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Stochastic vibration response of a sandwich beam with nonlinear adjustable visco-elastomer core and supported mass

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Abstract. The stochastic vibration response of the sandwich beam with the nonlinear adjustable visco-elastomer core and supported mass under stochastic support motion excitations is studied. The nonlinear dynamic properties of the visco-elastomer core are considered. The nonlinear partial differential equations for the horizontal and vertical coupling motions of the sandwich beam are derived. An analytical solution method for the stochastic vibration response of the nonlinear sandwich beam is developed. The nonlinear partial differential equations are converted into the nonlinear ordinary differential equations representing the nonlinear stochastic multi-degree-of-freedom system by using the Galerkin method. The nonlinear stochastic system is converted further into the equivalent quasi-linear system by using the statistic linearization method. The frequency-response function, response spectral density and mean square response expressions of the nonlinear sandwich beam are obtained. Numerical results are given to illustrate new stochastic vibration response characteristics and response reduction capability of the sandwich beam with the nonlinear visco-elastomer core and supported mass under stochastic support motion excitations. The influences of geometric and physical parameters on the stochastic response of the nonlinear sandwich beam are discussed, and the numerical results of the nonlinear sandwich beam are compared with those of the sandwich beam with linear visco-elastomer core.

Keywords: stochastic nonlinear vibration; sandwich beam; nonlinear visco-elastomer core; stochastic excitation; statistic linearization

1. Introduction

The structural vibration control is a significant research subject in engineering. The damped composite structures can effectively suppress the structural vibration response to excitations. Beams are important engineering structures or structural components so that damped sandwich beams have been studied extensively (Timoshenko *et al.* 1974, Yu 1962, Ditaranto 1965, Mead and Markus 1969, Yan and Dowell 1972, Rao and Nakra 1974, Rao 1977, Frostig and Baruch 1994, Li and Crocker 2005, Kovac *et al.* 1971, Xia and Lukasiewicz 1994, Daya *et al.* 2004, Mahmoudkhani *et al.* 2014, Rajagopal *et al.* 1986, Lee 1998, Baber *et al.* 1998, Jacques *et al.* 2010, Xi *et al.* 1986, etc.). The vibration equations of sandwich beams with nonadjustable viscoelastic cores described by complex moduli were derived. The dynamic characteristics, deterministic vibration response of the sandwich beams under external loading were analyzed (Yu 1962, Ditaranto 1965, Mead and Markus 1969, Yan and Dowell 1972, Rao and Nakra 1974, Rao 1977, Frostig and Baruch 1994, etc.). For certain large vibration of the sandwich beams, the geometric nonlinearity of elastic layers and small physical nonlinearity of viscoelastic layers were considered. The spatial modal expansion and temporal multiple scales and harmonic balance methods were used to obtain the periodic vibration response (Kovac *et al.* 1971, Xia and Lukasiewicz 1994, Daya *et al.* 2004, Mahmoudkhani *et al.* 2014, etc.). The finite element method for the sandwich beams with viscoelastic cores was developed to calculate the deterministic vibration response (Rajagopal *et al.* 1986, Lee 1998, Baber *et al.* 1998, Jacques *et al.* 2010, etc.). Several researches were given on the response estimation of the sandwich beams and plates with viscoelastic cores under external stationary random loading (Xi *et al.* 1986, Mahmoudkhani and Haddadpour 2013, etc.). However, only the single mode response amplitude of the sandwich plate with the small nonlinearity of viscoelastic cores under the special narrow-band phase random excitation was analyzed (Mahmoudkhani and Haddadpour 2013). Thus, the stochastic vibration of the nonlinear sandwich beams with viscoelastic cores needs to be study further by considering multiple modes and general random excitations.

Due to the randomness of environmental loadings, the dynamic adjustability of damped composite structures is required for vibration control. Smart materials such as magneto-rheological liquid and magneto-rheological visco-elastomer can supply the adjustable damping and stiffness

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for structures. In particular, the adjustment of the smart structural dynamics is achieved only by applied magnetic fields and the structural design is unchanged. Thus, the smart materials are the reasonable replacement of the nonadjustable viscoelastic cores in composite structures. The structural vibration suppression using magneto-rheological liquid dampers and the vibration response characteristics of linear sandwich beams with magneto-rheological liquid cores have been studied (Dyke *et al.* 1996, Spencer and Nagarajaiah 2003, Casciati *et al.* 2012, Hernandez *et al.* 2015, Rajamohan *et al.* 2010, etc.). Nevertheless, the magneto-rheological visco-elastomer has several advantages over the magneto-rheological liquid. For example, the magneto-rheological visco-elastomer improves the potential disadvantage of magnetic particle settlement in magneto-rheological liquid, and the magneto-rheological visco-elastomer is more suitable for composite-structural cores than the magneto-rheological liquid. Thus, the dynamic properties of the magneto-rheological visco-elastomer and the application of the adjustable magneto-rheological visco-elastomer to the structural vibration control have been studied (Bellan and Bossis 2002, Demchuk and Kuz'min 2002, Shen *et al.* 2004, Bose 2007, Koo *et al.* 2010, Kaleta *et al.* 2011, Ying *et al.* 2013, York *et al.* 2007, Hu and Wereley 2008, Hoang *et al.* 2011, Jung *et al.* 2011, Ying *et al.* 2015a, Ying and Ni 2016, etc.). The periodic vibration and adjustable stiffness of the linear sandwich beam with the magneto-rheological visco-elastomer core described by complex modulus were analyzed and tested (Zhou and Wang 2005, 2006, Choi *et al.* 2010, Hu *et al.* 2011). The vibration equations with temporal periodic coefficients of the linear sandwich beam with the magneto-rheological visco-elastomer core under periodic longitudinal loading were derived. The dynamic stability of the sandwich beam with the time-varying coefficients was analyzed (Dwivedy *et al.* 2009, Nayak *et al.* 2011). The micro-vibration response of the linear sandwich beam with the magneto-rheological visco-elastomer core under external stationary random loading was also estimated (Ni *et al.* 2011, Ying *et al.* 2015b). However, in all those researches, only the linear geometric and constitutive relations of the elastic layers and magneto-rheological visco-elastomer layer were considered. Since the adjustable visco-elastomer has the nonlinear dynamic behavior for certain large deformation (Ying *et al.* 2013), the nonlinear vibration of the sandwich beam with nonlinear adjustable visco-elastomer core needs to be studied. At present, only the random vibration of the nonlinear sandwich beam with the linear electro-rheological visco-elastomer core under the uniformly distributed band-limited Gaussian white noise excitation was studied, where the geometric nonlinearity of elastic layers was considered and the random response was estimated by the numerical method (Vaicaitis *et al.* 2008). Therefore, the nonlinear dynamic constitutive relation of the adjustable visco-elastomer needs to be considered, and the stochastic vibration of the nonlinear sandwich beam with the visco-elastomer core under general random excitations needs to be studied further to exhibit the difference of stochastic nonlinear and linear responses.

As a representative practical subject, the vibration-sensitive precise apparatuses require extremely stable operation environments. However, there exist inevitably environmental disturbances in random with wide frequency bands. Thus, the stochastic vibration control is very important to the vibration-sensitive apparatuses. The vibration-sensitive apparatus and its support structure can be modeled as a beam or plate with concentrated mass. The adjustable visco-elastomer can be used to construct a sandwich structure for vibration control. In the present paper, the stochastic vibration response of a sandwich beam with nonlinear adjustable visco-elastomer core and supported mass under stochastic support motion excitations is studied. The nonlinear dynamic properties of the visco-elastomer core are considered. Firstly, the nonlinear partial differential equations for the horizontal and vertical coupling motions of the sandwich beam with the supported mass are derived by the dynamic equilibrium, constitutive and geometric relations. Secondly, the nonlinear partial differential equations are converted into the nonlinear ordinary differential equations by using the Galerkin method, which represent a multi-degree-of-freedom system with the cubic nonlinear stiffness and damping subjected to stochastic excitations. Thirdly, the nonlinear stochastic system is converted into an equivalent quasi-linear system by using the statistic linearization method. The frequency-response function, response spectral density and mean square response expressions of the nonlinear sandwich beam are obtained. Finally, numerical results are given to illustrate new stochastic vibration response characteristics and response reduction capability of the sandwich beam with the (hard and soft) nonlinear visco-elastomer core and supported mass under stochastic support motion excitations. The numerical results are compared with those of the sandwich beam with linear visco-elastomer core to illustrate their stochastic response differences.

