end{aligned} \tag{15}$$

Introducing Eq. (15) into Eq. (14), the compatibility equation of a FG plate becomes

$$\nabla^4 f - E_1(w^2_{0,xy} - w_{0,xx}\, w_{0,yy}) = 0 \tag{16}$$

In this study we are concerned with the exact solution of Eqs. (12) and (16) for a simply supported FG plate. In this case, the proposed solutions of w and f respecting boundary conditions are assumed to be (Librescu and Lin 1997, Lin and Librescu 1998)

$$w = W\sin(\lambda_m x)\sin(\delta_n y) \tag{17a}$$

$$f = A_1\cos(2\lambda_m x) + A_2\cos(2\delta_n y) + A_3\sin(\lambda_m x)\sin(\delta_n y) + \frac{1}{2}N_{x0}y^2 + \frac{1}{2}N_{y0}x^2 \tag{17b}$$

where $\lambda_m = m\pi/a$, $\delta_n = n\pi/b$, m, n are odd numbers and W is amplitude of the deflection. The coefficients A_i (i = 1, 2, 3) are determined by substitution of Eqs. (17a), (17b) into Eq. (16) as

$$A_1 = \frac{E_1\delta_n^2}{32\lambda_m^2}W^2, \quad A_2 = \frac{E_1\lambda_m^2}{32\delta_n^2}W^2, \quad A_3 = 0 \tag{18}$$

Then, setting Eqs. (17a), (17b) into Eq. (12) and by employing the Galerkin method for the resulting equation yield

$$\begin{aligned}&\left(\left(D_1D_3 - D_2^2\right)\left(\lambda_m^2 + \delta_n^2\right)^3 + D_1D_4\left(\lambda_m^2 + \delta_n^2\right)^2\right)W + \frac{E_1}{16}\left(D_3\left(5\left(\lambda_m^4\delta_n^2 + \lambda_m^2\delta_n^4\right) + \lambda_m^6 + \delta_n^6\right) + D_4\left(\lambda_m^4 + \delta_n^4\right)\right)W^3 \\ &+ \left(D_3\left(\lambda_m^2 + \delta_n^2\right) + D_4\right)\times\left(N_{x0}\lambda_m^2 + N_{y0}\delta_n^2\right)W = 0\end{aligned} \tag{19}$$

4. Closed-form solution

Rectangular plates are generally classified in accordance with the type of support employed. Here, we are concerned with the exact solutions of Eq. (19) for a simply supported FG plate resting on elastic foundation.

4.1 Mechanical buckling

Consider a simply supported rectangular plate resting on elastic foundation with length a and width b which is subjected to in-plane loading in two directions

$$N_{x0} = -F_x h,\ \ N_{y0} = -F_y h \tag{20}$$

and Eq. (19) leads to

$$F_x = e_1^1\frac{W}{h(W)} \tag{21}$$

where

$$e_1^1 = \frac{(D_1D_3 - D_2^2)(\lambda_m^2 + \delta_n^2)^3 + D_1D_4(\lambda_m^2 + \delta_n^2)^2}{(\lambda_m^2 + \beta\ \delta_n^2)[D_3(\lambda_m^2 + \delta_n^2) + D_4]} + \frac{[K_1 + K_2a^2(\lambda_m^2 + \delta_n^2)]\ D_1}{a^4(\lambda_m^2 + \beta\ \delta_n^2)} \tag{22}$$

in which

$$\beta = F_y / F_x\ ,\ K_1 = \frac{k_1a^4}{D_1}\ ,\ K_2 = \frac{k_2a^2}{D_1} \tag{23}$$

4.2 Thermal buckling

A rectangular plate under thermal loads is examined in this part. To determine the critical stability temperature, the pre-buckling thermal loads should be found. Thus, solving the membrane form of the equilibrium equations and by employing the method presented by Meyers and Hyer (1991), the pre-buckling load resultants of FG plate exposed to the temperature distribution within the thickness are found to be

