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Experimental validation of FE model updating based on multi-objective optimization using the surrogate model

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Abstract. In this paper, finite element (FE) model updating based on multi-objective optimization with the surrogate model for a steel plate girder bridge is investigated. Conventionally, FE model updating for bridge structures uses single-objective optimization with finite element analysis (FEA). In the case of the conventional method, computational burden occurs considerably because a lot of iteration are performed during the updating process. This issue can be addressed by replacing FEA with the surrogate model. The other problem is that the updating result from single-objective optimization depends on the condition of the weighting factors. Previous studies have used the trial-and-error strategy, genetic algorithm, or user's preference to obtain the most preferred model; but it needs considerable computation cost. In this study, the FE model updating method consisting of the surrogate model and multi-objective optimization, which can construct the Pareto-optimal front through a single run without considering the weighting factors, is proposed to overcome the limitations of the single-objective optimization. To verify the proposed method, the results of the proposed method are compared with those of the single-objective optimization. The comparison shows that the updated model from the multi-objective optimization is superior to the result of single-objective optimization in calculation time as well as the relative errors between the updated model and measurement.

Keywords: FE model updating; surrogate model; multi-objective optimization; pareto-optimal front

1. Introduction

Infrastructures, such as bridges, roads, and tunnels affect the services and necessities essential to maintaining our lives. During the life-cycle, infrastructures have been exposed to unexpected loads due to earthquakes, wind and traffic. Also, deterioration of materials can cause the performance degradation. Therefore, structural condition assessment is important to accurately express the state of structures. In order to evaluate the performance of the infrastructure, numerical models have been widely used recently. A finite element (FE) method is one of the representative numerical models. Typically, the FE model is constructed based on a physical law (e.g., equations of motion). Therefore, the FE model can provide the predictive capability for simulating the physical system of interest (i.e., fatigue life prediction, failure probability).

It should be noted that the FE models are a concise and simple representation of the physical system in the mathematical and numerical expression. Uncertainties related to modal properties always exist. In addition, infrastructures are exposed to various loadings and external environments. These also result in additional uncertainties between real structure and FE model. As a result of the abovementioned reasons, FE models which can reflect real structures are not sufficient to represent the actual behavior. Therefore, FE models inherently have model parameters that cannot be directly known, so that they must be expected from measurement (parameter inference). This procedure is also known as FE model updating. FE model can represent the actual behavior of the infrastructure by FE model updating.

Typically, FE model updating is a procedure to minimize the discrepancy between model predictions and measurements. This discrepancy is referred to residual. Minimizing the residual can simply perform FE model updating. This is also known as residual minimization. In the residual minimization, multiple residuals are calculated according to weighting factors of the objective function. Weighting factors should be assigned based on the uncertainties related to numerical model and measurement.

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Therefore, assignment of the optimal weighting factors in the objective function is one of the most important issues in the residual minimization based FE model updating. However, assigning the optimal weighting factors in objective function have not been known. Generally, trial-and-error strategy, genetic algorithm, or user's preference have been used to impose the weighting factors (Kim and Park 2004). To avoid the limitation of the residual minimization based FE model updating, the multiple-objective function has been employed recently. From the multi-objective function, multiple alternative solutions which are called as 'Pareto-optimal front' or 'non-dominated solutions' are obtained. The multi-objective function does not need weighting factors to calculate the multiple objective functions compared with residual minimization. But, there is one limitation to apply the multi-objective function in FE model updating. That is a computational issue. In order to minimize the multiple objective and obtain Pareto-optimal front simultaneously, the optimization process consumes computational resource highly over the residual minimization based FE model updating.

The FE model is a very important part to minimize the systematic errors due to modeling simplifications (Goulet *et al*. 2014, Erdogan *et al*. 2013a), FE discretization errors (Steenackers and Guillaume. 2006), and the omission of structural components (Eamon and Nowak 2004). Bias in the model prediction can occur due to systematic errors (Erdogan *et al*. 2013b), and this can result in incorrect estimations of the updating parameters (Goulet and Smith 2013). When the sophisticated modeling or finer FE discretization with full geometric description (a high-fidelity model) is used to minimize systematic errors (i.e., modeling and discretization errors), the computational burden increases. Such a high fidelity FE model can require high computational burden from seconds to minutes for a simple analysis (e.g., modal analysis). For a single run, this would not be important; however, when the numerical simulation will be iterated, the computational cost would be significantly increased. This computational issue can be addressed by either high-performance computing (HPC) (such as parallel or distributed computing) or the use of a surrogate model. In the case of HPC, it is not always possible, owing to computational and message passing interface programming requirements. In such cases, the surrogate model can provide an alternative solution for addressing computational burden in FE model updating using a high-fidelity model (Jin and Jung 2016b).

The surrogate model has recently been drawing attention as a faster option for iterative FE analysis. Surrogate modeling is a method of emulating a computer simulation model in the form of mathematical/statistical approximations to fit the relationship between input and output. In reliability, the surrogate model is not a new approach. However, FE model updating with a surrogate model has been recently employed, especially in the field of civil engineering (Fen and Chen 2010). Some studies based on surrogate models have been conducted in field such as polynomial models, multilayer perception, and so on (Marwala 2007).

