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Mechanics based analytical approaches to predict nonlinear behaviour of LSCC beams

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Abstract. This paper presents the details of analytical studies carried out towards the prediction of flexural capacity and load-deflection behaviour of Laced Steel-Concrete Composite (LSCC) beams. Analytical expressions for flexural capacity of the beams are derived in accordance with the basic principles of conventional Reinforced Concrete (RC) beams, but incorporated with relevant modifications to account for the composite nature of the cross-section. The ultimate flexural capacity of the two LSCC beams predicted using the derived expressions is found to be approximately 20% lower than those obtained due to measurement from experiments. Further to these, two simple methods are also proposed on the basis of unit load method and equivalent steel beam method to determine the non-linear load-deflection response of the LSCC beams for monotonic loading. Upon validation of the proposed methods by comparing the predicted responses with those of experiments and finite element analysis, it is found that the methods are useful to find nonlinear response of such composite beams.

Keywords: steel-concrete composite construction; flexural capacity; equivalent steel beam method; unit load method

1. Introduction

Laced Steel-Concrete Composite (LSCC) system is a form of Steel-Concrete Composite (SCC) construction developed by the authors (Anandavalli *et al.* 2012). LSCC system comprises of thin steel cover plates provided with perforations, through which reinforcements are introduced and held in position with the help of cross rods and in filled with concrete between the cover plates as shown in Fig. 1. Reinforcing members are continuously bent rods known as lacing, which transfer the force between steel cover plate and concrete core. This system is devoid of welding due to particular arrangement of lacings being inserted through the perforations at appropriate places and made to stay intact by using the cross rods.

Preliminary studies have already been conducted to understand the basic response characteristics of LSCC beams under static or quasi-static loading. Experimental investigations on LSCC beams have been carried out by the authors (Anandavalli *et al.* 2012) and it has been observed that such beams possess large ductility and rotational capacity. This makes LSCC system suitable for structures which are subjected to shock loads such as due to blast/impact. Towards this, the support rotation of the tested LSCC beam specimens was found to be nearly 13° with maximum mid-span deflection of 170 mm (Anandavalli *et al.* 2012). Due to the conventional test set-up, the failure mechanism of the LSCC beam specimens could not be ascertained from the experiments.

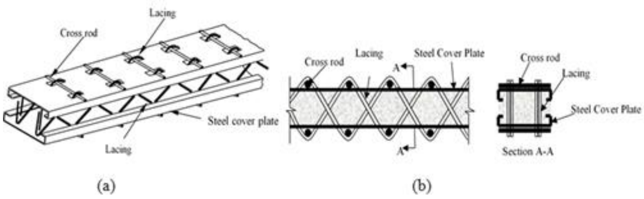

Fig. 1 (a) Isometric view of the LSCC configuration; (b) Cross-section of LSCC system

Finite element (FE) analysis of LSCC beam has also been carried out by the authors to determine the limiting deformation capacity and mode of failure (Thirumalaiselvi *et al.* 2016). FE model has proven to be effective in terms of predicting load-deflection response, post peak behaviour and failure mode of LSCC beam. Through detailed FE analysis, the beams are found to fail by yielding of the bottom cover plate. In spite of these, it is preferable to have simplified set of solutions which can be readily used to make a first-hand estimate of the member capacity and also the load-deflection response. This is the motivation to develop analytical solutions for obtaining flexural capacity of LSCC beams and to predict its load-deflection behaviour.

Few analytical solutions for estimation of moment of resistance and deflection for SCC elements are available (McKinley and Boswell 2002, Xie *et al.* 2007, Liew and Sohel 2009, Liew *et al.* 2009, Sohel *et al.* 2012). Double Skin Composite (DSC), a form of SCC construction consists of two steel plates and a group of shear stud connectors (welded to either top or bottom steel plate). In finding the position of neutral axis of DSC panel, it has

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been found that error in neglecting bending stiffness of plate about their own axes is less than 1%. The moment of resistance of DSC element has been determined by assuming linear stress distribution throughout depth of panel. Bending deflection has been calculated through pure bending theory and shear deflection has been obtained through the concept of effective shear modulus. However, deflections are relied upon for shear stiffness determination from experimental data (McKinley and Boswell 2002). Bi-steel is another form of SCC construction, consisting of two steel plates, a concrete core and a group of shear connectors (welded to both plates). Equivalent steel beam method has been adopted by Xie *et al.* (2007) to calculate the bending stresses and deflection due to bending, slip and shear. But, disadvantage is that only linear response of bi-steel has been ascertained. Liew and Sohel (2009) investigated a composite structure comprising of lightweight concrete core sandwiched in between two steel plates which are interconnected by J-hook connectors. Using the Eurocodes as basis of design, analytical (Sohel *et al.* 2012) methods to evaluate the flexural and shear resistance of this type of composite system has been developed. Flexural resistance and deflection are computed as given by McKinley and Boswell (2002) and plastic moment of resistance is determined by assuming rectangular plastic stress block.

Taking clue from all these, expressions for computing the flexural capacity of the LSCC beams are derived in accordance with the basic concept used in Reinforced Concrete (RC) beams after implementing suitable modifications to account for the composite nature of the geometry. Flexural capacity at yield point is computed by assuming linear stress distribution in LSCC section. Beyond yielding, flexural capacity is determined by assuming parabolic stress distribution for concrete in compression zone. In addition to this, analytical methods based on unit load and equivalent steel beam method are also proposed to determine the load-deflection response of LSCC beam. Conventional unit load method is suitably modified to obtain non-linear load-deflection response. Moment curvature relationship for LSCC beam is incorporated into equivalent steel beam method to determine the deflection response. The analytically calculated ultimate flexural capacity is found to be about 85% of that measured in experiments. Also, the load-deflection curves obtained using the proposed methods are found to reasonably match with those from experiment and FE analysis though to a different degree.

