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A new quasi-3D sinusoidal shear deformation theory for functionally graded plates

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Abstract. In this paper, a new quasi-3D sinusoidal shear deformation theory for functionally graded (FG) plates is proposed. The theory considers both shear deformation and thickness-stretching influences by a trigonometric distribution of all displacements within the thickness, and respects the stress-free boundary conditions on the upper and lower faces of the plate without employing any shear correction coefficient. The advantage of the proposed model is that it posses a smaller number of variables and governing equations than the existing quasi-3D models, but its results compare well with those of 3D and quasi-3D theories. This benefit is due to the use of undetermined integral unknowns in the displacement field of the present theory. By employing the Hamilton principle, equations of motion are obtained in the present formulation. Closed-form solutions for bending and free vibration problems are determined for simply supported plates. Numerical examples are proposed to check the accuracy of the developed theory.

Keywords: quasi 3D theory; bending; vibration; functionally graded plate

1. Introduction

Theoretical studies of mechanical response of structural components fabricated from functionally graded material (FGM) have taken considerable importance among the investigators. This is because of the huge potential of FGM in various engineering applications such as aerospace, mechanical, civil, automotive, electrical, biomedical etc (Fekrar *et al*. 2014, Celebi *et al*. 2016, Kar and Panda 2015, Bourada *et al*. 2015, Belkorissat *et al*. 2015, Barati and Shahverdi 2016, Beldjelili *et al*. 2016, Bellifa *et al*. 2017a, Bouafia *et al*. 2017). Although FGMs are basically employed for high temperature environment, its behavior at ambient condition is also necessary for its safety and reliability.

Since the shear deformation impacts are more considered in FGMs, shear deformation models such as first shear deformation theory (FSDT) and higher-order shear deformation theories (HSDTs) should be employed. The FSDT (Nguyen *et al*. 2008, Zhao *et al*. 2009, Hosseini-Hashemi *et al*. 2010, Hosseini-Hashemi *et al*. 2011a, Irschik 1993, Nosier and Fallah 2008, Saidi *et al*. 2011, Yang *et al*. 2009, Meksi *et al*. 2015, Mantari and Granados 2015, Hadji *et al*. 2016, Bellifa *et al*. 2016) produces reasonable results, but needs a shear correction coefficient that is difficult to assess correctly due to its dependency on many parameters incorporating geometry, boundary conditions, and loading conditions. The HSDTs (Reddy 2000, Ferreira *et al*. 2005, Pradyumna and Bandyopadhyay 2008, Hosseini-Hashemi *et al*. 2008b, Xiang *et al*. 2008, Bouderba *et al*. 2013, Tounsi *et al*. 2013, Bessaim *et al*. 2013, Tounsi *et al*. 2013, Ait Amar Meziane *et al*. 2014, Zidi *et al*. 2014, Taibi *et al*. 2015, Ait Yahia *et al*. 2015, Mahi *et al*. 2015, Bounouara *et al*. 2016, Hebali *et al*. 2016, Draiche *et al*. 2016, Bousahla *et al*. 2016, Saidi *et al*. 2016, Javed *et al*. 2016, Chikh *et al*. 2017, Bellifa *et al*. 2017b, Klouche *et al*. 2017, Hanifi Hachemi Amar *et al*. 2017, Sekkal *et al*. 2017, Menasria *et al*. 2017) do not need a shear correction coefficient, but their governing equations are more complicated than those of the FSDT. It should be indicated that the thickness stretching influence (i.e., ε_z=0) is neglected in both the FSDT and HSDTs by considering a constant deflection within the thickness of the plate. Although this supposition is justifiable for moderately thick FG structures, it is not appropriate for thick FG ones (Qian *et al*. 2004). The importance of the thickness stretching influence in FG plates has been demonstrated in the article presented by Carrera *et al*. (2011).

Quasi-3D theories are HSDTs in which the deflection is presented as a higher-order distribution within the thickness of the plate, and thus, thickness stretching influence is incorporated. One can found several quasi-3D models developed in the literature. For example, Kant and Swaminathan (2002) developed a quasi-3D theory with all displacement components expressed as a cubic variation

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within the thickness. The shear deformation theories proposed by Chen *et al*. (2009), Talha and Singh (2010), Reddy (2011), Neves *et al*. (2013) are based on a cubic distribution of in-plane displacements and a quadratic distribution of deflection. Ferreira *et al*. (2011), Bousahla *et al*. (2014), Hamidi *et al*. (2015) utilized the trigonometric functions for both in-plane and transverse displacement. Neves *et al*. (2012a, b) used the sinusoidal (Neves *et al*. 2012a) and hyperbolic (Neves *et al*. 2012b) functions for in-plane displacements, while the transverse displacement is modeled by the polynomial functions. The theories developed by Hebali *et al*. (2014), Belabed *et al*. (2014), Bennoun *et al*. (2016) are based on a hyperbolic variation of all displacement components. Benahmed *et al*. (2017) proposed a novel quasi-3D hyperbolic shear deformation theory for functionally graded thick rectangular plates on elastic foundation. Abualnour *et al*. (2018) proposed also a new quasi-3D trigonometric plate theory for free vibration analysis of advanced composite plates. Recently, a generalized formulation is proposed by Mantari and Guedes Soares (2012a) in which many hybrid quasi-3D models with six variables can be obtained. Although the hybrid quasi-3D models (Mantari and Guedes Soares, 2012a) contain six variables, they are still more complicated than the FSDT. Thus, a simple quasi-3D theory presented in the present investigation is necessary.

This present work aims to propose a simple quasi-3D theory with only five variables for bending and dynamic response of FG plates. The displacement field is presented based on a sinusoidal variation for all displacements. By considering integral terms in the in-plane displacements, the number of variables of the theory is reduced, thus saving computational time. Based on Hamilton principle, the equations of motion are obtained and solved for bending and dynamic problems of a simply supported plate. Numerical examples are proposed to check the accuracy of the present quasi-3D theory.

2. Mathematical formulation

As indicated above, the kinematic of the proposed quasi-3D theory is taken based on the sinusoidal distribution for all displacement components

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + f(z)\varphi_x(x,y,t) \tag{1a}$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + f(z)\varphi_y(x,y,t) \tag{1b}$$

$$w(x,y,z,t) = w_0(x,y,t) + g(z)\varphi_z(x,y,t) \tag{1c}$$

where u_0; v_0; w_0, φ_x, φ_y, and φ_z are six unknown displacements of the mid-plane of the plate, and $f(z)$ is a shape function showing the variation of the transverse shear strains and shear stresses within the thickness. In this work, the shape function is taken based on the sinusoidal function given by Touratier (1991) as

$$f(z) = \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right) \quad \text{and} \quad g(z) = \frac{df(z)}{dz} \tag{2a}$$

with h is the thickness of the plate. In this work, we consider that (Bouderba *et al*. 2016, Bourada *et al*. 2016, Boukhari *et al*. 2016, Besseghier *et al*. 2017, El-Haina *et al*. 2017, Fahsi *et al*. 2017, Khetir *et al*. 2017)

$$\varphi_x = \int \theta(x,y)dx \quad \text{and} \quad \varphi_y = \int \theta(x,y)dy \tag{2b}$$

Thus, the kinematic of the proposed can be expressed in a simpler form as

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + k_1\, f(z)\int \theta(x,y,t)\,dx \tag{3a}$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + k_2\, f(z)\int \theta(x,y,t)\,dy \tag{3b}$$

$$w(x,y,z,t) = w_0(x,y,t) + g(z)\varphi_z(x,y,t) \tag{3c}$$

The coefficients k_1 and k_2 depends on the geometry. It can be observed that the kinematic in Eq. (3) uses only five unknowns (u_0, v_0, w_0, θ andφ_z). The nonzero strains associated with the displacement field in Eq. (3) are

$$\begin{gathered}
\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} + z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix} + f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix}\\
\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} = g(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix}\\
\varepsilon_z = g'(z)\,\varepsilon_z^0
\end{gathered} \tag{4}$$

Where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} = \begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial x}\\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}\end{Bmatrix}, \quad \begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix} = \begin{Bmatrix}-\dfrac{\partial^2 w_0}{\partial x^2}\\ -\dfrac{\partial^2 w_0}{\partial y^2}\\ -2\dfrac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix} \tag{5a}$$

$$\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix} = \begin{Bmatrix}k_1\theta\\ k_2\theta\\ k_1\dfrac{\partial}{\partial y}\displaystyle\int\theta\,dx + k_2\dfrac{\partial}{\partial x}\int\theta\,dy\end{Bmatrix}$$

$$\begin{gathered}
\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} = \begin{Bmatrix}k_2\displaystyle\int\theta\,dy + \dfrac{\partial\varphi_z}{\partial y}\\ k_1\displaystyle\int\theta\,dx + \dfrac{\partial\varphi_z}{\partial x}\end{Bmatrix}\\
\varepsilon_z^0 = \varphi_z
\end{gathered} \tag{5b}$$

and

$$g'(z) = \frac{dg(z)}{dz} \tag{5c}$$

It can be observed from Eq. (4) that the transverse shear strains (γ_{xz}, γ_{yz}) are equal to zero at the upper ($z=h/2$) and lower ($z=-h/2$) surfaces of the plate. A shear correction coefficient is, hence, not required.