The conventional method of the surrogate model can proceed with trial and error based on different designs. In the conventional method, complicated response surfaces are also difficult to represent under local variations of response behaviors, since the conventional method makes samples spread out equally in parameter space. In addition, it is not enough to apply the identical training samples to all target outputs. To deal with these issues, the sequential surrogate model based on the Kriging surrogate model has been proposed. The sequential surrogate model can address the abovementioned difficulties of the conventional approach. One important advantage of the sequential surrogate model is the ability to statistically interpret the uncertainty in the prediction, so that this approach can use the measure of infill criteria and update a surrogate model by adding a new sample (Jin and Jung 2016a).

FE model updating with multi-objective optimization based on the surrogate model is proposed in this study. Jin and Jung 2016b have been performed FE model updating based on surrogate model in lab-scale structure. In this paper, the verification of the proposed method was performed in full-scale structure. To validate the proposed method, a steel plate girder bridge is selected as a test-bed bridge, and the ambient vibration test is performed to obtain the modal properties of the target bridge. From the field test, 4 natural frequencies (i.e., the 1st bending and torsion, the 2nd bending and torsion modes) are obtained. These frequencies are set as the calibration values (i.e., the 1st bending, 2nd bending and torsion) and the validation value (i.e., the 1st torsion) when FE model updating is conducted. The initial FE model is constructed by ANSYS APDL and updated using the obtained modal properties by single-objective function with various cases of weighting factors and the proposed multi-objective function. Finally, the updated results of MOF are compared those of SOF with regard to the most preferred model and the number of iterations.

2. Theoretical background

2.1 Weighted sum for residual minimization

In mathematics, the generalized minimal residual method is an iterative method for the numerical solution of a non-symmetric system of linear equations. The method approximates the solution by the vector in a Krylov space with minimal residual. FE model updating is a process of modifying updating parameters within determined boundary condition. It is a matter of mathematical optimization that finds residual-based objective functions numerically minimal, which fundamentally reflects the discrepancy between the FE derived predictions and the measured data. Traditionally, uncertainties are defined for the residual value between predicted and measured values. In most applications, when uncertainties associated with model simplifications are included, they have been modeled through assuming independent Gaussian distributions having a mean value of zero. Combining these uncertainties leads to symmetrical combined probability curves. While such an assumption may be reasonable for measurement errors such as noise and sensor resolution, it is rarely justifiable for uncertainties related to model simplifications that are common in structural engineering.

The conventional FE model updating employs an optimization technique with the single-objective function (SOF). General equation of SOF can be stated as follows (Jin *et al*. 2014)

$$min = \sum_{i=1}^{n} \omega_i F_i(\boldsymbol{X}) = \omega^T F(\boldsymbol{X}) \tag{1}$$

$$subject\ to\ g_j(\boldsymbol{X}) < 0, \qquad j = 1,2,\cdots \tag{2}$$

$$h_k(\boldsymbol{X}) = 0, \qquad k = 1,2,\cdots \tag{3}$$

$$\boldsymbol{X}^L \leq \boldsymbol{X} < \boldsymbol{X}^U, \qquad \sum \omega_i\,, \qquad \omega_i \geq 0 \tag{4}$$

where X is an updating parameter vector; $F_i(X)$ is the sub-objective function that returns a scalar value; n is the number of the sub-objective functions; ω_i is the corresponding weighting factor to each sub-objective functions; $g_i(X)$ and $h_j(X)$ are inequality and equality constraints, respectively; and X^L and X^U are lower and upper constraint vectors of the updating parameter vector, respectively. The weighting factors play an important role in balancing the significance of the sub-objective functions in the SOF, whereas they are generally selected by a user based on experience and expertise.

In the FE model updating method, the objective function is formulated in terms of the discrepancy between finite element and reference properties. When experimental modal properties (i.e., natural frequencies) are used as reference properties, the SOF can be formulated as the weighted sum of sub-objective function.

2.2 FE model updating based on multi-objective function

In multi-objective function (MOF), all objective function spaces are considered independently without assigning weighting factors. MOF aims to find a set of preferred solutions called as Pareto-optimal front or non-dominated solutions as shown in Fig 1. The practical solution can be found during the search of solutions, and the black line is considered as the feasible solution domain. In the feasible domain, non-dominated solutions, at which an objective function value cannot be improved without degradation of the other objective function values, constitute the Pareto-optimal front (Jin *et al*. 2014). Multi-objective function problem can be expressed as

$$min\big(F_1(\boldsymbol{X}), F_2(\boldsymbol{X}), \cdots, F_M(\boldsymbol{X})\big) \tag{5}$$

$$subject\ to\ g_j(\boldsymbol{X}) < 0, \qquad j = 1,2,\cdots \tag{6}$$

$$h_k(\boldsymbol{X}) = 0, \qquad k = 1,2,\cdots \tag{7}$$

$$\boldsymbol{X}^L \leq \boldsymbol{X} \leq \boldsymbol{X}^U \tag{8}$$

In contrast to SOF, the solution of MOF consists of sets of trade-offs between objectives. In the case of SOF, the various results can be obtained depending on the weighting factors in each objective function. The goal of MOF is to generate these trade-offs. Surveying all these trade-offs in particularly is important because it provides the system designer with the ability to understand and weigh the different choices available to them. Fig. 2 shows the comparison between SOF and MOF.

