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A multi-parameter optimization technique for prestressed concrete cable-stayed bridges considering prestress in girder

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Abstract. The traditional design procedure of a prestressed concrete (PC) cable-stayed bridge is complex and time-consuming. The designers have to repeatedly modify the configuration of the large number of design parameters to obtain a feasible design scheme which maybe not an economical design. In order to efficiently achieve an optimum design for PC cable-stayed bridges, a multi-parameter optimization technique is proposed. In this optimization technique, the number of prestressing tendons in girder is firstly set as one of design variables, as well as cable forces, cable areas and cross-section sizes of the girders and the towers. The stress and displacement constraints are simultaneously utilized to ensure the safety and serviceability of the structure. The target is to obtain the minimum cost design for a PC cable-stayed bridge. Finally, this optimization technique is carried out by a developed PC cable-stayed bridge optimization program involving the interaction of the parameterized automatically modeling program, the finite element structural analysis program and the optimization algorithm. A low-pylon PC cable-stayed bridge is selected as the example to test the proposed optimization technique. The optimum result verifies the capability and efficiency of the optimization technique, and the significance to optimize the number of prestressing tendons in the girder. The optimum design scheme obtained by the application can achieve a 24.03% reduction in cost, compared with the initial design.

Keywords: cable-stayed bridge; prestressed concrete; multi-parameter; optimization technique; prestressing tension

1. Introduction

Cable-stayed bridges are highly redundant structures, especially in modern cable-stayed bridges that a large number of cables are required for more slender main girder. The design procedure of those complicated structures becomes complex because the cables, the profile of the girders and the towers, and the prestressing tendons in the case of a prestressed concrete (PC) cable-stayed bridge are interactive and coupled to each other. Therefore, a reasonable configuration of those design parameters is always difficult to obtain.

The traditional design procedure of a cable-stayed bridge is generally performed in three stages. In the first stage, the profile of the girders and the towers is determined according to the experience of successfully designing schemes of similar cable-stayed bridges. The second stage, as well as the most key and complicated step in the whole design procedure of a cable-stayed bridge, includes the calculation of cable forces and cross-section areas of cables. The third stage, which is not included in the case of steel cable-stayed bridges, involves the configuration of the prestressing tendons in the PC girder. If the stresses and deformations obtained in the third stage, cannot meet the requirement of the design code, the design process must go back to the first stage and restart the whole design procedure again. This procedure will repeatedly continue until all the design criteria are satisfied. The final design strategy achieved by this traditional design procedure is just a feasible scheme and its reasonableness depends on the experience and skills of the designers. Since different designers will obtain different feasible schemes, a criterion is needed to evaluate which one is the best among those feasible design schemes.

Many studies have focused on the solution to improve the tedious, expensive and time-consuming design procedure. There are several methods which were proposed to determine the post-tensioning cable forces in cable-stayed bridges based on controlling the desired geometry of the final bridge state (Dan *et al*. 2014, Feng *et al*. 2017, Zhang *et al*. 2014, Cheng *et al*. 2004, Lee *et al*. 2008, Lonetti and Pascuzzo 2014, Modano *et al*. 2015).

Some researchers have attempted to apply optimization techniques on the design of cable-stayed bridges. In some available references, only the optimization of cable forces was studied. Sung *et al*. (2006) used the minimum strain energy theory to obtain the optimum post-tensioning cable forces of the bridge. The equality constraints for the restriction on the displacements of the tower and the inequality constraints for the limitation on the envelopes of the cable forces are both implemented in the optimization model. Fabbrocino *et al*. (2017) presented an approach to the optimum pre-tensioning design of composite cable-stayed bridges whose aim is to achieve a target bending moment distribution over the deck. Martins *et al*. (2015a) presented a multi-objective optimization algorithm to compute the cable forces of the concrete cable-stayed

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bridges to achieve the desired final geometry. Load history and geometry changes due to the construction sequence and the time-dependent effects of the concrete were included in the structural analysis. Martins *et al*. (2015b, 2016) developed this multi-objective optimization algorithm, additionally considering the geometrical nonlinearities in the structural analysis, to compute cable forces of concrete cable-stayed bridges. Hassan *et al*. (2010) aimed at obtaining the minimum vertical deflection of the deck and the horizontal deflection of the towers' top simultaneously by optimizing the post-tensioning cable forces. An optimization technique was presented, combining finite element analysis, B-spline curves, and an optimization algorithm. This optimized result improved the convergence speed, the accuracy of final solution and the performance of the optimization technique, and decreased the number of the design variables in the meantime.

Some researches referred to the optimization of cable cross-section areas. Hassan (2013) investigated an optimization technique to achieve the minimum weight of steel in stay cables under self-weight, initial post-tensioning cable forces, and live load cases. An optimization problem of cable cross section, considering constraints of cable stresses and deck displacements, was examined in the study of Baldomir *et al*. (2010).

There have been a few attempts to determine the optimum cross-section of girders and towers. Venkat *et al*. (2009) presented a system level optimization, considering the side to main span ratio, the tower height to bridge length ratio, and the crucial size of the girder and the tower cross-section as design variables. An approach combining genetic algorithm (GA) and support vector machine (SVM) has been adopted to minimize the cost of the cable-stayed bridge. Among all these optimization methods for cable-stayed bridges, only Simões and Negrão (2000) considered the comprehensive influence of cables, decks, and towers in the optimization problem. An optimization method for cable-stayed bridges with steel box-girder decks was presented. The deck and the tower cross-section size, the cable forces and areas, and the length of end block zone of cables in towers were set as the design variables. A multi-criteria approach was considered for the optimization itself, with constraints on maximum stresses, minimum stresses in stays and deflections under dead load condition.

Among all these above-mentioned studies, most of them focused on the cable-stayed bridge with the steel girder, a few of them aimed at the cable-stayed bridge with the concrete girder. Major differences between the steel and PC cable-stayed bridges are the consideration for the time-independent influence of the concrete and the prestress in the girder. Those two differences make it more difficult to optimize the concrete cable-stayed bridges compared with the steel cable-stayed bridges. Most of the current available studies related to PC cable-stayed bridges mainly concentrated on the cable force optimization, and few of them have referred to optimizing the size of the girders and the towers. The optimization of the prestress in girder of PC cable-stayed bridges has not yet been studied.