2. LSCC beam: structural concept

The strength of bond between steel cover plate and concrete core, transfer of force between the cover plates and concrete core are the two governing factors in the design of SCC system (Liew *et al.* 2009). Most importantly composite action requires sufficient transfer of load between the concrete and steel. In LSCC beams, the lacings are structurally integrated with the cover plates through cross rods, and their primary function is to resist both transverse and longitudinal shear and to provide resistance

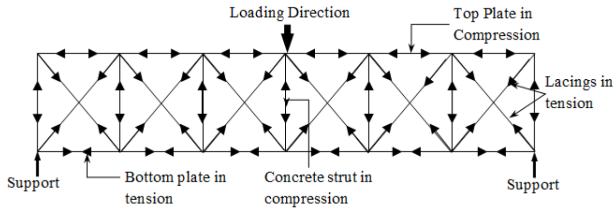
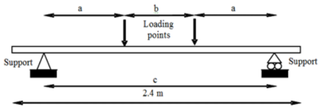


Fig. 2 Strut and tie model of LSCC system



Parameter	LSCC-45 beam (mm)	LSCC-60 beam (mm)
a	665	670
b	470	610
c	1800	1950

Fig. 3 Loading arrangement on LSCC beam

against outwards local buckling of the top plate when the beam is subject to loading. Lacings are the primary components that mainly transfer the load between the plates, even after the core concrete is completely disintegrated. Cross rod also plays a significant role in transferring forces and preventing local buckling of steel cover plates apart from holding the lacings in position. The steel cover plates confine the concrete and are used to prevent spalling of concrete core at failure.

In the strut and tie model of LSCC system (Anandavalli 2012) the bottom steel cover plate acts as tension member, top steel cover plate and the concrete above neutral axis are represented by the compression member. Inclined lacings which are connected to the top and bottom plates acts as tension member and resistance to compression is provided by virtual concrete strut as shown in Fig. 2.

3. Experimental investigations

Experimental investigations carried out on LSCC beam specimens by the authors were explained by Anandavalli *et al.* (2012) earlier. However, for the sake of completeness in reading, brief details about the experiments are given in this section. LSCC beam having cross section of 300 mm x 150 mm and length of 2.4 m is chosen for the present study. Cover plates are made of 3 mm thick cold formed steel provided with a lip of 50 mm on each side. Average yield and ultimate stress of cold formed steel are found to be 210 MPa and 300 MPa. Two test beam specimens, one with lacing angle of 45° (labelled as LSCC-45) and other with lacing angle of 60° (labelled as LSCC-60) are fabricated. The cover plate assembled with lacings and cross rods is shown in Fig. 1. Lacings and cross rods are made of mild steel. Average yield and ultimate stress of mild steel rods are found to be 400 MPa and 540 MPa. Diameter of lacing and cross rod is 8 mm and 10 mm respectively.

Two LSCC beam specimens are subjected to monotonically applied loading under a typical arrangement to understand their behaviour. The specimens are tested with simply supported boundary conditions. Schematic diagram of four point bending set-up is shown in Fig. 3. Test set-up is shown in Fig. 4. During experiment, both the beams are found to possess large deformation and ductility.

Fig. 4 Test set up

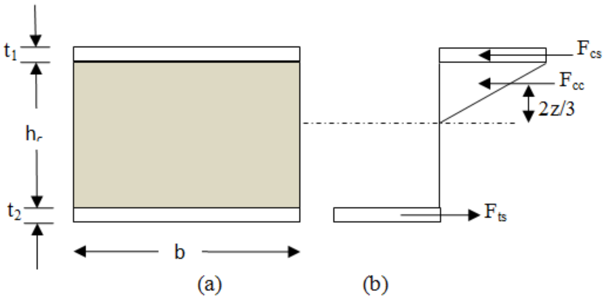


Fig. 5 LSCC section details (a) LSCC cross section ignoring lip portions; (b) Assumed linear stress distribution in section

However, the test was discontinued to avoid the risk of support rod slipping at such large deformation. As the experiment has been stopped abruptly due to safety issues, the ultimate capacity could not be ascertained. The readers are referred to Anandavalli *et al.* (2012) for more details about the experiments.

4. Analytical methods

A set of analytical methods are developed to determine the load-deflection responses and flexural capacity of LSCC beam elements having equal plate thickness. The validity of the analytical methods is verified by comparing the analytical results with those obtained from experiments (Anandavalli *et al.* 2012).

4.1 Flexural capacity of LSCC section

Fig. 5 shows the cross-section of LSCC beam of width '*b*'. The location of neutral axis, *z* and flexural capacity of LSCC sections, *M* are determined in consistent with the conventional RC theory, modified for the changes in basic cross-sectional property. In computing the location of the neutral axis of LSCC, concrete in tensile zone is assumed to be ineffective after cracking as in case of cracked theory of conventional RC. The flexural stiffness of thin steel plates about their own axes is assumed to be negligible (McKinley and Boswell 2002). Fully composite beam action is also assumed. In experiments, fully composite action is witnessed in such a way that slip at steel-concrete interface at the support regions is found to be insignificant. Thus, by assuming fully composite action, it is implied that the flexural capacity of beam is mainly due to the strength of the steel cover plates. It also has a corollary that the lacings contribute to the shear strength of the LSCC beams (Kwon 2008).

From Fig. 5, the position of neutral axis, *z*, is estimated by taking moments of area about the neutral axis

$$bt_1[z+t_1/2]+(b/m)(z^2/2)=bt_2[h_c-z+t_2/2] \quad (1)$$

where *b* is the width of the beam, t_1 is the thickness of top cover plate, t_2 is the thickness of bottom cover plate, h_c is the thickness of concrete core, m which equals E_s/E_c is the ratio between the elastic moduli of steel and concrete. Eq. (1) when rearranged in terms of quadratic equation.

$$z^2+2mz(t_1+t_2)-m(t_2^2-t_1^2+2t_2h_c)=0 \quad (2)$$

The only relevant solution to this equation is the positive value

$$z=-m(t_1+t_2)+\sqrt{m^2(t_1+t_2)^2-m(t_1^2-2t_2h_c-t_2^2)} \quad (3)$$

In computing the flexural capacity at yield point, it is assumed that the stress in the concrete in compression zone is linearly distributed in the LSCC section. The flexural capacity of LSCC section can be obtained by taking moments about the line of action of the concrete compressive force by (Fig. 5).

$$\begin{aligned} M &= F_{cs}\left(\frac{z}{3}+\frac{t_1}{2}\right)+F_{ts}(h_c-z/3+t_2/2) \\ &= \sigma_c bt_1\left(\frac{z}{3}+\frac{t_1}{2}\right)+\sigma_t bt_2(h_c-z/3+t_2/2) \end{aligned} \quad (4)$$

where F_{cs} is the compressive force in top cover plate (= $\sigma_c bt_1$), F_{ts} is the tensile force in bottom cover plate (= $\sigma_t bt_2$).

