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Vibration analysis of FG nanoplates with nanovoids on viscoelastic substrate under hygro-thermo-mechanical loading using nonlocal strain gradient theory

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Abstract. According to a generalized nonlocal strain gradient theory (NSGT), dynamic modeling and free vibrational analysis of nanoporous inhomogeneous nanoplates is presented. The present model incorporates two scale coefficients to examine vibration behavior of nanoplates much accurately. Porosity-dependent material properties of the nanoplate are defined via a modified power-law function. The nanoplate is resting on a viscoelastic substrate and is subjected to hygro-thermal environment and in-plane linearly varying mechanical loads. The governing equations and related classical and non-classical boundary conditions are derived based on Hamilton's principle. These equations are solved for hinged nanoplates via Galerkin's method. Obtained results show the importance of hygro-thermal loading, viscoelastic medium, in-plane bending load, gradient index, nonlocal parameter, strain gradient parameter and porosities on vibrational characteristics of size-dependent FG nanoplates.

Keywords: nanoporous plate; hygro-thermal environment; nonlocal strain gradient theory; four-variable plate theory

1. Introduction

Functionally graded materials (FGMs) are a new class of Composite Structures that is of great interest for engineering design and manufacture. These kinds of materials possess desirable properties for specific applications, particularly for aircrafts, space vehicles, optical, biomechanical, electronic, chemical, mechanical, shipbuilding and other engineering structures under stress concentration, high thermal and residual stresses. In general, FGMs are both macroscopically and microscopically heterogeneous advanced composites which are made for example from a mixture of ceramics and metals with continuous composition gradation from pure ceramic on one surface to full metal on the other. This is achieved by gradually varying the volume fraction of the constituent materials. Due to the importance and wide engineering applications of FGMs, the static, vibrational, thermo-mechanical and buckling analyses of FGM structures have been addressed by many investigators (Jabari *et al*. 2008, Chikh *et al*. 2016, Sobhy 2016).

The functionally graded (FG) plates are commonly used in thermal environments; they can buckle under thermal and mechanical loads (Bouderba *et al*. 2016). The classical plate theory (CPT) is usually used to carry out stability analysis of thin FG plates. This theory ignores the transverse shear deformation and assumes that the normal to the middle plane before deformation remains straight and normal to the middle surface after deformation. As a result, the classical plate theory overestimates the buckling load except for truly thin plates. The first-order shear deformation theory (FSDT), including the effects of transverse shear deformation was employed by some researches to analyze buckling behavior of moderately thick FG plates. The FSDT assumes a constant value of transverse shear strain through the thickness of the plate and requires shear correction factor to correct for the discrepancy between the actual transverse shear strain and the constant one. The shear correction factor, which is crucial to an accurate analysis, depends on geometric parameters, loadings, material and boundary conditions of the plate. Also in the FSDT, the cross-sectional warping is neglected as it is assumed that the plane sections remain plane (Bourada *et al*. 2015, Draiche *et al*. 2016). To overcome the drawbacks of these theories (i.e., CPT and FSDT), various higher-order plate theories have been proposed by assuming higher-order displacement fields (Mahi *et al*. 2015, Houari *et al*. 2016).

The increased applications of advanced composite materials in nano structural members have stimulated interest in the accurate prediction of the response characteristics of functionally graded (FG) nanoplates used in situations (Lee *et al*. 2006, Zalesak *et al*. 2016). Investigation of mechanical behavior of scale-free plates has been extensively conducted in the literature based on classical theories. However, these theories are impotent to describe the size effects on the nanostructures. This problem is resolved using the nonlocal elasticity theory of Eringen (1983) in which small size effects are considered by introducing an additional scale parameter. According to the nonlocal stress field theory, the stress state at a given point depends on the strain states at all points. The nonlocal elasticity theory has been broadly applied to examine the static and dynamic behaviors of nanoscale structures (Berrabah *et al*. 2013, Zenkour and Abouelregal 2014, Chakraverti and Behera 2015, Aissani *et al*. 2015). However, analysis and modeling of FG nanoplates are

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performed by various researchers (Natarajan *et al*. 2012, Daneshmehr and Rajabpoor 2014, Belkorissat *et al*. 2015, Ebrahimi and Barati 2016a, Sobhy 2015, Sobhy and Radwan 2017, Larbi Chaht *et al*. 2015).

Although nonlocal elasticity theory (NET) of Eringen is a suitable theory for modeling of nanostructure, it has some shortcomings due to neglecting stiffness-hardening mechanism reported in experimental works and strain gradient elasticity (Lam *et al*. 2003, Akgöz and Civalek 2015a,b,c, Mirsalehi *et al*. 2017). By using nonlocal strain gradient theory (NSGT), Lim *et al*. (2015) matched the dispersion curves of nanobeams with those of experimental data. They concluded that NSGT is more accurate for modeling and analysis of nanostructures by considering both stiffness reduction and enhancement effects. Application of NSGT in modeling and analysis of nanoscale structures have been examined by some researchers (Li *et al*. 2015, 2016, Li and Hu 2015, 2016, 2017, Ebrahimi and Barati 2017, Barati and Zenkour 2017).

In this paper, nonlocal strain gradient theory is employed to investigate damping vibration behavior of FG nanoplates under hygro-thermo-mechanical loading resting on viscoelastic medium using a refined four-variable plate theory. The theory introduces two scale parameters corresponding to nonlocal and strain gradient effects to capture both stiffness-softening and stiffness-hardening influences. Hamilton's principle is employed to obtain the governing equation of a nonlocal strain gradient FG nanoplate. These equations are solved via Galerkin's method to obtain the natural frequencies.The results show that vibrational behavior of the nanoplate are significantly influenced by the nonlocality, strain gradient parameter, viscoelastic medium, in-plane mechanical load, hygro-thermal loading, material composition, elastic medium and geometrical parameters. Obtained frequencies can be used as benchmark results in analysis of nanoplates modeled by nonlocal strain gradient theory.

2. Nonlocal strain gradient nanoplate model

The proposed nonlocal strain gradient theory (Ebrahimi and Barati 2017) takes into account both nonlocal stress field and the strain gradient effects by introducing two scale parameters. This theory defines the stress field as

$$\sigma_{ij} = \sigma_{ij}^{(0)} - \nabla \sigma_{ij}^{(1)} \tag{1}$$

in which the stresses $\sigma_{ij}^{(0)}$ and $\sigma_{ij}^{(1)}$ are corresponding to strain ε_{ij} and strain gradient $\nabla\varepsilon_{ij}$, respectively as

$$\sigma_{ij}^{(0)} = \int_V C_{ijkl}\alpha_0(x,x',e_0a)\varepsilon'_{kl}(x')dx' \tag{2a}$$

$$\sigma_{ij}^{(1)} = l^2\int_V C_{ijkl}\alpha_1(x,x',e_1a)\nabla\varepsilon'_{kl}(x')dx' \tag{2b}$$

in which C_{ijkl} are the elastic coefficients and e_0a and e_1a capture the nonlocal effects and l captures the strain gradient effects. When the nonlocal functions $\alpha_0(x,x',e_0a)$ and $\alpha_1(x,x',e_0a)$satisfy the developed conditions by Eringen,

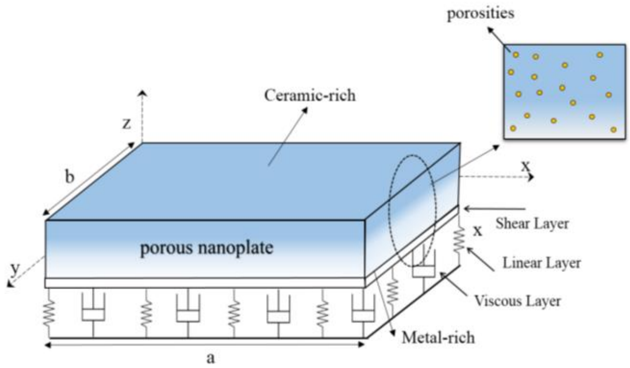


Fig. 1 Configuration of nanoporous inhomogeneous nanoplate on elastic substrate

the constitutive relation of nonlocal strain gradient theory has the following form

$$\begin{aligned}&[1-(e_1a)^2\nabla^2][1-(e_0a)^2\nabla^2]\sigma_{ij} = C_{ijkl}[1-(e_1a)^2\nabla^2]\varepsilon_{kl}\\&-C_{ijkl}\,l^2[1-(e_0a)^2\nabla^2]\nabla^2\varepsilon_{kl}\end{aligned} \tag{3}$$

in which ∇^2 denotes the Laplacian operator. Considering $e_1=e_0=e$, the general constitutive relation in Eq. (3) becomes

$$[1-(ea)^2\nabla^2]\sigma_{ij} = C_{ijkl}\,[1-l^2\nabla^2]\varepsilon_{kl} \tag{4}$$

To consider hygro-thermal effects Eq. (4) can be written as (Ebrahimi and Barati 2017)

$$[1-(ea)^2\nabla^2]\sigma_{ij} = C_{ijkl}[1-l^2\nabla^2](\varepsilon_{kl}-\gamma_{ij}T-\beta_{ij}C) \tag{5}$$

where γ_{ij} and β_{ij} are thermal and moisture expansion coefficients, respectively.

