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A consistent FEM-Vlasov model for laminated orthotropic beams subjected to moving load

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Abstract. In the study, dynamic behavior of laminated orthotropic beams on elastic foundation is investigated. Consistent model presented here combines the finite element solution of the system with SAP2000 software and the calculation of soil parameters with MATLAB software using Modified Vlasov Model type elastic foundation. For this purpose, a computing tool is coded in MATLAB which employs Open Application Programming Interface (OAPI) feature of SAP2000 to provide two-way data flow during execution. Firstly, an example is taken from the literature to demonstrate the accuracy of the consistent FEM-Vlasov Model. Subsequently, the effects of boundary conditions, subsoil depth, elasticity modulus of subsoil, slenderness ratio, velocity of moving load and lamination scheme on the behavior of laminated orthotropic beams on elastic foundation are investigated on a new numerical example. It can be concluded that it is really convenient to use OAPI feature of SAP2000 to model this complex behavior of laminated orthotropic beams on elastic foundation under moving load.

Keywords: moving load; elastic foundation; laminated orthotropic beam; OAPI

1. Introduction

Many modern engineering structures such as highways and airfield pavements are usually modeled as a plate resting on elastic foundation and they are subjected to traversing moving loads such as wheel loads from moving vehicles and planes. So, it is interesting and important for engineers to understand the dynamic behavior of the plates or beams on elastic foundations before structural designs. Therefore, numerous studies have been conducted on plates or beams on elastic foundations subjected to moving loads (Jaiswal and Iyengar 1993, Thambiratnam and Zhuge 1996, Huang and Thambiratnam 2002, Kim 2004, Kocaturk and Simsek 2006, Raftoyiannis, Avraam *et al*. 2012). On the other hand, the use of laminated composite materials in the structural industry is rapidly increasing owing to superior material properties of composites. Layered composite materials allow to be made high strength or stiffness sections for lower weight using various fiber orientations and lamination scheme. Laminated orthotropic beams or plates subjected to moving load have also been studied extensively (Kadivar and Mohebpour 1998, Kadivar and Mohebpour 1998, Abu-Hilal and Mohsen 2000, Abu Hilal and Zibdeh 2000, Lee and Yhim 2004, Aydogdu 2005, Aydogdu 2006, Malekzadeh *et al*. 2009, Malekzadeh Haghighi *et al*. 2010, Mohebpour *et al*. 2011, Kahya 2012, Kahya 2012, Malekzadeh *et al*. 2015, Malekzadeh and Monajjemzadeh 2015, Malekzadeh and Monajjemzadeh 2016). However, the studies combining laminated orthotropic material properties with subsoil effects are very limited (Kiral and Kiral 2009, Vosoughi *et al*. 2013). Studies on laminated orthotropic beams on elastic foundation using modified Vlasov model are not found in the literature.

In the study, dynamic behavior of laminated orthotropic beams on modified Vlasov type elastic foundation is investigated. For the analysis, SAP2000 (2008) and MATLAB (2009)software are used simultaneously. Using a computing tool developed in this study, soil parameters are calculated in MATLAB and finite element solution is performed in SAP2000. Two-way data flow during execution between SAP2000 and MATLAB is provided employing Open Application Programming Interface (OAPI) feature. After verifying the accuracy of the proposed model, the effects of boundary conditions, subsoil depth, elasticity modulus of subsoil, slenderness ratio, velocity of moving load and lamination scheme on the behavior of laminated orthotropic beams on elastic foundation are examined.

2. SAP2000 Open Application Programming Interface

The Open Application Programming Interface (OAPI) is a powerful tool that allows users to automate many of the processes required to build, analyze and design models and to obtain customized analysis and design results. It also allows users to link SAP2000 with third-party software, providing a path for two-way exchange of model information with other programs. Most major programming languages can be used to access SAP2000 through OAPI. This includes Visual Basic, Microsoft Excel and MATLAB (SAP2000 2008).