2. Nonlinear differential equations of sandwich beam with nonlinear visco-elastomer core

The vibration-sensitive apparatus and its support structure are modeled as a sandwich beam with concentrated mass for response characteristics analysis. Consider a simply supported horizontal sandwich beam with visco-elastomer core and a supported mass under stochastic support motion excitations, as shown in Fig. 1. The beam length and width are L and b, respectively. The two facial layers are linear elastic material and have the identical elastic modulus E_1, mass density ρ_1 and thickness h_1. The core layer is nonlinear visco-elastomer material and has the mass density ρ_2 and thickness h_2. The supported mass is fixed on the beam and has the mass $m\times bL$ (for simplicity in the final equation). Its size is very small compared with the beam length and then neglected. The supports have the identical vertical displacement w_0 which is a stochastic disturbance excitation.

The visco-elastomer core has the adjustable dynamic properties (for example, the magneto-rheological visco-elastomer has the damping and stiffness adjustable by

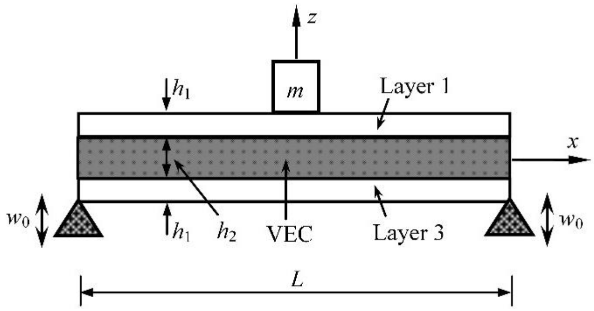

Fig. 1 Sandwich beam with visco-elastomer core (VEC) and supported mass (m) under support excitation (w_0)

applied magnetic fields). The visco-elastomer core layer is soft compared with the elastic facial layers, and the elastic modulus of the core layer is much smaller than E_1. However, the shearing deformation of the core layer is larger than the facial layers and then taken into account. For large shearing deformation, the shear stress τ_2 of the visco-elastomer core depends nonlinearly on the corresponding shear strain γ_2. The nonlinear dynamic stress-strain relation can be described by (Ying *et al.* 2013)

$$\tau_2 = \sum_{k=1}^{3} [G_{2k} \gamma_2{}^k + G_{ck} (\frac{\partial \gamma_2}{\partial t})^k] \tag{1}$$

where G_{2k} and G_{ck} are constant coefficients (adjustable by external action, for example, magneto-rheological visco-elastomer by applied magnetic fields), and t is time variable. In general, the nonlinear coefficients are smaller than linear coefficients. There are $G_{23}<G_{22}<G_{21}$ and $G_{c3}<G_{c2}<G_{c1}$. For certain large deformation, a single shear strain is determined by a shear stress.

For the sandwich beam, it is assumed that: (1) the two elastic facial layers and visco-elastomer core layer are respectively homogeneous and continuous, and the facial layer materials are isotropic; (2) the normal stress of the core layer is small and neglected; (3) the normal stresses of the facial layers in the axis-z direction are small and neglected; (4) the vertical displacement of the sandwich beam is invariant along the thickness; (5) the cross section of each facial layer is perpendicular to the facial layer axis line, and the cross section of the core layer is a plane in deformation; (6) the longitudinal and rotational inertias of the beam are small and neglected; (7) the interfaces between the facial layers and core layer are continuous all the time.

Based on above assumptions, the displacements and shear stresses on the interfaces between the facial layers and core layer are continuous. The vertical beam displacement relative to the supports is $w=w(x,t)$, where x is the horizontal coordinate. The horizontal displacements of the facial layers can be expressed as (Ni *et al.* 2011)

$$u_1(x, z_1, t) = u_{10}(x,t) - z_1 \frac{\partial w(x,t)}{\partial x} \tag{2}$$

$$u_3(x, z_3, t) = u_{30}(x,t) - z_3 \frac{\partial w(x,t)}{\partial x} \tag{3}$$

where u_{10} and u_{30} are the horizontal mid-layer displacements of the upper and lower facial layers (layer 1 and layer 3), respectively, z_1 and z_3 are the vertical local coordinates of the two facial layers. The horizontal displacements (u_{I1} and u_{I3}) on the two interfaces between the facial layers and core layer can be obtained by Eqs. (2) and (3). Thus the shear strain of the core layer is

$$\gamma_2 = \frac{u_{I1} - u_{I3}}{h_2} + \frac{\partial w}{\partial x} = \frac{u_{10} - u_{30}}{h_2} + \frac{h_a}{h_2} \frac{\partial w}{\partial x} \tag{4}$$

where $h_a=h_1+h_2$. By using Eq. (1), the shear stress of the core layer is expressed as

$$\tau_2(x,t) = \sum_{k=1}^{3} [G_{2k} (\frac{u_{10} - u_{30}}{h_2} + \frac{h_a}{h_2} \frac{\partial w}{\partial x})^k + G_{ck} (\frac{\dot{u}_{10} - \dot{u}_{30}}{h_2} + \frac{h_a}{h_2} \frac{\partial \dot{w}}{\partial x})^k] \tag{5}$$

where $\dot{u}_{10} = \partial u_{10} / \partial t$, $\dot{u}_{30} = \partial u_{30} / \partial t$ and $\dot{w} = \partial w / \partial t$.

The horizontal normal strains of the two facial layers can be obtained by the derivatives of displacements (2) and (3). The corresponding normal stresses of the upper and lower facial layers are

$$\sigma_1(x, z_1, t) = E_1 (\frac{\partial u_{10}}{\partial x} - z_1 \frac{\partial^2 w}{\partial x^2}) \tag{6}$$

$$\sigma_3(x, z_3, t) = E_1 (\frac{\partial u_{30}}{\partial x} - z_3 \frac{\partial^2 w}{\partial x^2}) \tag{7}$$

Based on the normal and shear stresses equilibrium of an element in the axis-x direction, the shear stresses of the upper and lower facial layers are expressed as (Mead and Markus 1969, Ni *et al.* 2011)

$$\tau_1(x, z_1, t) = E_1 (\frac{h_1}{2} - z_1) \frac{\partial^2 u_{10}}{\partial x^2} - E_1 (\frac{h_1^2}{8} - \frac{z_1^2}{2}) \frac{\partial^3 w}{\partial x^3} \tag{8}$$

$$\tau_3(x, z_3, t) = -E_1 (\frac{h_1}{2} + z_3) \frac{\partial^2 u_{30}}{\partial x^2} - E_1 (\frac{h_1^2}{8} - \frac{z_3^2}{2}) \frac{\partial^3 w}{\partial x^3} \tag{9}$$

According to the continuity of shear stresses (5), (8) and (9) on the interfaces between the facial layers and core layer, the differential equation for the horizontal displacement of the sandwich beam is obtained as follows

$$E_1 h_1 \frac{\partial^2 u}{\partial x^2} - \sum_{k=1}^{3} [G_{2k} (\frac{2u}{h_2} + \frac{h_a}{h_2} \frac{\partial w}{\partial x})^k + G_{ck} (\frac{2\dot{u}}{h_2} + \frac{h_a}{h_2} \frac{\partial \dot{w}}{\partial x})^k] = 0 \tag{10}$$

where $u=u_{10}=-u_{30}$. With taking into account the vertical inertia, the dynamic equilibrium equation of an element of the sandwich beam with the supported mass in the axis-z direction is given by

$$\sum_{i=1}^{3}\int_{h_i}\frac{\partial \tau_i}{\partial x}\mathrm{d}z_i-[\rho h_t+mL\delta(x-x_0)](\ddot{w}+\ddot{w}_0)=0 \tag{11}$$

where δ(·) is the Dirac delta function, x_0 is the horizontal coordinate of the mass, $\ddot{w}=\partial^2 w/\partial t^2$, $\ddot{w}_0=\partial^2 w_0/\partial t^2$, $\rho h_t=2\rho_1 h_1+\rho_2 h_2$ and $h_t=2h_1+h_2$. Substituting shear stresses (5), (8) and (9) into Eq. (11) yields the differential equation for the vertical beam displacement

$$\begin{aligned}&\frac{E_1h_1^3}{6}\frac{\partial^4 w}{\partial x^4}-E_1h_1^2\frac{\partial^3 u}{\partial x^3}-\sum_{k=1}^{3}[kG_{2k}\times\\&(\frac{2u}{h_2}+\frac{h_a}{h_2}\frac{\partial w}{\partial x})^{k-1}(2\frac{\partial u}{\partial x}+h_a\frac{\partial^2 w}{\partial x^2})+\\&kG_{ck}(\frac{2\dot{u}}{h_2}+\frac{h_a}{h_2}\frac{\partial \dot{w}}{\partial x})^{k-1}(2\frac{\partial \dot{u}}{\partial x}+h_a\frac{\partial^2 \dot{w}}{\partial x^2})]+\\&[\rho h_t+mL\delta(x-x_0)](\ddot{w}+\ddot{w}_0)=0\end{aligned} \tag{12}$$

