

# Improved block-wise MET for estimating vibration fields from the sensor

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**Abstract.** Modal expansion technique (MET) is a method to estimate the vibration fields of flexible structures by using eigenmodes of the structure and the signals of sensors. It is the useful method to estimate the vibration fields but has the truncation error since it only uses the limit number of the eigenmodes in the frequency of interest. Even though block-wise MET performed frequency block by block with different valid eigenmodes was developed, it still has the truncation error due to the absence of other eigenmodes. Thus, this paper suggested an improved block-wise modal expansion technique. The technique recovers the truncation errors in one frequency block by utilizing other eigenmodes existed in the other frequency blocks. It was applied for estimating the vibration fields of a cylindrical shell. The estimated results were compared to the vibration fields of the forced vibration analysis by using two indices: the root mean square error and parallelism between two vectors. These indices showed that the estimated vibration fields of the improved block-wise MET more accurately than those of the established METs. Especially, this method was outstanding for frequencies near the natural frequency of the highest eigenmode of each block. In other words, the suggested technique can estimate vibration fields more accurately by recovering the truncation errors of the established METs.

**Keywords:** modal expansion technique; vibration field; truncation error; modal assurance criterion; sensor placement optimization

## 1. Introduction

Estimation and reducing the structural vibration and noise on dynamic behavior is a sensitive issue because it is closely relate to human life and safety. For this reason, there are many studies to estimate the structural vibration and noise, and various techniques also have been developed. Hosoya *et al.* (2012) directly measures the vibration response of the structures having the pulsed-laser ablation excitation, Goncalves *et al.* (2013) investigates the non-linear vibration of a thin-walled column having large amplitude by experimental method. Au *et al.* (2011) predicts the vibration of the bridge having the vehicular loading by numerical simulation, Lee *et al.* (2012) predicts the vibration of the pipe with pulsating fluid based on the finite element method. The direct measurement method and the numerical analysis mentioned above are the typical methods to estimate the vibration and noise of the structure. The measurement using sensors directly are very accurate and reliable method, but needs too many sensors and expensive measurement instruments to obtain vibrations over the entire surface. Another approach, numerical analysis, can easily predict the vibration field but is less accurate because it does not consider the actual dynamic conditions. Thus, the hybrid method of the measurement

and the numerical analysis are suggested to estimate the structural vibration more accurate and efficient (Jung *et al.* 2011, Chen *et al.* 2012, Sun *et al.* 2014, Hadianfard *et al.* 2015).

Modal expansion technique (MET) is one of the hybrid methods to estimate vibration fields by using the measured signals of a few sensors and eigenmodes obtained from numerical analysis. The MET is based on the theory of modal superposition and calculates the modal contributions of the eigenmodes as the final output. The calculated modal contribution vectors are used to estimate a whole surface vibration without the direct measurement by using eigenmatrix. Avitabile *et al.* (2012) adapted this technique to get the dynamic strain of the structure. Wan *et al.* (2014) reconstructed the transient response of the structure having transient force based on the modal superposition and expansion technique. Iliopoulos *et al.* (2016) estimated the dynamic response of the offshore structure by using the limited number of the sensors. These researches have indicated that vibration responses using the MET are more efficient and accurate than those of the common numerical analysis because it utilizes the measured vibration signals obtained from the sensors. However, the MET has significant errors near the frequency of the highest eigenmode used in the MET calculation because it reproduces the vibration fields by using the finite number of the eigenmodes. Even if it estimates the vibration by using the some of the dominant eigenmodes (Jung *et al.* 2015), it still have the truncation error in the vibration responses. Block-wised modal expansion technique, which divides frequency of interest into several frequency blocks first, and

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then reproduces the vibration field by using the eigenmodes existed in one of the blocks, is also developed (Jung *et al.* 2016), but it still cannot completely recover the truncation error caused by the absence of other eigenmodes.

Accordingly, this paper suggests an improved block-wise modal expansion technique that can not only resolve the truncation error but also predict the vibration field more accurately at all over frequencies. It estimates the vibration fields of the cylindrical shell under dynamic excitation by using the established METs and the improved block-wise MET. Through the comparison, it shows the limits of the established techniques and demonstrates the advantages and excellence of the suggested technique.