In this study, OAPI features of SAP2000 is used interactively with a computing tool coded in MATLAB to

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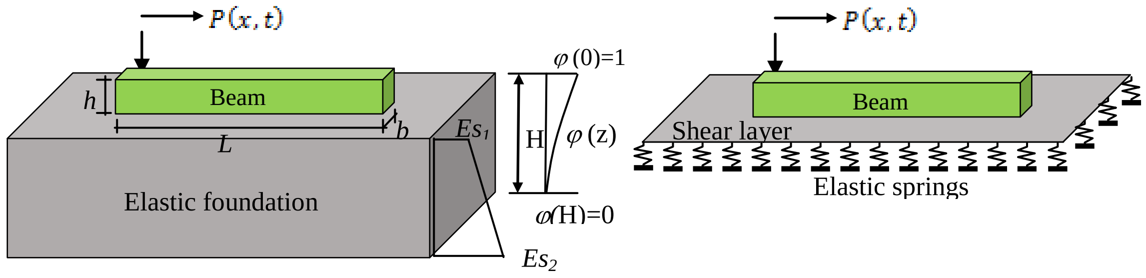


(a) 3D beam on elastic subsoil (b) Mathematical model

Fig. 1 A beam on three parameter elastic foundation

perform the analysis of laminated orthotropic beams on elastic foundations using Modified Vlasov Model.

3. Finite element model

Governing differential equation for a Timoshenko beam on elastic foundation subjected to moving load without damping is

$$KGA\left(\frac{\partial^2 w(x,t)}{\partial x^2}-\frac{\partial \phi(x,t)}{\partial x}\right)-\rho A\frac{\partial^2 w(x,t)}{\partial t^2}$$
$$-C_k w(x,t)+C_{2t}\frac{\partial^2 w(x,t)}{\partial x^2}+P(x,t)=0 \quad (1)$$

$$EI\frac{\partial^2 \phi(x,t)}{\partial x^2}+KGA\left(\frac{\partial w(x,t)}{\partial x}-\phi(x,t)\right)$$
$$-\rho I\frac{\partial^2 \phi(x,t)}{\partial t^2}=0 \quad (2)$$

where G, E, I, ρ and A are equivalent shear modulus, equivalent elasticity modulus, moment inertia, mass density and cross-section of the beam respectively. P depicts moving load. C_k and C_{2t} are subgrade reaction modulus and shear deformation parameter of the subsoil respectively. w and ϕ denote vertical and rotational displacement of the beam respectively. K is shear correction factor which is taken as 5/6 for a rectangular cross-section. The equivalent modulus of elasticity and shear modulus in the x and y direction are calculated from following equations according to Jones (1975).

$$\frac{1}{E}=\frac{1}{E_{11}}\cos^4\theta+\left(\frac{1}{G_{12}}-\frac{2\nu_{12}}{E_{11}}\right)\sin^2\theta\cos^2\theta+\frac{1}{E_{22}}\sin^4\theta \quad (3)$$

$$\frac{1}{G}=2\times\left[\frac{2}{E_{11}}+\frac{2}{E_{22}}+\frac{4\nu_{12}}{E_{11}}-\frac{1}{G_{12}}\right]\sin^2\theta\cos^2\theta$$
$$+\frac{1}{G_{12}}\left(\cos^4\theta+\sin^4\theta\right) \quad (4)$$

Newmark-β method is used for time integration of Eq.(1 and 2). Value of γ and β are taken as 0.5 and 0.25 respectively for Newmark-β method (Humar 1990). For $P(x,t)=0$, Eq. (1) returns the governing equation for a beam on elastic foundation subjected to a free vibration with no damping.

Subsoil reactions of a beam resting on a Modified Vlasov Model type elastic foundation may be given by

$$q_z=C_k w(x,t)-C_{2t}\frac{\partial^2 w(x,t)}{\partial x^2} \quad (5)$$

depending on the displacement function w of the subsoil surface. Soil parameters (C_k and C_{2t}) in above expression may be defined as

$$C_k=\int_0^{H_s}\frac{E_s b(1-\nu_s)}{(1+\nu_s)(1-2\nu_s)}\left(\frac{d\varphi(z)}{dz}\right)dz \quad (6)$$

$$C_{2t}=\int_0^{H_s}\frac{E_s b}{2(1+\nu_s)}\varphi(z)^2\,dz \quad (7)$$

where H_s, E_s and ν_s are depth, elasticity modulus and Poisson's ratio of the subsoil respectively. b denotes beam width.