In the current available optimization methods for PC cable-stayed bridges, the cable force and cable area optimization would lag behind the design of the girders and the towers. For example, if the parameters of the girders are changed to explore another feasible design scheme, another configuration of the cable forces and areas, and the prestress in the girders corresponding to the new parameters of the girders may be obtained. Therefore, a large number of feasible design schemes will be achieved according to the different heights of the girder. The designers have to rely on their experiences and skills to select a final design from a large number of feasible candidate schemes, which is still a challenging and complicated work. Owing to the complex structural behavior of the cable-stayed bridge, it is impossible to foresee the configuration for all design parameters. Therefore, a more automatic and reasonable optimization technique will be needed if more parameters are set as variables. When the optimum target is related to the cost, an economical design scheme for PC cable-stayed bridges will be obtained if the expense of all materials of the structure is considered.

This study outlines a multi-parameter optimization technique for PC cable-stayed bridges. In particular, several coupled parameters are taken into account, including comprehensive influence of all load-bearing elements, corresponding to the prestress, the girders, the towers, and the cables. The target is to obtain the minimum cost design for PC cable-stayed bridges, based on the objective function related to structural economy. In this optimization technique, the number of prestressing tendons in girder is firstly set as one of the design variables, as well as cable forces and areas, cross sizes of the girders and the towers. The stress and displacement constraints are simultaneously utilized to ensure the safety and serviceability of the structure. The optimization technique is carried out by a developed PC cable-stayed bridge optimization program involving the interaction of the parameterized automatically modeling program, the finite element structural analysis program and the optimization algorithm. Creep and shrinkage of concrete and cable-sage effect will also be taken into consideration in the structural analysis.

2. Structural analysis and modeling

2.1 Numerical simulation procedure

Since the stress and deformation constraints cannot be expressed in explicit function with design variables, the structural analysis should be executed in every optimization iteration to calculate the values of constraints. Some FE programs have this capability, such as Abaqus and Ansys. These FE programs are general for mechanical analysis, but they are not convenience for bridge structural analysis. In fact, it is not easy to consider the effects of concrete creep and shrinkage, and to calculate the loss of prestress. They cannot provide the influence lines loading method to determine the worst live load loading position. Moreover, it will cost a lot of time to setup and shutdown these programs when called by the main optimization program for each time, which reduces the optimization efficiency. Currently, there are several commercial programs available for bridge structural analysis, such as Midas and Doctor Bridge. However, these programs cannot provide second-developed interface for users. As independent systems, this kind of structural analysis programs cannot work as a subsystem to be called repeatedly, and cannot evaluate the constraints for the optimization algorithm.

Consequently, a reprogramed bridge structural analysis source program is needed to meet the requirement of structural analysis and seamlessly integrate into the main optimization program. A structural analysis code is developed in this work, based on the theory of the finite element theory. Besides the general mechanical analysis for structure, the developed code can analysis the effects of concrete creep and shrinkage, the loss of prestress, and can implement the influence lines loading.

Although the erection process of the cable-stayed bridge has impact on the stresses and deformations of the bridge, the main goal of the present work is to achieve the minimum cost for the bridge at operation state. Therefore, the erection process is not considered in the structural analysis and the algorithm will be improved in the future.

2.2 Design loads

In this study, the optimum design variables are evaluated, by the combined effect of loads, as follows:

- Self-weight of all structural components and appendages attached to the bridge, such as the bridge deck pavement.
- Traveling live load. The traveling live load is comprised of uniform load and one concentrated force. According to the second-class road load in China code (JTG D60-2004), the standard value of uniform load is 7.875 kN/m, and the standard value of concentrated force is 270 kN. The loading position of live load is determined by the influence lines loading method.
- Post-tensioning forces of stayed cables.
- Prestress in the concrete girder.
- Concrete creep and shrinkage.
- Cable-sag effect

Displacements and stress constraints should be computed with different loading conditions. Deflections and displacements must be considered under serviceability loading conditions. In the study, deflections and displacement constraints are computed under combination for short-term action effects, which is composed of the characteristic value of permanent action and the frequent value of variable action. The frequent value of traveling live load is 0.7, according to China code (JTG D60-2004). While, the stresses are obtained in elastic phase, and the partial safety factor and the combination factor for action effects are equal to 1.0, according to China code (JTG D60-2004).

2.3 Time-dependent effects

The stresses and deformations in concrete cable-stayed bridges are significantly influenced by the time-dependent material behavior of concrete (Yang *et al*. 2015). The total strain $\varepsilon_b(t)$ at time t of a concrete specimen can be written as the sum of the stress dependent strain ε_e and stress independent strain $\varepsilon_c(t)$

$$\varepsilon_b(t)=\varepsilon_e+\varepsilon_c(t) \tag{1}$$

where $\varepsilon_b(t)$ is the total strain at time t, ε_e is the instantaneous strain and $\varepsilon_c(t)$ is the creep strain.

The creep characteristic of concrete can generally be described by the creep coefficient $\varphi(t)$, which is written as

$$\varphi(t)=\varepsilon_c(t)/\varepsilon_e \tag{2}$$

The total shrinkage strain $\varepsilon_s(t,\tau)$ from age τ to age t is defined as

$$\varepsilon_s(t,\tau)=\varepsilon_{sk}(e^{-p\tau}-e^{-p\tau}) \tag{3}$$

where ε_{sk} is the terminal value of shrinkage deformation and p is the coefficient of increasing speed of shrinkage deformation.

The time t of the concrete creep and shrinkage is considered for 1500 days.