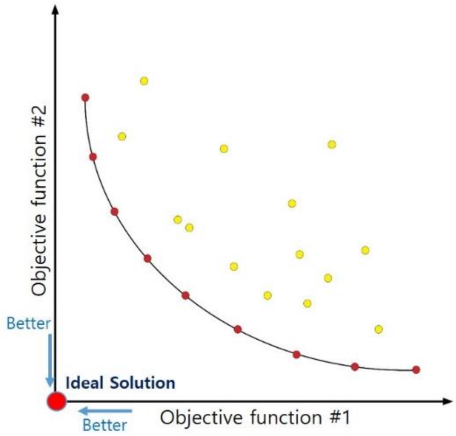


Fig. 1 Pareto-optimal front

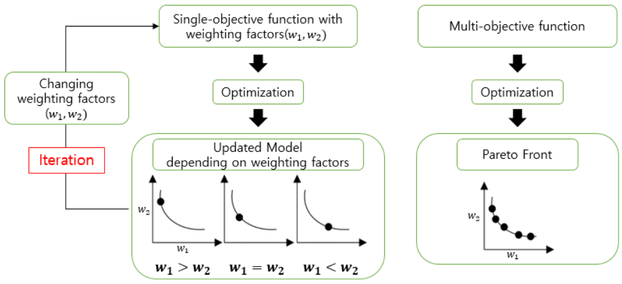


Fig. 2 Comparison SOF and MOF (Jin *et al*. 2014)

SOF which sets one case of weighting factor provides the single solution through a single run, while MOF provides a set of solution that, when all the objectives are considered, other solutions in the search space are better than them. This set is known as the Pareto-optimal front. The optimal solution in MOF can be defined from a mathematical concept of partial ordering. The Pareto-optimal front is shown in Fig. 1.

The Pareto optimal solution is the vectors of the solution from SOF. Deb (2008) and Zitzler *et al* (1999) demonstrated that one of the Pareto-optimal solutions is the solution of single objective optimization. The effect of different weighting factors is also shown. The main goal is to obtain the Pareto-optimal solutions and compare each optimal solution from FE model updating based on SOF. With enough time to update the FE model, MOF can be used and effective to acquire multiple solutions in a single optimization run.

In order to obtain the Pareto-optimal front, there are many related algorithms such as non-dominated sorting genetic algorithm-2 (NSGA-2), simulated annealing, particle swarm solver and so on. In this study, a multi-algorithm, genetically adaptive multi-objective optimization (AMALGAM) was used as an optimization algorithm (Vrugt and Robison 2007). Compared with the conventional optimization algorithms, this method runs multiple different search strategies simultaneously for population and adaptively updates the preference of each of these individual methods based on their reproductive success (Jeong 1996). It consists of 4 types of evolutionary algorithms: (1) Differential evolution (DE), (2) Adaptive Metropolis sampler (AMS), (3) Particle swarm optimizer (PSO), and (4) Genetic algorithm (GA). The goal is to determine values for modal parameters that provide the best possible solution to an objective function, or a set of optimal trade-off values in the case of two or more conflicting objectives (Vrugt 2016).

2.3 Surrogate model

In various scientific fields, mathematical models are used to describe processes that are very difficult to analysis, and these models are usually implemented with computer codes (Kennedy and O'Hagan 2001). As the computer performance improves and the efficiency of the FE program increases, the user can easily model the target structures and a more sophisticated modeling method is available (Catbas *et al*. 2013). In spite of the development of technology, users tend to construct the model simple to reduce the analysis time. When constructing FE model simple, it is insufficient to represent the real structure accurately. This indicates that poorly constructed FE model can result in systematic errors caused by modeling simplifications (Goulet *et al*. 2014). The simple FE models can lead to inaccurate prediction (Goulet and Smith 2013). To deal with this issue, two options are suggested. One is that using analytical solution is to validate the simplified model. The other is to construct FE model sophisticated. Generally, the former is not available. Therefore, to establish the sophisticated model would be better than to construct a simple model.

The more sophisticated FE model exists, the more analysis time is needed. In the case of a few times run, it is not important; however, many iterations are needed to analysis in finite-element analysis (FEA). Therefore, some applications become impractical under iterative analysis with an expensive computation model.

To address the computation burden, there are two approaches available. The first solution is to use high-performance computing and the other one is to use a fast surrogate of the computation model. The surrogate model has recently attracted considerable attentions as fast alternatives to the iterative FEA (Jones 2001, Forrester *et al*. 2008). It is also called the response surface or metal model. The surrogate model uses a mathematical/statistical model to match the input and output relationships. The procedure of constructing the surrogate model is as follows: (1) model parameters and target outputs are defined; (2) to generate training samples, the design of experiments is employed; (3) once training samples are generated, model simulation is performed to generate model outputs; and (4) based on the observed inputs and outputs, the surrogate model is constructed.