The integrals used in the above equations shall be resolved by a Navier type procedure and can be expressed as follows

$$\frac{\partial}{\partial y}\int \theta\, dx = A'\frac{\partial^2\theta}{\partial x\partial y},\quad \frac{\partial}{\partial x}\int \theta\, dy = B'\frac{\partial^2\theta}{\partial x\partial y}$$
$$\int \theta\, dx = A'\frac{\partial\theta}{\partial x},\quad \int \theta\, dy = B'\frac{\partial\theta}{\partial y} \tag{6}$$

where the coefficients A' and B' are considered according to the type of solution employed, in this case via Navier method. Therefore, A', B', k_1 and k_2 are expressed as follows

$$A' = -\frac{1}{\alpha^2},\ B' = -\frac{1}{\beta^2},\ k_1 = \alpha^2,\ k_2 = \beta^2 \tag{7}$$

where α and β are defined in expression (25b).

The constitutive relations of an FG plate can be expressed as

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\\ \tau_{xz}\\ \tau_{yz}\end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0\\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{66} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & C_{44}\end{bmatrix} \begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{Bmatrix} \tag{8}$$

where C_{ij} are the three-dimensional elastic constants defined by

$$C_{11} = C_{22} = C_{33} = \frac{(1-\nu)E(z)}{(1-2\nu)(1+\nu)}, \tag{9a}$$

$$C_{12} = C_{13} = C_{23} = \frac{\nu\, E(z)}{(1-2\nu)(1+\nu)}, \tag{9b}$$

$$C_{44} = C_{55} = C_{66} = \frac{E(z)}{2(1+\nu)}, \tag{9c}$$

with $E(z)$ and ν being Young's modulus and Poisson's ratio, respectively, of an FG plate.

In this work, two homogenization methods are employed for the calculation of the Young's modulus $E(z)$ namely: (1) the exponential distribution, and (2) the Mori-Tanaka scheme. For the exponential distribution, the Young's modulus is given as (Belabed *et al*. 2014, Zenkour *et al*. 2007)

$$E(z) = E_0 e^{p(0.5+z/h)} \tag{10}$$

where $E_b = E_0$ and $E_t = E_0 e^p$ present Young's modulus of the bottom and top surfaces of the FG plate, respectively, E_0 is Young's modulus of the homogeneous plate, and p is the non-negative variable coefficient (power-law exponent) which controls the material variation within the thickness of the plate.

For Mori-Tanaka scheme, the Young's modulus is given as (Benveniste 1987, Mori and Tanaka 1973)

$$E(z) = E_m + (E_c - E_m)\frac{V_c}{1 + V_m\left(\dfrac{E_c}{E_m} - 1\right)\dfrac{1+\nu}{3-3\nu}} \tag{11}$$

where subscripts m and c denote the metal and ceramic constituents, respectively, G is the shear modulus, and the volume fractions of the metal phase V_m and ceramic phase V_c are defined by

$$V_m = 1 - V_c \quad \text{and} \quad V_c = \left(\frac{1}{2} + \frac{z}{h}\right)^p \tag{12}$$

The effective density $\rho(z)$ is determined by employing the power-law distribution with Voigt's rule of mixtures as (Reddy 2000, Larbi Chaht *et al*. 2015, Ahouel *et al*. 2016, Mouffoki *et al*. 2017, Zidi *et al*. 2017, Zemri *et al*. 2015, Meksi *et al*. 2018)

$$\rho(z) = \rho_m + (\rho_c - \rho_m)V_c \tag{13}$$

Hamilton's principle is employed herein to deduce the equations of motion. The principle can be analytically expressed as

$$0 = \int_0^T (\delta U + \delta V - \delta K)\, dt \tag{14}$$

where δU is the variation of strain energy; δV is the variation of the external work done by external load applied to the plate; and δK is the variation of kinetic energy.

The variation of strain energy is expressed explicitly by

$$\begin{aligned}\delta U &= \int_V \left[\sigma_x\delta\varepsilon_x + \sigma_y\delta\varepsilon_y + \sigma_z\delta\varepsilon_z + \tau_{xy}\delta\gamma_{xy} + \tau_{yz}\delta\gamma_{yz} + \tau_{xz}\delta\gamma_{xz}\right]dV\\ &= \int_A \Big[N_x\delta\varepsilon_x^0 + N_y\delta\varepsilon_y^0 + N_z\delta\varepsilon_z^0 + N_{xy}\delta\gamma_{xy}^0 + M_x^b\delta k_x^b + M_y^b\delta k_y^b + M_{xy}^b\delta k_{xy}^b\\ &\quad + M_x^s\delta k_x^s + M_y^s\delta k_y^s + M_{xy}^s\delta k_{xy}^s + S_{yz}^s\delta\gamma_{yz}^s + S_{xz}^s\delta\gamma_{xz}^0\Big]dA = 0\end{aligned} \tag{15}$$

where A is the area of top surface and the stress resultants N, M, and S are expressed by

$$\begin{aligned}&\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} (1, z, f)\sigma_i\, dz,\quad (i = x, y, xy);\\ &N_z = \int_{-h/2}^{h/2} g'(z)\sigma_z\, dz \quad \text{and} \quad \left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} g(\tau_{xz}, \tau_{yz})dz\end{aligned} \tag{16}$$

Substituting Eq. (4) into Eq. (8) and the subsequent results into Eq. (16), the stress resultants can be written in terms of generalized displacements (u_0, v_0, w_0, θ, φ_z) as

$$\begin{Bmatrix}N_x\\ N_y\\ N_{xy}\\ M_x^b\\ M_y^b\\ M_{xy}^b\\ M_x^s\\ M_y^s\\ M_{xy}^s\\ N_z\end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & B_{11}^s & B_{12}^s & 0 & X_{13}\\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 & B_{12}^s & B_{22}^s & 0 & X_{23}\\ 0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 & B_{66}^s & 0\\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & D_{11}^s & D_{12}^s & 0 & Y_{13}\\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 & D_{12}^s & D_{22}^s & 0 & Y_{23}\\ 0 & 0 & B_{66} & 0 & 0 & D_{11} & 0 & 0 & D_{66}^s & 0\\ B_{11}^s & B_{12}^s & 0 & D_{11}^s & D_{12}^s & 0 & H_{11}^s & H_{12}^s & 0 & Y_{13}^s\\ B_{12}^s & B_{22}^s & 0 & D_{12}^s & D_{22}^s & 0 & H_{12}^s & H_{22}^s & 0 & Y_{23}^s\\ 0 & 0 & B_{66}^s & 0 & 0 & D_{66}^s & 0 & 0 & H_{66}^s & 0\\ X_{13} & X_{23} & 0 & Y_{13} & Y_{23} & 0 & Y_{13}^s & Y_{23}^s & 0 & Z_{33}\end{bmatrix} \begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial y}\\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x}\\ -\dfrac{\partial^2 w_0}{\partial x^2}\\ -\dfrac{\partial^2 w_0}{\partial y^2}\\ -2\dfrac{\partial^2 w_0}{\partial x\partial y}\\ k_1\,\theta\\ k_2\,\theta\\ (k_1 A' + k_2 B')\dfrac{\partial^2\theta}{\partial x\partial y}\\ \phi_z\end{Bmatrix} \tag{17a}$$