Eqs. (10) and (12) are two coupling nonlinear partial differential equations, which describe the horizontal and vertical motions of the sandwich beam with the supported mass under support motion excitations. In the case that the supported mass is fixed on the middle of the beam, there is x_0=0. For the simply supported beam, the displacement boundary conditions are given by (Ni *et al.* 2011)

$$w(\pm\frac{L}{2},t)=0,\quad \frac{\partial^2 w(\pm L/2,t)}{\partial x^2}=0,\quad \frac{\partial u(\pm L/2,t)}{\partial x}=0 \tag{13}$$

Introduce dimensionless coordinates and displacements as follows

$$y=\frac{x}{L},\quad y_0=\frac{x_0}{L},\quad \bar{u}=\frac{u}{W_a},\quad \bar{w}=\frac{w}{W_a},\quad \bar{w}_0=\frac{w_0}{W_a} \tag{14}$$

where W_a is the amplitude of the support motion w_0. The nonlinear differential equations of motion (12) and (10) and the boundary conditions (13) are transformed into

$$\begin{aligned}&\frac{E_1h_1^3}{6}\frac{\partial^4 \bar{w}}{\partial y^4}-E_1h_1^2L\frac{\partial^3 \bar{u}}{\partial y^3}-\sum_{k=1}^{3}[kG_{2k}L^3\times\\&(\frac{2W_a\bar{u}}{h_2}+\frac{h_aW_a}{h_2L}\frac{\partial \bar{w}}{\partial y})^{k-1}(2\frac{\partial \bar{u}}{\partial y}+\frac{h_a}{L}\frac{\partial^2 \bar{w}}{\partial y^2})+\\&kG_{ck}L^3(\frac{2W_a\dot{\bar{u}}}{h_2}+\frac{h_aW_a}{h_2L}\frac{\partial \dot{\bar{w}}}{\partial y})^{k-1}(2\frac{\partial \dot{\bar{u}}}{\partial y}+\\&\frac{h_a}{L}\frac{\partial^2 \dot{\bar{w}}}{\partial y^2})]+L^4[\rho h_t+m\delta(y-y_0)](\ddot{\bar{w}}+\ddot{\bar{w}}_0)\\&=0\end{aligned} \tag{15}$$

$$\begin{aligned}&E_1h_1\frac{\partial^2 \bar{u}}{\partial y^2}-\sum_{k=1}^{3}[G_{2k}L^2W_a^{k-1}\times\\&(\frac{2\bar{u}}{h_2}+\frac{h_a}{h_2L}\frac{\partial \bar{w}}{\partial y})^k+G_{ck}L^2W_a^{k-1}\times\\&(\frac{2\dot{\bar{u}}}{h_2}+\frac{h_a}{h_2L}\frac{\partial \dot{\bar{w}}}{\partial y})^k]=0\end{aligned} \tag{16}$$

$$\bar{w}(\pm 1/2,t)=0,\quad \frac{\partial^2 \bar{w}(\pm 1/2,t)}{\partial y^2}=0,\quad \frac{\partial \bar{u}(\pm 1/2,t)}{\partial y}=0 \tag{17}$$

3. Nonlinear stochastic vibration response based on spatial modal expansion and temporal statistic linearization

Under the homogeneous boundary conditions (17), the dimensionless displacements $\bar{u}$ and $\bar{w}$ of the sandwich beam can be expanded into

$$\bar{u}(y,t)=\sum_{i=1}^{N}p_i(t)\sin(2i-1)\pi y \tag{18}$$

$$\bar{w}(y,t)=\sum_{i=1}^{N}q_i(t)\cos(2i-1)\pi y \tag{19}$$

where p_i and q_i are the modal displacements, and N is an integer. According to the Galerkin method, substituting displacements (18) and (19) into Eqs. (15) and (16), multiplying the equations respectively by cos(2*j*-1)π*y* and sin(2*j*-1)π*y*, and integrating them with respect to *y* yield nonlinear ordinary differential equations for p_i and q_i. Note that the nonlinear terms and velocity terms of the horizontal displacement $\bar{u}$ (or p_i) are small and can be neglected. Then by eliminating the modal displacement p_i, the nonlinear ordinary differential equations for the modal displacement q_i corresponding to the dimensionless vertical beam displacement $\bar{w}$ can be obtained. They are rewritten in the following matrix form

$$\begin{aligned}&\mathbf{M}\ddot{\mathbf{Q}}+[\mathbf{C}+\mathbf{C}_{NL}(\dot{\mathbf{Q}}\dot{\mathbf{Q}}^{\mathrm{T}})]\dot{\mathbf{Q}}+\\&[\mathbf{K}+\mathbf{K}_{NL}(\mathbf{Q}\mathbf{Q}^{\mathrm{T}})]\mathbf{Q}=\mathbf{F}(t)\end{aligned} \tag{20}$$

where the modal displacement vector $\mathbf{Q}=[q_1, q_2, \ldots, q_N]^{\mathrm{T}}$, $\dot{\mathbf{Q}}=\mathrm{d}\mathbf{Q}/\mathrm{d}t$, $\ddot{\mathbf{Q}}=\mathrm{d}^2\mathbf{Q}/\mathrm{d}t^2$ and modal excitation vector $\mathbf{F}=-\ddot{\bar{w}}_0\mathbf{F}_C$. The modal mass matrix **M**, linear stiffness matrix **K**, nonlinear stiffness matrix $\mathbf{K}_{NL}$, linear damping matrix **C**, nonlinear damping matrix $\mathbf{C}_{NL}$ and vector $\mathbf{F}_C$ have elements as follows

$$M_{ij}=\rho h_tL^4\delta_{ij}+2mL^4\times\cos(2i-1)\pi y_0\cos(2j-1)\pi y_0,$$

$$K_{ij}=\frac{1}{6}(2i-1)^4\pi^4E_1h_1^3\delta_{ij}+\frac{(2i-1)^4\pi^4E_1G_{21}h_1h_a^2L^2}{(2i-1)^2\pi^2E_1h_1h_2+2G_{21}L^2}\delta_{ij},$$

$$K_{NL,ij}=\sum_{k=1}^{N}\sum_{l=1}^{N}\{\frac{3(2k-1)(2l-1)\pi^6E_1}{[E_1h_1h_2(2k-1)^2\pi^2+2G_{21}L^2]}\times\frac{G_{21}G_{23}h_1h_a^3W_a^2L^2/h_2}{[E_1h_1h_2(2l-1)^2\pi^2+2G_{21}L^2]}q_kq_l\times$$

$$[\frac{(2j-1)^4 G_{21}L^2 D_{T,ijkl}}{E_1 h_1 h_2 (2j-1)^2\pi^2 + 2G_{21}L^2} - (2l-1)^3 \times$$
$$(2j-1)D_{S,ijkl}]\} + \sum_{k=1}^{N}\sum_{l=1}^{N} D_{R,ijkl} q_k q_l \times$$
$$\frac{3(2k-1)^4(2l-1)(2j-1)\pi^6 E_1 G_{23} h_1 h_a^3 W_a^2}{4h_2[E_1 h_1 h_2 (2k-1)^2\pi^2 + 2G_{21}L^2]}$$