2.4 Cable-sag effects

Cable-stayed bridges are highly flexible and redundant structures. They behave intensely geometrical nonlinearity. The nonlinearity of a cable-stayed bridge is introduced from three main resources: the cable-sag effect, the beam-column effect and the large displacement effect. From 1960s, researchers of many countries have devoted themselves to the geometrical non-linear behavior of structures (Yang *et al*. 2010, Yang and Cai 2016). Most of them agree in that the cable-sag effect governed the geometrical nonlinearity. Ernst firstly proposed an equivalent, or Ernst modulus method, to calculate the decrease or loss of cable elasticity modulus caused by catenary effect under self-weight of cables (Ernst 1965). It is generally adopted to consider the nonlinearity of cables. The calculation of equivalent elasticity modulus is given by

$$E_{eq}=\frac{E_0}{1+\frac{\gamma^2 l^2}{12\sigma_0^2}\cdot E_0} \tag{4}$$

where E_{eq} represents the equivalent elasticity modulus, E_0 is the cable material effect of elasticity, γ is weight per unit volume, l is the length of horizontal projection and σ_0 is the tension stress in the cable.

3. Proposed optimization technique

3.1 Objective function

In this study, the objective is to find the minimum cost of the PC cable-stayed bridge. The cumulative material cost of the bridge is expressed as

$$F(X)=\sum_{i=1}^{n} C_i W_i \tag{5}$$

where X represents optimization variables, n is the type of materials, C_i and W_i respectively represent the price factor and the total weight of material i.

3.2 Design variables

As previously referred, in this study, the target is to optimize the cost of the PC cable-stayed bridge under design constraint conditions. The cost of a structure mainly consists of expenses of all structure components. Therefore, the parameters regarding the consumption of all types of main materials, such as cables, concrete and steel strand, should be set as the design variables due to these types of materials governing the cost of a cable-stayed bridge. The variables of the proposed optimization model are as follows:

- The height of the girder section. Since the stiffness of the deck is conclusively decided by the height of the girder, the height optimization for the girder is the one of the most effective way to improve the stress state and deflection state.
- The width of the tower cross-section along the longitudinal direction of the bridge. It is necessary to optimize the section of the tower because the tower occupies a considerable concrete usage.
- Tensioning forces of cables.
- The areas of cables. In general, the cable is the most expensive component of a cable-stayed bridge compared with other structural members. The optimum areas of the cables, corresponding to the optimum cable forces, may contribute the most to decrease the cost of a cable-stayed bridge.
- Prestressing tendons in each beam elements. Generally, the optimization of the prestress in the girder is a complex topological problem because of the difficulty of foreseeing the distribution of prestressing tendons in each girder (Kutylowski and Rasiak 2014). In order to transform it into a simpler quantity optimization problem, a solution is proposed. In the structural analysis model, at least one prestressing tendon is allocated at both top and bottom margin of every beam element for the need of calculation. That is to say, in the initial input for structural analysis model, at least one prestressing tendon is needed to be allocated at both the top and bottom margin of the girder region, even if the prestress is actually not needed. Similarly, in the final optimized result, the pretress is probably not needed in this girder region where one prestressing tendon is allocated.

3.3 Constrains

Since the optimum structure must satisfy the requirement of structural strength and stiffness, constrains are utilized to ensure the stresses and deformations meeting the design criteria. The constraints are divided into two types.

3.3.1 Stress constrains

The stress state of the girder is affected integrally by the cable forces, and locally by the prestress in the deck. As a result, the stress control is crucial. Stress constrains of the beam and the tower elements are given as follows

$$-[\sigma_c]\le[\sigma_m^n]\le[\sigma_t] \tag{6}$$

where $[\sigma_m^n]$ is the stress of section m at point n, whose value is positive if and only if it is a tension stress, $[\sigma_t]$ denotes the allowable tension stress and $[\sigma_c]$ stands for the allowable compressive stress.

3.3.2 Deformation constraints

The stiffness, as well as the strength, must be ensured in the design of the cable-stayed bridge, to prevent the deck from suffering a great displacement. The displacement constrains of the girder and the tower is given by

$$|\omega_{D\text{-}max}|\le\omega_{max} \tag{7}$$

$$|u_{T\text{-}max}|\le u_{max} \tag{8}$$

where $\omega_{D\text{-}max}$ is the maximum deflection of the beam elements, $u_{T\text{-}max}$ is the maximum displacement of the tower elements at longitudinal direction, ω_{max} is the allowable deflection of the girder and u_{max} is the allowable displacement of the tower in longitudinal direction.

3.4 Optimization algorithm

The variables are nonlinear to the stress and deformation constraints. The sequential quadratic programming (SQP) algorithm has been one of the most successful general methods in solving nonlinear constrained optimization problems (Carpentieri *et al*. 2015). In its many implemented forms, this method has been shown to be a very useful tool for solving nonlinear programs, especially where significant degree nonlinearity is presented (Boggs and Tolle 2000). The other reason of choosing the SQP algorithm is that it is an efficient optimization algorithm to solve the large-scale optimization problem for bridges.

In the optimization model of this study, the variables of the prestressing tendon number, the cable strand number and the cable force are integers. The area variables of the girder and the tower are transformed into integers, so that all the design variables are integers. For example, the initial width of the tower top section is 4.2 m, it is transformed into 42 when computing the gradients. Therefore, all the optimum variables are integers. Generally, SQP is a gradient-based method that is programmed for non-integer values. But the optimum variables in the manuscript are integers, thus an improved version of SQP is needed. The MISPQ program, a Fortran implementation of a trust region SQP algorithm for mixed-integer nonlinear programming (Exler *et al*. 2007, 2009), is adopted as the algorithm in this paper.