$$\begin{Bmatrix} S_{yz}^s \\ S_{xz}^s \end{Bmatrix} = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix} \begin{Bmatrix} k_2 B' \dfrac{\partial \theta}{\partial y} + \dfrac{\partial \varphi_z}{\partial y} \\ k_1 A' \dfrac{\partial \theta}{\partial x} + \dfrac{\partial \varphi_z}{\partial x} \end{Bmatrix} \tag{17b}$$

where

$$\left(A_{ij}, A_{ij}^s, B_{ij}, D_{ij}, B_{ij}^s, D_{ij}^s, H_{ij}^s\right) = \int_{-h/2}^{h/2} C_{ij}\left(1, g^2(z), z, z^2, f(z), z\, f(z), f^2(z)\right) dz \tag{18a}$$

$$\left(X_{ij}, Y_{ij}, Y_{ij}^s, Z_{ij}\right) = \int_{-h/2}^{h/2} \left(1, z, f(z), g'(z)\right) g'(z)\, C_{ij}\, dz \tag{18b}$$

The variation of the external work is expressed explicitly by

$$\delta V = -\int_A q \delta\, w_0 dA \tag{19}$$

where q is the transverse applied load.

The variation of kinetic energy is given by

$$\begin{aligned}
\delta K &= \int_V \left[\dot{u}\, \delta\, \dot{u} + \dot{v}\, \delta\, \dot{v} + \dot{w}\, \delta\, \dot{w}\right] \rho(z)\, dV \\
&= \int_A \{ I_0 \left[\dot{u}_0 \delta \dot{u}_0 + \dot{v}_0 \delta \dot{v}_0 + \dot{w}_0 \delta \dot{w}_0\right] + J_0 \left(\dot{\varphi}_z\, \delta\, \dot{w}_0 + \dot{w}_0\, \delta\, \dot{\varphi}_z\right) \\
&- I_1 \left(\dot{u}_0 \frac{\partial \delta \dot{w}_0}{\partial x} + \frac{\partial \dot{w}_0}{\partial x} \delta\, \dot{u}_0 + \dot{v}_0 \frac{\partial \delta \dot{w}_0}{\partial y} + \frac{\partial \dot{w}_0}{\partial y} \delta\, \dot{v}_0 \right) \\
&+ J_1 \left((k_1 A') \left(\dot{u}_0 \frac{\partial \delta \dot{\theta}}{\partial x} + \frac{\partial \dot{\theta}}{\partial x} \delta\, \dot{u}_0 \right) + (k_2 B') \left(\dot{v}_0 \frac{\partial \delta \dot{\theta}}{\partial y} + \frac{\partial \dot{\theta}}{\partial y} \delta\, \dot{v}_0 \right) \right) \\
&+ I_2 \left(\frac{\partial \dot{w}_0}{\partial x} \frac{\partial \delta\, \dot{w}_0}{\partial x} + \frac{\partial \dot{w}_0}{\partial y} \frac{\partial \delta\, \dot{w}_0}{\partial y} \right) + \\
&K_2 \left((k_1 A')^2 \left(\frac{\partial \dot{\theta}}{\partial x} \frac{\partial \delta\, \dot{\theta}}{\partial x} \right) + (k_2 B')^2 \left(\frac{\partial \dot{\theta}}{\partial y} \frac{\partial \delta\, \dot{\theta}}{\partial y} \right) \right) \\
&- J_2 \left((k_1 A') \left(\frac{\partial \dot{w}_0}{\partial x} \frac{\partial \delta\, \dot{\theta}}{\partial x} + \frac{\partial \dot{\theta}}{\partial x} \frac{\partial \delta\, \dot{w}_0}{\partial x} \right) + (k_2 B') \left(\frac{\partial \dot{w}_0}{\partial y} \frac{\partial \delta\, \dot{\theta}}{\partial y} + \frac{\partial \dot{\theta}}{\partial y} \frac{\partial \delta\, \dot{w}_0}{\partial y} \right) \right) \\
&+ K_0 \left(\dot{\varphi}_z \delta\, \dot{\varphi}_z \right) \} dA
\end{aligned} \tag{20}$$

where dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z)$ is the mass density expressed by Eq. (13); and (I_i, J_i, K_i) are mass inertias expressed by

$$\left(I_0, I_1, I_2\right) = \int_{-h/2}^{h/2} \left(1, z, z^2\right) \rho(z) dz \tag{21a}$$

$$\left(J_0, J_1, J_2\right) = \int_{-h/2}^{h/2} \left(g, f, z\, f\right) \rho(z) dz \tag{21b}$$

$$\left(K_0, K_2\right) = \int_{-h/2}^{h/2} \left(g^2, f^2\right) \rho(z) dz \tag{21c}$$

The equations of motion can be deduced by substituting the equations for δU, δV, and δK from Eqs. (15), (19), and (20) into Eq. (14), integrating by parts and collecting the coefficients of δu_0, δv_0, δw_0, $\delta\theta$, and $\delta\varphi_z$

$$\begin{aligned}
&\delta\, u_0 : \ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial x} + k_1 A'\, J_1 \frac{\partial \ddot{\theta}}{\partial x} \\
&\delta\, v_0 : \ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0 \ddot{v}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial y} + k_2\, B'\, J_1 \frac{\partial \ddot{\theta}}{\partial y} \\
&\delta\, w_0 : \ \frac{\partial^2 M_x^b}{\partial x^2} + 2 \frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + q = I_0 \ddot{w}_0 + I_1 \left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y} \right) \\
&- I_2 \nabla^2 \ddot{w}_0 + J_2 \left(k_1\, A' \frac{\partial^2 \ddot{\theta}}{\partial x^2} + k_2\, B' \frac{\partial^2 \ddot{\theta}}{\partial y^2} \right) + J_0\, \ddot{\varphi}_z \\
&\delta\, \theta : -k_1\, M_x^s - k_2\, M_y^s - (k_1 A' + k_2\, B') \frac{\partial^2 M_{xy}^s}{\partial x \partial y} + k_1\, A' \frac{\partial S_{xz}^s}{\partial x} + k_2\, B' \frac{\partial S_{yz}^s}{\partial y} = \\
&- J_1 \left(k_1\, A' \frac{\partial \ddot{u}_0}{\partial x} + k_2\, B' \frac{\partial \ddot{v}_0}{\partial y} \right) - K_2 \left((k_1\, A')^2 \frac{\partial^2 \ddot{\theta}}{\partial x^2} + (k_2\, B')^2 \frac{\partial^2 \ddot{\theta}}{\partial y^2} \right) \\
&+ J_2 \left(k_1\, A' \frac{\partial^2 \ddot{w}_0}{\partial x^2} + k_2\, B' \frac{\partial^2 \ddot{w}_0}{\partial y^2} \right) \\
&\delta\, \varphi_z : -N_z + \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} = J_0\, \ddot{w}_0 + K_0\, \ddot{\varphi}_z
\end{aligned} \tag{22}$$