## 2. Theory of the modal expansion technique

### 2.1 Common modal expansion technique

The equation of motion for the structure in a linear multi-degree-of-freedom system is as follows

$$(-\omega^2 \mathbf{M} + i\omega \mathbf{C} + \mathbf{K}) \mathbf{x}(\omega) = \mathbf{F}(\omega) \quad (1)$$

where  $\mathbf{M}$  is the mass matrix,  $\mathbf{C}$  is the damping matrix,  $\mathbf{K}$  is the stiffness matrix,  $\mathbf{F}$  is the external force, and  $\omega$  is the frequency. The vibration displacement  $\mathbf{x}(\omega)$  in Eq. (1) is defined from the theory of the modal superposition as follows

$$\mathbf{x}(\omega) = \phi_1 \mathbf{a}_1(\omega) + \phi_2 \mathbf{a}_2(\omega) + \phi_3 \mathbf{a}_3(\omega) + \dots = \sum_{r=1}^N \phi_r \mathbf{a}_r(\omega) \quad (2)$$

where  $\phi_r$  is the  $r$ -th eigenmode vector,  $\mathbf{a}_r$  is the modal participation factor of  $r$ -th eigenmode and  $N$  is the degree of freedom of the system.

The eigenmode vectors are calculated from an eigenvalue problem of Eq. (1) as follows

$$(\mathbf{K} - \omega_r^2 \mathbf{M}) \phi_r = 0 \quad (3)$$

where  $\omega_r$  is the  $r$ -th natural frequency. Because the eigenmode has the characteristic of the linear independence, the equation of the physical coordinates is converted into modal coordinates by using the  $r$ -th eigenmode as follows

$$\phi_r^T \mathbf{M} \phi_r \ddot{\mathbf{a}}_r(\omega) + \phi_r^T \mathbf{C} \phi_r \dot{\mathbf{a}}_r(\omega) + \phi_r^T \mathbf{K} \phi_r \mathbf{a}_r(\omega) = \phi_r^T \mathbf{F} \quad (4)$$

$$\ddot{\mathbf{a}}_r(\omega) + 2\zeta_r \omega_r \dot{\mathbf{a}}_r(\omega) + \omega_r^2 \mathbf{a}_r(\omega) = \phi_r^T \mathbf{F} \quad (5)$$

$\mathbf{a}_r$  refers to the  $r$ -th displacement in the modal coordinates and is defined as

$$\mathbf{a}_r = \frac{\phi_r^T \mathbf{F}}{(\omega_r^2 - \omega^2) + j2\zeta_r \omega_r \omega} \quad (6)$$

where  $\zeta_r$  is the  $r$ -modal damping ratio. If the vibration response can be approximated by the summation of  $n$  eigenmodes, the approximated response  $\mathbf{x}_N(\omega)$  is as represented below

$$\mathbf{x}(\omega) \approx \mathbf{x}_N(\omega) = \sum_{k=1}^n \phi_k \mathbf{a}_k(\omega) = \Phi_{Nn} \alpha_n(\omega) \quad (7)$$

In this equation,  $\Phi_{Nn}$  and  $\alpha_n(\omega)$  refer to the eigenmode matrix and modal contribution vector (or modal displacement), respectively. These are defined as follows

$$\Phi_{Nn} = [\phi_1, \phi_2, \phi_3, \dots, \phi_n] \quad (8)$$

$$\alpha_n(\omega) = \{\mathbf{a}_1(\omega), \mathbf{a}_2(\omega), \mathbf{a}_3(\omega), \dots, \mathbf{a}_n(\omega)\}^T \quad (9)$$

The MET is based on the vibration response approximation in Eq. (7). It is a method for estimating the modal contribution from some eigenmodes calculated by the eigenvalue problem of the numerical model and vibration signals of a few sensors attached to the surface of the structure. When the responses obtained from  $m$  sensors are defined as  $\mathbf{x}_m(\omega)$  and the eigenmode matrix composed of  $p$  eigenmodes of the measured  $m$  sensor locations is defined as  $\Phi_{mp}$ , the modal contribution vector of  $p$  eigenmodes  $\alpha_p(\omega)$  is calculated as

$$\mathbf{x}_m(\omega) = \Phi_{mp} \alpha_p(\omega) \quad (10)$$

$$\alpha_p(\omega) = (\Phi_{mp}^T \Phi_{mp})^{-1} \Phi_{mp}^T \mathbf{x}_m(\omega) = \Phi_{mp}^\dagger \mathbf{x}_m(\omega) \quad (11)$$