3. FG plate model based on neutral surface position

Consider a rectangular $(a\times b)$ porous nanoplate of uniform thickness h made of FGM as shown in Fig. 1. Also, different types of mechanical loads are illustrate in Fig. 2. A FG material can be specified by the variation in the volume fractions. Due to this variation, neutral axis of FG nanoplate may not coincide with its mid-surface which leads to bending-extension coupling. By using neutral axis, this coupling is eliminated. Based on the modified power-law model, Young' modulus E, density ρ, thermal expansion coefficient γ and moisture expansion coefficient β are described as

$$E(z) = (E_c-E_m)\left(\frac{z}{h}+\frac{1}{2}\right)^p + E_m - \frac{\alpha}{2}(E_c+E_m) \tag{6a}$$

$$\rho(z) = (\rho_c-\rho_m)\left(\frac{z}{h}+\frac{1}{2}\right)^p + \rho_m - \frac{\alpha}{2}(\rho_c+\rho_m) \tag{6b}$$

$$\gamma(z) = (\gamma_c-\gamma_m)\left(\frac{z}{h}+\frac{1}{2}\right)^p + \gamma_m - \frac{\alpha}{2}(\gamma_c+\gamma_m) \tag{6c}$$

$$\beta(z) = (\beta_c-\beta_m)\left(\frac{z}{h}+\frac{1}{2}\right)^p + \beta_m - \frac{\alpha}{2}(\beta_c+\beta_m) \tag{6d}$$

in which c and m denote the material properties of ceramic and metal phases, respectively and p is inhomogeneity or

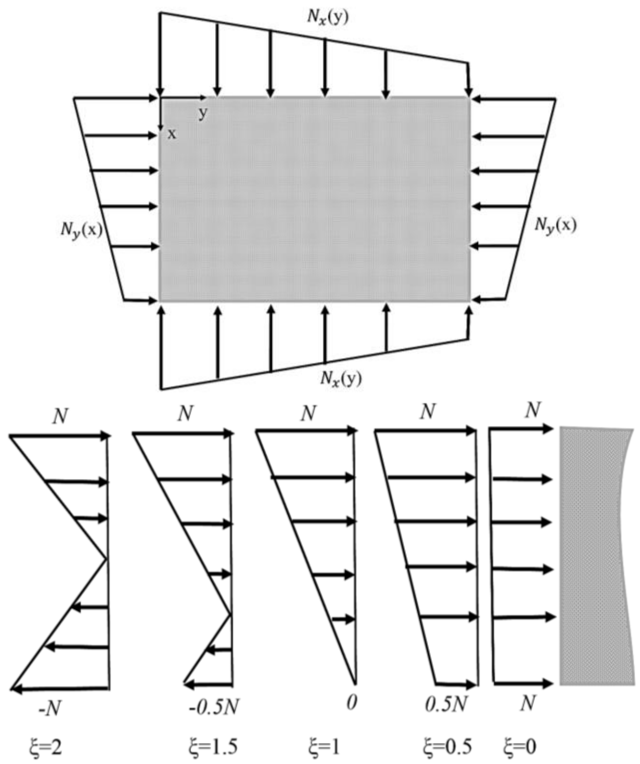


Fig. 2 Different cases of in-plane loads

power-law index. Also, α is the porosity volume fraction. The displacement field according to the four-variable plate model considering exact position of neutral surface can be expressed by

$$u_1(x,y,z,t) = u(x,y,t) - (z - z^*)\frac{\partial w_b}{\partial x} - [f(z) - z^{**}]\frac{\partial w_s}{\partial x} \tag{7a}$$

$$u_2(x,y,z,t) = v(x,y,t) - (z - z^*)\frac{\partial w_b}{\partial y} - [f(z) - z^{**}]\frac{\partial w_s}{\partial y} \tag{7b}$$

$$u_3(x,y,z,t) = w(x,y,t) = w_b(x,y,t) + w_s(x,y,t) \tag{7c}$$

where

$$z^* = \frac{\int_{-h/2}^{h/2} E(z)\, z dz}{\int_{-h/2}^{h/2} E(z)\, dz}, \quad z^{**} = \frac{\int_{-h/2}^{h/2} E(z)\, f(z) dz}{\int_{-h/2}^{h/2} E(z)\, dz} \tag{8}$$

Also, u and v are in-plane displacements and w_b and w_d denote the bending and shear transverse displacement, respectively. The shape function of transverse shear deformation is considered as

$$f(z) = -\frac{z}{4} + \frac{5z^3}{3h^2} \tag{9}$$

According to the present plate theory with four unknown, the nonzero strains are obtained as

$$\begin{aligned}
\varepsilon_x &= \frac{\partial u}{\partial x} - (z - z^*)\frac{\partial^2 w_b}{\partial x^2} - [f(z) - z^{**}]\frac{\partial^2 w_s}{\partial x^2} \\
\varepsilon_y &= \frac{\partial v}{\partial y} - (z - z^*)\frac{\partial^2 w_b}{\partial y^2} - [f(z) - z^{**}]\frac{\partial^2 w_s}{\partial y^2} \\
\gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2(z - z^*)\frac{\partial^2 w_b}{\partial x \partial y} - 2[f(z) - z^{**}]\frac{\partial^2 w_s}{\partial x \partial y} \\
\gamma_{yz} &= g(z)\frac{\partial w_s}{\partial y}, \quad \gamma_{xz} = g(z)\frac{\partial w_s}{\partial x}
\end{aligned} \tag{10}$$

Also, the extended Hamilton's principle express that

$$\int_0^t \delta(U - T + V)\, dt = 0 \tag{11}$$

in which t is time; U is strain energy, T is kinetic energy and V is work done by external forces. The first variation of the strain energy can be calculated as

$$\begin{aligned}
\delta U = \int_V (&\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{xx}^{(1)}\delta\nabla\varepsilon_{xx} + \sigma_{yy}\delta\varepsilon_{yy} + \sigma_{yy}^{(1)}\delta\nabla\varepsilon_{yy} \\
&+ \sigma_{xy}\delta\gamma_{xy} + \sigma_{xy}^{(1)}\delta\nabla\gamma_{xy} + \sigma_{yz}\delta\gamma_{yz} + \sigma_{yz}^{(1)}\delta\nabla\gamma_{yz} \\
&+ \sigma_{xz}\delta\gamma_{xz} + \sigma_{xz}^{(1)}\delta\nabla\gamma_{xz})\, dV
\end{aligned} \tag{12}$$

in which σ are the components of the stress tensor and ε_{ij} are the components of the strain tensor. Substituting Eqs. (8) and (10) into Eq.(12) yields

$$\begin{aligned}
\delta U = \int_0^a \int_0^b [&N_{xx}[\frac{\partial \delta u}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial \delta w}{\partial x}] - M_{xx}^b \frac{\partial^2 \delta w_b}{\partial x^2} - M_{xx}^s \frac{\partial^2 \delta w_s}{\partial x^2} \\
&+ N_{yy}[\frac{\partial \delta v}{\partial y} + \frac{\partial w}{\partial y}\frac{\partial \delta w}{\partial y}] - M_{yy}^b \frac{\partial^2 \delta w_b}{\partial y^2} - M_{yy}^s \frac{\partial^2 \delta w_s}{\partial y^2} \\
&+ N_{xy}(\frac{\partial \delta u}{\partial y} + \frac{\partial \delta v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial \delta w}{\partial y} + \frac{\partial w}{\partial y}\frac{\partial \delta w}{\partial x}) - 2M_{xy}^b \frac{\partial^2 \delta w_b}{\partial x \partial y} \\
&- 2M_{xy}^s \frac{\partial^2 \delta w_s}{\partial x \partial y} + Q_{yz}\frac{\partial \delta w_s}{\partial y} + Q_{xz}\frac{\partial \delta w_s}{\partial x}]\, dydx
\end{aligned} \tag{13}$$

in which

$$\begin{aligned}
N_{xx} &= \int_{-h/2}^{h/2} (\sigma_{xx}^0 - \nabla\sigma_{xx}^{(1)}) dz = N_{xx}^{(0)} - \nabla N_{xx}^{(1)} \\
N_{xy} &= \int_{-h/2}^{h/2} (\sigma_{xy}^0 - \nabla\sigma_{xy}^{(1)}) dz = N_{xy}^{(0)} - \nabla N_{xy}^{(1)} \\
N_{yy} &= \int_{-h/2}^{h/2} (\sigma_{yy}^0 - \nabla\sigma_{yy}^{(1)}) dz = N_{yy}^{(0)} - \nabla N_{yy}^{(1)} \\
M_{xx}^b &= \int_{-h/2}^{h/2} z(\sigma_{xx}^0 - \nabla\sigma_{xx}^{(1)}) dz = M_{xx}^{b(0)} - \nabla M_{xx}^{b(1)} \\
M_{xx}^s &= \int_{-h/2}^{h/2} f(\sigma_{xx}^0 - \nabla\sigma_{xx}^{(1)}) dz = M_{xx}^{s(0)} - \nabla M_{xx}^{s(1)} \\
M_{yy}^b &= \int_{-h/2}^{h/2} z(\sigma_{yy}^0 - \nabla\sigma_{yy}^{(1)}) dz = M_{yy}^{b(0)} - \nabla M_{yy}^{b(1)} \\
M_{yy}^s &= \int_{-h/2}^{h/2} f(\sigma_{yy}^0 - \nabla\sigma_{yy}^{(1)}) dz = M_{yy}^{s(0)} - \nabla M_{yy}^{s(1)} \\
M_{xy}^b &= \int_{-h/2}^{h/2} z(\sigma_{xy}^0 - \nabla\sigma_{xy}^{(1)}) dz = M_{xy}^{b(0)} - \nabla M_{xy}^{b(1)} \\
M_{xy}^s &= \int_{-h/2}^{h/2} f(\sigma_{xy}^0 - \nabla\sigma_{xy}^{(1)}) dz = M_{xy}^{s(0)} - \nabla M_{xy}^{s(1)} \\
Q_{xz} &= \int_{-h/2}^{h/2} g(\sigma_{xz}^0 - \nabla\sigma_{xz}^{(1)}) dz = Q_{xz}^{(0)} - \nabla Q_{xz}^{(1)} \\
Q_{yz} &= \int_{-h/2}^{h/2} g(\sigma_{yz}^0 - \nabla\sigma_{yz}^{(1)}) dz = Q_{yz}^{(0)} - \nabla Q_{yz}^{(1)}
\end{aligned} \tag{14a}$$