Substituting Eq. (17) into Eq. (22), the equations of motion of the present quasi-3D sinusoidal shear deformation theory can be written in terms of displacements (u_0, v_0, w_0, θ, φ_z) as

$$\begin{aligned}
&A_{11} d_{11} u_0 + A_{66}\, d_{22} u_0 + \left(A_{12} + A_{66}\right) d_{12} v_0 + X_{13} d_1 \varphi_z \\
&- B_{11} d_{111} w_0 - \left(B_{12} + 2B_{66}\right) d_{122} w_0 \\
&+ \left(B_{66}^s \left(k_1 A' + k_2\, B'\right)\right) d_{122} \theta + \left(B_{11}^s k_1 + B_{12}^s\, k_2\right) d_1 \theta \\
&= I_0 \ddot{u}_0 - I_1\, d_1 \ddot{w}_0 + J_1\, A' k_1 d_1 \ddot{\theta},
\end{aligned} \tag{23a}$$

$$\begin{aligned}
&A_{22}\, d_{22} v_0 + A_{66}\, d_{11} v_0 + \left(A_{12} + A_{66}\right) d_{12} u_0 + X_{23} d_2 \varphi_z \\
&- B_{22}\, d_{222} w_0 - \left(B_{12} + 2B_{66}\right) d_{112} w_0 \\
&+ \left(B_{66}^s \left(k_1 A' + k_2\, B'\right)\right) d_{112} \theta + \left(B_{22}^s k_2 + B_{12}^s k_1\right) d_2 \theta \\
&= I_0 \ddot{v}_0 - I_1\, d_2 \ddot{w}_0 + J_1\, B' k_2\, d_2 \ddot{\theta},
\end{aligned} \tag{23b}$$

$$\begin{aligned}
&B_{11}\, d_{111} u_0 + \left(B_{12} + 2B_{66}\right) d_{122} u_0 + \left(B_{12} + 2B_{66}\right) d_{112} v_0 \\
&+ B_{22}\, d_{222} v_0 + Y_{13}\, d_{11} \varphi_z + Y_{23}\, d_{22} \varphi_z \\
&- D_{11} d_{1111} w_0 - 2\left(D_{12} + 2D_{66}\right) d_{1122} w_0 - D_{22}\, d_{2222} w_0 \\
&+ \left(D_{11}^s\, k_1 + D_{12}^s\, k_2\right) d_{11} \theta + 2\left(D_{66}^s \left(k_1 A' + k_2 B'\right)\right) d_{1122} \theta \\
&+ \left(D_{12}^s k_1 + D_{22}^s k_2\right) d_{22} \theta + q \\
&= I_0 \ddot{w}_0 + I_1 \left(d_1 \ddot{u}_0 + d_2\, \ddot{v}_0\right) \\
&- I_2 \left(d_{11} \ddot{w}_0 + d_{22} \ddot{w}_0\right) + J_2 \left(k_1 A'\, d_{11} \ddot{\theta} + k_2\, B'\, d_{22} \ddot{\theta}\right) + J_0\, \ddot{\varphi}_z
\end{aligned} \tag{23c}$$

$$\begin{aligned}
&- \left(B_{11}^s k_1 + B_{12}^s k_2\right) d_1 u_0 - \left(B_{66}^s \left(k_1 A' + k_2 B'\right)\right) d_{122} u_0 \\
&- \left(B_{66}^s \left(k_1 A' + k_2 B'\right)\right) d_{112} v_0 - \left(B_{12}^s k_1 + B_{22}^s k_2\right) d_2 v_0 \\
&- k_1 Y_{13}^s \varphi_z - k_2 Y_{23}^s \varphi_z + \left(D_{11}^s k_1 + D_{12}^s k_2\right) d_{11} w_0 \\
&+ 2\left(D_{66}^s \left(k_1 A' + k_2 B'\right)\right) d_{1122} w_0 + \left(D_{12}^s k_1 + D_{22}^s k_2\right) d_{22} w_0 \\
&- H_{11}^s\, k_1^2\ \theta - H_{22}^s\ k_2^2\, \theta - 2\, H_{12}^s\, k_1\, k_2 \theta \\
&- \left(\left(k_1 A' + k_2 B'\right)^2 H_{66}^s\right) d_{1122} \theta \\
&+ A_{44}^s \left(k_2 B'\right)^2 d_{22} \theta + A_{55}^s\ \left(k_1\, A'\right)^2 d_{11} \theta \\
&+ A_{44}^s \left(k_2 B'\right) d_{22} \varphi_z + A_{55}^s \left(k_1 A'\right) d_{11} \varphi_z \\
&= -J_1 \left(k_1\, A' d_1 \ddot{u}_0 + k_2\, B' d_2 \ddot{v}_0\right) \\
&+ J_2 \left(k_1\, A' d_{11} \ddot{w}_0 + k_2\, B' d_{22} \ddot{w}_0\right) \\
&- K_2 \left(\left(k_1 A'\right)^2 d_{11} \ddot{\theta} + \left(k_2\, B'\right)^2 d_{22} \ddot{\theta}\right)
\end{aligned} \tag{23d}$$

$$\begin{aligned}
&- X_{13} d_1 u_0 - X_{23}\, d_2 v_0 - Z_{33} \varphi_z \\
&+ Y_{13}\, d_{11} w_0 + Y_{23}\, d_{22} w_0 \\
&+ (A_{44}^s - Y_{23}^s)\left(k_2 B'\right) d_{22} \theta + (A_{55}^s - Y_{13}^s)\ \left(k_1\, A'\right) d_{11} \theta \\
&+ A_{44}^s\, d_{22} \varphi_z + A_{55}^s\, d_{11} \varphi_z = J_0\, \ddot{\varphi}_z + K_0 \ddot{w}_0\, ,
\end{aligned} \tag{23e}$$

where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}, \quad d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l}$$
$$d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m} \quad d_i = \frac{\partial}{\partial x_i} \quad (i,j,l,m=1,2). \tag{24}$$

3. Analytical solutions

In this part, a simply supported rectangular plate is considered with length a and width b under transverse load q. Using the Navier solution procedure, the following expressions of displacements (u_0, v_0, w_0, θ, φ_z) are taken

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \\ \varphi_z \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn} e^{i\omega t}\cos(\alpha\, x)\sin(\beta\, y) \\ V_{mn} e^{i\omega t}\sin(\alpha\, x)\cos(\beta\, y) \\ W_{mn} e^{i\omega t}\sin(\alpha\, x)\sin(\beta\, y) \\ X_{mn} e^{i\omega t}\sin(\alpha\, x)\sin(\beta\, y) \\ Y_{mn} e^{i\omega t}\sin(\alpha\, x)\sin(\beta\, y) \end{Bmatrix} \tag{25a}$$

with

$$\alpha = m\pi / a \quad \text{and} \quad \beta = n\pi / b \tag{25b}$$

where $i = \sqrt{-1}$, (U_{mn}, V_{mn}, W_{mn}, X_{mn}, Y_{mn}) are the unknown maximum amplitudes of displacement, and ω is the frequency of vibration. The transverse load q is also expressed in the double-Fourier sine series as

$$q(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Q_{mn}\sin(\alpha\, x)\sin(\beta\, y) \tag{25c}$$

For the case of a sinusoidally distributed load, the coefficient $Q_{mn}=q_0$ indicates the intensity of the load at the plate center. Substituting Eq. (25) into Eq. (23), the analytical solutions can be determined by

$$\left(\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} \\ S_{12} & S_{22} & S_{23} & S_{24} & S_{25} \\ S_{13} & S_{23} & S_{33} & S_{34} & S_{35} \\ S_{14} & S_{24} & S_{34} & S_{44} & S_{45} \\ S_{15} & S_{25} & S_{35} & S_{45} & S_{55} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} & 0 \\ 0 & m_{22} & m_{23} & m_{24} & 0 \\ m_{13} & m_{23} & m_{33} & m_{34} & m_{35} \\ m_{14} & m_{24} & m_{34} & m_{44} & 0 \\ 0 & 0 & m_{35} & 0 & m_{55} \end{bmatrix} \right) \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \\ Y_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ Q_{mn} \\ 0 \\ 0 \end{Bmatrix} \tag{26}$$