$$D_{T,ijkl} = -\delta_{k+l+j-i-1,0} - \delta_{k+l-j+i-1,0} - \delta_{k+l-j-i,0} + \delta_{k-l+j+i-1,0} + \delta_{k-l+j-i,0} + \delta_{k-l-j+i,0} + \delta_{k-l-j-i+1,0},$$

$$D_{S,ijkl} = -\delta_{k+l+j-i-1,0} + \delta_{k+l-j+i-1,0} + \delta_{k+l-j-i,0} - \delta_{k-l+j+i-1,0} - \delta_{k-l+j-i,0} + \delta_{k-l-j+i,0} + \delta_{k-l-j-i+1,0}$$

$$D_{R,ijkl} = -\delta_{k+l+j-i-1,0} + \delta_{k+l-j+i-1,0} + \delta_{k+l-j-i,0} + \delta_{k-l+j+i-1,0} + \delta_{k-l+j-i,0} - \delta_{k-l-j+i,0} - \delta_{k-l-j-i+1,0},$$

$$C_{ij} = \frac{(2i-1)^4\pi^4 G_{c1} E_1 h_1 h_2 h_a L^2}{(2i-1)^2\pi^2 E_1 h_1 h_2 + 2G_{21}L^2}\delta_{ij},$$

$$C_{NL,ij} = \sum_{k=1}^{N}\sum_{l=1}^{N}\{\frac{3(2k-1)(2l-1)\pi^6 E_1}{[E_1 h_1 h_2 (2k-1)^2\pi^2 + 2G_{21}L^2]} \times$$
$$\frac{G_{21} G_{c3} h_1 h_a^3 W_a^2 L^2 / h_2}{[E_1 h_1 h_2 (2l-1)^2\pi^2 + 2G_{21}L^2]}\dot{q}_k \dot{q}_l \times$$
$$[\frac{(2j-1)^4 G_{21}L^2 D_{T,ijkl}}{E_1 h_1 h_2 (2j-1)^2\pi^2 + 2G_{21}L^2} - (2l-1)^3$$
$$(2j-1)D_{S,ijkl}]\} + \sum_{k=1}^{N}\sum_{l=1}^{N} D_{R,ijkl}\dot{q}_k \dot{q}_l \times$$
$$\frac{3(2k-1)^4(2l-1)(2j-1)\pi^6 E_1 G_{c3} h_1 h_a^3 W_a^2}{4h_2[E_1 h_1 h_2 (2k-1)^2\pi^2 + 2G_{21}L^2]},$$

$$F_{C,i} = \frac{4(-1)^{i+1}}{(2i-1)\pi}\rho h_t L^4 + 2mL^4\cos(2i-1)\pi y_0,$$

$$\delta_{ij} = \begin{cases}1, & i = j\\ 0, & i \neq j\end{cases}, \quad i, j = 1, 2, \cdots, N \tag{21}$$

Eq. (20) represents a nonlinear multi-degree-of-freedom system subjected to stochastic excitations, which is derived from the sandwich beam with the supported mass under support motion excitations. The system has the cubic nonlinear stiffness and damping. Its nonlinear stochastic response cannot be exactly obtained, but an approximate response can be obtained. The nonlinear system is subjected to external stochastic excitations and the system response has a single value for an excitation based on the visco-elastomer core nonlinearity. The statistic linearization method is just suitable for solving the stochastic nonlinear multi-degree-of-freedom system (Roberts and Spanos 1990, Caughey 1971, Socha and Soong 1991, Elishakoff and Falsone 1993). Based on the method, the equivalent quasi-linear system of the nonlinear system (20) can be expressed as

$$\mathbf{M}\ddot{\mathbf{Q}} + (\mathbf{C} + \mathbf{C}_{eq})\dot{\mathbf{Q}} + (\mathbf{K} + \mathbf{K}_{eq})\mathbf{Q} = \mathbf{F}(t) \tag{22}$$

where $\mathbf{K}_{eq}$ and $\mathbf{C}_{eq}$ are equivalent quasi-linear stiffness and damping matrices, respectively. The difference of the left side terms of Eqs. (22) and (20) is

$$\mathbf{\Delta} = \mathbf{C}_{eq}\dot{\mathbf{Q}} + \mathbf{K}_{eq}\mathbf{Q} - \mathbf{C}_{NL}\dot{\mathbf{Q}} - \mathbf{K}_{NL}\mathbf{Q} \tag{23}$$

The minimization of mean square difference $\mathrm{E}[\mathbf{\Delta}^T\mathbf{\Delta}]$ yields algebraic equations for the equivalent stiffness and damping as follows

$$\mathbf{K}_{eq}\mathrm{E}[\mathbf{Q}\mathbf{Q}^{\mathrm{T}}] + \mathbf{C}_{eq}\mathrm{E}[\dot{\mathbf{Q}}\mathbf{Q}^{\mathrm{T}}] - \mathrm{E}[(\mathbf{C}_{NL}\dot{\mathbf{Q}} + \mathbf{K}_{NL}\mathbf{Q})\mathbf{Q}^{\mathrm{T}}] = 0 \tag{24}$$

$$\mathbf{K}_{eq}\mathrm{E}[\mathbf{Q}\dot{\mathbf{Q}}^{\mathrm{T}}] + \mathbf{C}_{eq}\mathrm{E}[\dot{\mathbf{Q}}\dot{\mathbf{Q}}^{\mathrm{T}}] - \mathrm{E}[(\mathbf{C}_{NL}\dot{\mathbf{Q}} + \mathbf{K}_{NL}\mathbf{Q})\dot{\mathbf{Q}}^{\mathrm{T}}] = 0 \tag{25}$$

where E[·] denotes the expectation operation. The equivalent stiffness $\mathbf{K}_{eq}$ and damping $\mathbf{C}_{eq}$ can be obtained by solving Eqs. (24) and (25), which depend on the second and fourth moments of the system response. For a Gaussian stochastic excitation, the equivalent linear system response is also a Gaussian stochastic process. Thus Eqs. (24) and (25) lead to the equivalent stiffness and damping

$$\mathbf{K}_{eq} = \mathrm{E}[(\mathbf{C}_{NL}\dot{\mathbf{Q}} + \mathbf{K}_{NL}\mathbf{Q})\mathbf{Q}^{\mathrm{T}}]\mathrm{E}[\mathbf{Q}\mathbf{Q}^{\mathrm{T}}]^{-1} \tag{26}$$

$$\mathbf{C}_{eq} = \mathrm{E}[(\mathbf{C}_{NL}\dot{\mathbf{Q}} + \mathbf{K}_{NL}\mathbf{Q})\dot{\mathbf{Q}}^{\mathrm{T}}]\mathrm{E}[\dot{\mathbf{Q}}\dot{\mathbf{Q}}^{\mathrm{T}}]^{-1} \tag{27}$$

which depend finally on the second moment of the system response.

The stochastic vibration response of the equivalent linear system (22) can be estimated by using the power spectral density function. The frequency response function and response spectral density matrices of the system (22) are given by

$$\mathbf{H}(\omega) = \{\mathbf{K} + \mathbf{K}_{eq} + \mathrm{j}\omega(\mathbf{C} + \mathbf{C}_{eq}) - \omega^2\mathbf{M}\}^{-1} \tag{28}$$

$$\mathbf{S}_{\mathbf{Q}}(\omega) = \mathbf{H}(\omega)\mathbf{F}_C\mathbf{F}_C^{\mathrm{T}}\mathbf{H}^{*\mathrm{T}}(\omega)S_{\ddot{w}_0}(\omega) \tag{29}$$

where $\mathrm{j} = \sqrt{-1}$, ω is the vibration frequency, superscript $*$ denotes the complex conjugate, and $S_{\ddot{w}_0}(\omega)$ is the power spectral density of the support motion excitation $\ddot{w}_0(t)$. By using the expressions (19) and (29), the frequency response function of the dimensionless vertical displacement of the sandwich beam to the unit support excitation is obtained as

$$R_{\bar{w}}(\omega, y) = \mathbf{\Phi}^{\mathrm{T}}(y)\mathbf{H}(\omega)\mathbf{F}_C,$$
$$\mathbf{\Phi}(y) = [\cos\pi y, \cos 3\pi y, \cdots, \cos(2N-1)\pi y]^{\mathrm{T}} \tag{30}$$