Since the stress distribution throughout the depth of concrete is assumed to be linear, the stress in the top steel plate, σ_c can be expressed in terms of that in the tension plate, σ_t as

$$\begin{aligned} \frac{\sigma_c}{z+\frac{t_1}{2}} &= \frac{\sigma_t}{h_c-z+\frac{t_2}{2}} \\ \sigma_c &= \frac{\sigma_t}{\left(h_c-z+\frac{t_2}{2}\right)}\left(z+\frac{t_1}{2}\right) \end{aligned} \quad (5)$$

The point at which the bottom tensile plate starts yielding is considered as the yield point of the beam. At the stage of yielding of plate, $\sigma_t=\sigma_y$, hence from Eqs. (4)-(5), the flexural capacity of the LSCC section can be calculated as

$$\begin{aligned} M_y = {} & bt_1\left(\frac{z}{3}+\frac{t_1}{2}\right)\frac{\sigma_y}{\left(h_c-z+\frac{t_2}{2}\right)}\left(z+\frac{t_1}{2}\right) \\ & +\sigma_y bt_2\left(h_c-\frac{z}{3}+\frac{t_2}{2}\right) \end{aligned} \quad (6)$$

Beyond yielding of the bottom tensile plate, stress distribution of concrete in compression zone is non-linear. Thus the flexural capacity of LSCC section can be determined by assuming a parabolic stress distribution (Vakil 2012) for concrete in accordance with conventional RC theory (Fig. 6). The concrete beneath the neutral axis is

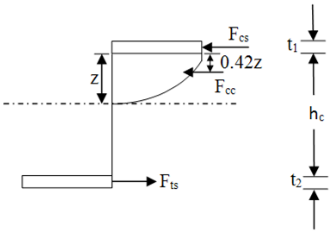


Fig. 6 Parabolic stress distribution assumed in the section after yielding

Table 1 Comparison of flexural capacity (kNm)

Beam	Predicted		Experiment		$(M_y)_{exp}/(M_y)_{ana}$	$(M_u)_{exp}/(M_u)_{ana}$
	At yield (Eq. (9))	At ultimate (Eq. (10))	At yield	At ultimate	At yield	At ultimate
LSCC-45	28.2	42.93	36.575	49.875	1.29	1.16
LSCC-60	28.2	42.93	40.2	53.6	1.45	1.25

assumed to be cracked and parabolic stress block of depth 'z' is considered.

In such case, to determine the location of neutral axis from top concrete surface, equilibrium condition is applied. Under equilibrium, total compressive force is equal to the total tensile force in the section. Thus equating the total compressive force to the total tensile force

$$F_{cs} + F_{cc} = F_{ts} \tag{7}$$

where F_{cs} is the compressive force in top cover plate ($=\sigma_c b t_1$), F_{ts} is the tensile force in bottom cover plate ($=\sigma_t b t_2$), F_{cc} is the compressive force in concrete ($=0.36 f_{ck} b z$).

The depth of neutral axis can be obtained as

$$z = \frac{(\sigma_t t_2 - \sigma_c t_1)}{0.36 f_{ck}} \tag{8}$$

Taking moments about the line of action of steel compressive force, F_{cs} the flexural capacity can be calculated as

$$\begin{aligned} M &= F_{ts}\left(h_c + \frac{t_1}{2} + \frac{t_2}{2}\right) - F_{cc}\left(\frac{z}{2} + \frac{t_1}{2}\right) \\ &= \sigma_t b t_2\left(h_c + \frac{t_1}{2} + \frac{t_2}{2}\right) - 0.36 f_{ck} z b\left(0.42z + \frac{t_1}{2}\right) \end{aligned} \tag{9}$$

After yielding of tensile cover plate, the concrete cracking continues to propagate upwards. As the crack propagates, neutral axis shifts towards the top cover plate. The ultimate flexural capacity of the LSCC beam is attained when the neutral axis reaches the bottom portion of the compression cover plate (i.e., z=0). Substituting z=0, in Eq. (9), the ultimate flexural capacity can be written as,

$$M_{ul} = \sigma_t b t_2\left(h_c + \frac{t_1}{2} + \frac{t_2}{2}\right) \tag{10}$$

Values of flexural capacity obtained at yield and ultimate stages using the above relations are compared with the experimental values in Table 1. It is observed that for LSCC-45 beam the ultimate flexural capacity predicted analytically is about 14% less than the experimentally observed values. Also for LSCC-60 beam, the predicted ultimate flexural capacity is approximately 20% less than the corresponding values due to the experiment. This difference could be due to the reason that contribution of the lip portions in resisting flexure is not taken into account. The comparison indicates that the derived expressions are useful to obtain a first order conservative estimate of the beam capacity.

4.2 Load-deflection behavior

Load-deflection response of structural members gives an indication of ductility possessed by them. In this study, two analytical methods are proposed based on unit load method and equivalent steel beam method to determine the deflection of LSCC flexural member under monotonic loading:

- Modified unit load method
- Extended equivalent steel beam method

Unit load method is suitably modified by equivalent linearization procedure to obtain non-linear load-deflection response. Bending and shear components of the deflection obtained using modified unit load method are explained in the subsequent sections. Equivalent steel beam method is also adopted to calculate the deflection of LSCC beam. Conventional equivalent steel beam method is used along with moment curvature relationships for the LSCC section, to predict non-linear load-deflection behaviour of LSCC beams.