where

$$\begin{aligned}
N_{ij}^{(0)} &= \int_{-h/2}^{h/2} (\sigma_{ij}^{(0)})dz, \quad N_{ij}^{(1)} = \int_{-h/2}^{h/2} (\sigma_{ij}^{(1)})dz \\
M_{ij}^{b(0)} &= \int_{-h/2}^{h/2} z(\sigma_{ij}^{b(0)})dz, \quad M_{ij}^{b(1)} = \int_{-h/2}^{h/2} z(\sigma_{ij}^{b(1)})dz \\
M_{ij}^{s(0)} &= \int_{-h/2}^{h/2} f(\sigma_{ij}^{s(0)})dz, \quad M_{ij}^{s(1)} = \int_{-h/2}^{h/2} f(\sigma_{ij}^{s(1)})dz \\
Q_{xz}^{(0)} &= \int_{-h/2}^{h/2} g(\sigma_{xz}^{i(0)})dz, \quad Q_{xz}^{(1)} = \int_{-h/2}^{h/2} g(\sigma_{xz}^{i(1)})dz \\
Q_{yz}^{(0)} &= \int_{-h/2}^{h/2} g(\sigma_{yz}^{i(0)})dz, \quad Q_{yz}^{(1)} = \int_{-h/2}^{h/2} g(\sigma_{yz}^{i(1)})dz
\end{aligned} \tag{14b}$$

in which (ij=xx, xy, yy). The first variation of the work done by applied forces can be written as

$$\delta V=\int_0^a\int_0^b(N_x^0\frac{\partial(w_b+w_s)}{\partial x}\frac{\partial\delta(w_b+w_s)}{\partial x}+N_y^0\frac{\partial(w_b+w_s)}{\partial y}\frac{\partial\delta(w_b+w_s)}{\partial y}$$
$$+2\delta N_{xy}^0\frac{\partial(w_b+w_s)}{\partial x}\frac{\partial(w_b+w_s)}{\partial y}-k_w(w_b+w_s)\delta(w_b+w_s)$$
$$-c_d\delta\frac{\partial(w_b+w_s)}{\partial t}+k_p(\frac{\partial(w_b+w_s)}{\partial x}\frac{\partial\delta(w_b+w_s)}{\partial x}$$
$$+\frac{\partial(w_b+w_s)}{\partial y}\frac{\partial\delta(w_b+w_s)}{\partial y}))dydx \tag{15a}$$

where N_x^0, N_y^0, N_{xy}^0 are in-plane applied loads; k_w, k_p and c_d are Winkler, Pasternak and damping constants. It is assumed that the nanoplate is subjected to the following in-plane load while shear loading is zero $N_{xy}^0=0$

$$N_x^0=N^T+N^H+N_x^M,\quad N_y^0=N^T+N^H+N_y^M$$
$$N_x^0=N(1-\xi\frac{y}{b}),\quad N_y^0=\eta N(1-\xi\frac{x}{a}) \tag{15b}$$

where hygro-thermal resultant can be expressed by

$$N^T=\int_{-h/2}^{h/2}\frac{E(z)}{1-\nu}\gamma(z)(T-T_0)\,dz$$
$$N^H=\int_{-h/2}^{h/2}\frac{E(z)}{1-\nu}\beta(z)(C-C_0)\,dz \tag{15c}$$

in which $C=\Delta C+C_0$ and $T=\Delta T+T_0$ are uniform moisture and temperature changes; C_0 and T_0 are reference moisture and temperature. Also, in-plane mechanical loads are expressed by

$$N_x^M=N(1-\xi\frac{y}{b}),\quad N_y^M=N(1-\xi\frac{x}{a}) \tag{15d}$$

The first variation of the kinetic energy can be written in the following form

$$\delta K=\int_0^a\int_0^b[I_0(\frac{\partial u}{\partial t}\frac{\partial\delta u}{\partial t}+\frac{\partial v}{\partial t}\frac{\partial\delta v}{\partial t}+\frac{\partial(w_b+w_s)}{\partial t}\frac{\partial\delta(w_b+w_s)}{\partial t})$$
$$-I_1(\frac{\partial u}{\partial t}\frac{\partial\delta w_b}{\partial x\partial t}+\frac{\partial w_b}{\partial x\partial t}\frac{\partial\delta u}{\partial t}+\frac{\partial v}{\partial t}\frac{\partial\delta w_b}{\partial y\partial t}+\frac{\partial w_b}{\partial y\partial t}\frac{\partial\delta v}{\partial t})$$
$$-I_3(\frac{\partial u}{\partial t}\frac{\partial\delta w_s}{\partial x\partial t}+\frac{\partial w_s}{\partial x\partial t}\frac{\partial\delta u}{\partial t}+\frac{\partial v}{\partial t}\frac{\partial\delta w_s}{\partial y\partial t}+\frac{\partial w_s}{\partial y\partial t}\frac{\partial\delta v}{\partial t})$$
$$+I_2(\frac{\partial w_b}{\partial x\partial t}\frac{\partial\delta w_b}{\partial x\partial t}+\frac{\partial w_b}{\partial y\partial t}\frac{\partial\delta w_b}{\partial y\partial t})+I_5(\frac{\partial w_s}{\partial x\partial t}\frac{\partial\delta w_s}{\partial x\partial t}+\frac{\partial w_s}{\partial y\partial t}\frac{\partial\delta w_s}{\partial y\partial t})$$
$$+I_4(\frac{\partial w_b}{\partial x\partial t}\frac{\partial\delta w_s}{\partial x\partial t}+\frac{\partial w_s}{\partial x\partial t}\frac{\partial\delta w_b}{\partial x\partial t}+\frac{\partial w_b}{\partial y\partial t}\frac{\partial\delta w_s}{\partial y\partial t}+\frac{\partial w_s}{\partial y\partial t}\frac{\partial\delta w_b}{\partial y\partial t})]dydx \tag{16}$$

in which

$$(I_0,I_1,I_2,I_3,I_4,I_5)=$$
$$\int_{-h/2}^{h/2}(1,z-z^*,(z-z^*)^2,f-z^{**},(z-z^*)(f-z^{**}),(f-z^{**})^2)\rho(z)dz \tag{17}$$

By inserting Eqs. (13)-(16) into Eq. (11) and setting the coefficients of δu, δv, δw_b and δw_s to zero, the following Euler-Lagrange equations can be obtained.

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=I_0\frac{\partial^2 u}{\partial t^2}-I_1\frac{\partial^3 w_b}{\partial x\partial t^2}-I_3\frac{\partial^3 w_s}{\partial x\partial t^2} \tag{18}$$

$$\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=I_0\frac{\partial^2 v}{\partial t^2}-I_1\frac{\partial^3 w_b}{\partial y\partial t^2}-I_3\frac{\partial^3 w_s}{\partial y\partial t^2} \tag{19}$$

$$\frac{\partial^2 M_x^b}{\partial x^2}+2\frac{\partial^2 M_{xy}^b}{\partial x\partial y}+\frac{\partial^2 M_y^b}{\partial y^2}-(N^T+N^H)\nabla^2(w_b+w_s)$$
$$-N_x^M(y)\frac{\partial^2(w_b+w_s)}{\partial x^2}-N_y^M(x)\frac{\partial^2(w_b+w_s)}{\partial y^2}-k_w(w_b+w_s)$$
$$-c_d\frac{\partial(w_b+w_s)}{\partial t}+k_p\nabla^2(w_b+w_s)=I_0\frac{\partial^2(w_b+w_s)}{\partial t^2}$$
$$+I_1(\frac{\partial^3 u}{\partial x\partial t^2}+\frac{\partial^3 v}{\partial y\partial t^2})-I_2\nabla^2(\frac{\partial^2 w_b}{\partial t^2})-I_4\nabla^2(\frac{\partial^2 w_s}{\partial t^2}) \tag{20}$$

$$\frac{\partial^2 M_x^s}{\partial x^2}+2\frac{\partial^2 M_{xy}^s}{\partial x\partial y}+\frac{\partial^2 M_y^s}{\partial y^2}+\frac{\partial Q_{xz}}{\partial x}+\frac{\partial Q_{yz}}{\partial y}-(N^T+N^H)\nabla^2(w_b+w_s)$$
$$-N_x^M(y)\frac{\partial^2(w_b+w_s)}{\partial x^2}-N_y^M(x)\frac{\partial^2(w_b+w_s)}{\partial y^2}-k_w(w_b+w_s)$$
$$-c_d\frac{\partial(w_b+w_s)}{\partial t}+k_p\nabla^2(w_b+w_s)=I_0\frac{\partial^2(w_b+w_s)}{\partial t^2}$$
$$+I_3(\frac{\partial^3 u}{\partial x\partial t^2}+\frac{\partial^3 v}{\partial y\partial t^2})-I_4\nabla^2(\frac{\partial^2 w_b}{\partial t^2})-I_5\nabla^2(\frac{\partial^2 w_s}{\partial t^2}) \tag{21}$$

The classical and non-classical boundary conditions can be obtained in the derivation process when using the integrations by parts. Thus, we obtain classical boundary conditions at x=0 or a and y=0 or b as

Specify un_x+vn_y or

$$N_x n_x^2+2n_x n_y N_{xy}+N_y n_y^2=0$$

Specify $-un_y+vn_x$ or

$$(N_y-N_x)n_x n_y+N_{xy}(n_x^2-n_y^2)=0$$

Specify w_b or

$$\left(\frac{\partial M_{xx}^b}{\partial x}+\frac{\partial M_{xy}^b}{\partial y}-I_1\frac{\partial^2 u}{\partial t^2}+I_2\frac{\partial^3 w_b}{\partial x\partial t^2}+I_4\frac{\partial^3 w_s}{\partial x\partial t^2}\right)n_x$$
$$+\left(\frac{\partial M_{yy}^b}{\partial y}+\frac{\partial M_{xy}^b}{\partial x}-I_1\frac{\partial^2 v}{\partial t^2}+I_2\frac{\partial^3 w_b}{\partial y\partial t^2}+I_4\frac{\partial^3 w_s}{\partial y\partial t^2}\right)n_y=0 \tag{22}$$