3.5 Optimization technique and procedure

Due to the significant computational efficiency of Fortran language, all code programs in this paper are implemented in Fortran. The optimization technique, combining the parameterized automatically modeling program, the finite element structural analysis program and the optimization algorithm, is described as follows:

1. Prescribe the starting values for the variables to be optimized, according to the preliminary design.
2. Build the initial finite element modeling files with the

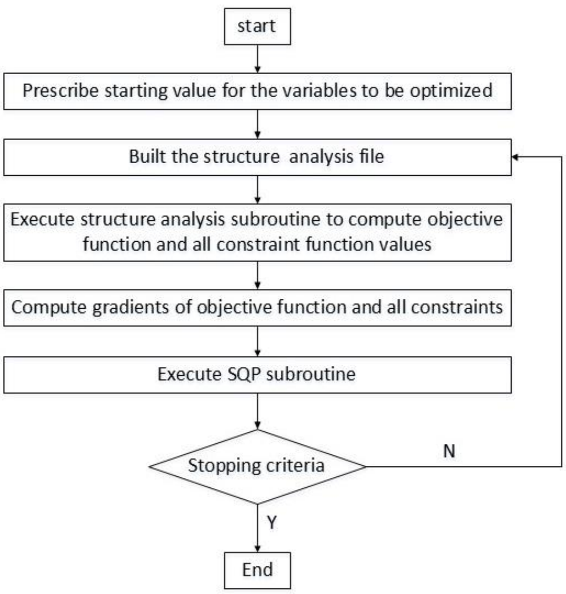


Fig. 1 Flow chat for the optimization procedure

parameterized automatically modeling program based on the preliminary design.

3. Compute the objective and all constraint function values for the first time. The constraint function values are obtained by the structure analysis.

4. Compute the gradients of the objective function and all constraints. Derivatives subject to the integer variables can be approximated using a difference formula.

5. Execute the SQP subroutine. If the internal stopping criteria are satisfied, the program terminates. Otherwise, call the structure analysis subroutine to compute the objective and the constraint function values for all variables with the modified values, and call SQP again.

The optimization procedure is summarized into a flow chart, as shown in Fig. 1.

4. Example and results

The low-pylon cable-stayed bridge behaves between the normal cable-stayed bridge and the continuous beam bridge. In the case of the low-pylon cable-stayed bridge, the cables provide an external prestressing assist for the girder, compared with the continuous beam bridge. Generally, a greater stiffness girder is needed compared with the normal cable-stayed bridges, because less number of cables is located to support the girder. At the same time, more prestressing tendons are needed. Therefore, it is significant to optimize the prestress in the girders of the low-pylon cable-stayed bridge.

4.1 Bridge description

A low-pylon cable-stayed bridge, with spans of 100.1 m and 100.1 m, is selected as the example to verify the optimization technique in this study. This low-pylon cable-stayed bridge is a real project in Zhejiang Province, China. The initial design of the bridge is taken as the initial

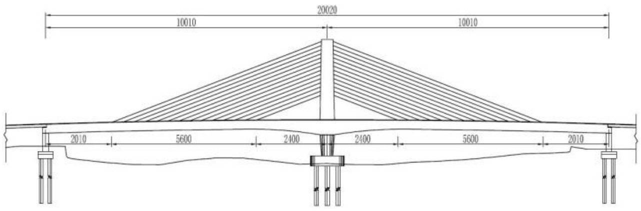


Fig. 2 Elevation of the example bridge (Unit: cm)

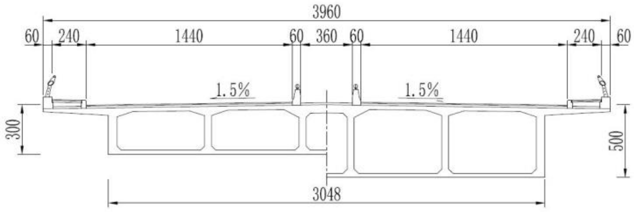
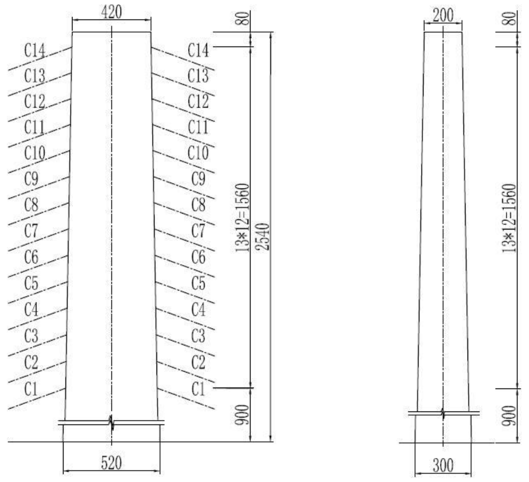


Fig. 3 Girder section of the initial design (Unit: cm)



(a) Longitudinal (b)Transverse

Fig. 4 Elevation of the tower (Unit: cm)

condition in the optimization program. The elevation of the bridge is presented in Fig. 2. Main components of the bridge are described as follows:

1. Girder: a variable cross-section PC box-girder design is adopted and shown in Fig. 3. The height of the box girder is varied from 3 m to 5 m in the region adjacent to the tower. The 39.6 m wide deck allows a configuration of 3 lanes in each direction. The grade of concrete for the girder and the tower is C50, and the standard value of strength for compressive stress and tension stress are, respectively, 32.4 MPa and 2.64 MPa. The standard value of strength for the prestressing tendons in the girder is 1860 MPa.

2. Tower: the single tower of the bridge is fixed with the deck. The size of rectangular section of the tower varies from top to bottom, as shown in Fig. 4.

3. Cables: the girder is supported by one single cable plane. Fourteen pairs of cables are anchored symmetrically at each side of the tower. Each cable consists of 73-φ15.2 steel strands and bears 800 t tension force. The standard value of the cable tensioning strength is 1860 MPa.

Main design parameters of the girder, the tower, and cables are listed in Table 1.