where  $\mathbf{T}$  is the transposed matrix and  $^\dagger$  is the generalized left inverse. In general, the left inverse problem must be an over-determinant problem to get reliable solution. In other words, the number of sensors  $m$  must be greater than the number of eigenmodes  $p$  to obtain a reliable modal contribution vector  $\alpha_p(\omega)$ . The modal contribution vector  $\alpha_p(\omega)$  identified in Eq. (11) is utilized to calculate the reconstructed vibration response  $\bar{\mathbf{x}}_N(\omega)$  as follows

$$\mathbf{x}_N(\omega) \approx \bar{\mathbf{x}}_N(\omega) = \Phi_{Np} \alpha_p(\omega) \quad (12)$$

### 2.2 Improved block-wise modal expansion technique

The common MET described in section 2.1 must satisfy the following equation to get reliable estimation results

$$m > p \quad (13)$$

It means that the vibration responses obtained from the MET are determined by  $m$  sensors and  $p$  eigenmodes. In other words, the estimated response has a truncation error due to the absence of eigendmoes over the  $(p+1)$ -th eigenmode. This is represented as follows

$$\mathbf{x}_N(\omega) = \Phi_{Np} \alpha_p(\omega) + \mathbf{O}(\Phi_{N(p+1)}) \quad (14)$$

In the above equation,  $\mathbf{O}(\Phi_{N(p+1)})$  refers to truncation error. According to Eqs. (13)~(14), the truncation error of the responses increases with the fewer sensors considered in the MET. This error especially increases close to the natural frequency of the highest eigenmode used in the MET. Accordingly, the present paper suggests an improved modal expansion technique that recovers the truncation error by

considering eigenmode vectors over the  $(p+1)$ -th eigenmode.

We first divides the frequency bandwidth of interest into several frequency blocks defined as  $\Omega_1, \Omega_2, \dots, \Omega_k$ . The frequency bandwidth of each frequency block is determined by the natural frequencies of the lowest and the highest valid eigenmode existed in each block. Then, the modal expansion technique is performed frequency block by block by using the valid eigenmodes in one of the blocks. This method is a block-wise modal expansion technique. The vibration field estimated by the block-wise MET, however, also still have the truncation error caused by the absence of the other eigenmodes. To recover this error, this paper suggested an improved block-wise MET by utilizing the other eigenmodes existed in the other blocks.

Let assume that a whole frequency band of interest divides into 3 frequency blocks ( $\Omega_1, \Omega_2, \Omega_3$ ) and the valid eigenmode matrix of each block defines as  $\Phi_{mp_1}, \Phi_{mp_2}, \Phi_{mp_3}$  respectively, then the improved block-wise MET follows the process given below in the first frequency block.

Modal contribution vectors of the valid eigenmodes can be calculated, and then reconstructed responses are predicted as follows

$$\alpha_{p_1}(\omega \in \Omega_1) = \Phi_{mp_1}^\dagger \mathbf{x}_m(\omega \in \Omega_1) \quad (15)$$

$$\overline{\mathbf{x}}_m(\omega \in \Omega_1) = \Phi_{mp_1} \alpha_{p_1}(\omega \in \Omega_1) \quad (16)$$

However, there are truncation errors between the measured responses  $\mathbf{x}_m(\omega \in \Omega_1)$  and the reconstructed responses  $\overline{\mathbf{x}}_m(\omega \in \Omega_1)$ . Thus, additional process to calculate the modal contribution vectors of the other eigenmodes in the other blocks to recover the truncation errors as follows

$$\begin{aligned} & \begin{Bmatrix} \alpha_{p_2}(\omega \in \Omega_1) \\ \alpha_{p_3}(\omega \in \Omega_1) \end{Bmatrix} \\ &= \Phi_{m[p_2, p_3]}^{\dagger\dagger} \left( \mathbf{x}_m(\omega \in \Omega_1) - \overline{\mathbf{x}}_m(\omega \in \Omega_1) \right) \end{aligned} \quad (17)$$

$$\begin{aligned} & \Phi_{m[p_2, p_3]}^{\dagger\dagger} \\ &= \left[ \Phi_{mp_2} \quad \Phi_{mp_3} \right]^T \left( \left[ \Phi_{mp_2} \quad \Phi_{mp_3} \right] \left[ \Phi_{mp_2} \quad \Phi_{mp_3} \right]^T \right)^{-1} \end{aligned} \quad (18)$$