$$N_{x0} = N_{y0} = -\frac{\Phi_1}{1-\nu} \tag{24}$$

and

$$\begin{aligned}\frac{\Phi_1}{1-\nu} = &\left[\frac{[(D_1D_3 - D_2^2)(\lambda_m^2 + \delta_n^2)^2 + D_1D_4(\lambda_m^2 + \delta_n^2)]}{D_3(\lambda_m^2 + \delta_n^2) + D_4} + \frac{[K_1 + K_2a^2(\lambda_m^2 + \delta_n^2)]\ D_1}{a^4(\lambda_m^2 + \delta_n^2)}\right] \\ &+ \left[\frac{E_1(1-\nu)(\lambda_m^4 + \delta_n^4)}{16\ (\lambda_m^2 + \delta_n^2)} + \frac{E_1(\lambda_m^4 + 2\ \nu\,\lambda_m^2\delta_n^2 + \delta_n^4)}{8(1+\nu)\ (\lambda_m^2 + \delta_n^2)}\right]W^2\end{aligned} \tag{25}$$

In this work, to study the influence of assumption type of temperature variation within the thickness on thermal stability behavior of FG plate resting on elastic foundation, three types of thermal loading within the plate thickness are considered.

4.2.1 Uniform temperature rise (UTR)

It is considered that the initial uniform temperature of the FG plate is T_i, and the temperature is uniformly raised to a final value T_f such that the plate buckles. The temperature change $\Delta T = T_f - T_i$ is considered to be independent from thickness variable. The thermal parameter Φ_1 is obtained from Eq. (9b), and substitution of the result into Eq. (25) yields

$$\begin{aligned}\Delta T_{Cr} &= e_1^2 \\ &= \frac{(1-\nu)\left[(D_1D_3 - D_2^2)(\lambda_m^2 + \delta_n^2)^2 + D_1D_4(\lambda_m^2 + \delta_n^2)\right]}{L\ [D_3(\lambda_m^2 + \delta_n^2) + D_4]} + \frac{[K_1 + K_2a^2(\lambda_m^2 + \delta_n^2)]\ D_1(1-\nu)}{a^4\ L\,(\lambda_m^2 + \delta_n^2)}\end{aligned} \tag{26}$$

where

$$L = \int_{-h/2}^{h/2}\alpha(z)E(z)dz \tag{27}$$

4.2.2 Linear temperature distribution through the thickness (LTD)

As an approximation, consider the following linear temperature variation along the thickness coordinate of the FG plate as

$$T(z) = \Delta T\left(\frac{z}{h} + \frac{1}{2}\right) + T_m, \quad \Delta T = T_c - T_m \tag{28}$$

Same as UTR procedure, the following expression for thermal buckling load is obtained

$$\Delta T_{Cr} = e_1^2 - \frac{T_m L}{H} \tag{29}$$

with

$$e_1^2 = \frac{(1-\nu)\left[(D_1 D_3 - D_2^2)(\lambda_m^2 + \delta_n^2)^2 + D_1 D_4(\lambda_m^2 + \delta_n^2)\right]}{H\,[D_3(\lambda_m^2 + \delta_n^2) + D_4]} + \frac{[K_1 + K_2 a^2(\lambda_m^2 + \delta_n^2)]\,D_1(1-\nu)}{a^4 H(\lambda_m^2 + \delta_n^2)} \tag{30}$$

and

$$H = \int_{-h/2_1}^{h/2} \alpha(z)\; E(z)\left(\frac{z}{h} + \frac{1}{2}\right) dz \tag{31}$$

4.2.3 Non-linear temperature distribution through the thickness (NTD)

The temperature field considered to be uniform over the plate surface but changing along the thickness direction due to heat conduction. In such a case, the temperature variation within the thickness can be determined by solving the steady-state heat transfer equation as

$$\frac{d}{dz}\left[K(z)\frac{dT}{dz}\right] = 0\,,\; T(z = -h/2) = T_m \quad \text{and} \quad T(z = h/2) = T_c \tag{32}$$

The differential Eq. (32) can be easily solved by employing the polynomial series. Thus, the temperature variation within the plate thickness is determined as

$$T(z) = T_m + \Delta T\,\frac{r\displaystyle\sum_{j=0}^{5}\frac{(-r^k\,{}^{K_{cm}}\!/\!{}_{K_m})^j}{jk+1}}{\displaystyle\sum_{j=0}^{5}\frac{({}^{-K_{cm}}\!/\!{}_{K_m})^j}{jk+1}} \tag{33}$$

where

$$r = \frac{(2z + h)}{2h} \quad \text{and} \quad \Delta T = T_c - T_m \tag{34}$$

Same as UTR procedure, the following expression for thermal buckling load is obtained