Specify w_s or

$$\left(\frac{\partial M_{xx}^s}{\partial x}+\frac{\partial M_{xy}^s}{\partial y}+Q_{xz}-I_3\frac{\partial^2 u}{\partial t^2}+I_4\frac{\partial^3 w_b}{\partial x\partial t^2}+I_5\frac{\partial^3 w_s}{\partial x\partial t^2}\right)n_x$$
$$+\left(\frac{\partial M_{yy}^s}{\partial y}+\frac{\partial M_{xy}^s}{\partial x}+Q_{yz}-I_3\frac{\partial^2 v}{\partial t^2}+I_4\frac{\partial^3 w_b}{\partial y\partial t^2}+I_5\frac{\partial^3 w_s}{\partial y\partial t^2}\right)n_y=0$$

Specify $\frac{\partial w_b}{\partial n}$ or $M_{xx}^b n_x^2+n_x n_y M_{xy}^b+M_{yy}^b n_y^2=0$

where $\frac{\partial()}{\partial n}=n_x\frac{\partial()}{\partial x}+n_y\frac{\partial()}{\partial y}$; n_x and n_y are the x and y-components of the unit normal vector on the nanoplate boundaries, respectively and the non-classical boundary conditions are

Specify $\frac{\partial^2 w_b}{\partial x^2}$ or $M_{xx}^{b(1)}=0$

Specify $\frac{\partial^2 w_b}{\partial y^2}$ or $M_{yy}^{b(1)}=0$

Specify $\frac{\partial^2 w_s}{\partial x^2}$ or $M_{xx}^{s(1)}=0$ (23)

Specify $\frac{\partial^2 w_s}{\partial y^2}$ or $M_{yy}^{s(1)}=0$

Based on the NSGT, the constitutive relations of presented higher order FG nanoplate can be stated as:

$$(1-\mu\nabla^2)\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\\ \sigma_{yz}\\ \sigma_{xz}\end{Bmatrix} = \frac{E(z)}{1-v^2}(1-\lambda\nabla^2)\begin{pmatrix}1 & v & 0 & 0 & 0\\ v & 1 & 0 & 0 & 0\\ 0 & 0 & (1-v)/2 & 0 & 0\\ 0 & 0 & 0 & (1-v)/2 & 0\\ 0 & 0 & 0 & 0 & (1-v)/2\end{pmatrix}\begin{Bmatrix}\varepsilon_x-\gamma\Delta T-\beta\Delta C\\ \varepsilon_y-\gamma\Delta T-\beta\Delta C\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} \tag{24}$$

Integrating Eq. (24) over the plate's cross-section area, one can obtain the force-strain and the moment-strain of the nonlocal refined FG plates can be obtained as follows

$$(1-\mu\nabla^2)\begin{Bmatrix}N_x\\ N_y\\ N_{xy}\end{Bmatrix} = A(1-\lambda\nabla^2)\begin{pmatrix}1 & v & 0\\ v & 1 & 0\\ 0 & 0 & (1-v)/2\end{pmatrix}\begin{Bmatrix}\frac{\partial u}{\partial x}\\ \frac{\partial v}{\partial y}\\ \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\end{Bmatrix} \tag{25}$$

$$(1-\mu\nabla^2)\begin{Bmatrix}M_x^b\\ M_y^b\\ M_{xy}^b\end{Bmatrix} = D(1-\lambda\nabla^2)\begin{pmatrix}1 & v & 0\\ v & 1 & 0\\ 0 & 0 & (1-v)/2\end{pmatrix}\begin{Bmatrix}-\frac{\partial^2 w_b}{\partial x^2}\\ -\frac{\partial^2 w_b}{\partial y^2}\\ -2\frac{\partial^2 w_b}{\partial x\partial y}\end{Bmatrix} + E(1-\lambda\nabla^2)\begin{pmatrix}1 & v & 0\\ v & 1 & 0\\ 0 & 0 & (1-v)/2\end{pmatrix}\begin{Bmatrix}-\frac{\partial^2 w_s}{\partial x^2}\\ -\frac{\partial^2 w_s}{\partial y^2}\\ -2\frac{\partial^2 w_s}{\partial x\partial y}\end{Bmatrix} \tag{26}$$

$$(1-\mu\nabla^2)\begin{Bmatrix}M_x^s\\ M_y^s\\ M_{xy}^s\end{Bmatrix} = E(1-\lambda\nabla^2)\begin{pmatrix}1 & v & 0\\ v & 1 & 0\\ 0 & 0 & (1-v)/2\end{pmatrix}\begin{Bmatrix}-\frac{\partial^2 w_b}{\partial x^2}\\ -\frac{\partial^2 w_b}{\partial y^2}\\ -2\frac{\partial^2 w_b}{\partial x\partial y}\end{Bmatrix} + F(1-\lambda\nabla^2)\begin{pmatrix}1 & v & 0\\ v & 1 & 0\\ 0 & 0 & (1-v)/2\end{pmatrix}\begin{Bmatrix}-\frac{\partial^2 w_s}{\partial x^2}\\ -\frac{\partial^2 w_s}{\partial y^2}\\ -2\frac{\partial^2 w_s}{\partial x\partial y}\end{Bmatrix} \tag{27}$$

$$(1-\mu\nabla^2)\begin{Bmatrix}Q_x\\ Q_y\end{Bmatrix} = A_{44}(1-\lambda\nabla^2)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{Bmatrix}\frac{\partial w_s}{\partial x}\\ \frac{\partial w_s}{\partial y}\end{Bmatrix} \tag{28}$$

in which

$$\begin{aligned}
&A=\int_{-h/2}^{h/2}\frac{E(z)}{1-v^2}dz,\ \ D=\int_{-h/2}^{h/2}\frac{E(z)(z-z^*)^2}{1-v^2}dz,\\
&E=\int_{-h/2}^{h/2}\frac{E(z)(z-z^*)(f-z^{**})}{1-v^2}dz\\
&F=\int_{-h/2}^{h/2}\frac{E(z)(f-z^{**})^2}{1-v^2}dz,\\
&A_{44}=\int_{-h/2}^{h/2}\frac{E(z)}{2(1+v)}g^2dz
\end{aligned} \tag{29}$$

The governing equations in terms of the displacements for a NSGT refined four-variable FG nanoplate can be derived by substituting Eqs. (25)-(28), into Eqs. (18)-(21) as follows

$$\begin{aligned}
&A(1-\lambda\nabla^2)\left(\frac{\partial^2 u}{\partial x^2}+\frac{1-v}{2}\frac{\partial^2 u}{\partial y^2}+\frac{1+v}{2}\frac{\partial^2 v}{\partial x\partial y}\right)\\
&+(1-\mu\nabla^2)\left(-I_0\frac{\partial^2 u}{\partial t^2}+I_1\frac{\partial^3 w_b}{\partial x\partial t^2}+I_3\frac{\partial^3 w_s}{\partial x\partial t^2}\right)=0
\end{aligned} \tag{30}$$

$$\begin{aligned}
&A(1-\lambda\nabla^2)\left(\frac{\partial^2 v}{\partial y^2}+\frac{1-v}{2}\frac{\partial^2 v}{\partial x^2}+\frac{1+v}{2}\frac{\partial^2 u}{\partial x\partial y}\right)\\
&+(1-\mu\nabla^2)\left(-I_0\frac{\partial^2 v}{\partial t^2}+I_1\frac{\partial^3 w_b}{\partial y\partial t^2}+I_3\frac{\partial^3 w_s}{\partial y\partial t^2}\right)=0
\end{aligned} \tag{31}$$

$$\begin{aligned}
&-D(1-\lambda\nabla^2)\left(\frac{\partial^4 w_b}{\partial x^4}+2\frac{\partial^4 w_b}{\partial x^2\partial y^2}+\frac{\partial^4 w_b}{\partial y^4}\right)\\
&-E(1-\lambda\nabla^2)\left(\frac{\partial^4 w_s}{\partial x^4}+2\frac{\partial^4 w_s}{\partial x^2\partial y^2}+\frac{\partial^4 w_s}{\partial y^4}\right)\\
&+(1-\mu\nabla^2)\Bigg(-I_0\frac{\partial^2 (w_b+w_s)}{\partial t^2}-I_1\left(\frac{\partial^3 u}{\partial x\partial t^2}+\frac{\partial^3 v}{\partial y\partial t^2}\right)\\
&+I_2\nabla^2\left(\frac{\partial^2 w_b}{\partial t^2}\right)+I_4\nabla^2\left(\frac{\partial^2 w_s}{\partial t^2}\right)-(N^T+N^H)\nabla^2(w_b+w_s)\\
&-N_x^M(y)\frac{\partial^2 (w_b+w_s)}{\partial x^2}-N_y^M(x)\frac{\partial^2 (w_b+w_s)}{\partial y^2}\\
&-\left(k_w+\frac{\partial}{\partial t}c_d\right)(w_b+w_s)+k_P\nabla^2(w_b+w_s)\Bigg)=0
\end{aligned} \tag{32}$$

$$\begin{aligned}
&-E(1-\lambda\nabla^2)\left(\frac{\partial^4 w_b}{\partial x^4}+2\frac{\partial^4 w_b}{\partial x^2\partial y^2}+\frac{\partial^4 w_b}{\partial y^4}\right)\\
&-F(1-\lambda\nabla^2)\left(\frac{\partial^4 w_s}{\partial x^4}+2\frac{\partial^4 w_s}{\partial x^2\partial y^2}+\frac{\partial^4 w_s}{\partial y^4}\right)\\
&+A_{44}(1-\lambda\nabla^2)\left(\frac{\partial^2 w_s}{\partial x^2}+\frac{\partial^2 w_s}{\partial y^2}\right)+(1-\mu\nabla^2)\Bigg(-I_0\frac{\partial^2 (w_b+w_s)}{\partial t^2}\\
&-I_3\left(\frac{\partial^3 u}{\partial x\partial t^2}+\frac{\partial^3 v}{\partial y\partial t^2}\right)+I_4\nabla^2\left(\frac{\partial^2 w_b}{\partial t^2}\right)+I_5\nabla^2\left(\frac{\partial^2 w_s}{\partial t^2}\right)\\
&-(N^T+N^H)\nabla^2(w_b+w_s)-N_x^M(y)\frac{\partial^2 (w_b+w_s)}{\partial x^2}\\
&-N_y^M(x)\frac{\partial^2 (w_b+w_s)}{\partial y^2}-\left(k_w+\frac{\partial}{\partial t}c_d\right)(w_b+w_s)\\
&+k_p\nabla^2(w_b+w_s)\Bigg)=0
\end{aligned} \tag{33}$$