The processes in the second and the third frequency block are given as follow

$$\alpha_{p_2}(\omega \in \Omega_2) = \Phi_{mp_2}^\dagger \mathbf{x}_m(\omega \in \Omega_2) \quad (19)$$

$$\overline{\mathbf{x}}_m(\omega \in \Omega_2) = \Phi_{mp_2} \alpha_{p_2}(\omega \in \Omega_2) \quad (20)$$

$$\begin{aligned} & \begin{Bmatrix} \alpha_{p_1}(\omega \in \Omega_2) \\ \alpha_{p_3}(\omega \in \Omega_2) \end{Bmatrix} \\ &= \Phi_{m[p_1, p_3]}^{\dagger\dagger} \left( \mathbf{x}_m(\omega \in \Omega_2) - \overline{\mathbf{x}}_m(\omega \in \Omega_2) \right) \end{aligned} \quad (21)$$

$$\alpha_{p_3}(\omega \in \Omega_3) = \Phi_{mp_3}^\dagger \mathbf{x}_m(\omega \in \Omega_3) \quad (22)$$

$$\overline{\mathbf{x}}_m(\omega \in \Omega_3) = \Phi_{mp_3} \alpha_{p_3}(\omega \in \Omega_3) \quad (23)$$

$$\begin{aligned} & \begin{Bmatrix} \alpha_{p_1}(\omega \in \Omega_3) \\ \alpha_{p_2}(\omega \in \Omega_3) \end{Bmatrix} \\ &= \Phi_{m[p_1, p_2]}^{\dagger\dagger} \left( \mathbf{x}_m(\omega \in \Omega_3) - \overline{\mathbf{x}}_m(\omega \in \Omega_3) \right) \end{aligned} \quad (24)$$

In other words, the modal participation factor of the one block is calculated from the measured vibration signals and the valid eigenmodes in that block, and then the residue or truncation error is recovered by utilizing the other eigenmodes existed in the other blocks.

Accordingly, the reconstructed vibration response  $\overline{\mathbf{x}}_N(\omega)$  derived by the improved block-wise MET is as follows

$$\begin{aligned} \mathbf{x}_N(\omega) &\approx \overline{\mathbf{x}}_N(\omega) \\ &= \Phi_{Np_1} \alpha_{p_1}(\omega) + \Phi_{Np_2} \alpha_{p_2}(\omega) + \Phi_{Np_3} \alpha_{p_3}(\omega) \end{aligned} \quad (25)$$

### 3. Numerical analysis and verification

#### 3.1 Numerical model

The numerical model of a cylindrical shell under clamped boundary conditions on both sides was considered, as shown in Fig. 1. This cylindrical shell was 2 m in diameter, 4 m in height, and 0.003 m in thickness. Table 1 presents the material properties of the structure. The point in Fig. 1 indicates the position of the vibration exciting force, which had a unit force from 0 to 400 Hz.

#### 3.2 Sensor placement optimization for vibration response acquisition

The MET utilizes the eigenmodes of the structure and the vibration responses to calculate the modal contributions of the eigenmodes. The eigenmodes can easily be calculated

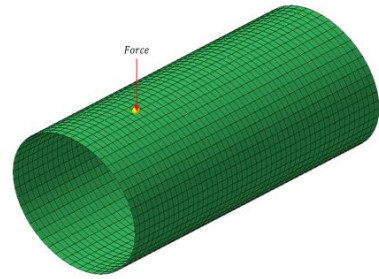


Fig. 1 Numerical analysis model of a cylindrical shell

Table 1 Material properties of cylindrical shell

Properties	Value
Young's modulus	210 GPa
Poisson ratio	0.3
Mass density	7850 kg m <sup>-3</sup>
Structural damping	0.005