The spectral density function of the dimensionless vertical displacement response of the sandwich beam is

$$S_{\bar{w}}(\omega, y) = \mathbf{\Phi}^{\mathrm{T}}(y)\mathbf{S}_{\mathbf{Q}}(\omega)\mathbf{\Phi}(y) \tag{31}$$

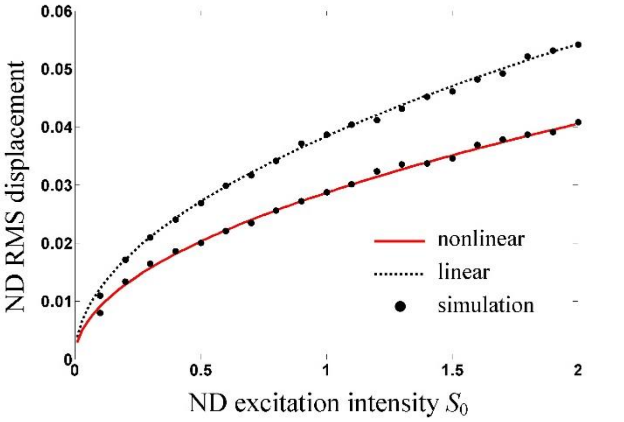

Fig. 2 Non-dimensional (ND) RMS displacement ($\overline{w}$) responses of the sandwich beam versus stochastic excitation intensity (S_0) for nonlinear visco-elastomer core (G_{23}=0.02G_{21}, G_{c3}=0.02G_{c1}) and linear visco-elastomer core (G_{23}=0, G_{c3}=0) (dots: numerical simulation)

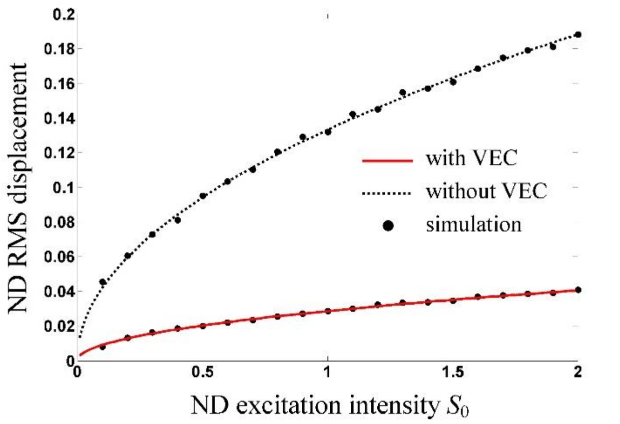

Fig. 3 Non-dimensional (ND) RMS displacement ($\overline{w}$) responses of the sandwich beam with visco-elastomer core (VEC) (G_{23}=0.02G_{21}, G_{c3}=0.02G_{c1}) and beam without VEC versus stochastic excitation intensity (S_0) (dots: numerical simulation)

The response statistics of the sandwich beam subjected to stochastic excitations can be estimated by using the spectral density function (31). For example, the mean square displacement response is expressed as

$$\mathrm{E}[\overline{w}^2(y)] = \int_{-\infty}^{\infty} S_{\overline{w}}(\omega, y)\mathrm{d}\omega \tag{32}$$

4. Numerical results

To show numerically the nonlinear stochastic vibration response, consider a sandwich beam with nonlinear magneto-rheological visco-elastomer core and a supported mass under stochastic support motion excitation. It has basic parameter values as follows: L=4 m, h_1=5 cm, h_2=20 cm, ρ_1=3×10^3 kg/m^3, ρ_2=1.2×10^3 kg/m^3, m=80 kg/m^2, E_1=10 GPa, G_{21}=2 MPa, G_{23}=0.02G_{21}, G_{c1}=0.003 MPa·s, G_{c3}=0.02G_{c1}, x_0=0, W_a=1 (Ni *et al.* 2011). The support excitation is a zero-mean Gaussian stochastic process with the following power spectral density

$$S_{\ddot{w}_0}(\omega) = \frac{1+4\zeta_g^2(\omega/\omega_g)^2}{[1-(\omega/\omega_g)^2]^2 + 4\zeta_g^2(\omega/\omega_g)^2} S_0 \tag{33}$$

where the dimensionless excitation intensity S_0=1.0, constants ω_g=23 rad/s and ζ_g=0.3. The number N in expansion (19) is determined based on the convergence of displacement responses. Numerical results on the equivalent frequency response and root-mean-square (RMS) response of the dimensionless vertical displacement at the sandwich beam midpoint (y=0) are obtained and shown in Figs. 2-20.

4.1 Displacement response for sandwich beam with hard-nonlinear visco-elastomer core

The visco-elastomer core with hard nonlinearity (G_{23}>0, G_{c3}>0) is considered firstly. Fig. 2 shows the non-dimensional RMS displacement ($\overline{w}$) responses of the sandwich beam varying with the non-dimensional stochastic excitation intensity (S_0) for the nonlinear visco-elastomer core (G_{23}=0.02G_{21}, G_{c3}=0.02G_{c1}) and linear visco-elastomer core (G_{23}=0, G_{c3}=0). The RMS displacement responses obtained by the numerical simulation are also given, which validate the results obtained by the proposed analysis method. The numerical simulation is conducted as follows: samples of the stochastic excitation are firstly produced according to the power spectral density (33), the stochastic responses of nonlinear system (20) with (19) are calculated by using the Runge-Kutta algorithm and then the response statistics are estimated. It is seen that the RMS displacement response of the nonlinear sandwich beam is smaller than that of the linear sandwich beam (for example, RMS values are 0.0385 for linear case and 0.0287 for nonlinear case under S_0=1), and the response difference increases with the stochastic excitation intensity. Thus, the RMS response of the sandwich beam can be overestimated by the linear model. Fig. 3 shows the non-dimensional RMS displacement ($\overline{w}$) responses of the sandwich beam with the visco-elastomer core (G_{23}=0.02G_{21}, G_{c3}=0.02G_{c1}) and the corresponding beam without the visco-elastomer core, which are validated by the numerical simulation. The RMS displacement response of the visco-elastomer sandwich beam is much smaller than that of the beam without the visco-elastomer core. Thus, the RMS beam response can be reduced remarkably by using the nonlinear visco-elastomer core (for example, RMS value descends from 0.1331 to 0.0287 for S_0=1. The relative reduction is 78.4%). The reduction increases with the stochastic excitation intensity (S_0).

Figs. 4 and 5 illustrate that the non-dimensional RMS displacement ($\overline{w}$) responses of the sandwich beam with the nonlinear visco-elastomer core (G_{23}=0.02G_{21}, G_{c3}=0.02G_{c1}) increase with the non-dimensional stochastic excitation intensity (S_0) for different facial layer thicknesses h_1 and core layer thicknesses h_2, respectively. The RMS displacement response of the nonlinear sandwich beam decreases as the thicknesses h_1 and h_2 increase. The response reduction increases with the stochastic excitation intensity.