where

$$\begin{gathered} s_{11} = \alpha^2 B_{11} + \beta^2 A_{66}, \quad s_{12} = \alpha\beta(A_{12} + A_{66}), \\ s_{13} = -\alpha^3 B_{11} - \alpha\beta^2 (B_{12} + 2B_{66}) \\ s_{14} = -\alpha(k_1 B_{11}^s + k_2 B_{12}^s) + \alpha\beta^2 B_{66}^s (k_1 A' + k_2 B') \\ s_{15} = -\alpha X_{13}, \quad s_{22} = \alpha^2 A_{66} + \beta^2 A_{22} \\ s_{23} = -\alpha^2 \beta(B_{12} + 2B_{66}) - \beta^3 B_{22}, \\ s_{24} = -\beta(k_1 B_{12}^s + k_2 B_{22}^s) + \alpha^2 \beta(k_1 A' + k_2 B') B_{66}^s \\ s_{25} = -\beta X_{23}, \quad s_{33} = \alpha^4 D_{11} + \beta^4 D_{22} + 2\alpha^2\beta^2 (D_{12} + 2D_{66}) \\ s_{34} = \alpha^2 k_1 D_{11}^s + (k_2 \alpha^2 + k_1 \beta^2) D_{12}^s + \beta^2 k_2 D_{22}^s \\ -2\alpha^2\beta^2 (k_1 A' + k_2 B') D_{66}^s \\ s_{44} = {k_1}^2 H_{11}^s + {k_2}^2 H_{22}^s + 2k_1 k_2 H_{12}^s \\ + \alpha^2\beta^2 (k_1 A' + k_2 B')^2 H_{66}^s \\ + \alpha^2 (k_1 A')^2 A_{55}^s + \beta^2 (k_2 B')^2 A_{44}^s \\ s_{35} = \alpha^2 Y_{13} + \beta^2 Y_{23} \\ s_{45} = k_1 Y_{13}^s + k_2 Y_{23}^s + \alpha^2 k_1 A' A_{55}^s + \beta^2 k_2 B' A_{44}^s \\ s_{55} = \alpha^2 A_{55}^s + \beta^2 A_{44}^s + Z_{33} \\ m_{13} = -\alpha\, I_1, \quad m_{11} = m_{22} = I_0 \\ m_{14} = \alpha\, k_1 A' J_1, \quad m_{23} = -\beta\, I_1 \\ m_{24} = \beta\, k_2 B' J_1, \quad m_{33} = I_0 + I_2(\alpha^2 + \beta^2) \\ m_{34} = -J_2 (k_1 A' \alpha^2 + k_2 B' \beta^2) \\ m_{44} = K_2 ((k_1 A')^2 \alpha^2 + (k_2 B')^2 \beta^2) \\ m_{35} = J_0 \end{gathered} \tag{27}$$

4. Numerical results

4.1 Results for bending investigation

Consider a simply supported FG plate subjected to sinusoidal loads. The effective Young's modulus $E(z)$ is considered to change exponentially within the thickness of the plate (Eq. (10)). The variation of the exponential function $V(z)=e^{p(0.5+z/h)}$ across the thickness of the plate is demonstrated in Fig. 1 for different values of p. Poisson's ratio is supposed to be constant ν=0.3. For convenience, the following dimensionless forms are employed

$$\begin{gathered} \bar{u}(z) = \frac{10E_0 h^3}{q_0 a^4} u\left(0, \frac{b}{2}, z\right), \bar{w}(z) = \frac{10E_0 h^3}{q_0 a^4} u\left(\frac{a}{2}, \frac{b}{2}, z\right) \\ \bar{\sigma}_x(z) = \frac{h^2}{q_0 a^2}\sigma_x\left(\frac{a}{2}, \frac{b}{2}, z\right) \bar{\sigma}_y(z) = \frac{h^2}{q_0 a^2}\sigma_y\left(\frac{a}{2}, \frac{b}{2}, z\right) \\ \bar{\tau}_{xy}(z) = \frac{10h^2}{q_0 a^2}\tau_{xy}(0,0,z) \; \bar{\tau}_{xz}(z) = \frac{h}{q_0 a}\tau_{xz}\left(0, \frac{b}{2}, z\right) \\ \bar{\tau}_{yz}(z) = \frac{h}{q_0 a}\tau_{yz}\left(\frac{a}{2}, 0, z\right) \end{gathered} \tag{28}$$

The non-dimensional displacement and stress are shown in Tables 1, 2, 3, and 4 for various values of aspect ratio

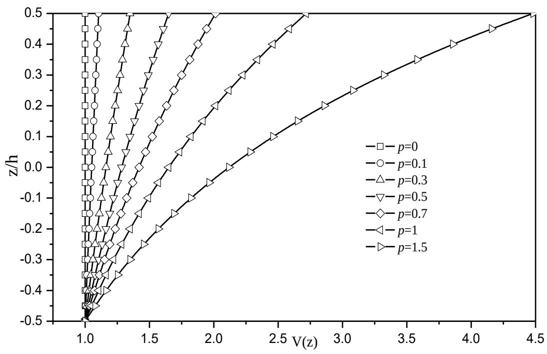

Fig. 1 The exponential variation function $V(z)$ along the thickness of an EG rectangular plate

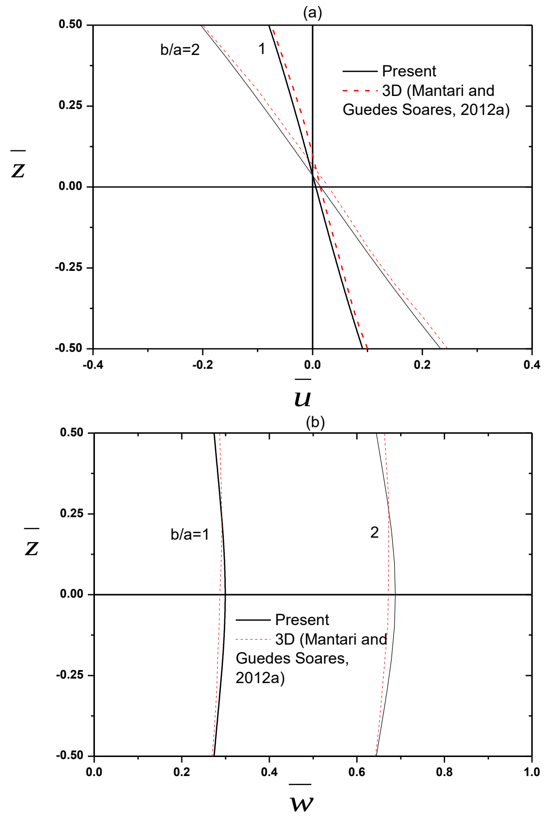

Fig. 2 Variation of dimensionless displacement and stresses through the thickness of plates (*a/h*=4, *p*=0.5)

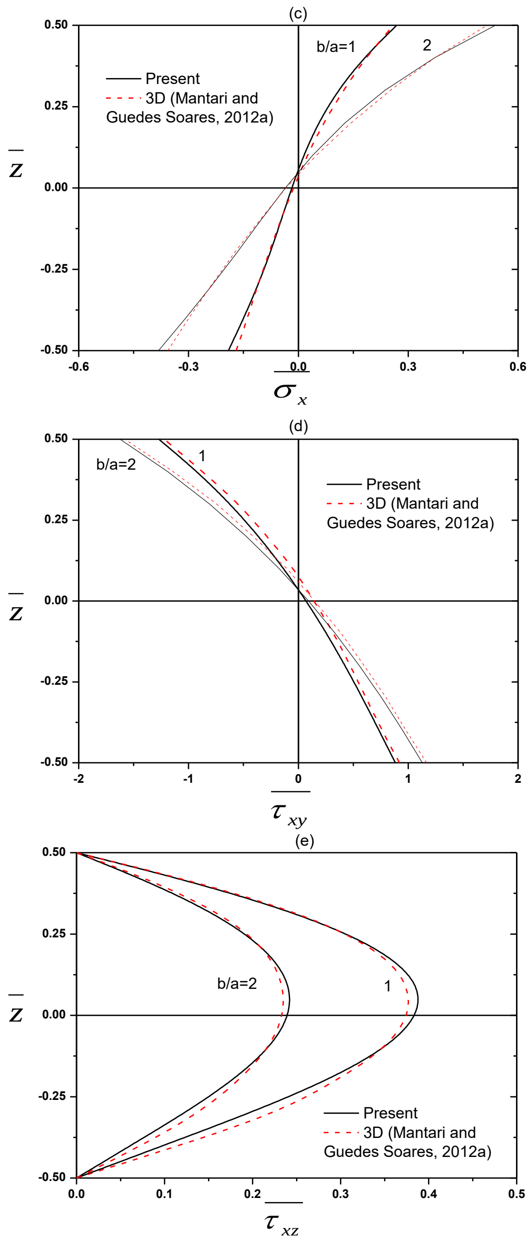

Fig. 2 Continued

b/a, thickness ratio *a/h*, and material parameter *p*. The through thickness distributions of the non-dimensional displacements and stresses are also presented in Fig. 2 for a thick FG plate with *a/h*=4 and *p*=0.5. The computed results are compared with the exact 3D (Zenkour 2007) and quasi-3D solutions (Mantari and Guedes Soares 2012a, 2013, Thai *et al*. 2014, Zenkour 2007). It should be indicated that the quasi-3D solutions (Zenkour 2007, Mantari and Guedes Soares 2013) are based on a trigonometric variation of both in-plane and transverse displacements, while the quasi-3D solutions (Mantari and Guedes Soares 2012a) are based on a cubic variation of in-plane displacements and a parabolic variation of transverse displacement within the thickness. In addition, the results of HSDT (Mantari and Guedes Soares 2012b) are also given to demonstrate the importance of introducing the thickness-stretching influence. The HSDT solution (Mantari and Guedes Soares 2012b) is based on a trigonometric variation of in-plane displacements and a constant transverse displacement across the thickness (i.e., thickness-stretching effect is neglected, ε_z=0).