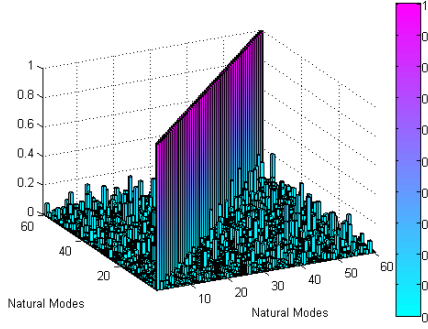


Fig. 2 MAC calculated by the reconstructed eigenvectors corresponding to the optimal sensor positions

from the eigenvalue problem of the finite element model of the cylindrical shell based on Eq. (3). The vibration responses are obtained from the sensors attached to the structure surface. However, the sensor placement is very important because the signals of the sensors reflect the dynamic behavior of the structure. Thus, the sensors should be attached to positions that can represent the dynamic characteristics and the shapes of the eigenmodes properly. For this reason, we use the sensor placement optimization technique. These are various sensor placement optimization techniques, but this paper utilizes modal assurance criterion (MAC), which is proceeded and used widely (Yi *et al.* 2011, Jung *et al.* 2014, Jung *et al.* 2015, Yi *et al.* 2015), as objective function.

$$\text{Minimize } F(X) = \sum_{i,j=1}^n \text{MAC}_{ij} \quad (26)$$

$$\text{MAC}_{ij} = \left| \phi_i^* \phi_j \right|^2 \left[ \left( \phi_i^* \phi_i \right) \left( \phi_j^* \phi_j \right) \right]^{-1} \quad (27)$$

The MAC, which is most widely used to check the correlation between two eigenmodes, is defined in Eq. (26), where  $\phi_i$  is the  $i$ -th eigenmode vector,  $\phi_j$  is the  $j$ -th eigenmode vector, and  $*$  the conjugate transpose. The MAC has a value from zero to unity. It is unity when two eigenmodes are exactly the same, while it is zero when both are orthogonal and have no correlation. The off-diagonal term of the MAC when  $i$  is not equal to  $j$  is theoretically zero because of the orthogonality of two different eigenmodes. Nevertheless, it can have a value greater than zero when the MAC is calculated by using the reconstructed eigenmode vector, which consists of values at the sensor attachment points. Accordingly, an objective function that minimizes the summation of the off-diagonal term of the MAC was used. The 1<sup>st</sup> to 60<sup>th</sup> eigenmodes were used to calculate the MAC. The 60<sup>th</sup> natural frequency was 495 Hz. A total of 30 sensor positions that were well-arranged to represent the vibration fields at the frequency bandwidth of interest from 0 to 400 Hz were extracted. Fig. 2 shows the MAC calculated from the reconstructed eigenmode vectors corresponding to the 30 optimal sensors. Fig. 3 indicates the optimal sensor positions of the cylindrical shell. The MET was performed by using the vibration responses of these optimal sensors and the eigenmodes calculated from the

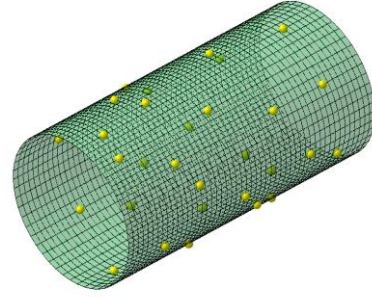


Fig. 3 Optimal sensor positions of the cylindrical shell

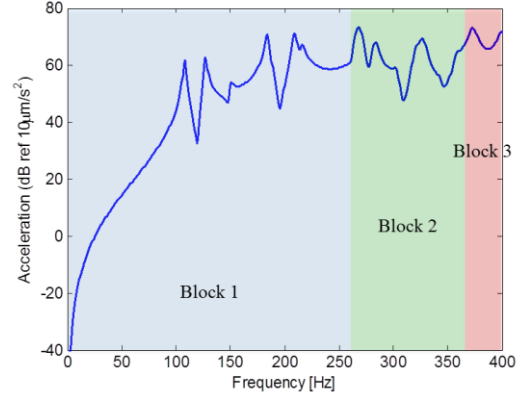


Fig. 4 The frequency blocks for the improved block-wise MET

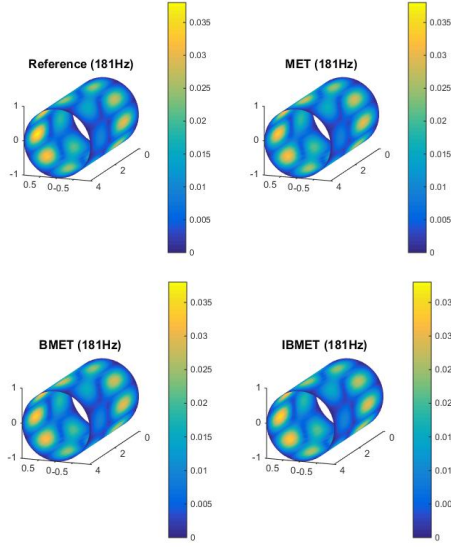
eigenvalue problem of the finite element cylindrical shell model.