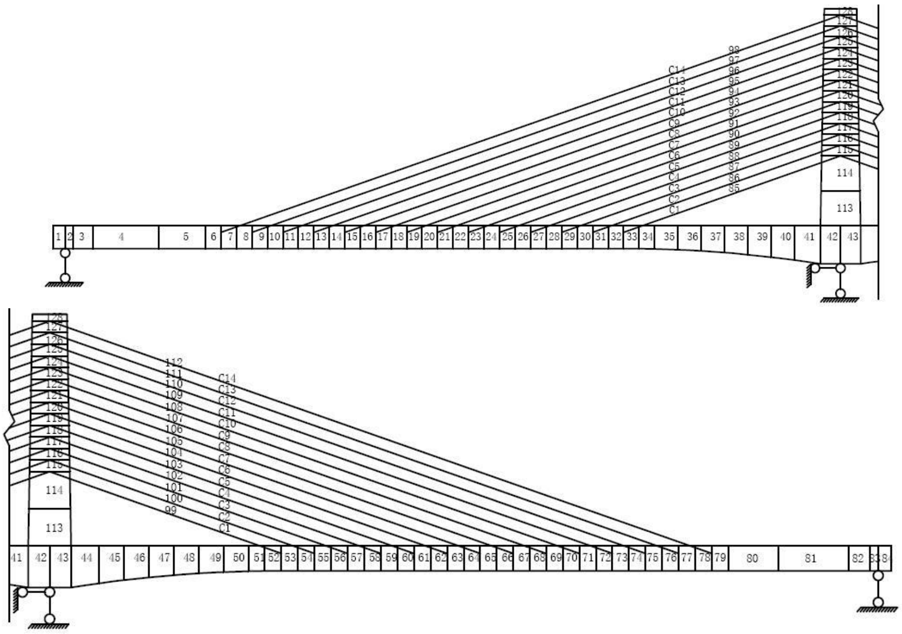


Fig. 5 Finite element model of the example bridge

Table 1 Main design parameters for the bridge

Member	E(MPa)	A(mm^2)	Ixx(mm^4)	Iyy(mm^4)	Izz(mm^4)
Girder section away from the tower	3.45×10^4	2.38×10^7	9.87×10^{13}	3.37×10^{13}	1.89×10^{15}
Girder section adjacent to the tower	3.45×10^4	4.20×10^7	4.20×10^7	1.55×10^{14}	3.01×10^{15}
Tower section at top	3.45×10^4	8.40×10^6	7.85×10^{12}	1.23×10^{13}	2.80×10^{12}
Tower section at bottom	3.45×10^4	1.56×10^7	2.99×10^{13}	3.52×10^{13}	1.17×10^{13}
Cable	1.95×10^5	1.02×10^4	0	0	0

4.2 Finite element model

The two-dimensional finite element model of the bridge is shown in Fig.5. The girders and towers are modeled with beam elements, and the stay cables are modeled with cable elements. There are totally 84 girder elements, 28 cable elements, and 16 tower elements.

4.3 Optimization model for the example

Making use of the geometric symmetry of the structure, the number of variables and constraints is halved. The optimization model of the bridge is described as follows:

4.3.1 Design variables

There are 116 design variables considered for the example bridge with the following meanings and sequences:

$X(1)$: the height of box girder adjacent to tower H_1;

$X(2)$: the height of box girder away from tower H_2;

$X(3)$: the width of the tower cross-section at the top of tower TH_1;

$X(4)$: the width of the cross-section at the bottom of tower TH_2;

$X(5)$~$X(18)$: cable stress S_1~S_{14};

$X(19)$~$X(32)$: the number of steel strands for each cable NC_1~NC_{14};

$X(33)$~$X(74)$: the number of prestressing tendons at upper margin of the girder;

$X(75)$~$X(116)$: the number of prestressing tendons at bottom margin of the girder.

According to the deck and tower profile of some previous designs for low-pylon cable-stayed bridges in the world, the upper and lower bounds of the first four variables are given as follows

$$L/42 \le H_1 \le L/14 \tag{9}$$

$$L/77 \le H_2 \le L/27 \tag{10}$$

$$2.4\text{ m} \le TH_1 \le 7.1\text{ m} \tag{11}$$

$$1.3\text{m} \le TH_2 \le 3.7\text{ m} \tag{12}$$

where L is the main span of the bridge.

In order to make the cable perform its functions properly, the tensioning force for the cable should not be too great or too small. In the case of the low-pylon cable-stayed bridge, the cables work as an external prestressing system and the upper limit of cable stress is set as 0.6Rb in China code (JTG/T D65-01-2007) (greater than the upper limit 0.4Rb for normal cable-stayed bridges, where Rb is the standard value of cable tensioning strength). Eventually, 0.55Rb is prescribed as the upper limit for cable stresses in this example, considering an improvement for safety coefficient. The minimum stress in stay cables is set as 0.3 Rb to prevent cable from suffering too small stress state. Therefore, the limitation of cable stresses for the bridge is given by

$$558\ \text{MPa} \le S_i \le 1023\ \text{MPa} \quad i=1,2,\ldots,14 \tag{13}$$

Considering the product model of the stay cables in the market, the range of the number for each cable is

$$22 \le NC_i \le 151,\ \ i=1,2,\ldots,14 \tag{14}$$

Since the section size of the girder limit the maximum number of prestressing tendons, the range of prestressing tendon number is

$$1 \le NP_i \le 100 \quad i=1,2,\ldots,84 \tag{15}$$

4.3.2 Constraints

There are 7 constraints considered for the example bridge with the following meanings and sequences:

G(1): the maximum stress of the upper and bottom of the left and right section for the beam and tower elements;

G(2): the minimum stress of the upper and bottom of the left and right section for the beam and tower elements;

G(3): the maximum deflection of the girders;

G(4): the longitudinal displacement constraint at top of the tower;

G(5): the maximum cable stress;

G(6): the minimum cable stress;

G(7): the width of the tower cross-section at top of the tower must less than that at the bottom of the tower.