In this study, the Kriging surrogate model is used. The Kriging surrogate model is a statistical approximation of a function as a realization of the Gaussian process, also known as Gaussian process model (GPM). Fig. 3 shows the

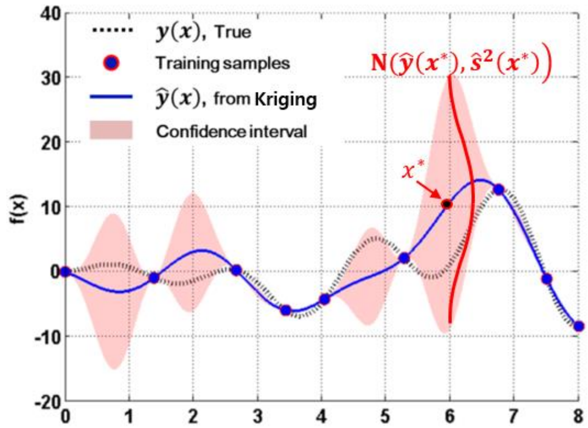


Fig. 3 Kriging with training samples (Jin and Jung 2016a)

Kriging model in one-dimensional space (Jin and Jung 2016a). The unknown prediction $y(x^*)$ is treated as the Gaussian process random variable based on training samples (x') (Kennedy and O'Hagan 2001). Therefore, the unknown prediction $y(x^*)$ is a combination of the deterministic and stochastic component.

$$y(x^*) = \mu(x^*) + Z(x^*)\ with\ Z(x^*) \sim GP\big(0, C(x^*, x')\big) \quad (9)$$

where $\mu(x^*)$ is a deterministic component to capture the global trend $Z(x^*)$ is the stochastic component to capture the local variants and is assumed to be the Gaussian process with zero mean and covariance $C(x^*, x')$.

The spatial correlation among the training samples derives the covariance function of GPM. In the deterministic simulations, the prediction $y(x)$ being modeled is smooth and continuous over the parameter space. That is, when the two samples $y(x^1)$ and $y(x^2)$ will have a comparable value if the distance between two samples is getting closer. This property can be demonstrated statistically that the predictions are correlated with a spatial distance. Therefore, the covariance between any samples $C(x^*, x')$ derived from the correlation, as

$$C(x^*, x') = \sigma^2 \psi(x, x') \quad (10)$$

where the variance σ^2 provides overall dispersion relative to the mean of the Gaussian process, and $\psi(\cdot,\cdot)$ denotes the spatial correlation matrix.

A typical choice of the correlation matrix $\psi(\cdot,\cdot)$ is the *k*-dimensional Gaussian correlation function as

$$\psi^{ij} = exp\left(-\sum_{p=1}^{k} \theta_p \left\| x_p^i - x_p^j \right\|^{m_p}\right) = corr[y(x^i), y(x^j)] \quad (11)$$

where subscript 'p' denotes the dimension of sample *x*, the superscript 'i' and 'j' indicate the 'i'-th and 'j'-th sample, respectively. And $\left\| x_p^i - x_p^j \right\|^{m_p}$ is the relative distance measure between two samples in a parameter space with *m*-norm. the Gaussian correlation function contains parameters corresponding to each dimension (θ_p and m_p). These parameters determine how fast the correlation decays in each dimension, and they reflect the significant importance of each dimension (Forrester *et al*. 2008). The main reason

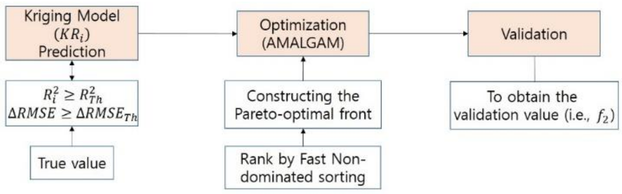


Fig. 3 Flowchart of the proposed method

of using Eq. (11) is to express the various shapes of the spatial correlation. To reduce the computational complexity, Eq. (11) can be expressed as a Euclidean distance ($m_p = 2$).

2.4 The proposed method

In the proposed approach, FE model updating is processed through multi-objective optimization with the surrogate model. To construct the Kriging surrogate model, initial samples and validation samples are generated with upper and lower bounds on parameters. These are used as inputs to run FE analysis to obtain the outputs. Then, initial Kriging model is constructed. By using the initial Kriging surrogate model, the validation sample outputs are predicted. Validation measurements such as R^2 and $RMSE$ are evaluated (Jin and Jung 2016). When the stopping criteria are met, the process can be stopped. After constructing Kriging model, the AMALGAM algorithm is used as the MOF algorithm (Vrugt and Robison 2007). The algorithm is initiated by an initial population, drawn randomly from some prior ranges using, for instance, Latin hypercube sampling. Then, each parent is assigned a rank using the fast non-dominated sorting algorithm of Deb *et al*. (2002). Instead of implementing a single operator for reproduction, four different recombination methods (i.e., (1) DE, (2) AMS, (3) PSO, and (4) GA) can be simultaneously generated the offspring. After the offspring has been created, the parents and children are combined. And the objective functions are ranked using the non-dominated sorting algorithm. After constructing the Pareto-optimal front, the validation process is performed.