### 3.3 Application of the modal expansion technique

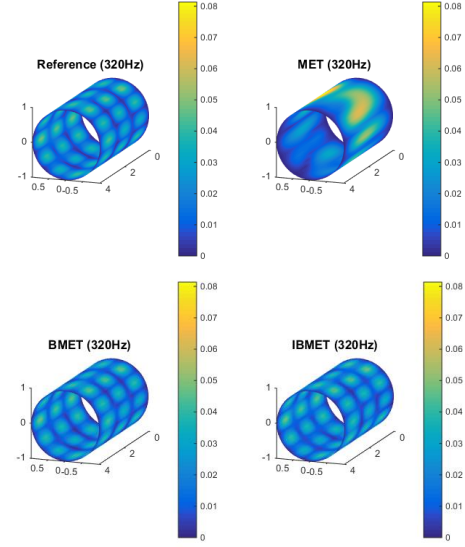
The common, the block-wise and the improved block-wise MET were used to estimate the vibration fields of the cylindrical shell under dynamic conditions. Forced vibration analysis of the cylindrical shell was conducted to obtain reference results for the comparison and verification of the MET estimation results.

To calculate the modal contributions of the eigenmodes by using the METs, the number of sensors  $m$  was set to 30, and the number of eigenmodes  $p$  was set to 20. These values satisfied Eq. (13), which defines the relationship between  $m$  and  $p$ . The natural frequency of the 20<sup>th</sup> eigenmode was 269 Hz. With the block-wise METs, the frequency bandwidth of interest was divided into three blocks, as shown in Fig. 4, and each block had 20 valid eigenmodes. Fig. 5 shows the surface normal acceleration fields of the reference, the common MET and the block-wise METs at the 7<sup>th</sup>, 14<sup>th</sup>, 21<sup>st</sup>, and 28<sup>th</sup> natural frequencies. Fig. 6 indicates the surface normal acceleration responses at two points of the three different techniques.

Figs. 5-6 indicate that the estimated responses of the common MET showed good agreement below the natural frequency (269 Hz) corresponding to the highest eigenmode used in the MET calculation, but had considerable errors over that frequency. On the other hand, the estimated responses of the block-wise METs were very similar to the reference results at all of frequencies. Especially, the

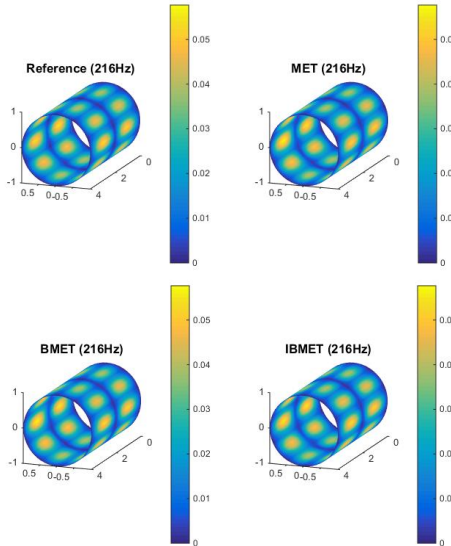


(a) Vibration field at 7th natural frequency (181Hz)



(d) Vibration field at 28th natural frequency (320Hz)

Fig. 5 Continued



(b) Vibration field at 14th natural frequency (215Hz)