It can be seen that the computed results are in excellent agreement with 3D and quasi-3D solutions, particularly with those given by Mantari and Guedes Soares (2012a, 2013). The proposed quasi-3D theory provides the same results to those of the quasi-3D sinusoidal theory (Zenkour 2007). It should be noted that the proposed theory is even simpler than the quasi-3D sinusoidal theory (Zenkour 2007) because in the present theory only five unknowns are used while in the theory of Zenkour (2007) we find six unknowns. Since the proposed quasi-3D theory and other quasi-3D theories introduce the thickness-stretching influence, their results are very close to each other. Meanwhile, the HSDT (Mantari and Guedes Soares, 2012b), which neglects this effect, provides inaccurate result and slightly overestimates the transverse displacement especially for very thick plates (i.e., *a/h*=2, see Tables 1, 3). The errors in the HSDT are reduced with increasing the thickness ratio a/h. In general, the proposed quasi-3D theory is highly accurate and comparable to 3D solution even in the case of very thick plates, e.g., *a/h*=2. It is worth indicating that the proposed theory consists of five unknowns, while the number of variables in the HSDT (Mantari and Guedes Soares 2012b) and other quasi-3D theories (Mantari and Guedes Soares 2012a, 2013, Zenkour 2007) is five and six, respectively. Consequently, it may be concluded that the developed quasi-3D theory is not only more accurate than the HSDT having the same five

Table 1 Dimensionless deflection $\overline{w}_0(0)$ of plates (a/h=2)

b/a	Theory	p					
		0.1	0.3	0.5	0.7	1.0	1.5
6	3D (Zenkour 2007)	1.6377	1.4885	1.3518	1.2269	1.0593	0.8261
	Quasi-3D (Zenkour 2007)	1.6294	1.4731	1.3307	1.2010	1.0282	0.7906
	Quasi-3D[(a)]	1.6365	1.4795	1.3364	1.2062	1.0333	0.7939
	Quasi-3D (Thai *et al*. 2014)	1.6367	1.4796	1.3365	1.2063	1.0327	0.7939
	Present	1.6294	1.4731	1.3307	1.2010	1.0282	0.7906
	HSDT[(b)]	1.7347	1.5688	1.4182	1.2815	1.1003	0.8500
5	3D (Zenkour 2007)	1.6095	1.4601	1.3261	1.2035	1.0391	0.8102
	Quasi-3D (Zenkour 2007)	1.5983	1.4449	1.3052	1.1780	1.0086	0.7754
	Quasi-3D[(a)]	1.6053	1.4513	1.3109	1.1832	1.0135	0.7787
	Quasi-3D (Thai *et al*. 2014)	1.6054	1.4513	1.3110	1.1833	1.0130	0.7787
	Present	1.5983	1.4449	1.3052	1.1780	1.0086	0.7754
	HSDT[(b)]	1.7025	1.5397	1.3919	1.2576	1.0798	0.8340
4	3D (Zenkour 2007)	1.5515	1.4101	1.2807	1.1624	1.0035	0.7824
	Quasi-3D (Zenkour 2007)	1.5435	1.3954	1.2605	1.1376	0.9740	0.7487
	Quasi-3D[(a)]	1.5504	1.4017	1.2661	1.1427	0.9788	0.7520
	Quasi-3D (Thai *et al*. 2014)	1.5505	1.4018	1.2662	1.1428	0.9783	0.7520
	Present	1.5435	1.3954	1.2605	1.1376	0.9740	0.7487
	HSDT[(b)]	1.6458	1.4885	1.3455	1.2157	1.0437	0.8060
3	3D (Zenkour 2007)	1.4430	1.3116	1.1913	1.0812	0.9334	0.7275
	Quasi-3D (Zenkour 2007)	1.4354	1.2977	1.1722	1.0579	0.9057	0.6962
	Quasi-3D[(a)]	1.4421	1.3037	1.1776	1.0628	0.9104	0.6993
	Quasi-3D[(c)]	1.4419	1.3035	1.1774	1.0626	0.9096	0.6991
	Quasi-3D (Thai *et al*. 2014)	1.4422	1.3038	1.1777	1.0629	0.9098	0.6993
	Present	1.4354	1.2977	1.1722	1.0579	0.9057	0.6962
	HSDT[(b)]	1.5341	1.3784	1.2540	1.1329	0.9725	0.7506
2	3D (Zenkour 2007)	1.1945	1.0859	0.9864	0.8952	0.7727	0.6017
	Quasi-3D (Zenkour 2007)	1.1880	1.0740	0.9701	0.8755	0.7494	0.5758
	Quasi-3D[(a)]	1.1941	1.0795	0.9750	0.8799	0.7538	0.5786
	Quasi-3D[(c)]	1.1938	1.0793	0.9748	0.8797	0.7530	0.5785
	Quasi-3D (Thai *et al*. 2014)	1.1942	1.0796	0.9751	0.8800	0.7532	0.5786
	Present	1.1880	1.0740	0.9701	0.8755	0.7494	0.5758
	HSDT[(b)]	1.2776	1.1553	1.0441	0.9431	0.8093	0.6238
1	3D (Zenkour 2007)	0.5769	0.5247	0.4766	0.4324	0.3727	0.2890
	Quasi-3D (Zenkour 2007)	0.5731	0.5181	0.4679	0.4222	0.3612	0.2771
	Quasi-3D[(a)]	0.5779	0.5224	0.4718	0.4257	0.3649	0.2794
	Quasi-3D[(c)]	0.5776	0.5222	0.4716	0.4255	0.3640	0.2792
	Quasi-3D (Thai *et al*. 2014)	0.5780	0.5225	0.4719	0.4258	0.3642	0.2794
	Present	0.5731	0.5181	0.4679	0.4222	0.3612	0.2771
	HSDT[(b)]	0.6363	0.5752	0.5195	0.4687	0.4018	0.3079

[(a)] Mantari and Guedes Soares (2013); [(b)] Mantari and Guedes Soares (2012a);[(c)] Mantari and Guedes Soares (2012c)

unknowns, but also comparable with the quasi-3D theories having more number of variables.

4.2 Results for free vibration investigation

The accuracy of the developed quasi-3D theory is also checked through the dynamic analysis. Consider a simply supported Al/ZrO_2 plate made from a mixture of a metal (Al) and a ceramic (ZrO_2). Young's modulus and density of the metal are E_m=70 GPa and ρ_m=2702 kg/m^3, respectively, and those of ceramic are E_c=200 GPa and ρ_c=5700 kg/m^3, respectively. Poisson's ratio is considered to be constant and equal to 0.3. The effective Young's modulus is estimated using the power-law distribution with Mori-Tanaka scheme (Eq. (11)). However, the effective density $\rho(z)$ is determined using the power-law variation with Voigt's rule of mixtures as shown in Eq. (13).