When second parameter of subsoil, C_{2t}, equals to zero, equations of beam on Winkler type foundation are obtained. Winkler model is widely used for the analysis and design of beams on elastic foundation, but subsoil under the beam is defined as independent, closely spaced, discrete and linearly elastic springs. Main deficiency of the Winkler model is that it disregards shear deformation within the subsoil assuming no interaction between the adjacent springs. In reality, the subsoil is a continuous medium. The deficiency of Winkler model is eliminated by using various type of shear layer in the classical two-parameter foundation models like Pasternak model, Hetenyi model and Vlasov model etc. But difficulty of these models is to establish a relationship between soil type and soil parameters. Vallabhan *et al.* (1991) introduced another parameter, γ, as a function of the vertical deformation profile within the subsoil, and named the model as Modified Vlasov model. The advantage of this model is the elimination of the necessity to determine the values of soil parameters, C_k and C_{2t}, arbitrarily because these values can be computed as a function of a new parameter, γ using an iterative procedure.

$\varphi(z)$ in Eq. (6) and Eq. (7) is the mode shape function to describe the relationship between the vertical displacement of the subsoil and beam. Mode shape function $\varphi(z)$ may be given depending on the subsoil surface vertical deformation parameter (γ) as below, Fig. 1.

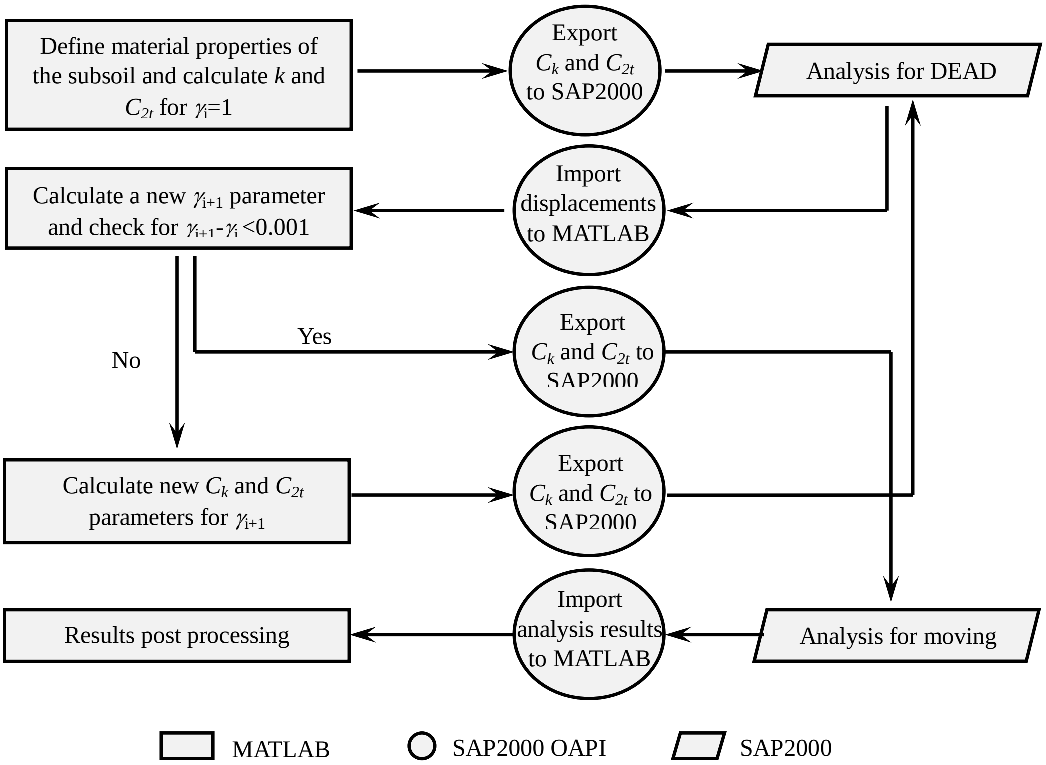


Fig. 2 Flowchart for analysis of beams on elastic foundation by OAPI

$$\varphi(z)=\frac{Sinh\,\gamma\left(1-\frac{z}{H_s}\right)}{Sinh\,\gamma} \tag{8}$$

$$\left(\frac{\gamma}{H_s}\right)^2=\frac{1-2\nu_s}{2(1-\nu_s)}\frac{\int_{-\infty}^{+\infty}\left(\frac{\partial w}{\partial x}\right)^2dx}{\int_{-\infty}^{+\infty}w^2dx} \tag{9}$$

As can be seen in Eq. (9) the value of γ varies with the displacement of the beam and the depth of the subsoil. Therefore, the variables w, C_k, C_{2t}, H and γ are all connected to each other for a beam on elastic foundation. So, the solution of this complex soil-structure interaction problem can be performed using an iterative technique.