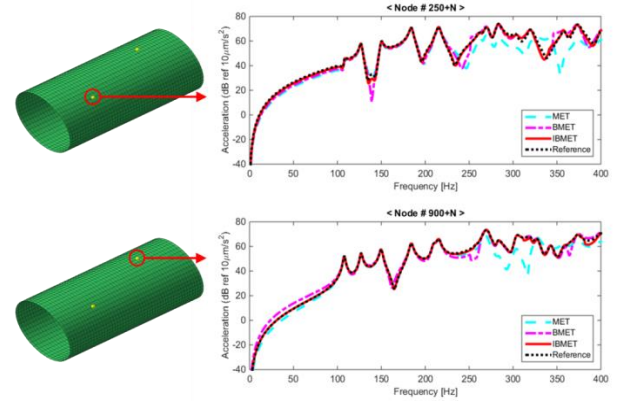
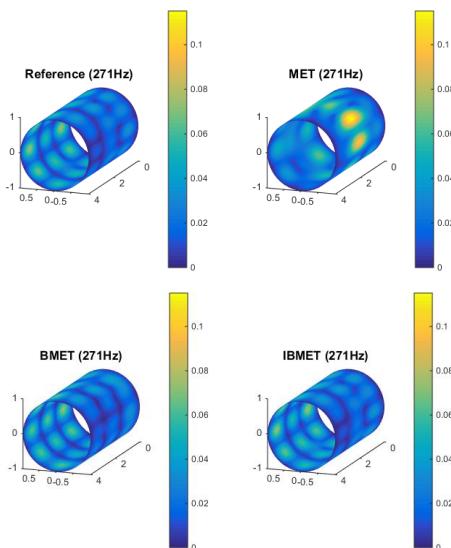


Fig. 6 Comparison of the frequency responses obtained by reference and METs



(c) Vibration field at 21st natural frequency (271Hz)

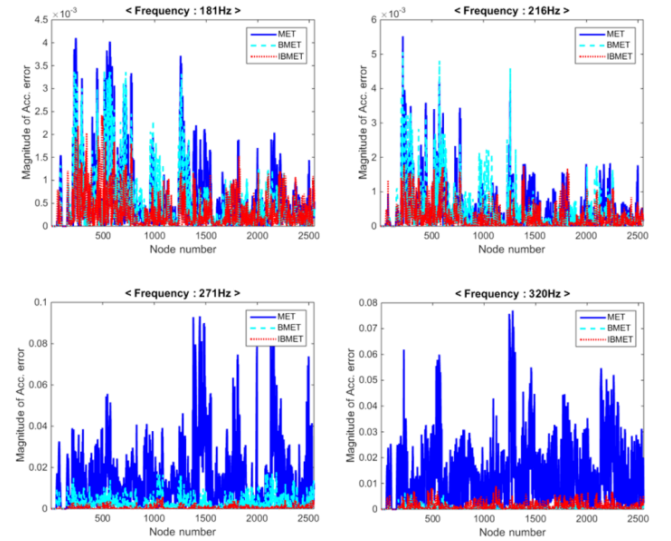
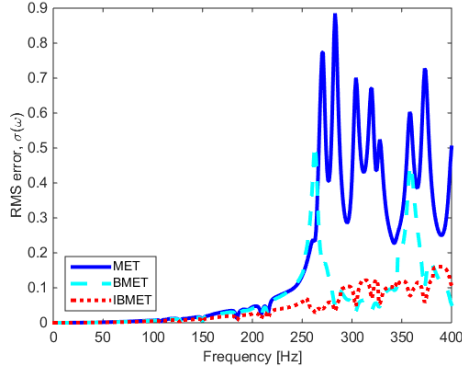


Fig. 7 Nodal absolute errors normalized by the maximum reference acceleration between reference and METs at 7th (151Hz), 14th (215Hz), 21st (271Hz), and 28th (320Hz) natural frequencies

Fig. 5 Comparison of the vibration fields obtained by reference and METs



Fig. 8 RMS errors index,  $\sigma(\omega)$  between reference and METs

improved block-wise MET showed the best agreement.

Fig. 7 shows the nodal absolute errors normalized by the maximum reference acceleration between the reference results and the normal acceleration responses of the common MET, the block-wise METs at the 7<sup>th</sup>, 14<sup>th</sup>, 21<sup>st</sup>, and 28<sup>th</sup> natural frequencies of the cylindrical shell. The errors between the reference and the common MET below 269 Hz corresponding to the 7<sup>th</sup> (181 Hz) and 14<sup>th</sup> (216 Hz) natural frequencies were not large. However, those over 269 Hz corresponding to the 21<sup>st</sup> (271 Hz) and 28<sup>th</sup> (320 Hz) natural frequencies were significantly large. In contrast, the errors between the reference and the block-wise METs were small at those natural frequencies. In this case, the improved block-wise MET also showed the best prediction. In other words, the improved block-wise MET estimated the vibration fields more accurately than the other METs.