The developed program allows the users to select the limit of tension and compression stress for concrete. The users can choose the stress limit according to the code criterion or a stricter requirement. 60% of the standard value of strength of concrete compressive stress is prescribed as the allowable compressive stress according to the China code (JTG/T D65-01-2007). The structural analysis result of the model illustrates that only a few beam elements near the bearings bear tension stress. In order to obtain a full prestressed girder, a large number of materials will be cost to eliminate the tension stress. Therefore, 20% of the standard value of strength of concrete tension stress is prescribed as the allowable tension stress to improve the optimal efficiency. The upper and lower stress limit of the concrete is given by

$$-19.44\ \text{MPa} \le \sigma_m^n \le 0.53\ \text{MPa} \tag{16}$$

The allowable value of the deflection in China code (JTG/T D65-01-2007) for each beam elements of the girder is given by

$$\Delta_i \le L/500 = 20.02\ \text{cm} \quad i=1,2,\ldots,42 \tag{17}$$

The allowable value of the longitudinal displacement in China code (JTG/T D65-01-2007) for the elements at top of the tower is given by

$$\Delta_i \le H/500 = 5.05\ \text{cm} \tag{18}$$

where *H* is the height of the tower.

4.3.3 Optimization model

The optimization model of the example is as follows. All design variables are transformed into integers:

1. Objective function

$$F(X) = 5538 \cdot W_1 + 178.5 \cdot W_2 + 16202.4 \cdot W_3 \tag{19}$$

Table 2 Cost factor of each member

Member	Prestressing tendon	C50 concrete	Cable
Cost (Chinese yuan/t)	5538	178.5	16202.4

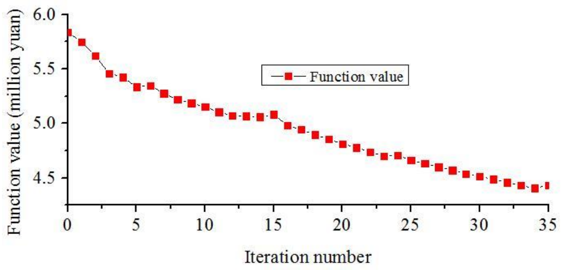


Fig. 6 Objective function value reduction corresponding to iteration

Table 3 Objective function values

Design scheme	Initial design (million Chinese yuan)	Optimum result (million Chinese yuan)	Save
Objective function value	5.838	4.435	24.03%

where W_1, W_2, and W_3 represent the weight (*t*) of the prestressing tendons, the concrete and the cables, respectively.

The cost factor of each member shown in Table 2 is offered by the design institute of the bridge.

2. Variable bounds:

$$24 \le X(1) \le 71$$
$$13 \le X(2) \le 45$$
$$24 \le X(3) \le 71$$
$$13 \le X(4) \le 37$$
$$558 \le X(i) \le 1023 \qquad i=5,6\ldots18$$
$$22 \le X(i) \le 151 \qquad i=19,20\ldots32$$
$$1 \le X(i) \le 100 \qquad i=33,34\ldots116$$

3. Constraints:

$$19.44 + G(1) \ge 0$$
$$0.53 - G(2) \ge 0$$
$$200.2 - G(3) \ge 0$$
$$50.8 - G(4) \ge 0$$
$$1023.0 + G(5) \ge 0$$
$$-G(6) - 558.0 \ge 0$$
$$G(7) \ge 0$$

The optimum result is obtained at iteration 35. The computing procedure costs about 145 minutes. The comparison of the design variables for the initial design and the optimum results are listed in Tables 3-6.

4.4 Results after optimization

4.4.1 The cost

The objective function value reduction corresponding to iteration is shown in Fig. 6. The objective function, which means the cost of the bridge, is reduced after each iteration. Table 3 shows the cost of the optimum design scheme is 4.435 million Chinese yuan, which saves 24.03% cost of the initial design 5.838 million Chinese yuan. The optimum design scheme is more economic structure.

Table 4 Cross-section size of the girder and the tower

Item	Initial design(m)	Optimum result(m)
Height of the girder near side span	3.0	1.4
Height of the girder adjacent to the tower	5.0	2.4
Width of section at bottom of the tower	5.2	3.4
Width of section at top of the tower	4.2	3.2

Table 5 Number of the prestressing tendon for each element

Element No.	Initial design		Optimum result		Element No.	Initial design		Optimum result	
	Upper	Bottom	Upper	Bottom		Upper	Bottom	Upper	Bottom
1	6	36	6	1	22	1	60	1	40
2	6	36	1	16	23	1	60	1	40
3	6	36	1	29	24	1	60	1	40
4	6	48	1	49	25	4	48	1	28
5	6	72	1	51	26	4	48	1	28
6	6	72	1	72	27	9	36	1	16
7	6	72	1	81	28	9	36	1	17
8	6	72	2	78	29	13	36	1	19
9	6	72	1	66	30	13	36	1	19
10	6	72	1	66	31	42	24	6	7
11	1	72	1	56	32	42	24	6	7
12	1	72	1	56	33	24	12	6	1
13	1	72	1	52	34	24	12	6	1
14	1	72	1	52	35	30	1	12	1
15	1	72	1	52	36	36	1	18	1
16	1	72	1	52	37	42	1	24	1
17	1	72	1	52	38	52	1	35	1
18	1	72	1	52	39	62	1	44	1
19	1	60	1	40	40	72	1	54	1
20	1	60	1	40	41	82	1	63	1
21	1	60	1	40	42	82	1	63	1

4.4.2 Cross-section size of the girder and the tower

It is obtained from Table 4 that, the height of the girder section adjacent to the tower reduces from 5.0 m to 2.4 m, and the height of the girder section away from the tower reduces from 3.0 m to 1.4 m. The width of the section at top of the tower reduces from 4.2 m to 3.2 m, and the width of the section at bottom of the tower reduces from 5.2 m to 3.4 m. Since the height of the girder and the width of tower cross-section are reduced, the concrete weight reduces by 25.7%, from 16243.8 t before optimization to 12070.0 t after optimization.

4.4.3 Prestressing tendons

The number of the prestressing tendons for most girder elements after optimization is less than that for the initial design, except several girder elements near the backstay, as shown in Table 5 and Fig. 7. The pretressing tendon is actually not needed in this girder region where one prestressing tendon is allocated. After optimization, the weight of the prestressing tendons can reduce from 128.0 t to 90.4 t, meaning a 29.4% reduction compared with the initial design.