4. Solution procedure

In this section, Galerkin's method is implemented to solve the governing equations of nonlocal strain gradient based FG nanoplates. Thus, the displacement field can be calculated as

$$u=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}U_{mn}\frac{\partial X_m(x)}{\partial x}Y_n(y)e^{i\omega_n t} \tag{34}$$

$$v=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}V_{mn}X_m(x)\frac{\partial Y_n(y)}{\partial y}e^{i\omega_n t} \tag{35}$$

$$w_b=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{bmn}X_m(x)Y_n(y)e^{i\omega_n t} \tag{36}$$

$$w_s=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{smn}X_m(x)Y_n(y)e^{i\omega_n t} \tag{37}$$

where (U_{mn}, V_{mn}, W_{bmn}, W_{smn}) are the unknown coefficients; ω_n is the natural frequency and the functions X_m and Y_n satisfy the boundary conditions. The classical and non-classical boundary condition based on the present plate model are

$$\begin{aligned} & w_b = w_s = 0, \\ & \frac{\partial^2 w_b}{\partial x^2} = \frac{\partial^2 w_s}{\partial x^2} = \frac{\partial^2 w_b}{\partial y^2} = \frac{\partial^2 w_s}{\partial y^2} = 0 \\ & \frac{\partial^4 w_b}{\partial x^4} = \frac{\partial^4 w_s}{\partial x^4} = \frac{\partial^4 w_b}{\partial y^4} = \frac{\partial^4 w_s}{\partial y^4} = 0 \end{aligned} \tag{38}$$

By substituting Eqs. (34)-(37) into Eqs. (30)-(33), and using the Galerkin's method, one obtains

$$\left\{ \{K\} + i\varpi_n [C] + \varpi_n{}^2 [M] \right\} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{bmn} \\ W_{smn} \end{Bmatrix} = 0 \tag{39}$$

in which

$$\begin{aligned} k_{1,1} &= A(\int_0^b\int_0^a (\frac{\partial^3 X_m}{\partial x^3} Y_n \frac{\partial X_m}{\partial x} Y_n) dxdy - \lambda(\int_0^b\int_0^a (\frac{\partial^5 X_m}{\partial x^5} Y_n \frac{\partial X_m}{\partial x} Y_n) dxdy \\ &+ \int_0^b\int_0^a (\frac{\partial^3 X_m}{\partial x^3} \frac{\partial^2 Y_n}{\partial y^2} \frac{\partial X_m}{\partial x} Y_n) dxdy)) + A\frac{1-\nu}{2}(\int_0^b\int_0^a (\frac{\partial X_m}{\partial x} \frac{\partial^2 Y_n}{\partial y^2} \frac{\partial X_m}{\partial x} Y_n) dxdy \\ &- \lambda(\int_0^b\int_0^a (\frac{\partial^3 X_m}{\partial x^3} \frac{\partial^2 Y_n}{\partial y^2} \frac{\partial X_m}{\partial x} Y_n) dxdy + \int_0^b\int_0^a (\frac{\partial X_m}{\partial x} \frac{\partial^4 Y_n}{\partial y^4} \frac{\partial X_m}{\partial x} Y_n) dxdy)) \end{aligned} \tag{40}$$

$$\begin{aligned} k_{1,2} &= A\frac{1+\nu}{2}(\int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} \frac{\partial Y_n}{\partial y} X_m \frac{\partial Y_n}{\partial y}) dxdy \\ &- \lambda(\int_0^b\int_0^a (\frac{\partial^4 X_m}{\partial x^4} \frac{\partial Y_n}{\partial y} X_m \frac{\partial Y_n}{\partial y}) dxdy + \int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} \frac{\partial^3 Y_n}{\partial y^3} X_m \frac{\partial Y_n}{\partial y}) dxdy)) \end{aligned} \tag{41}$$

$$\begin{aligned} k_{2,1} &= A\frac{1+\nu}{2}(\int_0^b\int_0^a (\frac{\partial X_m}{\partial x} \frac{\partial^2 Y_n}{\partial y^2} \frac{\partial X_m}{\partial x} Y_n) dxdy \\ &- \lambda(\int_0^b\int_0^a (\frac{\partial^3 X_m}{\partial x^3} \frac{\partial^2 Y_n}{\partial y^2} \frac{\partial X_m}{\partial x} Y_n) dxdy + \int_0^b\int_0^a (\frac{\partial X_m}{\partial x} \frac{\partial^4 Y_n}{\partial y^4} \frac{\partial X_m}{\partial x} Y_n) dxdy)) \end{aligned} \tag{42}$$

$$\begin{aligned} k_{2,2} &= A(\int_0^b\int_0^a (X_m \frac{\partial^3 Y_n}{\partial y^3} X_m \frac{\partial Y_n}{\partial y}) dxdy \\ &- \lambda(\int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} \frac{\partial^3 Y_n}{\partial y^3} X_m \frac{\partial Y_n}{\partial y}) dxdy + \int_0^b\int_0^a (X_m \frac{\partial^5 Y_n}{\partial y^5} X_m \frac{\partial Y_n}{\partial y}) dxdy)) \\ &+ A\frac{1-\nu}{2}(\int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} \frac{\partial Y_n}{\partial y} X_m \frac{\partial Y_n}{\partial y}) dxdy \\ &- \lambda(\int_0^b\int_0^a (\frac{\partial^4 X_m}{\partial x^4} \frac{\partial Y_n}{\partial y} X_m \frac{\partial Y_n}{\partial y}) dxdy + \int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} \frac{\partial^3 Y_n}{\partial y^3} X_m \frac{\partial Y_n}{\partial y}) dxdy)) \end{aligned} \tag{43}$$

$$\begin{aligned} k_{2,3} = k_{3,2} &= -E(\int_0^b\int_0^a (\frac{\partial^4 X_m}{\partial x^4} Y_n X_m Y_n) dxdy \\ &+ 2\int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} \frac{\partial^2 Y_n}{\partial y^2} X_m Y_n) dxdy + \int_0^b\int_0^a (X_m \frac{\partial^4 Y_n}{\partial y^4} X_m Y_n) dxdy \\ &- \lambda(\int_0^b\int_0^a (\frac{\partial^6 X_m}{\partial x^6} Y_n X_m Y_n) dxdy + 3\int_0^b\int_0^a (\frac{\partial^4 X_m}{\partial x^4} \frac{\partial^2 Y_n}{\partial y^2} X_m Y_n) dxdy \\ &+ 3\int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} \frac{\partial^4 Y_n}{\partial y^4} X_m Y_n) dxdy + \int_0^b\int_0^a (X_m \frac{\partial^6 Y_n}{\partial y^6} X_m Y_n) dxdy)) \end{aligned} \tag{44}$$

$$\begin{aligned} k_{3,3} &= -D(\int_0^b\int_0^a (\frac{\partial^4 X_m}{\partial x^4} Y_n X_m Y_n) dxdy \\ &+ 2\int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} \frac{\partial^2 Y_n}{\partial y^2} X_m Y_n) dxdy + \int_0^b\int_0^a (X_m \frac{\partial^4 Y_n}{\partial y^4} X_m Y_n) dxdy \\ &- \lambda(\int_0^b\int_0^a (\frac{\partial^6 X_m}{\partial x^6} Y_n X_m Y_n) dxdy + 3\int_0^b\int_0^a (\frac{\partial^4 X_m}{\partial x^4} \frac{\partial^2 Y_n}{\partial y^2} X_m Y_n) dxdy \\ &+ 3\int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} \frac{\partial^4 Y_n}{\partial y^4} X_m Y_n) dxdy + \int_0^b\int_0^a (X_m \frac{\partial^6 Y_n}{\partial y^6} X_m Y_n) dxdy)) \\ &- k_w(\int_0^b\int_0^a (X_m Y_n X_m Y_n) dxdy - \mu(\int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} Y_n X_m Y_n) dxdy \\ &+ \int_0^b\int_0^a (X_m \frac{\partial^2 Y_n}{\partial y^2} X_m Y_n) dxdy)) - N_x^0(y)[1 - \mu(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})][\frac{\partial^2 X_m}{\partial x^2} Y_n \\ &+ \frac{\partial^2 X_m}{\partial x^2} Y_n] - N_y^0(x)[1 - \mu(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})][\frac{\partial^2 Y_n}{\partial x^2} X_m + \frac{\partial^2 Y_n}{\partial x^2} X_m] \\ &- (N^T + N^H - K_p)(\int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} Y_n X_m Y_n) dxdy + \int_0^b\int_0^a (X_m \frac{\partial^2 Y_n}{\partial y^2} X_m Y_n) dxdy \\ &- \mu(\int_0^b\int_0^a (\frac{\partial^4 X_m}{\partial x^4} Y_n X_m Y_n) dxdy + 2\int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} \frac{\partial^2 Y_n}{\partial y^2} X_m Y_n) dxdy \\ &+ \int_0^b\int_0^a (X_m \frac{\partial^4 Y_n}{\partial y^4} X_m Y_n) dxdy)) \end{aligned} \tag{45}$$