In this study, the 1st torsion mode is set as the validation value to compare the updated models between SOF and MOF. The advantages of the AMALGAM algorithm are as follows: (1) by facilitating direct information exchange between individual algorithms, the method merges the strengths of different search strategies to increase the speed of evolution toward the Pareto-optimal solutions, and (2) by adaptively changing preferences to individual search algorithms during the course of the optimization, the method should adapt quickly to the specific difficulties and peculiarities of the optimization at hand. The flowchart of FE model updating proposed in this study is shown in Fig. 3.

3. Experimental validation

In this chapter, the proposed method is briefly demonstrated and verified using field test data. A field test

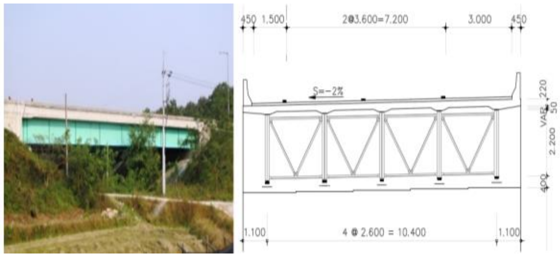


Fig. 4 Samseung Bridge and its cross section

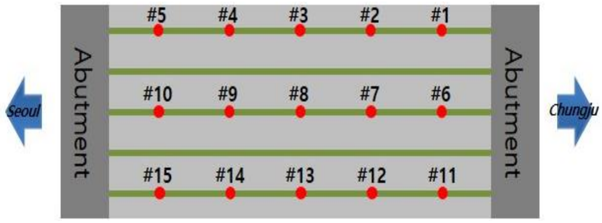


Fig. 5 Sensor deployment

was conducted in a steel plate girder bridge with a single span. It is located in Chung-bu inland expressway, where many proposed techniques are carried out and there are restrictions on vehicle traffic. In the steel plate girder bridge, the ambient vibration test was performed to obtain the natural frequency of a target bridge. The obtained data are set as calibration values and validation value to compare the proposed method (i.e., MOF) and the conventional method (i.e., SOF).

3.1 Target bridge: steel plate girder bridge

The target bridge investigated for the test is a steel plate girder bridge called the Samseung bridge. It is a single span with 5 girders and 9 cross girders including 3 diaphragms. It is 40.0 m long and 12.6 m wide and also skewed 2%, as shown in Fig. 4.

3.2 Ambient vibration test and modal analysis

The dynamic responses were obtained by performing the ambient vibration test. The obtained dynamic responses were used for the target output and validation value when the FE model updating was performed and after optimization. A total of 15 accelerometers were arranged as shown in Fig. 5 and were used to obtain the dynamic responses. The data acquisition system was composed of the computer (NI PXI-1000B), the multifunction DAQ (NI USB-6353), a sensor signal conditioner (PCB model 481a), and 15 accelerometers (PCB 393B12). The data was set to 100 Hz sampling frequency and collected for 120 minutes. Fig. 6 indicates the accelerometer data of the 1st array of the sensor (No.1 to No.5). From the ambient vibration data, modal identifications, which are stochastic subspace identification (SSI) and cross power spectral density (CPSD), were used to figure out the modal properties such as natural frequencies and mode shapes of the target bridge. Fig. 7 shows the data of the SSI chart and the CPSD of the target bridge. Several mode shapes were identified and the related mode shapes are depicted in Fig. 8.

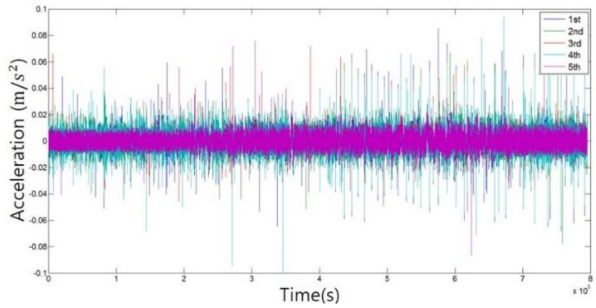
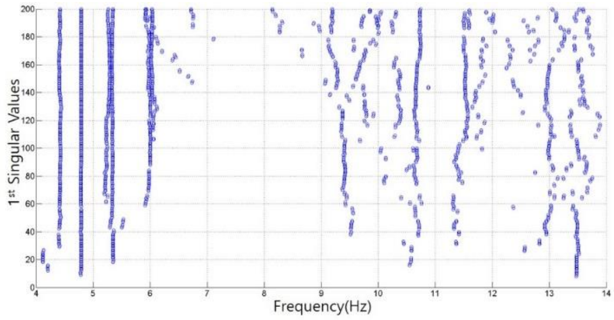
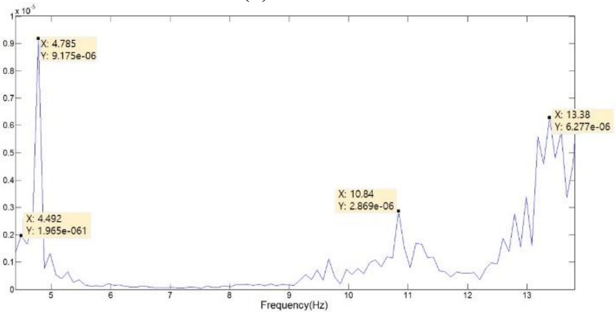