$$\Delta T_{Cr} = e_1^2 - \frac{T_m L}{G} \tag{35}$$

with

$$e_1^2 = \frac{(1-\nu)\left[(D_1 D_3 - D_2^2)(\lambda_m^2 + \delta_n^2)^2 + D_1 D_4(\lambda_m^2 + \delta_n^2)\right]}{G\,[D_3(\lambda_m^2 + \delta_n^2) + D_4]} + \frac{[K_1 + K_2 a^2(\lambda_m^2 + \delta_n^2)]\,D_1(1-\nu)}{a^4 G(\lambda_m^2 + \delta_n^2)} \tag{36}$$

and

$$G = \frac{\displaystyle\sum_{j=0}^{5}\frac{({}^{-K_{cm}}\!/\!{}_{K_m})^j}{jk+1}\left[\frac{E_m\alpha_m}{jk+2} + \frac{E_m\alpha_{cm} + E_{cm}\alpha_m}{(j+1)k+2} + \frac{E_{cm}\alpha_{cm}}{(j+2)k+2}\right]}{\displaystyle\sum_{j=0}^{5}\frac{({}^{-K_{cm}}\!/\!{}_{K_m})^j}{jk+1}} \tag{37}$$

Table 1 Comparison of non-dimensional critical buckling load $\tilde{N}$ of a simply supported thin homogeneous square plate resting on elastic foundations (a/h=1000)

Theory	(K_w, K_s)			
	(0,0)	(0,100)	(100,0)	(100,100)
CPT (Lam *et al.* 2000)	4.00000	18.92[a]	5.027	19.17[a]
FSDT (Akhavan *et al.* 2009)	3.99998	18.9151[a]	5.02658	19.1717[a]
FSDT (Sobhy 2013)	3.99998	18.91506[a]	5.02658	19.17171[a]
(Yaghoobi and Fereidoon 2014)	3.99990	18.91400[a]	5.02650	19.17200
Present	3.99999	18.91513[a]	5.02659	19.17178[a]

[a]Mode for plate is (m,n)=21

5. Results and discussion

In this section, numerical examples are examined and discussed for checking the accuracy of the proposed formulation in determining the mechanical and thermal stability loads. Analytical solutions are determined by employing the Navier solution for simply supported FG plates resting on elastic foundation. Critical buckling loads are determined and the comparison is carried out with the existing results. For numerical results, an Al/Al_2O_3 plate composed of aluminum (as metal) and alumina (as ceramic) is considered. The Young's modulus, thermal conductivity and coefficient of thermal expansion are E_m=70 GPa, α_m=23×10^{-6}/°C, K_m=204 W/m and those of alumina are E_c=380 GPa, α_c=7.4×10^{-6}/°C, K_c=10.4, Kc=10.4 W/mK, respectively. The Poisson's ratio of the plate is considered to be constant within the thickness and equal to 0.3 (Javaheri and Eslami 2002b, Lanhe 2004, Yaghoobi and Fereidoon 2014). For convenience, the following non-non-dimensional quantities are used in presenting the numerical results in tabular form

$$K_w = \frac{k_1 a^4}{D_c},\; K_s = \frac{k_2 a^2}{D_c},\; \hat{N} = \frac{N_{cr} b^2}{D_c},$$
$$\overline{N} = \frac{N_{cr} a^2}{E_m h^3},\; \tilde{N} = \frac{N_{cr} a^2}{\pi^2 D_c},\; D_m = \frac{E_m h^3}{12(1-\nu^2)}, \tag{38}$$
$$D_c = \frac{E_c h^3}{12(1-\nu^2)}$$

5.1 Comparisons for mechanical buckling

Example 1: The non-dimensional critical buckling loads $\tilde{N}$ of simply supported thin homogeneous square plate without or resting on elastic foundations are given in Table 1. The computed results are compared with those reported by Lam *et al.* (2000) based on CPT, Akhavan *et al.* (2009) and sobhy (2013) based on FSDT and Yaghoobi and Fereidoon (2014) based on Reddy's theory. It is mentioned that the solutions of Lam *et al.* (2000) are obtained via the Green's function. Good agreement is observed between the proposed theory and the published ones. Also, Table 2