4.2.1 Modified unit load method

Deflection determination using unit load method is well established procedure. Conventional unit load method is not capable of predicting non-linear load-deflection behaviour of structural members. But actually, the response of LSCC member subjected to monotonic loading is non-linear. Hence, in order to predict the non-linear response of LSCC beam, a modification is proposed to the unit load method. Modification is made by adopting moment curvature relationships for the LSCC section. The moment curvature relationship is obtained using the idealised stress-strain characteristics of concrete (as per IS 456:2000) and stress strain values of cold formed steel. The moments and corresponding curvatures are calculated for the following stages of response:

Stage (i): Cracking of concrete

Stage (ii): When the strain in tensile cover plate reaches strain corresponding to stress of 0.8 times f_y, where f_y is the yield stress of the cold formed steel

Stage (iii): When the strain in tensile steel cover plate reaches strain corresponding to stress of f_y

Stage (iv): When the strain in concrete core reaches a value of 0.0035

Stage (v): When the strain in tensile steel cover plate reaches a value of 0.02

Until cracking, the transformed moment of inertia, 'I_t', is used for computing the moment and the corresponding curvature. The cracking moment 'M_{cr}' is given by

$$M_{cr} = \frac{f_{cr} I_t}{(D/2)} \tag{11}$$

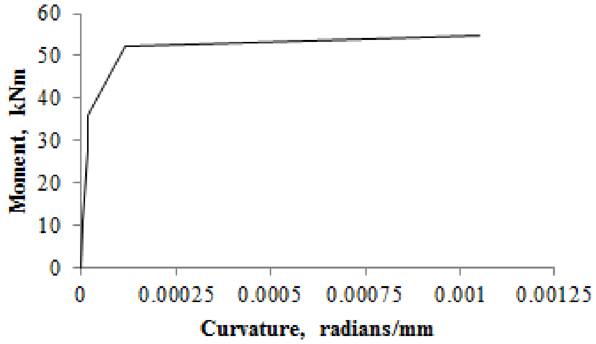


Fig. 7 Moment-curvature curve of LSCC beam

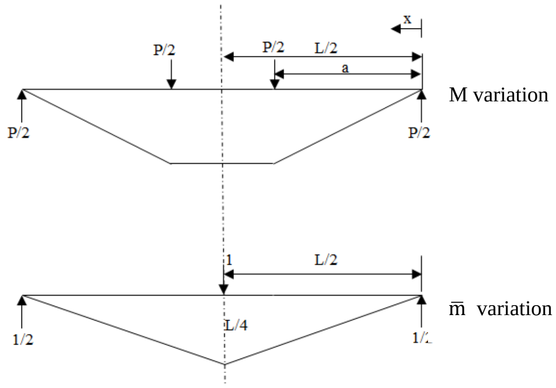


Fig. 8 *M* and $\overline{m}$ variation along the length of beam

where, f_{cr} is the modulus of rupture which equals $0.7\sqrt{(f_{ck})}$, f_{ck} is the characteristic strength of concrete, I_t is the transformed moment of inertia and *D* is the overall depth.

The curvature, Φ_{cr} is given by

$$\Phi_{cr} = \frac{M_{cr}}{E_c I_t} \tag{12}$$

where E_c is the modulus of elasticity of concrete.

The values for the moments and the corresponding curvatures at other stages were computed using linearity of strain profile across section and equilibrium of tensile and compressive forces. Then, moment is obtained by the following equation

$$M = Tj_d \tag{13}$$

where *M* is the moment at any stage, *T* is the total tensile force and j_d is the lever arm.

The curvature is computed using the relation

$$\Phi = \frac{\varepsilon_p}{z} \tag{14}$$

where *z* is the distance from the location of neutral axis to concrete top surface, ε_p is the strain in steel cover plate

The moment-curvature relation obtained for the LSCC beam is shown in Fig. 7.

Deflection due to shear and flexure is uncoupled, and the total deflection is the sum of deflections due to flexure (δ_f) and shear (δ_s).

Deflection due to flexure

Based on unit load method, the maximum deflection at the mid span is obtained using the relation

$$\delta_f = \int_0^L \frac{M\overline{m}}{EI} dx \tag{15}$$

$$\delta_f = \int_0^L \Phi\overline{m}\, dx \tag{16}$$

where *M* is the moment at a distance ‘*x*’ in the beam due to applied load, $\overline{m}$ is the moment at same section due to unit load at beam mid span, EI is the flexural stiffness of beam.

The bending moment variation along the span of LSCC beam due to applied loading as well as unit load is shown in Fig. 8. In conventional unit load method, from the relation given in Eq. (15), only linear response can be obtained.

Equivalent curvature corresponding to particular moment level, computed using a procedure similar to equivalent linearization (Anandavalli *et al*. 2005) is written as

$$\Phi' = \frac{2A_{m\phi}}{M} \tag{17}$$

which is twice the area of the multi-segmental moment curvature diagram up to the moment level under consideration divided by the moment, forgiven element.

Hence the equation for computation of deflection at particular stage is obtained as

$$\delta_f = \int_0^a \frac{\Phi'}{2a} x^2\, dx + \int_a^{L/2} \frac{\Phi'}{2} x\, dx \tag{18}$$

Integrating Eq. (18), the value of δ_f can be obtained as

$$\delta_f = \frac{\Phi'}{48}(3L^2 - 4a^2) \tag{19}$$

where a is the distance between support and loading point.

Deflection due to shear

Based on unit load method, the maximum deflection due to shear at the mid span is computed using the relation

$$\delta_s = \int_0^L \frac{Vv}{GA} dx \tag{20}$$

where *V* is the shear at a distance ‘*x*’ in the beam due to applied load which equals P/2 (Fig. 9), *v* is the shear at same section due to unit load at mid span which is 1/2. GA is the shear stiffness of the LSCC beam. Shear stiffness is computed using the procedure proposed by Anandavalli *et al*. (2005) and is explained below.