Fig. 6 Obtained ambient vibration data (1st array of the sensor)

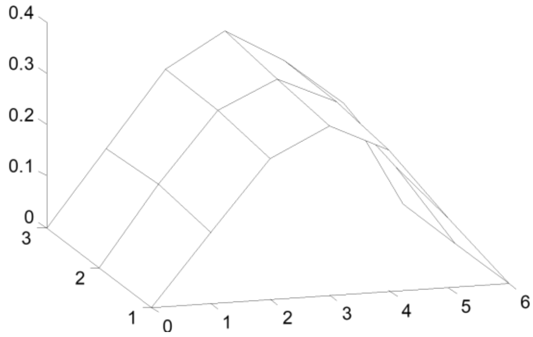


(a) SSI chart

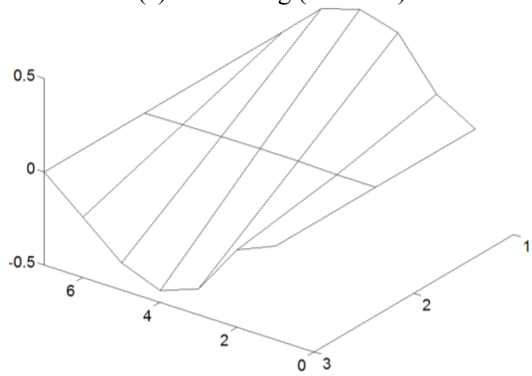


(b) CPSD

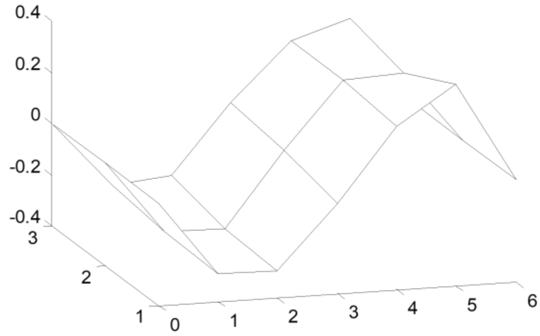
Fig. 7 SSI chart and CPSD of Samseung Bridge



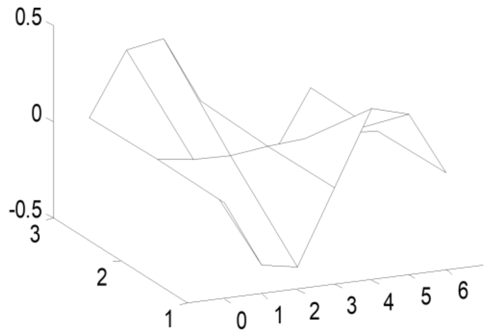
(a) 1st bending (4.419Hz)



(b) 1st torsion (4.787Hz)



(c) 2nd bending (10.683Hz)



(d) 2nd torsion (13.483Hz)

Fig. 8 Identified mode shapes (the 1st and 2nd bending and torsion modes)

4. FE model updating

FE model updating is conducted in two cases: (1) single-objective optimization and (2) multi-objective optimization. Two cases are based on the surrogate model. From the field test, four modes are obtained. In this study, three mode shapes (i.e., the 1st bending, and the 2nd bending & torsion) are set as the calibration values and the 1st torsion mode is set as the validation value.

4.1 Initial FE model

The initial FE model was constructed by using ANSYS APDL as shown in Fig. 9. The initial FE model is composed of 131,909 elements with shell and beam elements considering the related geometry and structural details of the target bridge. Slab, barrier, girders and cross girders except the diaphragms were modelled by shell elements. The diaphragms were only modelled by beam element. Based on the drawing of the target bridge, boundary conditions of each end represent roller and hinge, respectively. The natural frequencies of the initial FE model are compared with the data of experiment results in Table 1. As seen from the table, the errors of natural frequencies range from 7 to 16 %. The smallest error is 7.29% in the 1st bending mode, the biggest error is 16.35% in the 2nd torsion. Inferring from the revealed discrepancies of the natural frequencies, the initial FE model requires model updating to represent the existing bridge accurately.

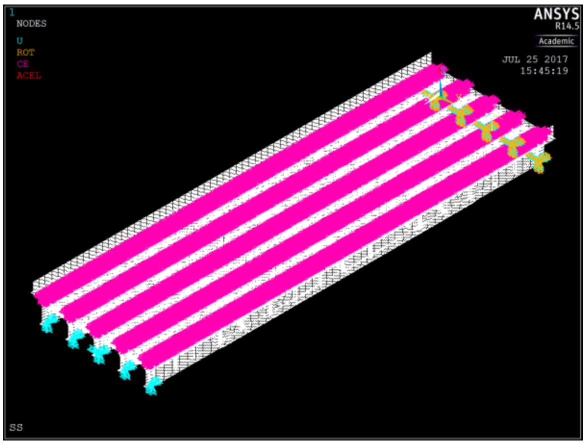


Fig. 9 Initial FE model

Table 1 Comparison the natural frequencies

Mode	f^{FEM}	f^{EXP}	Error (%)
1st bending	4.097	4.419	7.29
1st torsion	4.404	4.787	8.00
2nd bending	9.539	10.683	10.71
2nd torsion	11.284	13.483	16.31