Table 2 Comparison of non-dimensional critical buckling load $\hat{N}$ of a simply supported homogeneous plate under in-plane compression and resting on elastic foundations

a/b	(K_w, K_s)	Theory	a/h			
			5	10	100	1000
0.5	(0,0)	FSDT (*)	54.3207	59.6629	61.6641	61.6848
		FSDT (**)	54.0859	59.5887	61.6633	61.6848
		HSDT (***)	54.0737	59.5856	61.6633	61.6848
		Present	54.0802	59.5871	61.6633	61.6848
	(100,10)	FSDT (*)	144.6952	150.1910	152.1930	152.2130
		FSDT (**)	144.6140	150.1170	152.1920	152.2130
		HSDT (***)	144.6022	150.1141	152.1918	152.2133
		Present	144.6087	150.1156	152.1917	152.2132
	(1000,100)	FSDT (*)	643.5000[b]	686.1710[a]	704.3860[a]	704.5890[a]
		FSDT (**)	641.380[b]	685.567[a]	704.378[a]	704.589[a]
		HSDT (***)	640.9782[b]	685.5369[a]	704.3775[a]	704.5888[a]
		Present	641.2294[b]	685.5529[a]	704.3776[a]	704.5887[a]
1	(0,0)	FSDT (*)	32.4414	37.4477	39.457	39.4782
		FSDT (**)	32.2398	37.3753	39.4562	39.4782
		HSDT (***)	32.2276	37.3721	39.4562	39.4782
		Present	32.2343	37.3737	39.4561	39.4781
	(100,10)	FSDT (*)	55.0289[a]	67.5798	69.5891	69.6103
		FSDT (**)	54.6116[a]	67.5074	69.5883	69.6103
		HSDT (***)	54.5692[a]	67.5042	69.5883	69.6103
		Present	54.5945[a]	67.5058	69.5882	69.6103
	(1000,100)	FSDT (*)	174.9760[b]	204.6510[a]	211.9610[a]	212.0140[a]
		FSDT (**)	174.391[b]	204.416[a]	211.928[a]	212.014[a]
		HSDT (***)	174.2676[b]	204.4040[a]	211.9285[a]	212.0145[a]
		Present	174.3451[b]	204.4105[a]	211.9285[a]	212.0144[a]
2	(0,0)	FSDT (*)	19.2255[b]	32.4414[a]	39.3930[a]	39.4776[a]
		FSDT (**)	19.0400[b]	32.2398[a]	39.3897[a]	39.4775[a]
		HSDT (***)	18.9794[b]	32.2276[a]	39.3896[a]	39.4775[a]
		Present	19.0182[b]	32.2343[a]	39.3896[a]	39.4775[a]
	(100,10)	FSDT (*)	22.7476[c]	37.5182[b]	45.0262[a]	45.1108[a]
		FSDT (**)	22.6778[c]	37.8581[b]	45.0229[a]	45.1108[a]
		HSDT (***)	22.5785[c]	37.8358[b]	45.0228[a]	45.1108[a]
		Present	22.6434[c]	37.8486[b]	45.0228[a]	45.1107[a]
	(1000,100)	FSDT (*)	–	72.8290[c]	85.0953[b]	85.2563[b]
		FSDT (**)	52.2276[d]	72.4117[c]	85.0889[b]	85.2562[b]
		HSDT (***)	50.0214[d]	72.3694[c]	85.0887[b]	85.2562[b]
		Present	50.1233[d]	72.3946[c]	85.0887[b]	85.2562[b]

(*)(Akhavan *et al.* 2009),(**)(Sobhy 2013),(** *)(Yaghoobi and Fereidoon 2014)
[a]Mode for plate is $(m, n) = (2, 1)$
[b]Mode for plate is $(m, n) = (3, 1)$
[c]Mode for plate is $(m, n) = (4, 1)$
[d]Mode for plate is $(m, n) = (5, 1)$

Table 3 Comparison of non-dimensional critical buckling load $\bar{N}$ of a simply supported FG plate resting on elastic foundations ($a/b=1$, $a/h=10$)