$$\begin{aligned} k_{4,4} &= -F(\int_0^b\int_0^a (\frac{\partial^4 X_m}{\partial x^4} Y_n X_m Y_n) dxdy \\ &+ 2\int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} \frac{\partial^2 Y_n}{\partial y^2} X_m Y_n) dxdy + \int_0^b\int_0^a (X_m \frac{\partial^4 Y_n}{\partial y^4} X_m Y_n) dxdy \\ &- \lambda(\int_0^b\int_0^a (\frac{\partial^6 X_m}{\partial x^6} Y_n X_m Y_n) dxdy + 3\int_0^b\int_0^a (\frac{\partial^4 X_m}{\partial x^4} \frac{\partial^2 Y_n}{\partial y^2} X_m Y_n) dxdy \\ &+ 3\int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} \frac{\partial^4 Y_n}{\partial y^4} X_m Y_n) dxdy + \int_0^b\int_0^a (X_m \frac{\partial^6 Y_n}{\partial y^6} X_m Y_n) dxdy)) \\ &+ A_{44}(\int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} Y_n X_m Y_n) dxdy + \int_0^b\int_0^a (X_m \frac{\partial^2 Y_n}{\partial y^2} X_m Y_n) dxdy \\ &- \lambda(\int_0^b\int_0^a (\frac{\partial^4 X_m}{\partial x^4} Y_n X_m Y_n) dxdy + 2\int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} \frac{\partial^2 Y_n}{\partial y^2} X_m Y_n) dxdy \\ &+ \int_0^b\int_0^a (X_m \frac{\partial^4 Y_n}{\partial y^4} X_m Y_n) dxdy) - k_w(\int_0^b\int_0^a (X_m Y_n X_m Y_n) dxdy \\ &- \mu(\int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} Y_n X_m Y_n) dxdy + \int_0^b\int_0^a (X_m \frac{\partial^2 Y_n}{\partial y^2} X_m Y_n) dxdy)) \\ &- N_x^0(y)[1 - \mu(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})][\frac{\partial^2 X_m}{\partial x^2} Y_n + \frac{\partial^2 X_m}{\partial x^2} Y_n] \\ &- N_y^0(x)[1 - \mu(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})][\frac{\partial^2 Y_n}{\partial x^2} X_m + \frac{\partial^2 Y_n}{\partial x^2} X_m] \\ &- (N^T + N^H - K_p)(\int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} Y_n X_m Y_n) dxdy \\ &+ \int_0^b\int_0^a (X_m \frac{\partial^2 Y_n}{\partial y^2} X_m Y_n) dxdy - \mu(\int_0^b\int_0^a (\frac{\partial^4 X_m}{\partial x^4} Y_n X_m Y_n) dxdy \\ &+ 2\int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} \frac{\partial^2 Y_n}{\partial y^2} X_m Y_n) dxdy + \int_0^b\int_0^a (X_m \frac{\partial^4 Y_n}{\partial y^4} X_m Y_n) dxdy)) \end{aligned} \tag{46}$$

$$\begin{aligned} m_{1,1} &= +I_0(\int_0^b\int_0^a (\frac{\partial X_m}{\partial x} Y_n \frac{\partial X_m}{\partial x} Y_n) dxdy \\ &- \mu(\int_0^b\int_0^a (\frac{\partial^3 X_m}{\partial x^3} Y_n \frac{\partial X_m}{\partial x} Y_n) dxdy + \int_0^b\int_0^a (\frac{\partial X_m}{\partial x} \frac{\partial^2 Y_n}{\partial y^2} \frac{\partial X_m}{\partial x} Y_n) dxdy)) \end{aligned} \tag{47}$$

$$\begin{aligned} m_{1,2} &= +I_0(\int_0^b\int_0^a (X_m \frac{\partial Y_n}{\partial y} X_m \frac{\partial Y_n}{\partial y}) dxdy \\ &- \mu(\int_0^b\int_0^a (X_m \frac{\partial^3 Y_n}{\partial y^3} X_m \frac{\partial Y_n}{\partial y}) dxdy + \int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} \frac{\partial Y_n}{\partial y} X_m \frac{\partial Y_n}{\partial y}) dxdy)) \end{aligned} \tag{48}$$

$$\begin{aligned} m_{3,1} &= -I_1(\int_0^b\int_0^a (\frac{\partial X_m}{\partial x} Y_n \frac{\partial X_m}{\partial x} Y_n) dxdy \\ &- \mu(\int_0^b\int_0^a (\frac{\partial^3 X_m}{\partial x^3} Y_n \frac{\partial X_m}{\partial x} Y_n) dxdy + \int_0^b\int_0^a (\frac{\partial X_m}{\partial x} \frac{\partial^2 Y_n}{\partial y^2} \frac{\partial X_m}{\partial x} Y_n) dxdy)) \end{aligned} \tag{49}$$

$$\begin{aligned} m_{4,1} &= -I_3(\int_0^b\int_0^a (\frac{\partial X_m}{\partial x} Y_n \frac{\partial X_m}{\partial x} Y_n) dxdy \\ &- \mu(\int_0^b\int_0^a (\frac{\partial^3 X_m}{\partial x^3} Y_n \frac{\partial X_m}{\partial x} Y_n) dxdy + \int_0^b\int_0^a (\frac{\partial X_m}{\partial x} \frac{\partial^2 Y_n}{\partial y^2} \frac{\partial X_m}{\partial x} Y_n) dxdy)) \end{aligned} \tag{50}$$

$$\begin{aligned} m_{3,2} &= -I_1(\int_0^b\int_0^a (X_m \frac{\partial Y_n}{\partial y} X_m \frac{\partial Y_n}{\partial y}) dxdy \\ &- \mu(\int_0^b\int_0^a (X_m \frac{\partial^3 Y_n}{\partial y^3} X_m \frac{\partial Y_n}{\partial y}) dxdy + \int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} \frac{\partial Y_n}{\partial y} X_m \frac{\partial Y_n}{\partial y}) dxdy)) \end{aligned} \tag{51}$$

$$\begin{aligned} m_{4,2} &= -I_3(\int_0^b\int_0^a (X_m \frac{\partial Y_n}{\partial y} X_m \frac{\partial Y_n}{\partial y}) dxdy \\ &- \mu(\int_0^b\int_0^a (X_m \frac{\partial^3 Y_n}{\partial y^3} X_m \frac{\partial Y_n}{\partial y}) dxdy + \int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} \frac{\partial Y_n}{\partial y} X_m \frac{\partial Y_n}{\partial y}) dxdy)) \end{aligned} \tag{52}$$

$$\begin{aligned} m_{3,3} &= +I_0(\int_0^b\int_0^a (X_m Y_n X_m Y_n) dxdy - \mu(\int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} Y_n X_m Y_n) dxdy \\ &+ \int_0^b\int_0^a (X_m \frac{\partial^2 Y_n}{\partial y^2} X_m Y_n) dxdy)) - I_2(\int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} Y_n X_m Y_n) dxdy \\ &+ \int_0^b\int_0^a (X_m \frac{\partial^2 Y_n}{\partial y^2} X_m Y_n) dxdy - \mu(\int_0^b\int_0^a (\frac{\partial^4 X_m}{\partial x^4} Y_n X_m Y_n) dxdy \\ &+ 2\int_0^b\int_0^a (\frac{\partial^2 X_m}{\partial x^2} \frac{\partial^2 Y_n}{\partial y^2} X_m Y_n) dxdy + \int_0^b\int_0^a (X_m \frac{\partial^4 Y_n}{\partial y^4} X_m Y_n) dxdy)) \end{aligned} \tag{53}$$

$$
\begin{aligned}
m_{3,4} = m_{4,3} = &+I_0\Big(\int_0^b\int_0^a (X_mY_nX_mY_n)dxdy - \mu\Big(\int_0^b\int_0^a \Big(\frac{\partial^2 X_m}{\partial x^2}Y_nX_mY_n\Big)dxdy \\
&+\int_0^b\int_0^a \Big(X_m\frac{\partial^2 Y_n}{\partial y^2}X_mY_n\Big)dxdy\Big)\Big) - I_4\Big(\int_0^b\int_0^a \Big(\frac{\partial^2 X_m}{\partial x^2}Y_nX_mY_n\Big)dxdy \\
&+\int_0^b\int_0^a \Big(X_m\frac{\partial^2 Y_n}{\partial y^2}X_mY_n\Big)dxdy - \mu\Big(\int_0^b\int_0^a \Big(\frac{\partial^4 X_m}{\partial x^4}Y_nX_mY_n\Big)dxdy \\
&+2\int_0^b\int_0^a \Big(\frac{\partial^2 X_m}{\partial x^2}\frac{\partial^2 Y_n}{\partial y^2}X_mY_n\Big)dxdy + \int_0^b\int_0^a \Big(X_m\frac{\partial^4 Y_n}{\partial y^4}X_mY_n\Big)dxdy\Big)\Big)
\end{aligned} \tag{54}
$$

$$
\begin{aligned}
m_{4,4} = &+I_0\Big(\int_0^b\int_0^a (X_mY_nX_mY_n)dxdy - \mu\Big(\int_0^b\int_0^a \Big(\frac{\partial^2 X_m}{\partial x^2}Y_nX_mY_n\Big)dxdy \\
&+\int_0^b\int_0^a \Big(X_m\frac{\partial^2 Y_n}{\partial y^2}X_mY_n\Big)dxdy\Big)\Big) - I_5\Big(\int_0^b\int_0^a \Big(\frac{\partial^2 X_m}{\partial x^2}Y_nX_mY_n\Big)dxdy \\
&+\int_0^b\int_0^a \Big(X_m\frac{\partial^2 Y_n}{\partial y^2}X_mY_n\Big)dxdy - \mu\Big(\int_0^b\int_0^a \Big(\frac{\partial^4 X_m}{\partial x^4}Y_nX_mY_n\Big)dxdy \\
&+2\int_0^b\int_0^a \Big(\frac{\partial^2 X_m}{\partial x^2}\frac{\partial^2 Y_n}{\partial y^2}X_mY_n\Big)dxdy + \int_0^b\int_0^a \Big(X_m\frac{\partial^4 Y_n}{\partial y^4}X_mY_n\Big)dxdy\Big)\Big)
\end{aligned} \tag{55}
$$

$$
\begin{aligned}
&c_{3,3} = c_{3,4} = c_{4,3} = c_{4,4} = -c_d\Big(\int_0^b\int_0^a (X_mY_nX_mY_n)dxdy \\
&-\mu\Big(\int_0^b\int_0^a \Big(\frac{\partial^2 X_m}{\partial x^2}Y_nX_mY_n\Big)dxdy + \int_0^b\int_0^a \Big(X_m\frac{\partial^2 Y_n}{\partial y^2}X_mY_n\Big)dxdy\Big)\Big) \\
&c_{1,1} = c_{1,2} = c_{2,1} = c_{2,2} = c_{2,3} = c_{2,4} = c_{4,2} = 0
\end{aligned} \tag{56}
$$