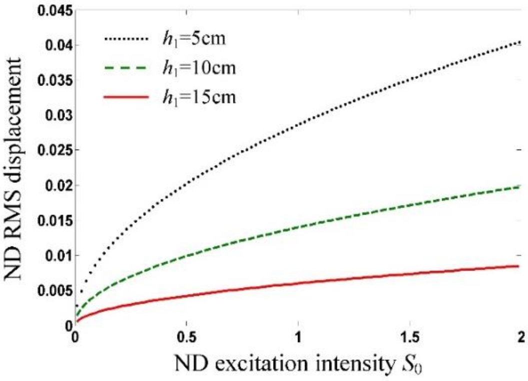

Fig. 4 Non-dimensional (ND) RMS displacement ($\overline{w}$) responses of the sandwich beam versus stochastic excitation intensity (S_0) for different facial layer thicknesses h_1 (G_{23}=0.02G_{21}, G_{c3}=0.02G_{c1})

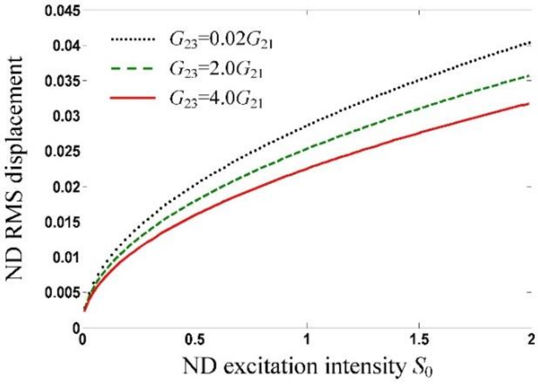

Fig. 7 Non-dimensional (ND) RMS displacement ($\overline{w}$) responses of the sandwich beam versus stochastic excitation intensity (S_0) for different nonlinear stiffness coefficients G_{23} of the core layer (G_{21}=2MPa, G_{c3}=0.02G_{c1})

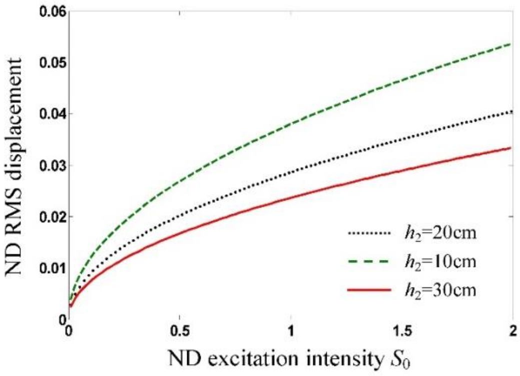

Fig. 5 Non-dimensional (ND) RMS displacement ($\overline{w}$) responses of the sandwich beam versus stochastic excitation intensity (S_0) for different core layer thicknesses h_2 (G_{23}=0.02G_{21}, G_{c3}=0.02G_{c1})

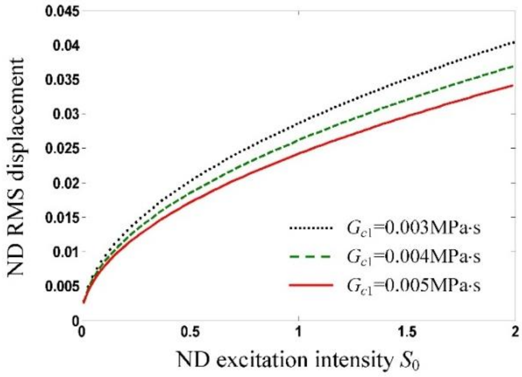

Fig. 8 Non-dimensional (ND) RMS displacement ($\overline{w}$) responses of the sandwich beam versus stochastic excitation intensity (S_0) for different linear damping coefficients G_{c1} of the core layer (G_{23}=0.02G_{21}, G_{c3}=60Pa·s^3)

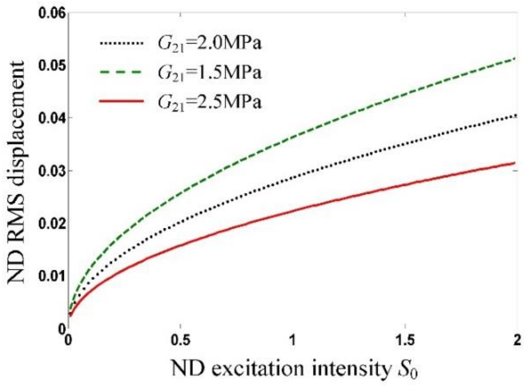

Fig. 6 Non-dimensional (ND) RMS displacement ($\overline{w}$) responses of the sandwich beam versus stochastic excitation intensity (S_0) for different linear stiffness coefficients G_{21} of the core layer (G_{23}=0.04MPa, G_{c3}=0.02G_{c1})

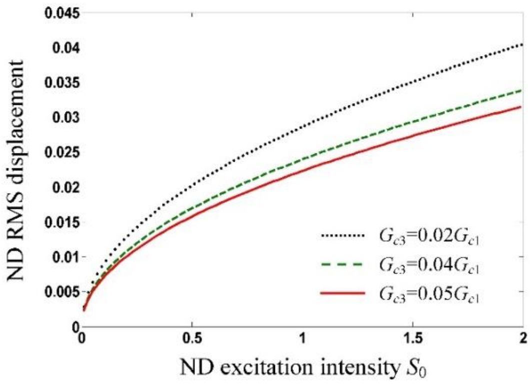

Fig. 9 Non-dimensional (ND) RMS displacement ($\overline{w}$) responses of the sandwich beam versus stochastic excitation intensity (S_0) for different nonlinear damping coefficients G_{c3} of the core layer (G_{23}=0.02G_{21}, G_{c1}=0.003MPa·s)

Figs. 6, 7, 8 and 9 illustrate that the non-dimensional RMS displacement ($\overline{w}$) responses of the sandwich beam with the nonlinear visco-elastomer core increase with the non-dimensional stochastic excitation intensity (S_0) for different linear stiffness coefficients G_{21} (G_{23}=0.04MPa, G_{c3}=0.02G_{c1}), nonlinear stiffness coefficients G_{23} (G_{21}=2MPa, G_{c3}=0.02G_{c1}), linear damping coefficients G_{c1} (G_{23}=0.02G_{21}, G_{c3}=60Pa·s^3) and nonlinear damping coefficients G_{c3} (G_{23}=0.02G_{21}, G_{c1}=0.003MPa·s) of the core layer, respectively. The RMS displacement response of the nonlinear sandwich beam decreases as the stiffness coefficients G_{21}, G_{23} and damping coefficients G_{c1} and G_{c3}

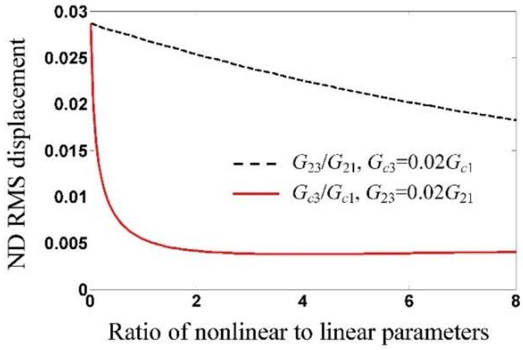

Fig. 10 Non-dimensional (ND) RMS displacement ($\overline{w}$) responses of the sandwich beam versus ratios of nonlinear to linear stiffness coefficients (G_{23}/G_{21}, G_{c3}=0.02G_{c1}) and nonlinear to linear damping coefficients (G_{c3}/G_{c1}, G_{23}=0.02G_{21}) of the core layer

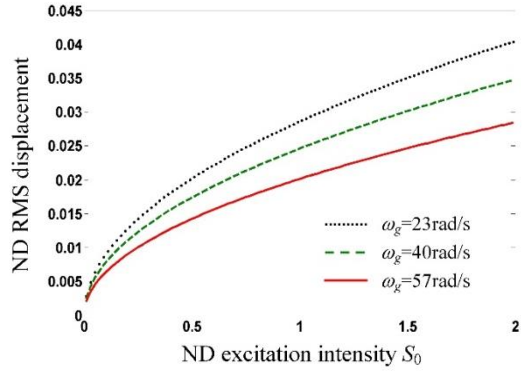

Fig. 12 Non-dimensional (ND) RMS displacement ($\overline{w}$) responses of the sandwich beam versus stochastic excitation intensity (S_0) for different excitation spectral density parameters ω_g (G_{23}=0.02G_{21}, G_{c3}=0.02G_{c1})

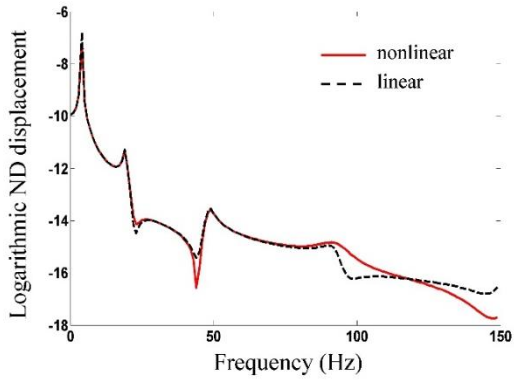

Fig. 11 Logarithmic non-dimensional (ND) RMS displacement ($\overline{w}$) responses of the sandwich beam versus vibration frequency (ω) for nonlinear visco-elastomer core (G_{23}=0.02G_{21}, G_{c3}=0.02G_{c1}) and linear visco-elastomer core (G_{23}=0, G_{c3}=0)