β	(K_w, K_s)	Theory	k					
			0	0.5	1	2	5	10
0	(0, 0)	Present	18.5793	12.1234	9.3394	7.2627	6.0329	5.4475
		HSDT(*)	18.5785	12.1229	9.3391	7.2631	6.0353	5.4528
	(100, 10)	Present	21.3386	14.8828	12.0987	10.0220	8.7922	8.1992
		HSDT(*)	21.3379	14.8823	12.0985	10.0224	8.7947	8.2122
	(1000, 100)	Present	40.6510[a]	31.4625[a]	27.4333[a]	24.3459[a]	22.3527[a]	21.4341[a]
		HSDT(*)	40.6477[a]	31.4605[a]	27.4319[a]	24.3470[a]	22.3602[a]	21.4516[a]
1	(0, 0)	Present	9.2896	6.0617	4.6697	3.6313	3.0164	2.7168
		HSDT(*)	9.2893	6.0615	4.6695	3.6315	3.0177	2.7264
	(100, 10)	Present	10.6693	7.4414	6.0493	5.0110	4.3961	4.0995
		HSDT(*)	10.6689	7.4411	6.0492	5.0112	4.3973	4.1061
	(1000, 100)	Present	23.0864	19.8584	18.4664	17.4281	16.8132	16.5210
		HSDT(*)	23.0860	19.8582	18.4663	17.4283	16.8144	16.5232

(*)(Yaghoobi and Fereidoon, 2014)
[a] Mode for plate is $(m, n) = (2, 1)$

shows the non-dimensional buckling loads $\hat{N}$ of simply supported homogeneous plate under uniaxial compression. The computed results are compared with those found by Akhavan *et al.* (2009) and sobhy (2013) based on FSDT and Yaghoobi and Fereidoon (2014) based on Reddy's theory. Good agreement can be observed for different values of foundation coefficients, K_w and K_s, aspect ratio a/b and thickness ratio h/a.

Example 2: Table 3 provides the non-dimensional critical buckling loads $\bar{N}$ of simply supported Al/Al_2O_3 square plate for different values of material index k and foundation parameters K_w and K_s. The predicted non-dimensional critical buckling loads are compared with those reported by Yaghoobi and Fereidoon (2014). In this table, two different loading cases are considered, and six arbitrary values of the material index k are taken. Three combinations of foundation parameters, K_w and K_s are also considered. It can be seen that the results calculated using the present model are in good agreement with those reported by Yaghoobi and Fereidoon (2014) for all loading types, material index, and foundation parameters. It should be signaled that in this example the dimensionless foundation coefficients, K_w and K_s are k_1a^4/D_M and k_2a^2/D_M respectively.

5.2 Comparisons for thermal stability

Example 3: In order to verify the thermal buckling solutions obtained in this study, the critical stability temperature difference, ΔT_{cr}, for FG plates resting on elastic foundations for the UTR, LTD and NTD are illustrated in Tables 4-6, respectively. It can be seen from Tables 4-6 that there is a very good agreement between the proposed theory (with four variables) and other HSDTs (with five variables). The significant differences between the results of HSDTs and those of CPT, is due to the shear deformation influence which is neglected by CPT.

Table 4 Comparison of critical buckling temperature difference $\Delta T_{cr}\times10^{-3}$ of square FG plate resting