Also, non-dimensional parameters are defined as

$$
\begin{aligned}
&\hat{\omega} = \omega a\sqrt{\frac{\rho_c}{E_c}},\ K_w = \frac{k_w a^4}{D_c}, K_p = \frac{k_p a^2}{D_c}, \\
&C_d = c_d\frac{a^2}{\sqrt{\rho h D_c}}, \bar{N} = N\frac{a^2}{D_c}, D_c = \frac{E_c h^3}{12(1-v_c^2)}
\end{aligned} \tag{57}
$$

Finally, setting the coefficient matrix to zero gives the natural frequencies. The function X_m for simply-supported boundary conditions is defined by

$$
\begin{aligned}
X_m(x) &= \sin(\lambda_m x) \\
\lambda_m &= \frac{m\pi}{a}
\end{aligned} \tag{58}
$$

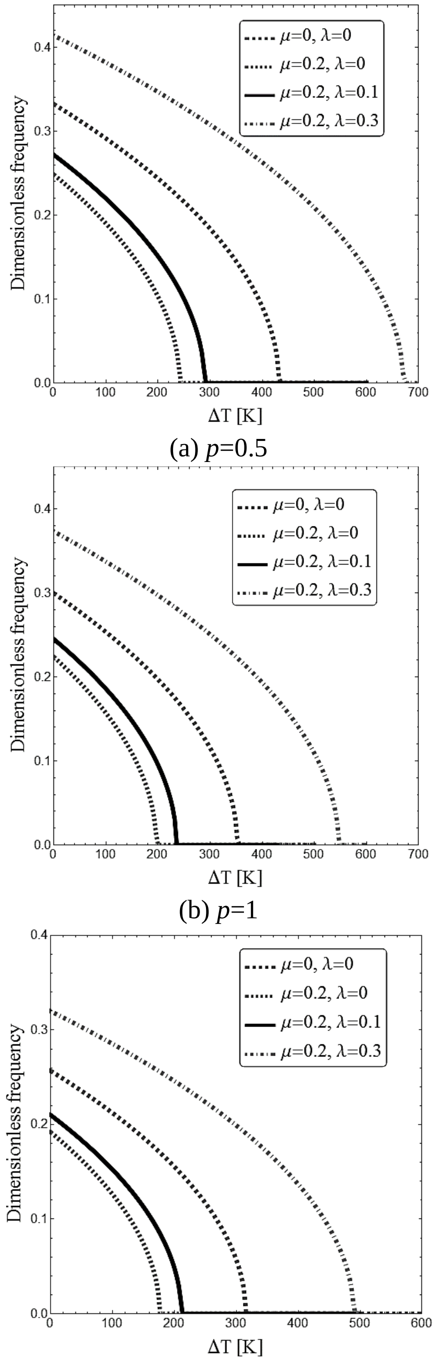
The function Y_n can be obtained by replacing *x*, *m* and *a*, respectively by *y*, *n* and *b*.

5. Numerical results and discussions

This section is devoted to study the hygro-thermo-mechanical vibration behavior of nonlocal strain gradient FG nanoplates on viscoelastic substrate based on a four-variable shear deformation theory. The model introduces two scale coefficients related to nonlocal and strain gradient effects for more accurate analysis of FG nanoplates. The exactness of obtained vibration frequencies via four-variable plate model are verified with those of classical plate theory (CPT) obtained by Natarajan *et al*. (2012) using finite element method and the results are tabulated in Table 1. It is noticeable that presented Galerkin's solution as well as higher order plate model can accurately predict vibrational behavior of FG nanoplates. The length of

Table 1 Comparison of non-dimensional fundamental natural frequency $\hat{\omega} = \omega h\sqrt{\rho_c / G_c}$ of FG nanoplates with simply-supported boundary conditions (*p*=5)

a/h	*μ*	*a/b*=1		*a/b*=2	
		Natarajan *et al*. (2012)	present	Natarajan *et al*. (2012)	present
10	0	0.0441	0.043823	0.1055	0.104329
	1	0.0403	0.04007	0.0863	0.085493
	2	0.0374	0.037141	0.0748	0.074174
	4	0.0330	0.032806	0.0612	0.060673
20	0	0.0113	0.011256	0.0279	0.027756
	1	0.0103	0.010288	0.0229	0.022722
	2	0.0096	0.009534	0.0198	0.019704
	4	0.0085	0.008418	0.0162	0.016110



(a) *p*=0.5

(b) *p*=1

(c) *p*=5

Fig. 3 Variation of dimensionless frequency of perfect nanoplate versus temperature rise for different nonlocal and strain gradient parameters (*a/h*=15, K_w=0, K_p=0, Δ*C*=0%)

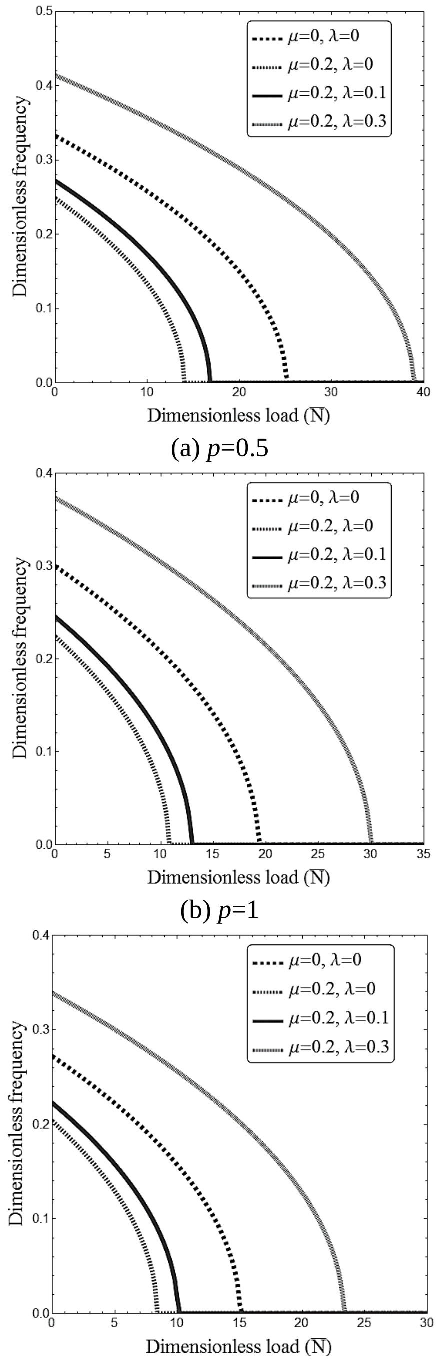


Fig. 4 Variation of dimensionless frequency versus dimensionless load for different nonlocal and strain gradient parameters (a/h=15, ξ=1, K_w=0, K_p=0, ΔT=0)

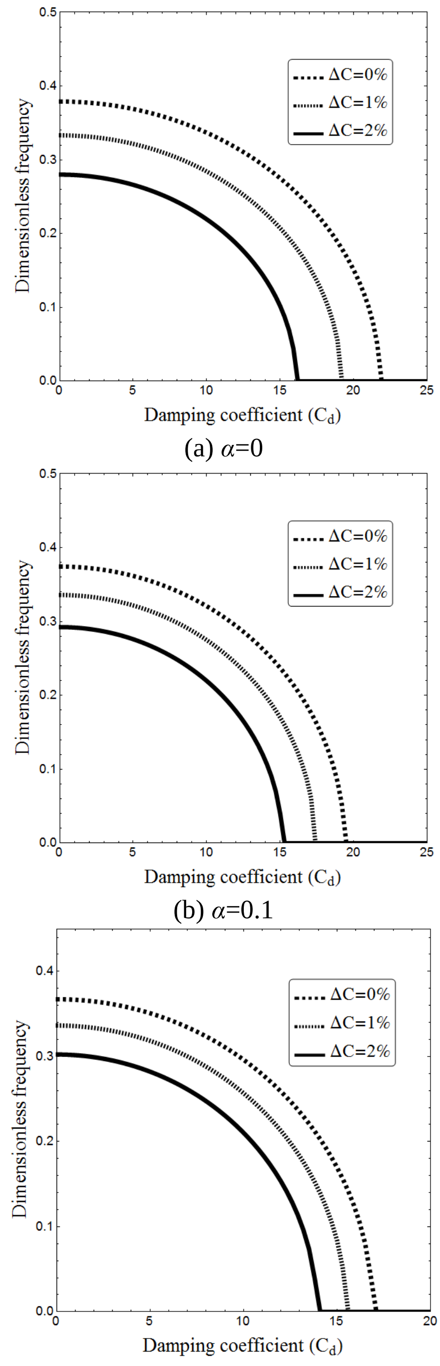
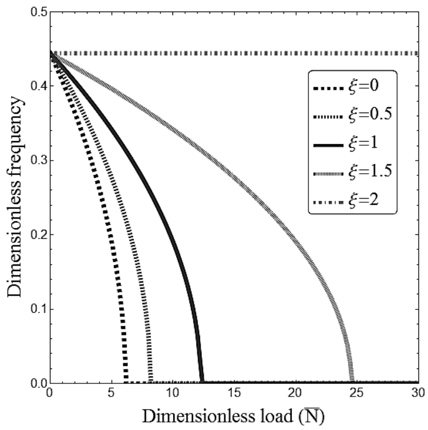