Table 2 Updating parameters

Updating parameter	Initial value	Lower bound	Upper bound
Elastic modulus of slab	24 GPa	0.7	1.5
Elastic modulus of girder	211.6 GPa	0.7	1.5
Elastic modulus of cross girder	211.6 GPa	0.7	1.5
Normal stiffness	2e8 N/mm	0.1	2.5
Sticking stiffness	2e8 N/mm	0.1	2.5

4.2 Updating parameters for FE model updating

Selecting updating parameters is an important part of FE model updating. Mathematically, if the estimation of too many parameters is attempted, then the problem may appear ill-conditioned or underdetermined because the observations are limited in vibration testing (Brownjohn *et al*. 2001). Therefore, it is preferred to select the updating parameters as small as users can. In the case of the target bridge, uncertainties were caused by the properties of slab, girder, cross girder and boundary conditions. When considering the effects of aging and damage, five updating parameters were selected (i.e., elastic moduli of slab, girder and cross girder, and normal/sticking stiffness of boundary conditions). Table 2 indicates the initial values of the updating parameters and their upper and lower bounds.

4.3 FE model updating with multi-objective optimization

The initial FE model was updated by using MOF. In the case of MOF, the Pareto-optimal front can be obtained through a single run without assigning weighting factors in objective function. The objective functions, which were used in FE model updating, are as follows

Table 3 Details of updating in MOF

Parameters	Values	Parameters	Values
Population size	500	No. run	1
Generation	200	No. iteration	100,000

$$\mathrm{F}_i = \left(\frac{f_i^{exp} - f_i^{FEM}}{f_i^{exp}}\right)^2 \tag{12}$$

where 'i' indicates the 'i'-th natural frequencies. f_i^{exp} and f_i^{FEM} indicate 'i'-th experiment and numerical model natural frequencies, respectively. Table 3 represents the details of FE model updating with MOF.

In the FE model updating method, basically, the target output (f^{FEM}) is found to be the closest to the measured value (f^{exp}), but since there may be uncertainty in the measured value, it can be dangerous to set the most preferred model to the case where the objective function (or the relative error between the target output and the measured value) is minimum. The proposed method based on multi-objective optimization can be more effectively applied in such cases because it is able to provide multiple solutions (i.e., the Pareto optimal front) in a single optimization run. Thus, it would be more effective to use the updated models through the multi-optimization in the condition assessment of bridges. This is one of the crucial advantages of the proposed method from a practical point of view.

Fig. 10 shows the improvement of the 3 target outputs (i.e., f_1, f_3 and f_4) and one for the validation value (i.e., f_2). The square symbol represents the mean value and two diamonds are maximum and minimum values in the proposed approach. Also, the red dotted line indicates the measurement data. According to Table 1, all the initial values of target outputs are well below the measured values. After FE model updating, however, all the updated models were improved in target outputs as shown in Fig. 10. Also, in the case of the validation value, the values of f_2 were well distributed covering the measurement data. This means that the proposed FE model updating is able to give multiple feasible solutions with a reasonable computational cost.

Fig. 11 shows the distribution of the updated parameters in MOF. To match the measurement data from the initial FE model, the values of updating parameters were increased because the natural frequencies of the initial FE model are lower than the natural frequencies of measurement. Since all target outputs after FE model updating were higher than those in the initial FE model, the updating parameters tended to move slightly toward the upper bound to be stiff; however, some parameters (e.g., elastic moduli of slab and cross girder and normal stiffness) are widely spread in all regions in MOF. To compare the updated values in more detail, the Pareto-optimal front was examined in the next section (see Fig. 12).