k	Theory	(K_w, K_s)=(0,0)			(K_w, K_s)=(10,0)			(K_w, K_s)=(10,40)		
		a/h=5	a/h=10	a/h=20	a/h=5	a/h=10	a/h=20	a/h=5	a/h=10	a/h=20
0	Present	5.58461	1.61875	0.42153	5.76015	1.66263	0.43251	9.22515	2.52888	0.64907
	CPT [(a)]	6.83964	1.70991	0.42748	7.01519	1.75380	0.43845	10.4801	2.62005	0.65501
	FSDT [(b)]	5.58069	1.61862	0.42153	5.75623	1.66251	0.43251	9.22123	2.52876	0.64907
	HSDT [(a)]	5.58344	1.61868	0.42154	5.75899	1.66257	0.43251	9.22398	2.52882	0.64907
	TPT [(a)]	5.58556	1.61882	0.42154	5.76109	1.66270	0.73252	9.22610	2.52896	0.64908
	HSDT [(c)]	5.58344	1.61868	0.42154	5.75898	1.66257	0.43251	9.22398	2.52882	0.64907
1	Present	2.67201	0.75842	0.19626	2.83562	0.79932	0.20649	6.06519	1.60671	0.40834
	CPT [(a)]	3.17751	0.79438	0.19859	3.34112	0.83528	0.20882	6.57068	1.64267	0.41067
	FSDT [(b)]	2.67039	0.75837	0.19626	2.83400	0.79928	0.20649	6.06356	1.60667	0.40834
	HSDT [(a)]	2.67153	0.75840	0.19627	2.83515	0.79930	0.20649	6.06470	1.60669	0.40835
	TPT [(a)]	2.67241	0.75845	0.19627	2.83603	0.79935	0.20649	6.06558	1.60674	0.40834
	HSDT [(c)]	2.67153	0.75840	0.19627	2.84312	0.79930	0.20649	6.06470	1.60669	0.40834
5	Present	2.27221	0.67903	0.17851	2.49898	0.73573	0.19268	6.97530	1.85481	0.47245
	CPT [(a)]	2.90629	0.72657	0.18164	3.13305	0.78326	0.19582	7.60938	1.90234	0.47559
	FSDT [(b)]	2.35948	0.68678	0.17905	2.58625	0.74347	0.19322	7.06257	1.86255	0.47299
	HSDT [(a)]	2.27501	0.67931	0.17854	2.50179	0.73600	0.19271	6.97810	1.85508	0.47248
	TPT [(a)]	2.27131	0.67895	0.17851	2.49808	0.73564	0.19268	6.97440	1.85472	0.47245
	HSDT [(c)]	2.27501	0.67931	0.17854	2.50178	0.73600	0.19271	6.97810	1.85508	0.47248
10	Present	2.24045	0.69133	0.18312	2.53163	0.75655	0.19913	7.58372	2.01957	0.51488
	CPT [(a)]	2.98770	0.74693	0.18673	3.24365	0.81091	0.20273	8.29575	2.07394	0.51848
	FSDT [(b)]	2.36822	0.70108	0.18373	2.62416	0.76507	0.19972	7.67626	2.02809	0.51548
	HSDT [(a)]	2.27678	0.69269	0.18314	2.53273	0.75668	0.19914	7.58483	2.01970	0.51490
	TPT [(a)]	2.27551	0.69254	0.18313	2.53146	0.75653	0.19913	7.58356	2.01955	0.51489
	HSDT [(c)]	2.27679	0.69269	0.18314	2.53273	0.75668	0.19914	7.58483	2.01970	0.51490

[(a)] (Zenkour and Sobhy 2011)
[(b)] (Yaghoobi and Torabi 2013)
[(c)] (Yaghoobi and Fereidoon 2014)

Table 5 Comparison of critical buckling temperature difference $\Delta T_{cr}\times10^{-3}$ of square FG plate resting on elastic foundation under LTD

k	Theory	(K_w, K_s)=(0,0)			(K_w, K_s)=(10,0)			(K_w, K_s)=(10,10)		
		a/h=5	a/h=10	a/h=20	a/h=5	a/h=10	a/h=20	a/h=5	a/h=10	a/h=20
0	Present	11.15922	3.22750	0.83307	11.51030	3.31527	0.85502	18.44031	5.04777	1.28814
	CPT [(a)]	13.66929	3.40982	0.84496	14.02036	3.49759	0.86690	20.95037	5.23009	1.30002
	FSDT [(b)]	11.15138	3.22725	0.83307	11.50246	3.31502	0.85501	18.43246	5.04752	1.28814
	HSDT [(a)]	11.15688	3.22736	0.83307	11.50796	3.31513	0.85501	18.43797	5.04764	1.28814
	TPT [(a)]	11.16112	3.22764	0.83309	11.51220	3.31541	0.85503	18.44220	5.04791	1.28816
	HSDT [(c)]	11.15688	3.22736	0.83307	11.50796	3.31513	0.85501	18.43797	5.04764	1.28814
1	Present	5.00190	1.41302	0.35871	5.30874	1.48973	0.37789	11.36568	3.00396	0.75645
	CPT [(a)]	5.94993	1.48045	0.36308	6.25678	1.55716	0.38226	12.31372	3.07140	0.76082
	FSDT [(b)]	4.99885	1.41292	0.35871	5.30570	1.48964	0.37789	11.36263	3.00387	0.75645
	HSDT [(a)]	5.00099	1.41297	0.35871	5.30784	1.48968	0.37789	11.36477	3.00391	0.75645
	TPT [(a)]	5.00264	1.41307	0.35872	5.30948	1.48978	0.37789	11.36642	3.00402	0.75645
	HSDT [(c)]	5.00099	1.41297	0.35871	5.30784	1.48968	0.37789	11.36477	3.00391	0.75645