Table 5 presents the non-dimensional fundamental frequency $\overline{\omega}$ of square plates for different values of thickness ratio and gradient index. The non-dimensional

Table 2 Dimensionless deflection $\overline{w}_0(0)$ of plates (a/h=4)

b/a	Theory	p					
		0.1	0.3	0.5	0.7	1.0	1.5
6	3D (Zenkour 2007)	1.1714	1.0622	0.9633	0.8738	0.7550	0.5919
	Quasi-3D (Zenkour 2007)	1.1668	1.0551	0.9535	0.8611	0.7382	0.5697
	Quasi-3D[(a)]	1.1703	1.0583	0.9563	0.8636	0.7403	0.5713
	Quasi-3D (Thai *et al*. 2014)	1.1703	1.0583	0.9563	0.8636	0.7403	0.5713
	Present	1.1668	1.0551	0.9535	0.8611	0.7382	0.5697
	HSDT[(b)]	1.1920	1.0789	0.9767	0.8844	0.7623	0.5955
5	3D (Zenkour 2007)	1.1459	1.0391	0.9424	0.8548	0.7386	0.5790
	Quasi-3D (Zenkour 2007)	1.1414	1.0321	0.9327	0.8423	0.7221	0.5573
	Quasi-3D[(a)]	1.1448	1.0352	0.9355	0.8448	0.7242	0.5588
	Quasi-3D (Thai *et al*. 2014)	1.1448	1.0352	0.9354	0.8448	0.7242	0.5588
	Present	1.1414	1.0321	0.9327	0.8423	0.7221	0.5573
	HSDT[(b)]	1.1663	1.0556	0.9556	0.8653	0.7458	0.5825
4	3D (Zenkour 2007)	1.1012	0.9985	0.9056	0.8215	0.7098	0.5564
	Quasi-3D (Zenkour 2007)	1.0968	0.9918	0.8963	0.8094	0.6939	0.5355
	Quasi-3D[(a)]	1.1001	0.9948	0.8989	0.8118	0.6959	0.5370
	Quasi-3D (Thai *et al*. 2014)	1.1001	0.9948	0.8989	0.8118	0.6959	0.5370
	Present	1.0968	0.9918	0.8963	0.8094	0.6939	0.5355
	HSDT[(b)]	1.1211	1.0147	0.9186	0.8317	0.7169	0.5599
3	3D (Zenkour 2007)	1.0134	0.9190	0.8335	0.7561	0.6533	0.5121
	Quasi-3D (Zenkour 2007)	1.0094	0.9127	0.8248	0.7449	0.6385	0.4927
	Quasi-3D[(a)]	1.0124	0.9155	0.8272	0.7470	0.6404	0.4941
	Quasi-3D[(c)]	1.0124	0.9155	0.8272	0.7470	0.6404	0.4941
	Quasi-3D (Thai *et al*. 2014)	1.0124	0.9155	0.8272	0.7470	0.6404	0.4941
	Present	1.0094	0.9127	0.8248	0.7449	0.6385	0.4927
	HSDT[(b)]	1.0325	0.9345	0.8459	0.7659	0.6601	0.5154
2	3D (Zenkour 2007)	0.8153	0.7395	0.6707	0.6085	0.5257	0.4120
	Quasi-3D (Zenkour 2007)	0.8120	0.7343	0.6635	0.5992	0.5136	0.3962
	Quasi-3D[(a)]	0.8145	0.7365	0.6655	0.6009	0.5151	0.3973
	Quasi-3D[(c)]	0.8145	0.7365	0.6655	0.6009	0.5151	0.3973
	Quasi-3D (Thai *et al*. 2014)	0.8145	0.7365	0.6655	0.6009	0.5151	0.3973
	Present	0.8120	0.7343	0.6635	0.5992	0.5136	0.3962
	HSDT[(b)]	0.8325	0.7534	0.6819	0.6173	0.5319	0.4150
1	3D (Zenkour 2007)	0.3490	0.3167	0.2875	0.2608	0.2253	0.1805
	Quasi-3D (Zenkour 2007)	0.3475	0.3142	0.2839	0.2563	0.2196	0.1692
	Quasi-3D[(a)]	0.3486	0.3152	0.2848	0.2571	0.2203	0.1697
	Quasi-3D[(c)]	0.3486	0.3152	0.2848	0.2571	0.2203	0.1697
	Quasi-3D (Thai *et al*. 2014)	0.3486	0.3152	0.2848	0.2571	0.2203	0.1697
	Present	0.3475	0.3142	0.2839	0.2563	0.2196	0.1692
	HSDT[(b)]	0.3602	0.3259	0.2949	0.2668	0.2295	0.1785

[(a)] Mantari and Guedes Soares (2013); [(b)] Mantari and Guedes Soares (2012a);[(c)] Mantari and Guedes Soares (2012c)

frequency is defined by $\overline{\omega} = \omega h\sqrt{\rho_m / E_m}$. The results of the proposed quasi-3D sinusoidal shear deformation theory are compared with the results of the HSDT of Benachour *et al*. (2011) and quasi-3D shear deformation theory of Matsunaga (2008), Neves *et al*. (2012), Belabed *et al*. (2014), Alijani and Amabili (2014) and three dimensional exact solution of Vel and Batra (2004). It can be observed from Table 5 that, the results of the proposed quasi-3D theory are in good agreement with the results of other quasi-3D theories. The small difference between the present 2D and quasi-3D shear deformation theory results is due to the neglecting the thickness stretching effect.

5. Conclusions

A quasi-3D sinusoidal shear deformation theory is proposed for bending and dynamic analysis of FG plates. The formulation contains five variables, but considers both shear deformation and thickness-stretching influences

Table 3 Dimensionless stress $\bar{\sigma}$ $(h/2)$ of plates (a/h=2)

b/a	Theory	p					
		0.1	0.3	0.5	0.7	1.0	1.5
6	3D (Zenkour 2007)	0.2943	0.3101	0.3270	0.3451	0.3746	0.4305
	Quasi-3D (Zenkour 2007)	0.2912	0.3118	0.3339	0.3573	0.3955	0.4679
	Quasi-3D[(a)]	0.2763	0.2954	0.3159	0.3378	0.3737	0.4416
	Quasi-3D (Thai *et al*. 2014)	0.2759	0.2951	0.3155	0.3374	0.3730	0.4411
	Present	0.2912	0.3118	0.3339	0.3573	0.3955	0.4679
	HSDT[(b)]	0.2187	0.2345	0.2512	0.2690	0.2980	0.3498
5	3D (Zenkour 2007)	0.2967	0.3128	0.3299	0.3483	0.3782	0.4350
	Quasi-3D (Zenkour 2007)	0.2935	0.3144	0.3366	0.3603	0.3988	0.4719
	Quasi-3D[(a)]	0.2789	0.2983	0.3191	0.3412	0.3776	0.4461
	Quasi-3D (Thai *et al*. 2014)	0.2786	0.2980	0.3187	0.3408	0.3768	0.4456
	Present	0.2935	0.3144	0.3366	0.3603	0.3988	0.4719
	HSDT[(b)]	0.2219	0.2378	0.2548	0.2729	0.3024	0.3549
4	3D (Zenkour 2007)	0.3008	0.3173	0.3349	0.3537	0.3844	0.4426
	Quasi-3D (Zenkour 2007)	0.2974	0.3186	0.3412	0.3653	0.4045	0.4786
	Quasi-3D[(a)]	0.2834	0.3032	0.3243	0.3469	0.3839	0.4537
	Quasi-3D (Thai *et al*. 2014)	0.2830	0.3028	0.3239	0.3465	0.3832	0.4532
	Present	0.2974	0.3186	0.3412	0.3653	0.4045	0.4786
	HSDT[(b)]	0.2272	0.2435	0.2610	0.2795	0.3097	0.3634
3	3D (Zenkour 2007)	0.3081	0.3252	0.3436	0.3633	0.3953	0.4562
	Quasi-3D (Zenkour 2007)	0.3042	0.3261	0.3493	0.3741	0.4143	0.4904
	Quasi-3D[(a)]	0.2912	0.3118	0.3337	0.3571	0.3954	0.4673
	Quasi-3D[(c)]	0.2920	0.3118	0.3337	0.3582	0.3963	0.4688
	Quasi-3D (Thai *et al*. 2014)	0.2909	0.3127	0.3333	0.3567	0.3947	0.4668
	Present	0.3042	0.3261	0.3493	0.3741	0.4143	0.4904
	HSDT[(b)]	0.2368	0.2539	0.2721	0.2914	0.3230	0.3788
2	3D (Zenkour 2007)	0.3200	0.3385	0.3583	0.3796	0.4142	0.4799
	Quasi-3D (Zenkour 2007)	0.3146	0.3376	0.3620	0.3880	0.4300	0.5092
	Quasi-3D[(a)]	0.3042	0.3261	0.3495	0.3743	0.4148	0.4905
	Quasi-3D[(c)]	0.3049	0.3269	0.3503	0.3752	0.4155	0.4918
	Quasi-3D (Thai *et al*. 2014)	0.3040	0.3259	0.3492	0.3740	0.4142	0.4901
	Present	0.3146	0.3376	0.3620	0.3880	0.4300	0.5092
	HSDT[(b)]	0.2539	0.2723	0.2919	0.3128	0.3469	0.4064
1	3D (Zenkour 2007)	0.3103	0.3292	0.3495	0.3713	0.4067	0.4741
	Quasi-3D (Zenkour 2007)	0.2955	0.3181	0.3421	0.3675	0.4085	0.4851
	Quasi-3D[(a)]	0.2924	0.3147	0.3383	0.3633	0.4041	0.4785
	Quasi-3D[(c)]	0.2927	0.3149	0.3385	0.3636	0.4039	0.4790
	Quasi-3D (Thai *et al*. 2014)	0.2924	0.3146	0.3382	0.3632	0.4034	0.4783
	Present	0.2955	0.3181	0.3421	0.3675	0.4085	0.4851
	HSDT[(b)]	0.2943	0.3101	0.3270	0.3451	0.3746	0.4305