4.4 Comparison between MOF and SOF

The updated models obtained from the proposed method

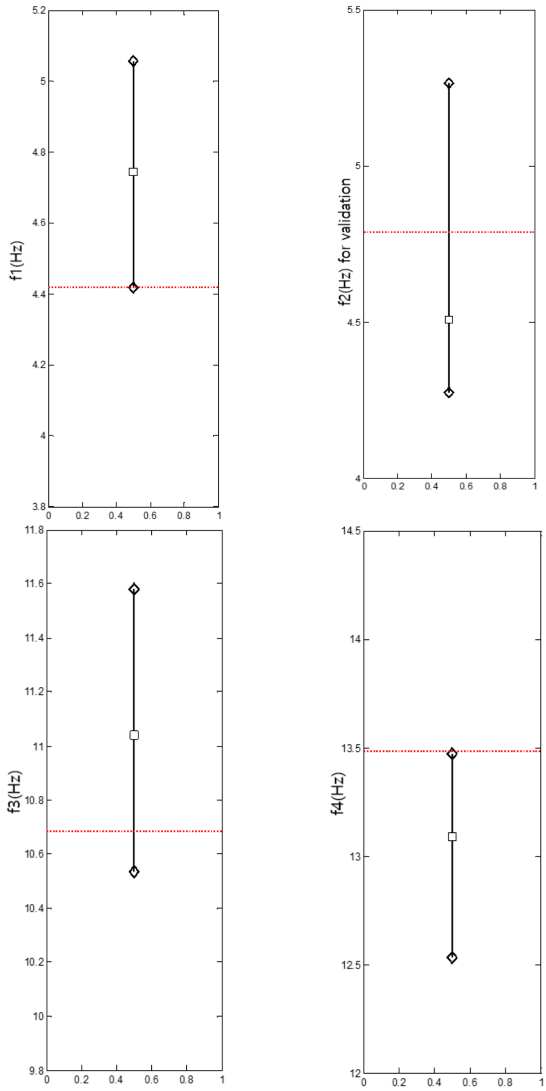


Fig. 10 Updating results of MOF

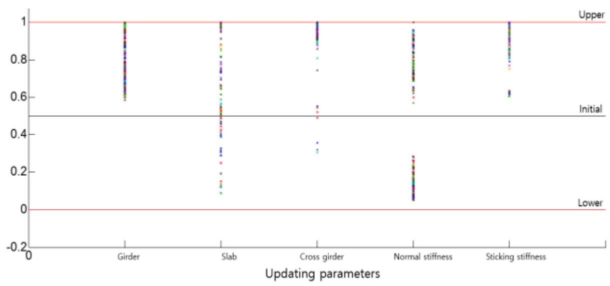


Fig. 11 Distribution of updated parameters in MOF

(i.e., MOF) were compared with those obtained from the conventional method (i.e., SOF) in order to verify the effectiveness of the proposed method in regard to the calculation time as well as the relative errors. In the case of the calibration values (i.e., f_1, f_3 and f_4), both methods result in accurately updated values compared with measurement; however, the validation value (i.e., f_2) shows that SOF has a large relative error compared with MOF. In the case of MOF, the validation value is also distributed upper and lower bound of measurement data. In the case of updating parameters, some parameters of each method are widely

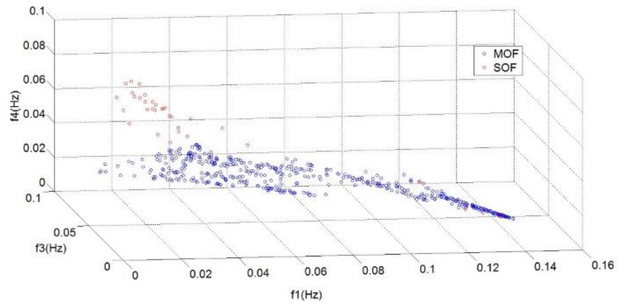


Fig. 12 Pareto-optimal front

Table 4 The most preferred model

Mode	f^{EXP}	SOF	Error (%)	MOF	Error (%)
1st bending	4.419	4.517	-2.22	4.499	-1.81
2nd bending	10.683	10.853	-1.59	10.974	-2.72
2nd torsion	13.483	12.732	5.57	12.847	4.72
1st torsion	4.787	4.329	9.57	4.993	-4.30

spread in all regions in the cases of SOF and MOF. To compare the results of SOF and MOF, the Pareto-optimal front is examined as shown in Fig. 12. The distribution of updated models can be described in Fig. 12. In the case of SOF (i.e., red cross symbols), updated solutions cannot express the Pareto-optimal front well compared with MOF. On the other hand, the results of MOF (i.e., blue circle symbols) can show the non-dominated solution in each objective function.

In Table 4, the most preferred models obtained from SOF and MOF are represented. Even in the case of the most preferred models, it can be seen that MOF is superior to SOF in the validation value as well as in the calibration values. Also, to obtain the most preferred model, SOF needs 229,950 iterations, while MOF needs only 100,000 iterations. In this study, 46 cases of weighting factors are considered in SOF. To obtain the validation value similar to MOF, thus, more conditions of weighting factors should be considered in SOF, resulting in more calculation burden.

5. Conclusions

In this study, FE model updating based on multi-objective optimization using the surrogate model was proposed and performed in full-scale structure. The proposed method is able to address the limitations of the conventional method such as computational burden due to many FE analysis and difficulty in finding an optimal weighting factor. To validate the effectiveness of the proposed method, the initial FE model of an in-service steel plate girder bridge was constructed by ANSYS APDL and its modal properties were obtained through the ambient vibration test for the FE model updating. According to the results of the FE model updating, it was confirmed that most of the updated models were well improved in target outputs and some of the updating parameters are well distributed. Most of the updated models were improved in target output. Especially, in the case of the validation value (i.e., f_2), its distribution was well spread covering the measurement data. Also, through comparison between the proposed method (i.e., MOF) and the existing method (i.e., SOF), it was demonstrated that the proposed method made a better Pareto optimal front than the existing method, and it was confirmed that the number of iterations required in the proposed method was much smaller than that in the existing method (i.e., 100,000 iterations vs. 299,950 iterations). In addition, it was found that the proposed method could construct an excellent model with a much smaller relative error even in finding the most preferred model.

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