The resistance mechanism in a LSCC member is already discussed by adopting strut and tie concept. In accordance with the established RC theory, in LSCC beams, lacings (shear reinforcements) contribute significantly to the overall shear resistance. Neglecting the small contribution due to the concrete in the form of shear friction and aggregate interlock, the entire shear is assumed to be resisted by the shear force ‘*V*’ developed in the lacings. Hence

$$V = 2n\, A_s f_s \sin(\alpha) \tag{21}$$

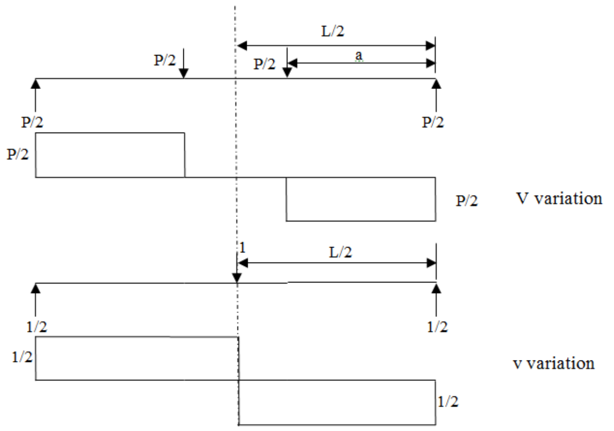


Fig. 9 V and v variation along the length of beam

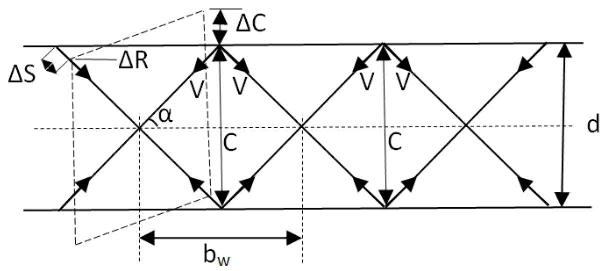


Fig. 10 Shear deformation

where A_s is the cross sectional area of steel lacings, f_s is the stress in lacings steel, n is the number of lacings crossing the section and α is the angle of lacing.

Shear deformation (ΔV) over the depth, d is the sum of change in length of concrete strut (ΔC) and resultant change in length of lacings (ΔR). Change in length of lacing, ΔS is given by

$$\Delta S = \text{original length x } \varepsilon_s = \frac{d}{\sin(\alpha)}\left(\frac{f_s}{E_s}\right) = \frac{V}{2\text{ n } A_s \sin(\alpha)}\left(\frac{d}{E_s \sin(\alpha)}\right) \tag{22}$$

where ε_s is the strain in lacing steel and E_s is the modulus of elasticity of lacing steel.

The resultant change in length of lacing, ΔR is given by

$$\Delta R = \sqrt{2}\Delta S = \frac{Vd}{\sqrt{2}\text{ n } A_s \sin^2(\alpha) E_s} \tag{23}$$

The force in strut is given by

$$C = V = 2\text{ n } A_s f_s \sin(\alpha) \tag{24}$$

Change in length of the strut, ΔC is given by

$$\Delta C = \frac{\text{stress in concrete}}{E_c}\text{ original length} = \frac{V}{d\, b_w E_c}\, d \tag{25}$$

where b_w is the effective width of LSCC beam.

Modulus of elasticity of cracked concrete

$$(EI)_{eff} = E_c\left(\frac{1}{12}bD^3\right)$$

$$E_c = \left[\frac{12(EI)_{eff}}{bD^3}\right] = \left[\frac{12\left(\frac{M}{\Phi}\right)}{bD^3}\right] \tag{26}$$

where b is the width of LSCC beam and D is the depth of LSCC beam.

Thus, the shear deformation over a distance, d is given by

$$\Delta V = \Delta R + \Delta C$$

$$\left(\frac{Vd}{GA}\right) = \frac{Vd}{\sqrt{2}\text{ n } A_s \sin^2(\alpha) E_s} + \frac{V}{b_w E_c} \tag{27}$$

Hence, shear stiffness can be written as

$$\frac{1}{GA} = \left[\frac{1}{\sqrt{2}\text{n } A_s \sin^2(\alpha) E_s} + \frac{1}{d\, b_w E_c}\right] \tag{28}$$

Substituting this in Eq. (20), the value of δ_s can be obtained as

$$\delta_s = \int_0^L Vv\left[\frac{1}{\sqrt{2}\text{n } A_s \sin^2(\alpha) E_s} + \frac{1}{d\, b_w E_c}\right]dx \tag{29}$$

Thus, the total deflection, δ at the mid span of the beam is given by $\delta = \delta_f + \delta_s$.

4.2.2 Extended equivalent steel beam method

Equivalent steel beam method has been recommended by Xie *et al*. (2007) for calculating the bending stresses and deflection of steel-concrete-steel (SCS) sandwich beams. According to the recommendation, deflection is calculated using the equivalent steel section with steel modulus E_s. The method does not provide means for calculating the non-linear response of SCS sandwich beams since it has been assumed that the steel and concrete are elastic. But in practice, the member will be in inelastic state at an early stage itself due to cracking of concrete. Hence, in the present study, equivalent steel beam method is extended with the aim of capturing non-linear load-deflection response of LSCC beams under bending. The basis of extended equivalent steel beam method is to make use of moment curvature relationships in determining the deflection response of LSCC beams.

Fig. 11 shows the cross-section of LSCC beam of width 'b', top and bottom steel plate of thicknesses 't_1' and 't_2' respectively and concrete core of thickness 'h_c'. In accordance with the equivalent steel beam method, the tensile strength of concrete is assumed to be negligible and steel cover plates are assumed to be continuously connected to the concrete. Width of steel plate is retained and that of concrete core is assumed to be reduced in proportion to the modular ratio 'm', which is equal to E_s/E_c as shown in Fig. 11. Deflection is calculated for the equivalent section with steel modulus E_s.