[(a)] Mantari and Guedes Soares (2013); [(b)] Mantari and Guedes Soares (2012a);[(c)] Mantari and Guedes Soares (2012c)

without the need for any shear correction factor. Equations of motion obtained from the Hamilton principle are analytically solved for bending and dynamic problems of a simply supported plate. By employing undetermined integral unknowns in displacement field, the number of variables of the theory is diminished and the computational time is thus reduced. The following main points may be drawn from the current work:

• The results computed by the present theory are in an excellent agreement with 3D solutions even for the caseof very thick plates with a/h=2.
• The proposed quasi-3D theory contains five unknowns, but provides results comparable with those computed by the existing quasi-3D models having more number of variables.
• The thickness-stretching influence is more pronounced

Table 4 Dimensionless stress $\bar{\sigma}$ (h/2) of plates (a/h=4)

b/a	Theory	*p*					
		0.1	0.3	0.5	0.7	1.0	1.5
6	3D (Zenkour 2007)	0.2181	0.2321	0.2470	0.2628	0.2886	0.3373
	Quasi-3D (Zenkour 2007)	0.2369	0.2520	0.2683	0.2857	0.3144	0.3699
	Quasi-3D[(a)]	0.2127	0.2255	0.2393	0.2544	0.2795	0.3294
	Quasi-3D (Thai *et al*. 2014)	0.2121	0.2249	0.2387	0.2537	0.2787	0.3285
	Present	0.2369	0.2520	0.2683	0.2857	0.3144	0.3699
	HSDT[(b)]	0.2010	0.2149	0.2298	0.2455	0.2711	0.3192
5	3D (Zenkour 2007)	0.2206	0.2348	0.2498	0.2659	0.2920	0.3413
	Quasi-3D (Zenkour 2007)	0.2391	0.2545	0.2710	0.2886	0.3176	0.3737
	Quasi-3D[(a)]	0.2152	0.2283	0.2424	0.2577	0.2832	0.3337
	Quasi-3D (Thai *et al*. 2014)	0.2147	0.2277	0.2418	0.2570	0.2825	0.3328
	Present	0.2391	0.2545	0.2710	0.2886	0.3176	0.3737
	HSDT[(b)]	0.2037	0.2178	0.2329	0.2488	0.2747	0.3235
4	3D (Zenkour 2007)	0.2247	0.2392	0.2546	0.2710	0.2977	0.3482
	Quasi-3D (Zenkour 2007)	0.2429	0.2586	0.2754	0.2934	0.3230	0.3800
	Quasi-3D[(a)]	0.2196	0.2330	0.2475	0.2633	0.2894	0.3411
	Quasi-3D (Thai *et al*. 2014)	0.2190	0.2324	0.2469	0.2626	0.2887	0.3402
	Present	0.2429	0.2586	0.2754	0.2934	0.3230	0.3800
	HSDT[(b)]	0.2082	0.2226	0.2380	0.2544	0.2808	0.3307
3	3D (Zenkour 2007)	0.2319	0.2469	0.2629	0.2800	0.3077	0.3602
	Quasi-3D (Zenkour 2007)	0.2493	0.2656	0.2831	0.3017	0.3323	0.3911
	Quasi-3D[(a)]	0.2272	0.2414	0.2666	0.2731	0.3004	0.3540
	Quasi-3D[(c)]	0.2286	0.2429	0.2583	0.2749	0.3024	0.3563
	Quasi-3D (Thai *et al*. 2014)	0.2267	0.2408	0.2560	0.2725	0.2997	0.3532
	Present	0.2496	0.2656	0.2831	0.3017	0.3323	0.3911
	HSDT[(b)]	0.2162	0.2312	0.2472	0.2642	0.2917	0.3435
2	3D (Zenkour 2007)	0.2431	0.2591	0.2762	0.2943	0.3238	0.3797
	Quasi-3D (Zenkour 2007)	0.2588	0.2761	0.2946	0.3143	0.3464	0.4079
	Quasi-3D[(a)]	0.2395	0.2550	0.2715	0.2894	0.3187	0.3756
	Quasi-3D[(c)]	0.2407	0.2563	0.2730	0.2909	0.3204	0.3776
	Quasi-3D (Thai *et al*. 2014)	0.2391	0.2545	0.2710	0.2888	0.3181	0.3749
	Present	0.2588	0.2761	0.2946	0.3143	0.3464	0.4079
	HSDT[(b)]	0.2294	0.2454	0.2624	0.2805	0.3097	0.3647
1	3D (Zenkour 2007)	0.2247	0.2399	0.2562	0.2736	0.3018	0.3588
	Quasi-3D (Zenkour 2007)	0.2346	0.2510	0.2684	0.2870	0.3171	0.3739
	Quasi-3D[(a)]	0.2237	0.2391	0.2554	0.2729	0.3014	0.3556
	Quasi-3D[(c)]	0.2244	0.2398	0.2563	0.2738	0.3024	0.3567
	Quasi-3D (Thai *et al*. 2014)	0.2235	0.2398	0.2551	0.2726	0.3010	0.3551
	Present	0.2346	0.2510	0.2684	0.2870	0.3171	0.3739
	HSDT[(b)]	0.2164	0.2316	0.2477	0.2649	0.2927	0.3451

[(a)] Mantari and Guedes Soares (2013); [(b)] Mantari and Guedes Soares (2012a);[(c)] Mantari and Guedes Soares (2012c)

Table 5 Comparison of non-dimensional fundamental frequencies $\bar{\varpi}$

Method	ε_z	*p*=0			*p*=1			*a/h*=5	
		$a/h=\sqrt{10}$	*a/h*=10	*a/h*=5	$a/h=\sqrt{10}$	*a/h*=10	*a/h*=5	$a/h=\sqrt{10}$	*a/h*=10
Benachour *et al*. (2011)	=0	4.6220	5.7600	5.6750	6.1800	6.3200	5.6225	5.6375	5.6650
Matsunaga (2008)	≠0	4.6582	5.7769	5.7123	6.1932	6.3390	5.6599	5.6757	5.7020
Neves *et al*. (2012)	≠0	-	-	5.4825	5.9600	6.1200	5.4950	5.5300	5.5625
Belabed *et al*. (2014)	≠0	4.6591	5.7800	5.4800	5.9700	6.1200	5.5025	5.5350	5.5625
Alijani and Amabili (2014)	≠0	4.6606	5.7769	5.4796	5.9578	6.1040	5.4919	5.5279	5.5633
Vel and Batra (2004)	≠0	4.6582	5.7769	5.4806	5.9609	6.1076	5.4923	5.5285	5.5632
Present	≠0	4.6743	5.7874	5.4921	5.9788	6.1279	5.5134	5.5456	5.5725

for thick plates and it needs to be considered in the modeling.

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