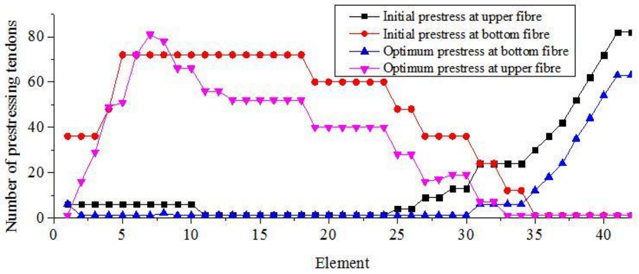


Fig. 7 Prestress configuration in the girder for half structure of the bridge

Table 6 Cable areas and forces

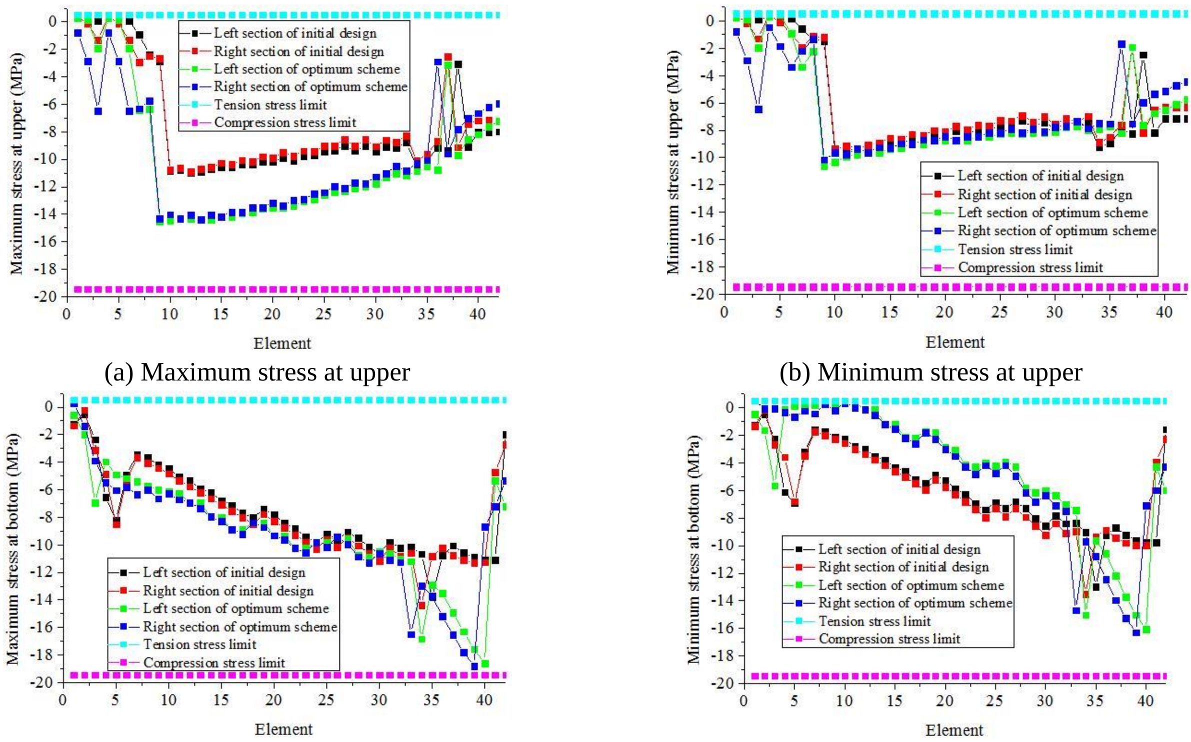
Cable No.	Initial design			Optimum result		
	Number of strands(φ15.2)	Area (mm^2)	Initial force (*t*)	Number of strands(φ15.2)	Area (mm^2)	Initial force (*t*)
C1	73	10015.6	800	52	7134.4	629
C2	73	10015.6	800	53	7271.6	670
C3	73	10015.6	800	52	7134.4	671
C4	73	10015.6	800	52	7134.4	657
C5	73	10015.6	800	53	7271.6	698
C6	73	10015.6	800	53	7271.6	698
C7	73	10015.6	800	54	7408.8	712
C8	73	10015.6	800	52	7134.4	685
C9	73	10015.6	800	54	7408.8	712
C10	73	10015.6	800	57	7820.4	751
C11	73	10015.6	800	58	7957.6	772
C12	73	10015.6	800	65	8918	891
C13	73	10015.6	800	75	10290	1029
C14	73	10015.6	800	63	8643.6	864
Cable weight (*t*)	137.6			109.6		

4.4.4 Cable areas and forces

The uniform 73-φ15.2 steel strands are adopted in the initial design, and the total cable weight for the initial design is 137.6 t. Compared with the initial uniform 800 t tensioning forces for every cable, the optimum cable initial tensioning forces are different to each cable. In general, the forces of the long cables are large than the short cables. Due to different cable forces, the number of the strands of each cable is not uniform. The cable areas and forces of the initial and optimum designs are listed in Table 6. The total cable weight for the optimum result is 109.6 t, meaning a 20.3% reduction compared with the initial design.

4.5 Structural analysis for the initial design and the optimum scheme

The stresses of the girder and tower elements for the initial and the optimum design schemes, are depicted in Fig. 8 and 9. In these figures, the symbol "-" for the stress represents the compression stress. It is obtained that the maximum and minimum stresses of the girder and the tower are almost compression stresses. Tension stress only appears in a few beam elements near the bearings. The compression stresses of the girder and the tower for the