The maximum deflection at the centre of a beam in a two point loading system can be calculated by using the relation

$$\delta = \frac{M_{max}}{24E_s I}(3L^2 - 4a^2) \tag{30}$$

where L is the span of the beam; M_{max} is the maximum moment at the mid-span due to concentrated loads, applied

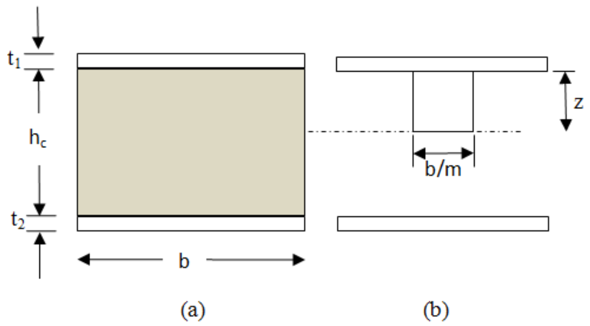


Fig. 11 LSCC section details (a) LSCC cross section ignoring lip portions; (b) Equivalent steel section

at a distance '*a*' from the support; E_s is the modulus of elasticity of steel; and I is the effective moment of inertia of the beam section after cracking.

Using the relation given above only linear load-deflection response of member can be obtained (Xie *et al*. 2003). To capture the non-linear deflection of LSCC beams, curvature corresponding to particular load level is adopted in determining the deflection. Moments and corresponding curvatures at different stages (as explained in previous section) for LSCC section is evaluated using the stress-strain model for concrete proposed by Hognestad (1995) and stress-strain values of cold formed steel. Depth of neutral axis, z is obtained iteratively by equating the compression and tension forces. Then maximum moment, $M_{\max}$ is calculated from the derived neutral axis depth.

Curvature, Φ is given by

$$\Phi = \frac{M_{max}}{E_s I} \tag{31}$$

Substituting this in Eq. (30), δ can be written as

$$\delta = \frac{\Phi}{24}(3L^2 - 4a^2) \tag{32}$$

Using above relation, deflection can be obtained at different stages of loading by making use of curvatures corresponding to that particular load level. This provided means to obtain the non-linear load-deflection response of LSCC beams. To illustrate the effectiveness of extended equivalent steel beam method, for LSCC-45 beam, deflection is obtained by using both the equivalent and extended equivalent steel beam methods and compared with the experimental values as shown in Fig. 12.

It is observed that equivalent steel beam method is capable of predicting only the linear response of LSCC beam (till a load level of about 65 kN) and fails in the non-linear range. However, the performance of extended equivalent steel beam method is found to be quite satisfactory in the entire load range.

4.2.3 Load-deflection response

As the load-deflection response of LSCC beams is non-linear, mid-span deflection corresponding to two-thirds the peak load and yield of the bottom cover plate is considered for validating the proposed analytical methods (Roberts *et al*. 1996). Predicted deflection values are presented in Table 2 along with the corresponding measured values. In spite of complex factors governing the behaviour of LSCC beams (concrete cracking, buckling of cover plates), the analytical results are seen to follow the trend of experimental results. As generally expected, deflection due to shear is found to be negligible when compared with that due to flexure. Among the two analytical methods, the results from extended equivalent steel beam method are found to be on higher side because of which this method is ideal for use in preliminary design verification of LSCC beams.

For LSCC-45 beam, deflection curves obtained using extended equivalent steel beam method and modified unit load method are presented in Fig. 13 along with those obtained from experiment and FE analysis. In the initial elastic region up to a load of about 70 kN, it is observed that the deflection obtained using both the extended equivalent steel beam method and modified unit load method coincides well with the those measured from experiment. Among

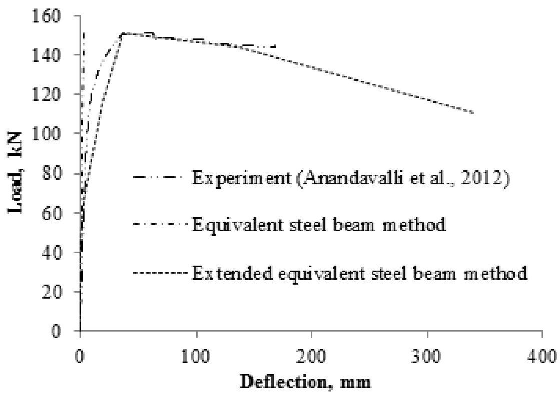


Fig. 12 Load-deflection behaviour of LSCC-45 beam due to equivalent steel beam method

Table 2 Comparison of the deflection of LSCC beam

Beam	Load (kN)	Predicted (mm): Modified unit load method δ_B	δ_S	δ_{total}	Predicted (mm): Extended equivalent steel beam method	Experiment (mm)	FE analysis (mm)
LSCC-45	$2P_u/3$=100	3.5	1.3	4.8	13.8	6.0	8.59
	P_y=110	4.9	1.4	6.3	16.9	7.6	10.8
LSCC-60	$2P_u/3$=106.7	4.4	1.2	5.6	15.9	9.91	9.92
	P_y=120	10.7	1.6	12.3	20.9	13	13.1

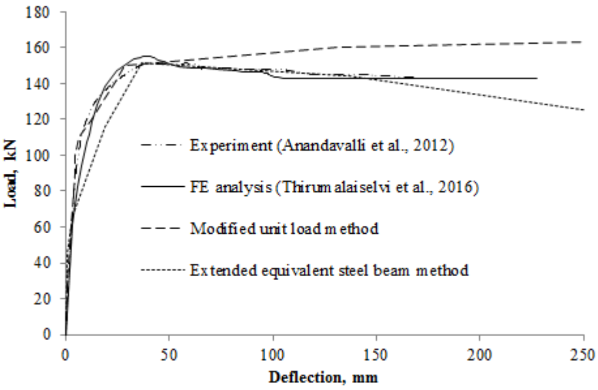


Fig. 13 Load-deflection behaviour of LSCC-45 beam

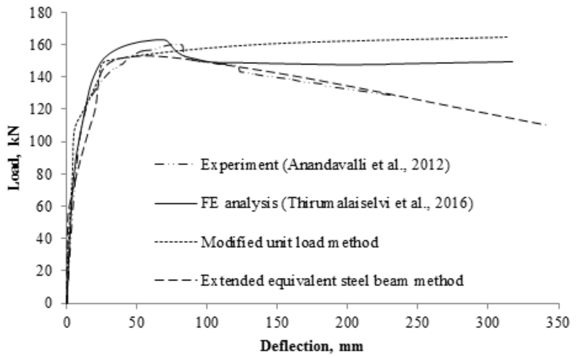


Fig. 14 Load-deflection behaviour of LSCC-60 beam

Table 3 Parameters considered in the study

Parameters	Abbreviation	Values
Plate thickness, mm	PT	2,3,4,5
Angle of lacing, degees	LA	30,45,60

these two methods, the extended equivalent steel beam method is found to perform better. Between 70 kN and 150 kN, it is seen that the extended equivalent steel beam method depicts flexible behaviour for the beam when compared to the experiment. The critical post peak behaviour shows minor deviation from the experimental values. Beyond 70 kN, load-deflection curve obtained from modified unit load method matches with experimental curve though with little discrepancy till peak load is reached. However, this method is found to be unsuitable to know the softening response of the beam.