For this purpose, initially a computing tool is developed using MATLAB to make two-way data flow with the finite element model developed in SAP2000. As is known, the modulus of subgrade reaction, C_k, which is the only soil parameter used in Winkler Model is represented by elastic area springs in SAP2000. The interaction between the springs is ignored assuming each spring is acting independently. A Shell-Layered/Nonlinear element with unit thickness is connected at the top of the springs to take the interaction between the springs into account. One of the main features of the SAP2000-OAPI is to provide data transfer and control of a structural model by different third-party applications simultaneously. A computing tool is developed in MATLAB and used to determine the soil parameters, C_k and C_{2t} in terms of γ iteratively. Therefore, γ is initially set equal to one and subgrade reaction, C_k, and soil shear parameter, C_{2t}, are calculated. Then, the structure-soil system is analyzed to find the surface displacements of the foundation which are the output of the structural model created by SAP2000. A comparison between the new value of γ and previously calculated γ is then made. If the difference between the two successive γ values is within a prescribed tolerance, the analysis is terminated. Otherwise, another iteration is performed and the process is repeated until convergence is obtained. Solution procedure is given in Fig. 2.

4. Numerical verification

An example solved before by Kahya (2012) and Reddy (1997) is selected to validate presented consistent model, and the some numerical results are compared with those of their studies. Material properties for frequency analysis are as follows:

$$E_1=172.5\times10^6\ \text{kN/m}^2,\ E_2=6.9\times10^6\ \text{kN/m}^2$$

$$G_{12}=3.45\times10^6\ \text{kN/m}^2,\ \nu_{12}=0.25,\ \rho=1578\ \text{kg/m}^3$$

Non-dimensional fundamental frequencies of various laminated orthotropic beams for different ratio of beam length to beam thickness are compared with those of Kahya (2012) and Reddy (1997) in Table 1. Ratio of beam width to beam thickness (b/h) equals 1 and non-dimensional frequency parameter is calculated with the equation of $\hat{\omega}_1=\omega_1(L^2/h)\sqrt{\rho/E_2}$. Results show that three different solutions are in good agreement with each other and accuracy of the presented method is clear.

Kahya (2012) presents center displacement of the beam subjected to moving load for various lamination scheme in the same study. Material properties for moving load

Table 1 Nondimensional fundamental frequencies ($\hat{\omega}_1$) for various laminated beams

Boundary conditions				
SS		CC		
Reddy (1997)	Present Study	Kahya (2012)	Reddy (1997)	Present Study
11.635	11.201	17.212	17.212	15.852
13.430	13.260	25.327	25.327	24.211
14.210	14.203	31.899	31.899	31.849
2.771	2.815	6.134	5.761	6.098
2.829	2.841	6.372	6.260	6.368
2.848	2.849	6.455	6.450	6.462
10.488	10.764	16.522	14.837	15.605
12.434	12.550	24.010	22.672	23.348
13.334	13.340	29.942	29.857	29.949
3.663	3.709	7.125	7.61	8.290
3.739	3.801	7.352	8.275	9.236
3.765	3.887	7.430	8.526	9.860

Table 2 The effect of the load speed and stacking order on the response of simply supported laminated beams

Beam	Ref.	Nondimensional velocity ($\alpha=c/c_{cr}$)					
		0.0	0.2	0.4	0.6	0.8	1.0
$[0/90]_s$	Kahya (2012)	0.00136	0.00141	0.00216	0.00228	0.00221	0.00205
	P.Study	0.00137	0.00140	0.00215	0.00228	0.00219	0.00205
[0/90/0/90]	Kahya (2012)	0.00249	0.00260	0.00395	0.00423	0.00406	0.00379
	P.Study	0.00220	0.00227	0.00347	0.00371	0.00356	0.00332
$[90/0]_s$	Kahya (2012)	0.00587	0.00620	0.00942	0.01007	0.00975	0.00896
	P.Study	0.00576	0.00608	0.00920	0.00989	0.00956	0.00885
$[45/-45]_s$	Kahya (2012)	0.01296	0.01380	0.02087	0.02243	0.02172	0.01996
	P.Study	0.01043	0.01107	0.01676	0.01801	0.01735	0.01612

analysis are as follows:

$$E_1=144.8\times10^6\ \text{kN/m}^2,\ E_2=9.65\times10^6\ \text{kN/m}^2,\ G_{12}=4.14\times10^6\ \text{kN/m}^2,\ \nu_{12}=0.30,\ \rho=1389.23\ \text{kg/m}^3$$