Table 5 Continued

5	Present	3.90254	1.16021	0.29867	4.29288	1.25780	0.32306	11.99794	3.18406	0.80463
	CPT [(a)]	4.99396	1.24204	0.30405	5.38430	1.33962	0.32845	13.08936	3.26588	0.81002
	FSDT [(b)]	4.05274	1.17354	0.29959	4.44308	1.27113	0.32399	12.14814	3.19739	0.80555
	HSDT [(a)]	3.90735	1.16069	0.29871	4.29770	1.25827	0.32310	12.00275	3.18453	0.80467
	TPT [(a)]	3.90098	1.16006	0.29866	4.29132	1.25765	0.32306	11.99637	3.18391	0.80462
	HSDT [(c)]	3.90735	1.16069	0.29871	4.29770	1.25827	0.32310	12.00275	3.18453	0.80467
10	Present	4.02381	1.21841	0.31566	4.47735	1.33180	0.34401	13.43104	3.56850	0.90341
	CPT [(a)]	5.28555	1.31474	0.32204	5.73910	1.42813	0.35039	14.69174	3.66629	0.90993
	FSDT [(b)]	4.18778	1.23350	0.31672	4.64132	1.34688	0.34506	13.59396	3.58504	0.90460
	HSDT [(a)]	4.02576	1.21864	0.31568	4.47930	1.33203	0.34403	13.43194	3.57019	0.90357
	TPT [(a)]	4.02350	1.21837	0.31566	4.47705	1.33176	0.34401	13.42969	3.56992	0.90355
	HSDT [(c)]	4.02576	1.21864	0.31568	4.47930	1.33203	0.34403	13.43194	3.57019	0.90357

[(a)] (Zenkour and Sobhy 2011)
[(b)] (Yaghoobi and Torabi 2013)
[(c)] (Yaghoobi and Fereidoon 2014)

Table 6 Comparison of critical buckling temperature difference $\Delta T_{cr} \times 10^{-3}$ of square FG plate resting on elastic foundation under NTD

k	Theory	(K_w, K_s)=(0,0)			(K_w, K_s)=(10,0)			(K_w, K_s)=(10,10)		
		a/h=5	a/h=10	a/h=20	a/h=5	a/h=10	a/h=20	a/h=5	a/h=10	a/h=20
0	Present	11.16922	3.22750	0.83307	11.51030	3.31527	0.85502	18.44031	4.04777	1.28814
	CPT [(a)]	13.66929	3.40982	0.84496	14.02036	3.49759	0.86690	20.95037	5.23009	1.30002
	FSDT [(b)]	11.15138	3.22725	0.83307	11.50246	3.31502	0.85501	18.43246	5.04752	1.28814
	HSDT [(a)]	11.15688	3.22736	0.83307	11.50796	3.31513	0.85501	18.43797	5.04764	1.28814
	TPT [(a)]	11.16112	3.22764	0.83309	11.51220	3.31541	0.85503	18.44220	5.04791	1.28816
	HSDT [(c)]	11.15688	3.22736	0.83307	11.50796	3.31513	0.85501	18.43797	5.04764	1.28814
1	Present	6.94309	1.96140	0.49792	7.36903	2.06788	0.52454	15.77661	4.16978	1.05002
	CPT [(a)]	8.25905	2.05500	0.50399	8.68499	2.16148	0.53061	17.09257	4.26338	1.05608
	FSDT [(b)]	6.93886	1.96127	0.49792	7.36479	2.06775	0.52454	15.77238	4.16965	1.05002
	HSDT [(a)]	6.94183	1.96133	0.49792	7.36777	2.06781	0.52455	15.77535	4.16971	1.05002
	TPT [(a)]	6.94412	1.96147	0.49793	7.37005	2.06796	0.52455	15.77763	4.16985	1.05003
	HSDT [(c)]	6.94183	1.96133	0.49792	7.36777	2.06781	0.52455	15.77535	4.16971	1.05002
5	Present	4.88065	1.45101	0.37353	5.36883	1.57305	0.40404	15.00050	3.98211	1.00630
	CPT [(a)]	6.24563	1.55334	0.38026	6.73381	1.67538	0.41077	16.37004	4.08444	1.01304
	FSDT [(b)]	5.06851	1.46768	0.37468	5.55669	1.58972	0.40519	15.19291	3.99878	1.00745
	HSDT [(a)]	4.88668	1.45160	0.37357	5.37486	1.57364	0.40408	15.01109	3.98270	1.00635
	TPT [(a)]	4.87871	1.45082	0.37082	5.36688	1.57286	0.40403	15.00311	3.98192	1.00629
	HSDT [(c)]	4.88668	1.45160	0.37357	5.37486	1.57364	0.40408	15.01109	3.98270	1.00635
10	Present	4.65068	1.40823	0.36484	5.17488	1.53928	0.40784	15.52226	4.12613	1.04432
	CPT [(a)]	6.10899	1.51957	0.37221	6.63320	1.65062	0.40497	16.98057	4.23746	1.05169
	FSDT [(b)]	4.84020	1.42567	0.36606	5.36440	1.55672	0.39882	15.71178	4.14356	1.04553
	HSDT [(a)]	4.65293	1.40849	0.36486	5.17714	1.53954	0.39763	15.52451	4.12639	1.04434
	TPT [(a)]	4.65033	1.40818	0.36864	5.17453	1.53923	0.39760	15.52191	4.12608	1.04432
	HSDT [(c)]	5.65293	1.40849	0.36486	5.17714	1.53954	0.39763	15.52451	4.12639	1.04434