Deflection responses of LSCC-60 beam obtained using extended equivalent steel beam method and modified unit load method are presented in Fig. 14 along with those obtained from the experiment and FE analysis. The results due to experiment is taken as reference in the comparative study. The comparison indicates the excellent performance of extended equivalent steel beam method in predicting the entire load-deflection response of the beam. In the initial elastic region up to around 70 kN, the curve is found to coincide with that of experiment. The response between 70 kN and 150 kN predicts a mild flexible behaviour for the beam. Post peak softening response also matches quite well with that of experiment. On the other hand, modified unit load method predicts stiffer behaviour for the beam besides failing to capture the softening response in the post-peak range.

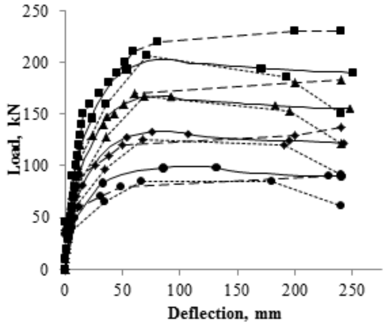
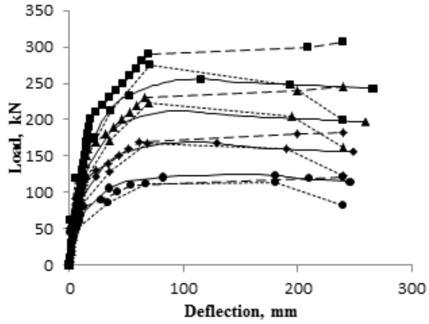
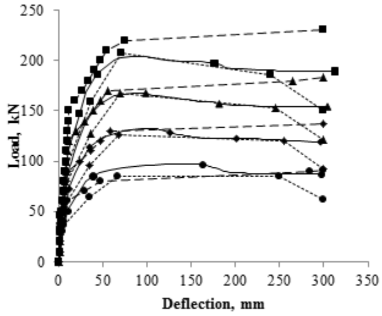
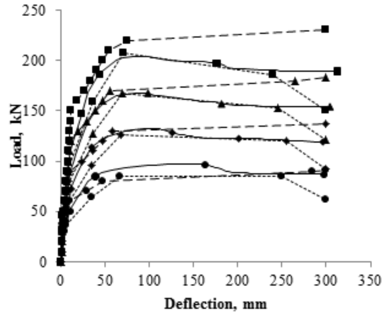
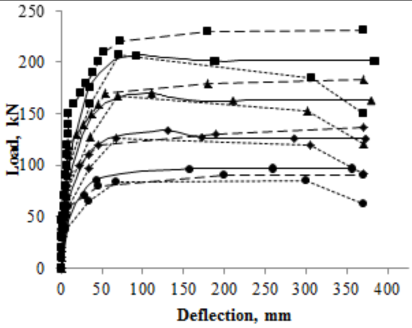
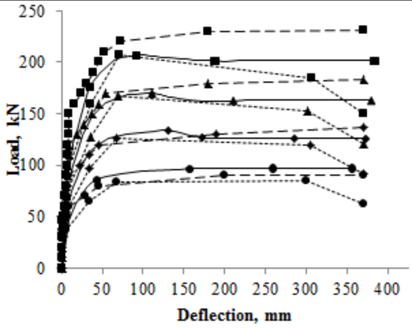
5. Validation studies

The improved analytical methods proposed in the paper are based on basic principles of mechanics, though with simplifying assumptions. Before accommodating these methods into use, it is proposed to understand the performance of the methods by comparing their prediction against that of another source, like an experiment. However, as already seen, laboratory experiments may not reveal the complete response of LSCC beams because of constraints in measuring their large deflection. As a possible solution, extensive parametric studies are carried out to generate the results for comparison. The results of such studies can also be used for comprehensive assessment of the influence of the model parameters such as the beam geometry (length and width of beam, angle and diameter of lacing, thickness of cover plates) and concrete grade on the response. Parameters considered and their corresponding values are given in Table 3.

For conducting the parametric studies, the validated FE model of LSCC beam reported by the authors earlier (Thirumalaiselvi *et al.* 2016) is used. Brief details about FE model are explained here for the sake of continuity in reading while interested readers can refer to the source for more details. Solid, shell and link elements are used to represent the concrete core, steel plates and shear connectors respectively. Stress-strain curve of concrete in compression including strain softening is defined using empirical relationship proposed by Attard and Setunge (1996). Linear stress–strain relationship is used for concrete in tension up to concrete cracking stress. To consider the post-cracking resistance in tension, the stiffening model proposed by Guo and Zhang (1987) is adopted. Concrete damaged plasticity model is adopted to represent complete inelastic behaviour including damage characteristics of concrete both in tension and compression. Plasticity based model is used to describe the non-linear behaviour of steel with nominal stress-strain values obtained from the coupon tensile tests results (Anandavalli 2012). Mechanical interaction between the cover plate and in-filled concrete is modelled by surface to surface contact interactions using friction formulation in tangential direction and hard contact in normal direction. The classical Coulomb model is employed to define friction between the surfaces. Lacings are embedded in concrete and also restrained by cross rods at intersections. The analysis is terminated when the tensile strain in bottom cover plate reach the failure strain of steel. The response of the beams for two-point loading is critically studied by the authors (Thirumalaiselvi *et al.* 2016) for different set of parameters such as plate thickness, concrete grade and angle of lacing. Parametric study carried out by them reveals the fact that lacing angle and plate thickness significantly influence the capacity of LSCC beams.