Fig. 5 Dimensionless frequency of FG nanoplate versus damping coefficient for various porosity volume fractions (a/h=10, p=1, ΔT=10, K_w=5, K_p=0.5, μ=0.2, λ=0.1)

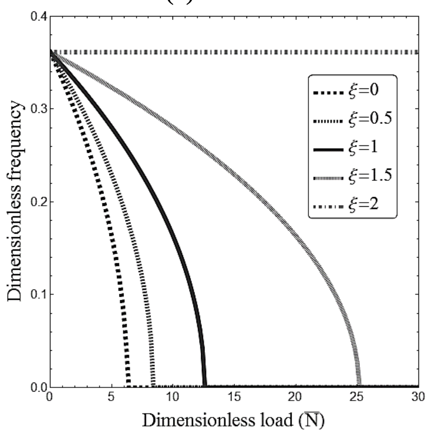
nanoplate is assumed as a=10 nm. Also, material properties of nanoplate (alumina and aluminum) are considered as:

$$E_c = 380\,GPa,\ \rho_c = 3800\,kg/m^3,\ \upsilon_c = 0.3,$$
$$\gamma_c = 7\times10^{-6}\,1/{}^{0}C,\ \beta_c = 0.001\,(wt.\%\,H_2o)^{-1}$$
$$E_m = 70\,GPa,\ \rho_m = 2707\,kg/m^3,\ \upsilon_m = 0.3,$$
$$\gamma_m = 23\times10^{-6}\,1/{}^{0}C,\ \beta_m = 0.44\,(wt.\%\,H_2o)^{-1}$$

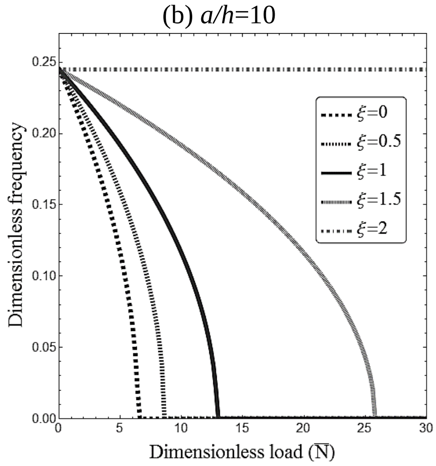
In Figs. 3 and 4, the variation of non-dimensional frequency of a FG nanoplate respectively versus thermal and mechanical loading is presented for different nonlocal (μ), stain gradient (λ) parameters and inhomogeneity index (p) when a/h=15, K_w=0 and K_p=0. When μ=λ=0, the natural frequencies according to the classical plate model are rendered. However, at λ=0 the frequencies of a nanoplate based on nonlocal elasticity theory (NET) without strain gradient effects are obtained. It is observed that increase of temperature or in-plane mechanical load yields reduction in both rigidity and natural frequencies of FG nanoplate. At a certain temperature and in-plane mechanical load, the natural frequency of nanoplate becomes zero. At this critical



(a) *a/h*=8



(b) *a/h*=10



(c) *a/h*=15

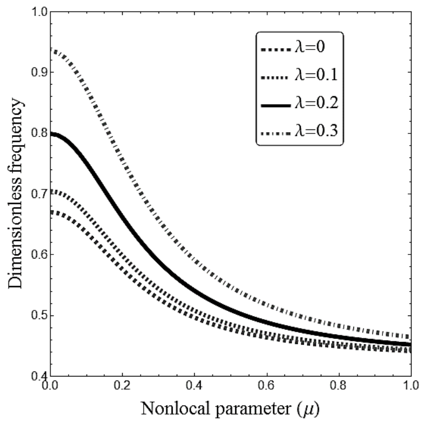
Fig. 6 Variation of dimensionless frequency versus dimensionless load for different load factors and side-to-thickness ratios (p=1, ΔT=0, ΔC=0%, C_d=0, μ=0.2, λ=0.1)

load, the nanoplate is buckled and doesn't oscillate. It is found that natural frequencies and critical buckling loads of FG nanoplates are significantly influenced by the value of nonlocal and strain gradient parameters. In fact, nonlocal parameter introduces a stiffness-softening mechanism, while strain gradient parameter provides a stiffness-hardening mechanism. In other words, increasing nonlocal parameter leads to smaller frequencies and critical temperatures. In contrast, increasing strain gradient parameter yields larger frequencies and critical temperatures. When $\lambda<\mu$, obtained frequency is smaller than that of nonlocal elasticity theory. However, at $\lambda>\mu$ obtained frequencies becomes larger than nonlocal elasticity theory. In respite of the significance, such conclusions are not reported in previous investigations on vibration of nanoplates. It is suggested that both nonlocal and strain gradient effects should be considered for more accurate analysis of nanoplates. Also, all these observations are affected by the gradation of material properties or inhomogeneity index (p). In fact, increase of inhomogeneity index (p) is proportional to higher metal constituent which leads to smaller frequencies and critical buckling temperatures.

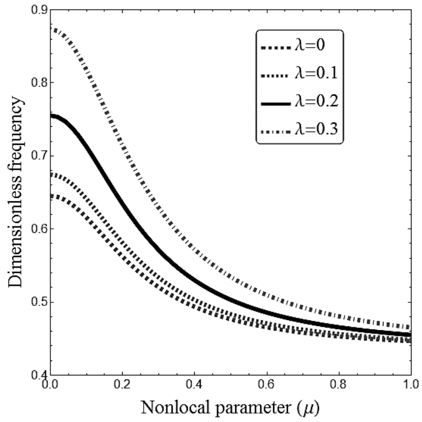
Effects of hygro-thermal loading and porosities on damping vibration behavior of nonlocal strain gradient FG nanoplates at a/h=10, p=1, ΔT=10, K_w=5, K_p=0.5, μ=0.2 and λ=0.1 are plotted in Fig. 5. It should be pointed out that increase of damping coefficient degrades the plate stiffness and natural frequencies reduce until a critical point in which the frequencies become zero. At this point, the nanoplate is critically damped and does not oscillate. It is well-known that hygro-thermal loadings degrade the plate stiffness an affect significantly the performance of structures. It is seen that increase of moisture concentration (ΔC) leads to smaller dimensionless frequencies for every value of temperature change. However, temperature increase leads to lower frequencies at a fixed moisture concentration rise. So, natural frequency of a nanoplate decreases significantly when it is subjected to a severe hygro-thermal environment. Accordingly, increase of moisture concentration and temperature leads to smaller critical damping coefficients. Also, it can be reported that porosities inside the material lead to smaller frequencies by reducing the stiffness of nanoplate. Therefore, a porous FG nanoplate has lower critical damping coefficients than a perfect one. These observations are consistent with the previous studies on FG macro scale structures.

Fig. 6 illustrates the variation of dimensionless frequency of nonlocal strain gradient FG nanoplate with respect to dimensionless load for different load factors (ξ) when p=1, μ=0.2, λ=0.1. It is clear that in-plane bending load degrade the plate stiffness and affect significantly the performance of structures. It is seen that increase of load factor leads to enlargement of dimensionless frequencies. So, critical buckling load shifts to the right. This is due to the fact that with increase of load factor, the resultant of in-plane load reduces. It is also seen that nanoplates with higher side-to-thickness ratios have larger vibration frequencies. Accordingly, a nanoplate with higher side-to-thickness ratios has higher critical buckling load.

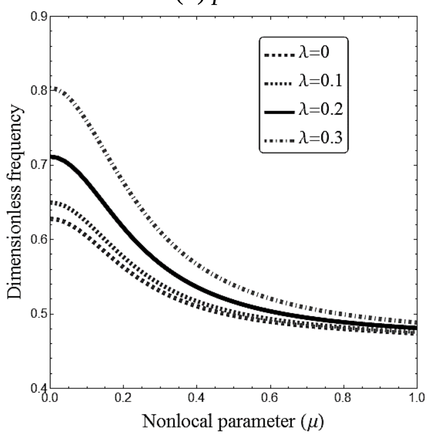
Another study on the aviation of natural frequency of hygro-thermally affected FG nanoplates with respect to nonlocal and strain gradient parameters is conducted in Fig. 7 when a/h=10, ΔT=50, ΔC=1%, α=0.05, C_d=0, K_w=25 and K_p=10. It is clear that natural frequency of FG nanoplate reduces with the increase of nonlocal parameter for every value of strain gradient parameter. But, vibration frequency increases at a fixed nonlocal parameter and inhomogeneity index. Due to the lack of a strain gradient parameter in previous vibration analyses of nanoplates, only the softening effect due to nonlocality was concluded.



(a) p=5



(b) p=1



(c) p=5

Fig. 7 Variation of dimensionless frequency of porous nanoplates versus nonlocal parameter for different strain gradient parameters (a/h=10, ΔT=50, ΔC=1%, K_w=25, K_p=10)

Therefore, the material instability and heterogeneous deformation due to strain gradient could not be considered within the framework of the nonlocal elasticity theory.

6. Conclusions

In this paper, nonlocal strain gradient theory is employed to investigate damping vibration behavior of FG nanoplates under hygro-thermo-mechanical loading resting on viscoelastic medium using a refined four-variable plate theory. The theory introduces two scale parameters corresponding to nonlocal and strain gradient effects to capture both stiffness-softening and stiffness-hardening influences. Hamilton's principle is employed to obtain the governing equation of a nonlocal strain gradient FG nanoplate. These equations are solved via Galerkin's method to obtain the natural frequencies. It is observed that natural frequency of FG nanoplate reduces with increase of nonlocal parameter. In contrast, natural frequency increases with increase of length scale parameter which highlights the stiffness-hardening effect due to the strain gradients. Also, increase of damping coefficient degrades the plate stiffness and natural frequencies reduce until a critical point in which the frequencies become zero. It is seen that porosities inside the material provides lower critical damping coefficients. Also, when the in-plane bending load factor increase, the resultant of applied load decreases leading to increment in vibration frequencies. All these observations are affected by the hygro-thermal loading which decreases the plate stiffness and decreases the natural frequencies.

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