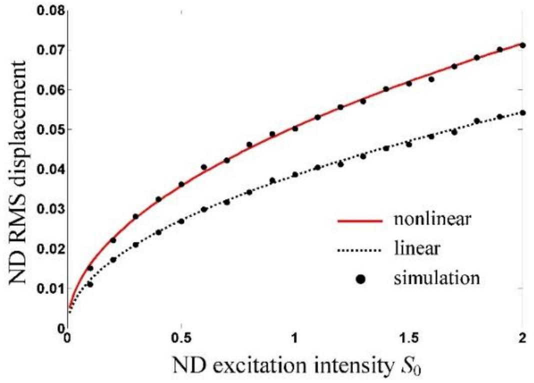

Fig. 13 Non-dimensional (ND) RMS displacement ($\overline{w}$) responses of the sandwich beam versus stochastic excitation intensity (S_0) for nonlinear visco-elastomer core (G_{23}=–0.01G_{21}, G_{c3}=–0.01G_{c1}) and linear visco-elastomer core (G_{23}=0, G_{c3}=0) (dots: numerical simulation)

increase. The response reduction increases with the stochastic excitation intensity. Fig. 10 shows further the non-dimensional RMS displacement ($\overline{w}$) responses of the nonlinear sandwich beam varying with the ratios of nonlinear to linear stiffness coefficients (G_{23}/G_{21}, G_{c3}=0.02G_{c1}) and nonlinear to linear damping coefficients (G_{c3}/G_{c1}, G_{23}=0.02G_{21}) of the core layer under the non-dimensional stochastic excitation intensity S_0=1. It is seen that the RMS displacement response of the sandwich beam decreases monotonously as the ratio of nonlinear to linear stiffness coefficients G_{23}/G_{21} increases. However, the RMS displacement response of the sandwich beam decreases rapidly for small ratio of nonlinear to linear damping coefficients and then becomes steady as the ratio of nonlinear to linear damping coefficients G_{c3}/G_{c1} increases. Thus, the RMS response of the sandwich beam can be reduced effectively by a small nonlinear damping (G_{c3}) of the visco-elastomer core.

Fig. 11 shows the logarithmic non-dimensional RMS displacement ($\overline{w}$) responses of the sandwich beam varying with vibration frequency (ω) for the nonlinear visco-elastomer core (G_{23}=0.02G_{21}, G_{c3}=0.02G_{c1}) and linear visco-elastomer core (G_{23}=0, G_{c3}=0). The first three resonant frequencies of the sandwich beam are 3.78, 19.25 and 48.61 Hz. The resonant frequency responses are large and decrease with the order number rising. Fig. 12 shows the non-dimensional RMS displacement ($\overline{w}$) responses of the nonlinear sandwich beam varying with the stochastic excitation intensity (S_0) for different excitation spectral density parameters ω_g (G_{23}=0.02G_{21}, G_{c3}=0.02G_{c1}). The RMS displacement response of the sandwich beam decreases as the dominant excitation frequency (ω_g) runs away from the resonant frequency (for example, 23 rad/s=3.66 Hz to 40 rad/s=6.36 Hz, 57 rad/s=9.07 Hz).

4.2 Displacement response for sandwich beam with soft-nonlinear visco-elastomer core

The visco-elastomer core with soft nonlinearity (G_{23}<0, G_{c3}<0) is considered secondly. Fig. 13 shows the non-dimensional RMS displacement ($\overline{w}$) responses of the sandwich beam varying with the non-dimensional stochastic excitation intensity (S_0) for the nonlinear visco-elastomer core (G_{23}=–0.01G_{21}, G_{c3}=–0.01G_{c1}) and linear visco-elastomer core (G_{23}=0, G_{c3}=0), which are validated by the numerical simulation. It is seen that the RMS displacement

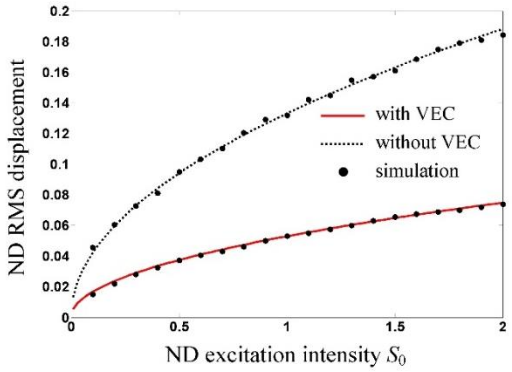

Fig. 14 Non-dimensional (ND) RMS displacement ($\overline{w}$) responses of the sandwich beam with visco-elastomer core (VEC) (G_{23}=–0.01G_{21}, G_{c3}=–0.01G_{c1}) and beam without VEC versus stochastic excitation intensity (S_0) (dots: numerical simulation)

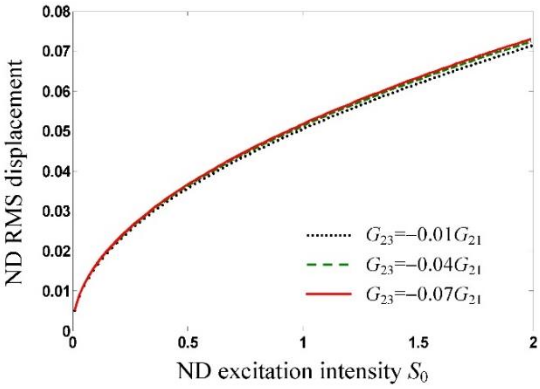

Fig. 16 Non-dimensional (ND) RMS displacement ($\overline{w}$) responses of the sandwich beam versus stochastic excitation intensity (S_0) for different nonlinear stiffness coefficients G_{23} of the core layer (G_{21}=2MPa, G_{c3}=–0.01G_{c1})

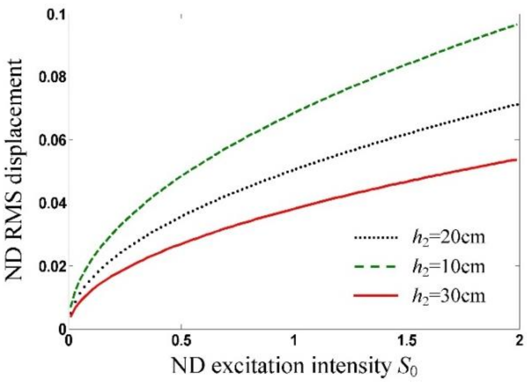

Fig. 15 Non-dimensional (ND) RMS displacement ($\overline{w}$) responses of the sandwich beam versus stochastic excitation intensity (S_0) for different core layer thicknesses h_2 (G_{23}=–0.01G_{21}, G_{c3}=–0.01G_{c1})

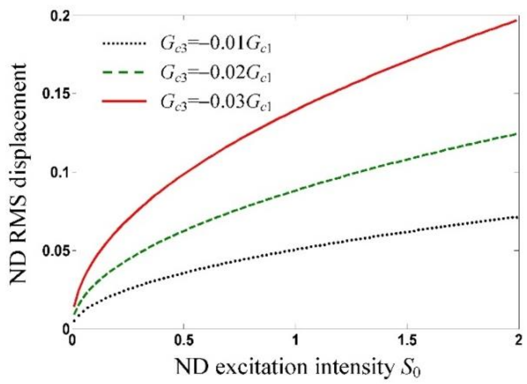

Fig. 17 Non-dimensional (ND) RMS displacement ($\overline{w}$) responses of the sandwich beam versus stochastic excitation intensity (S_0) for different nonlinear damping coefficients G_{c3} of the core layer (G_{23}=–0.01G_{21}, G_{c1}=0.003MPa·s)

response of the sandwich beam with the soft nonlinear core is larger than that of the linear sandwich beam (for example, RMS values are 0.0384 for linear case and 0.0506 for nonlinear case under S_0=1), and the response difference increases with the stochastic excitation intensity. Thus, the RMS response of the sandwich beam can be underestimated by the linear model. However, Fig. 14 illustrates that the non-dimensional RMS displacement ($\overline{w}$) response of the sandwich beam with the soft nonlinear visco-elastomer core (G_{23}=–0.01G_{21}, G_{c3}=–0.01G_{c1}) is much smaller than that of the beam without the visco-elastomer core. Thus, the RMS beam response can be reduced remarkably by using the nonlinear visco-elastomer core (for example, RMS value descends from 0.1331 to 0.0506 for S_0=1. The relative reduction is 62.0%).