This is taken into account while evaluating the performance of the proposed analytical methods. Thus the response of LSCC beams defined by different values of cover plate thickness (PT) and lacing angle (LA) given in Table 3 are obtained by the proposed analytical methods. In addition, with the aim to derive a deeper understanding of the performance of these methods, the analysis is repeated for another beam whose length and width are chosen as 1.8 m and 400 mm but retaining the depth as 150 mm. The analytical methods are proposed to capture the load-deflection response of the LSCC beams. However, these methods do not include a termination criterion to know the maximum deflection of the beam. Therefore, initially, an approximate value for the rotation capacity of the beams is calculated depending on the lacing angle. Then the load-deflection response of the beams is obtained by applying the analytical methods up to maximum deflection defined by the rotation capacity of the beams. Analytically predicted

Table 4 Load-deflection response of 2.4 m long LSCC beam

Lacing Angle	Width, *b* (mm)	
	300	400
30°		
45°		
60°		

——— FE analysis ········ Extended equivalent steel beam method -------- Modified unit load method

● 2 mm ◆3 mm ▲ 4 mm ■5 mm

load-deflection response is compared with that of the FE analysis in Tables 4 and 5 for LSCC beams of length 2.4 m and 1.8 m respectively.

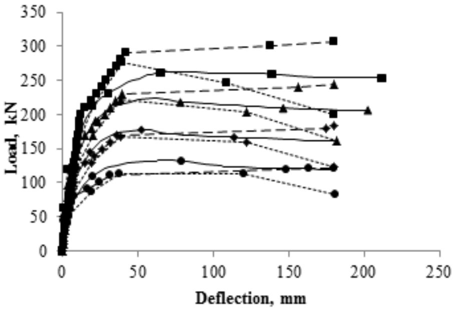
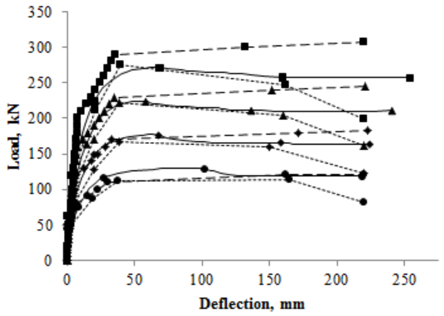
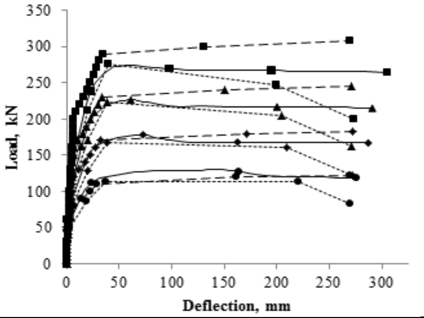
From the comparison made, it is observed that extended equivalent steel beam method makes better prediction of the response in the entire range of interest. This is particularly observed to be true in case of beams with higher capacities such as the ones made of thicker cover plates. Further, it is once again confirmed that this method is able to trace the post-peak softening response of the beam for all the cases analysed while the modified unit load method is not that successful.

It needs to be emphasized that analytical solutions are obtained by employing simplifications and the consequent limitations of these solutions should be realized. The maximum deflection calculated using the analytical methods is based on the approximate estimate of the support rotation of the beams which depends on the lacing angle. Also, it is observed from FE analysis that the ultimate deflection of LSCC beams varies with the thickness of steel cover plate. As this variation is only marginal, it is thought that the influence of plate thickness can be ignored while using the simple analytical methods. With this background, reasonable results are achieved for various configurations of LSCC beams and the analytical solutions provide acceptable first estimate of the response of LSCC beams. This is specifically valid as long as the peak load and the pattern of load-deflection behaviour is of prime importance.

6. Conclusions

Laced Steel-Concrete Composite (LSSC) beams are reported to possess unusually large ductility due to the specific assembly of steel cover plates with the continuous lacings by using short cross rods. An attempt is made in the present paper to estimate the load-deflection behaviour of LSCC beams through simple analytical methods. Initially,

Table 5 Load-deflection response of 1.8 m long LSCC beam with width 300 mm

<table>
<tr><th>Lacing Angle</th><th>Response</th></tr>
<tr><td>30°</td><td>
</td></tr>
<tr><td>45°</td><td>
</td></tr>
<tr><td>60°</td><td>
</td></tr>
<tr><td colspan="2">—— FE analysis ------ Extended equivalent steel beam method ········ Modified unit load method
● 2 mm ◆ 3 mm ▲ 4 mm ■ 5 mm</td></tr>
</table>

an analytical expression is derived based on the conventional reinforced concrete theory to find the flexural capacity of the beams. The derived expression is found to make a conservative estimate of the flexural capacity by a maximum of about 20% which makes it ideal for first-hand estimation of the beam capacity during preliminary design.

Further to these, two methods, based on i) unit load method and ii) equivalent steel beam method, are developed to trace the non-linear load-deflection response of LSCC beams for monotonic loading. In equivalent steel beam method, the beam deflection is calculated by introducing moment curvature relationship derived for LSCC beam section. On the other hand, the modification in unit load method is due to replacement with the curvature computed using equivalent linearization procedure to predict the load-deflection behaviour of LSCC beams. A comparison among the two methods denote that extended equivalent steel beam method overestimates the deflection values while modified unit load method underestimates the same compared to the experimentally measured and those obtained from extensive parametric studies using validated FE model. The beam deflection values obtained by using modified unit load method are found to be only grossly correlating with those from experiments and FE analysis. Mainly, post peak response could not be predicted using this method. Thus, extended equivalent steel beam method is found to provide acceptable load-deflection response of the beams including the critical post-peak behaviour. The proposed simple methods are particularly useful in the light of difficulties experienced in conducting laboratory experiments on LSCC beams due to their unusually large deformation capacity.

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