(a) Maximum stress at upper (b) Minimum stress at upper

(c) Maximum stress at bottom (d) Minimum stress at bottom

Fig. 8 The stresses of beam elements

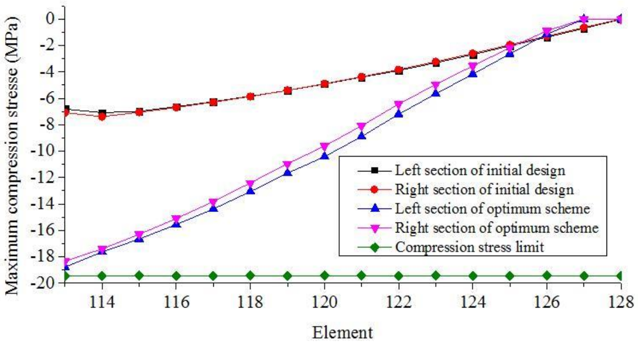
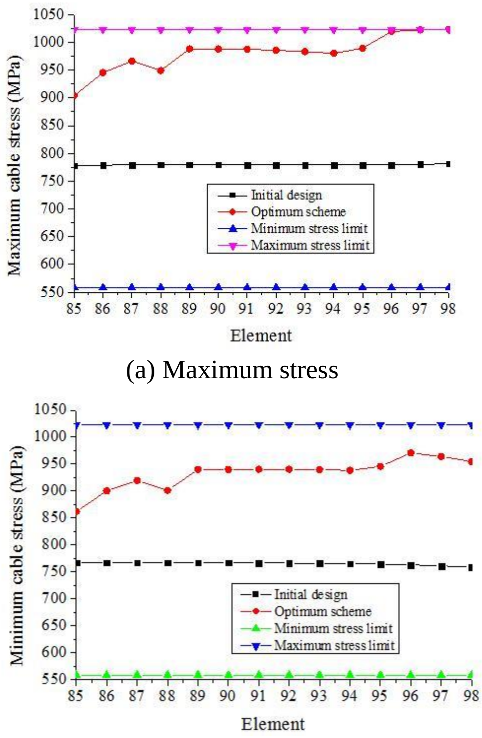


Fig. 9 Maximum compression stresses of tower elements



(a) Maximum stress

(b) Minimum stress

Fig. 10 The stresses of cable elements

optimum scheme are larger than those for the initial design due to the reduction of the section of the girder and the tower, as well as the reduction of the prestressing tensions. The maximum compression stresses of the girder for the initial design and optimum scheme are 14.37 MPa and 18.80 MPa, respectively. The maximum tension stresses of the girder are 0.44 MPa and 0.53 MPa, respectively for the initial design and the optimum scheme. The maximum compression stresses of the tower are 7.40 MPa and 18.84 MPa, for the initial design and the optimum scheme respectively. The tension and compression stresses for all girder and tower elements of the initial design and the optimum scheme can meet the stress constraints set by Eq. (16). The maximum compression stresses of the girder and the tower nearly reach the compression stress limit. The tension stresses of the elements between the bearing and the backstay are close to the tension stress limit, shown in Fig. 8(d). The maximum tension stress of the girder reaches the tension stress limit, which indicates that the tension stress of the girder is an active constraint.

The maximum and minimum stresses of the cables are shown in Fig. 10. The stresses of the optimum scheme are larger than those of the initial design due to the reduction of the cable area. It can be concluded that all the maximum

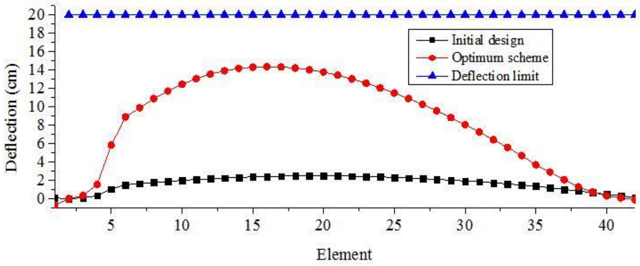


Fig. 11 Deflections of beam elements

and minimum cable stresses for the initial design and the optimum scheme can meet the requirement from Eq. (13). Most of the cable stresses are close to the maximum stress limit, shown in Fig. 10(b). The maximum stress of the cable is 1023MPa, which reaches the maximum stress limit. Therefore, the maximum stress of the cable is also an active constraint in this optimum example.

The deflections of the girder for the initial design and the optimum scheme are shown in Fig. 11. Since the stiffness of the girder for the optimum scheme is less than that for the initial design due to the reduction of the section, the deflections of the girder after optimization increase. The maximum deflection of the optimum scheme is 14.37 cm, which is much larger than the maximum deflection 2.49 cm of the initial design. Although the deflections of the girder after optimization increase, they are still within the allowable limit value of 20.02 cm (see Eq. (17)). The maximum longitudinal displacement at top of the tower is 4.31 cm under the non-symmetrical live load, less than the allowable limit value of 5.08 cm (see Eq. (18)).

The structural analysis results indicate that the optimum design scheme is a safe, economic and reasonable structure. In this optimum example, the tension stress of the girder and the maximum stress of the cable are active constraints. The optimization technique proposed in this paper is feasible and effective.

5. Conclusions

- A multi-parameter optimization technique considering prestress in girder is proposed in this study, aiming at obtaining the minimum cost design for PC cable-stayed bridges. The number of prestressing tendons in girder is firstly set as one of design variables, as well as cable forces, cable areas and cross-section sizes of the girders and the towers.
- The multi-parameter optimization technique is carried out by a developed PC cable-stayed bridge optimization program involving the interaction of the parameterized automatically modeling program, the finite element structural analysis program and the optimization algorithm. The concrete creep and shrinkage effect and the cable-sage effect are considered in the structural analysis.
- The multi-parameter optimization technique is applied in a low-pylon PC cable-stayed bridge. Due to the reduction of the number of the prestressing tendons, and the section of the girder, the tower and the cables, the optimum design scheme can achieve a 24.03% reduction in cost, compared with the initial design. The 29.4% reduction of the prestressing tendons weight indicates that it is significant to optimize the prestress in the girder. The optimum result verifies the capability and efficiency of the multi-parameter optimization technique.
- The structural analysis results for the optimum scheme indicate that the stresses and the deformations meet the constraint conditions, i.e. these values are within the allowable limit. Therefore, the optimum scheme produces a safe, economic and reasonable structure. In this optimum example, the tension stress of the girder and the maximum stress of the cable are active constraints. The optimization technique proposed in this study is feasible and effective.
- The erection process of the cable-stayed bridge is not taken into consideration in this optimization technique. The algorithm will be improved in the future.

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