[(a)] (Zenkour and Sobhy 2011)
[(b)] (Yaghoobi and Torabi 2013)
[(c)] (Yaghoobi and Fereidoon 2014)

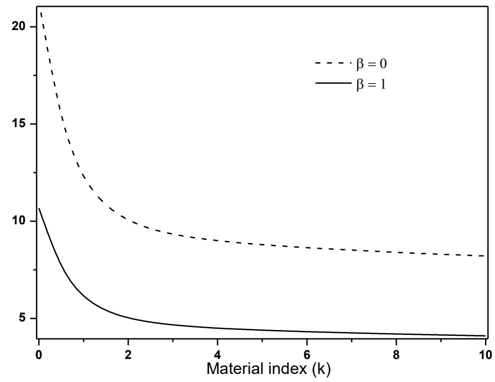

Fig. 1 Effect of the gradient index on the non-dimensional critical buckling load $\overline{N}$ of a square FG plate resting on elastic foundations. ($a/h=10$, $K_w=100$, $K_s=10$)

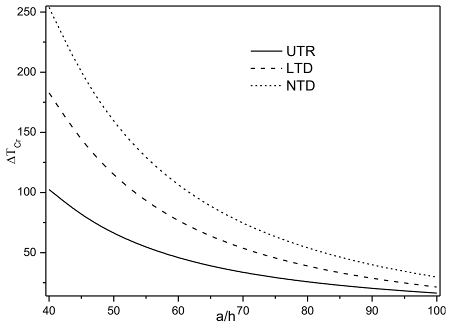

Fig. 2 Effect of the side-to-thickness ratio on the ΔT_{cr} of a square FG plate resting on elastic foundations. ($K_w=100$, $K_s=10$, $k=1$)

In Fig. 1, the effect of the material index (*k*) on the non-dimensional critical buckling loads $\overline{N}$ of a square FG plate resting on elastic foundations is investigated. It can be observed from this figure that the non-dimensional critical buckling load initially decreases, and then the variation of curves is not significant by increasing in the value of the material index.

Fig. 2 presents the variation of the ΔT_{cr} versus the variation of the *a*/*h* for all three types of thermal loads. From this figure, it can be seen that ΔT_{cr} is highest for NTD compare with two other thermal loads. Moreover, with increasing the plate thickness ratio, the ΔT_{cr} decreases.

6. Conclusions

The mechanical and thermal stability behaviors of FG plates resting on elastic foundation are studied analytically by employing the hyperbolic shear deformation and stress function concept. Various numerical examples are examined to prove the accuracy and efficacy of the proposed formulation. Results show that the proposed theory can be comparable with the existing HSDTs with a larger number of unknowns. Because of the interesting features of the proposed theory, the present findings will be a useful benchmark for assessing the reliability of other future plate models.

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