To evaluate quantitatively, two indices were used in this work: the root mean square (RMS) error and the parallelism between two response vectors.

The RMS error  $\sigma(\omega)$  is defined as follows

$$\sigma(\omega) = \sqrt{\sum_{k=1}^M \left( \frac{y_k(\omega) - \bar{y}_k(\omega)}{\sqrt{2}} \right)^2} \quad (28)$$

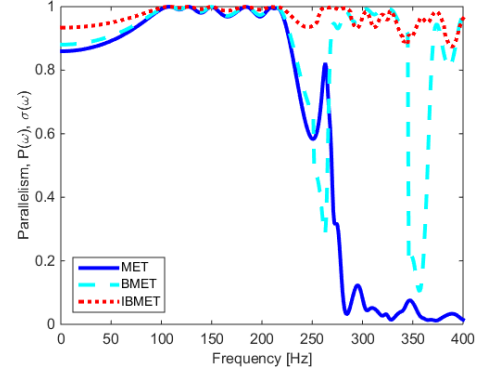
where  $y_k(\omega)$  is the normal acceleration responses of the reference,  $\bar{y}_k(\omega)$  is the normal acceleration response of the common or the block-wise METs,  $k$  is the node number, and  $M$  is the total number of nodes of the cylindrical shell model. The RMS errors are close to zero when the responses of the METs are similar to the reference results.

The parallelism between two response vectors  $P(\omega)$  is defined as follows

$$P(\omega) = \frac{|\overline{\mathbf{V}_{\text{refer}}(\omega)} \cdot \overline{\mathbf{V}_{\text{verif}}(\omega)}|}{\|\overline{\mathbf{V}_{\text{refer}}(\omega)}\| \|\overline{\mathbf{V}_{\text{verif}}(\omega)}\|} \quad (29)$$

where  $\overline{\mathbf{V}_{\text{refer}}(\omega)}$  is the response vector of the reference and  $\overline{\mathbf{V}_{\text{verif}}(\omega)}$  is the response vector of the common or the block-wise METs. This index is close to unity when the response vectors of the METs are similar and parallel to the reference vectors.

Figs. 8-9 indicate the RMS error and parallelism, respectively, as calculated from Eqs. (28)-(29). The

Fig. 9 Parallelism index,  $P(\omega)$  between reference and METs

estimated results of the improved block-wise MET were more similar to the reference results than those of the other METs. This was especially evident at frequencies over 269 Hz, which was the limit frequency of the common MET. For example, the RMS error of the improved block-wise MET was closer to zero than that of the other METs at all over frequencies. In the case of the parallelism, the value of the improved block-wise MET was closer to unity than that of the other METs at all over frequencies. In other words, the improved block-wise MET was better at estimating the vibration fields of the cylindrical shell than the established METs in both cases.

#### 4. Conclusions

This paper estimated the vibration fields of the cylindrical shell by using the modal expansion technique (MET), and showed the weakness of the established METs. The common MET and the block-wise MET have the truncation errors to reproduce the vibration responses since it only uses the finite number of the eigenmodes existed in one of the blocks. This leads to the inaccurate estimation of the vibration fields. Thus, this paper suggested an improved block-wise MET to recover the weakness of the established METs. The improved block-wise MET divides the frequency bandwidth of interest into several of segment called the block, and recovers the truncation error in the block by using the other eigenmodes in the other blocks.

The estimated vibration results of the improved block-wise MET were compared to the results of the established METs and the reference results obtained from the forced vibration analysis for the verification by using two indices: the RMS error and parallelism between two response vectors. The vibration fields of the improved block-wise MET were more accurate than those of the other METs for all frequencies. The responses of the established METs had significant errors near the natural frequency of the highest eigenmode of the block, but the responses of the improved block-wise MET is practically accurate at all frequencies. In other words, the responses of the established METs calculated by the superposition of  $p$  eigenmodes cannot represent the actual responses because of the absence of over  $(p+1)$ -th eigenmodes, and this leads to significant errors in the vibration field estimation. However, the

improved block-wise MET estimates the vibration responses more accurately because it utilizes valid eigenmodes existed in the other blocks to reduce the truncation error in the present block.

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