Fig. 15 illustrates that the non-dimensional RMS displacement ($\overline{w}$) response of the sandwich beam with the soft nonlinear visco-elastomer core (G_{23}=–0.01G_{21}, G_{c3}=–0.01G_{c1}) decreases as the core layer thicknesses h_2 increases. The response reduction increases with the stochastic excitation intensity (S_0). Figs. 16 and 17 show the non-dimensional RMS displacement ($\overline{w}$) responses of the sandwich beam with the soft nonlinear visco-elastomer core for different nonlinear stiffness coefficients G_{23} (G_{21}=2MPa, G_{c3}=–0.01G_{c1}) and nonlinear damping coefficients G_{c3} (G_{23}=–0.01G_{21}, G_{c1}=0.003MPa·s) of the core layer, respectively. The effect of the nonlinear damping coefficient on the RMS displacement response is much larger than that of the nonlinear stiffness coefficient. The RMS displacement response of the sandwich beam increases as the nonlinear damping coefficient decreases. The response increment increases with the stochastic excitation intensity. Fig. 18 shows the non-dimensional RMS displacement ($\overline{w}$) responses of the soft nonlinear sandwich beam varying complicatedly with the ratios of nonlinear to linear stiffness coefficients (G_{23}/G_{21}, G_{c3}=–0.02G_{c1}) and nonlinear to linear damping coefficients (G_{c3}/G_{c1}, G_{23}=–0.02G_{21}) of the core layer under the non-dimensional stochastic excitation intensity S_0=1. It is seen that the RMS displacement response of the sandwich beam increases in fluctuation as the ratio G_{23}/G_{21} of nonlinear to linear stiffness coefficients decreases. The fluctuation can be caused by the increased soft nonlinearity corresponding to local periodic instability enlarged. However, the RMS displacement response of the sandwich beam has several peak values for certain ratios G_{c3}/G_{c1} of nonlinear to linear damping coefficients. Thus,

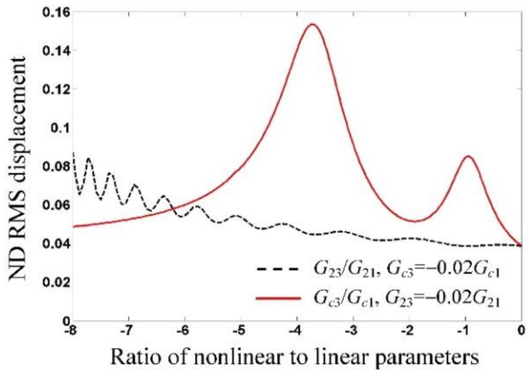

Fig. 18 Non-dimensional (ND) RMS displacement ($\overline{w}$) responses of the sandwich beam versus ratios of nonlinear to linear stiffness coefficients (G_{23}/G_{21}, G_{c3}=–0.02G_{c1}) and nonlinear to linear damping coefficients (G_{c3}/G_{c1}, G_{23}=–0.02G_{21}) of the core layer

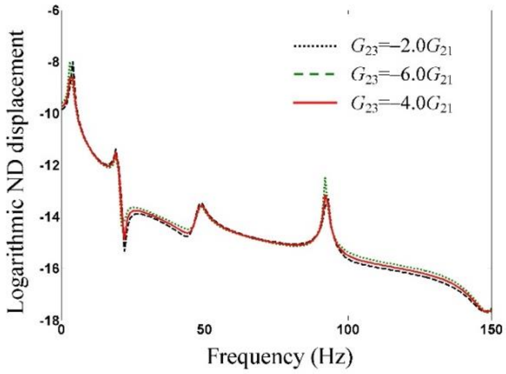

Fig. 19 Logarithmic non-dimensional (ND) RMS displacement ($\overline{w}$) responses of the sandwich beam versus vibration frequency (ω) for different nonlinear stiffness coefficients G_{23} of the core layer (G_{c3}=–0.02G_{c1})

the damping coefficient of the soft nonlinear core corresponding to the region around the response peaks needs to be avoided for the response reduction of the sandwich beam.

Fig. 19 shows the logarithmic non-dimensional RMS displacement ($\overline{w}$) responses of the sandwich beam varying with vibration frequency (ω) for different soft nonlinear stiffness coefficients G_{23} (G_{c3}=–0.02G_{c1}) of the core layer. It is seen that the high-order resonant frequency responses can be larger than the low-order resonant frequency responses (for example, the fourth response peak is higher than the third response peak) for the soft nonlinear case. However, Fig. 20 illustrates that the non-dimensional RMS displacement ($\overline{w}$) response of the soft nonlinear sandwich beam decreases as the dominant excitation frequency (ω_g) runs away from the resonant frequency (for example, 23 rad/s to 40 rad/s, 57 rad/s) (G_{23}=–0.01G_{21}, G_{c3}=–0.01G_{c1}).

5. Conclusions

The stochastic vibration response of the sandwich beam with the nonlinear adjustable visco-elastomer core and

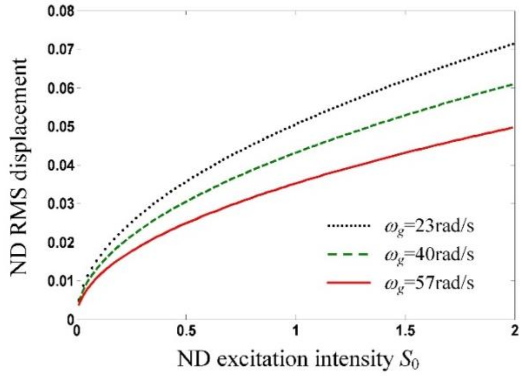

Fig. 20 Non-dimensional (ND) RMS displacement ($\overline{w}$) responses of the sandwich beam versus stochastic excitation intensity (S_0) for different excitation spectral density parameters ω_g (G_{23}=–0.01G_{21}, G_{c3}=–0.01G_{c1})

supported mass under stochastic support motion excitations has been studied. The nonlinear dynamic properties of the visco-elastomer core have been considered. The nonlinear partial differential equations for the horizontal and vertical coupling motions of the sandwich beam with the supported mass have been derived by the dynamic equilibrium, constitutive and geometric relations. An analytical solution method for the stochastic vibration response of the nonlinear sandwich beam has been developed based on the Galerkin method and statistic linearization method. The multi-degree-of-freedom system with the cubic nonlinear stiffness and damping subjected to stochastic excitations has been obtained by using the Galerkin method. The nonlinear stochastic system has been converted into the equivalent quasi-linear system by using the statistic linearization method. The frequency-response function, response spectral density and mean square response expressions of the nonlinear sandwich beam have been obtained. The developed analysis method is applicable to sandwich beams with arbitrary high-power nonlinear cores under arbitrary stationary stochastic excitations.

Numerical results illustrate that (1) the RMS response of the sandwich beam with the nonlinear visco-elastomer core under stochastic excitations can be overestimated for the hard nonlinearity and underestimated for the soft nonlinearity by using the linear core model; (2) the RMS response of the sandwich beam under stochastic excitations can be reduced remarkably by using the hard or soft nonlinear visco-elastomer core; (3) the RMS response of the stochastic nonlinear sandwich beam decreases as the facial layer thickness, core layer thickness, core stiffness and damping increase; (4) in the hard nonlinear case, the RMS response of the stochastic sandwich beam decreases rapidly for small ratio of nonlinear to linear damping coefficients and then becomes steady as the damping ratio increases, and however, in the soft nonlinear case, the RMS response of the stochastic sandwich beam has several large peak values for certain ratios of nonlinear to linear damping coefficients; (5) the high-order resonant frequency responses are smaller than the low-order resonant frequency responses for the hard nonlinear sandwich beam, and however, the high-order resonant frequency responses can be larger than the low-order resonant frequency responses for the soft nonlinear sandwich beam. The above results are valuable for the stochastic vibration control design of sandwich beams with nonlinear visco-elastomer core.

Acknowledgments

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