Table 2 shows the displacement of simply supported laminated beams for various load velocity and lamination scheme. All displacements are in mm. α is the non-dimensional velocity and is defined as $\alpha=c/c_{cr}$ where $c_{cr}=\omega_1 L/\pi$ is the critical velocity. Slenderness ratio (L/h) is considered as 15. As seen from the table, results obtained from the presented method are very close to those obtained by Kahya (2012).

5. Laminated orthotropic beams on three parameter elastic foundation

In this numerical example, a beam resting on three parameter elastic foundation subjected to a moving concentrated load is considered. Width and span of the beam are 25 cm and 500 cm respectively. The slenderness ratio (L/h) is considered as 20. Four different boundary conditions are considered such as simply supported at each end (SS), clamped supported at each end (CC), clamped supported at one end and simply supported at the other end (CS) and beam on freely resting on elastic foundation (FF). The maximum displacement and first five frequencies of 4-layer angle-ply laminated orthotropic beams are examined. The laminates are constructed by a material with the following mechanical properties:

$$E_1=172.5\times10^6\ \text{kN/m}^2,\ E_2=6.9\times10^6\ \text{kN/m}^2,$$
$$G_{12}=3.45\times10^6\ \text{kN/m}^2,\ \nu_{12}=0.25,\ \rho=1578\ \text{kg/m}^3$$

The beam has four equally thick layers. Also, the beam is subjected to concentrated moving load with an intensity of 250 kN. Three-parameter elastic foundation model is

Table 3 The effect of the boundary condition and lamination scheme on the response soil parameters

Boundary conditions		γ	C_k (kN/m^3)	C_{2t} (kN/m^2)
CC	$[0/90]_s$	2.08486	20085.64	29712.36
	[0/90/0/90]	2.11708	20245.27	29411.01
	$[90/0]_s$	2.18100	20571.71	28821.97
	$[45/-45]_s$	2.21343	20742.11	28527.83
CS	$[0/90]_s$	1.96449	19519.57	30863.15
	[0/90/0/90]	1.99034	19637.04	30612.80
	$[90/0]_s$	2.05741	19952.31	29971.40
	$[45/-45]_s$	2.09694	20145.12	29598.99
SS	$[0/90]_s$	1.85680	19055.17	31923.23
	[0/90/0/90]	1.88169	19158.88	31675.89
	$[90/0]_s$	1.94632	19438.36	31040.14
	$[45/-45]_s$	1.98953	19633.33	30620.61
FF	$[0/90]_s$	1.14908	17054.95	39170.48
	[0/90/0/90]	1.26447	17258.17	38008.99
	$[90/0]_s$	1.43067	17632.07	36295.98
	$[45/-45]_s$	1.51224	17852.32	35448.48

Table 4 First five frequencies for various boundary conditions and lamination scheme

Boundary conditions	Beam	Frequencies (Hertz)				
		f_1	f_2	f_3	f_4	f_5
CC	$[0/90]_s$	37.671	71.862	113.387	157.505	201.932
	0/90/0/90	33.954	63.728	101.978	144.248	188.067
	$[90/0]_s$	30.155	52.470	83.728	119.926	160.139
	$[45/-45]_s$	29.160	48.273	74.191	108.094	146.384
CS	$[0/90]_s$	33.175	67.058	109.930	155.450	200.395
	0/90/0/90	30.552	58.579	97.124	140.452	185.395
	$[90/0]_s$	28.122	48.305	78.040	114.441	155.075
	$[45/-45]_s$	27.465	44.227	68.757	100.348	138.025
SS	$[0/90]_s$	29.415	61.897	106.202	153.248	199.678
	0/90/0/90	28.024	53.238	92.216	136.176	182.785
	$[90/0]_s$	26.553	44.545	72.865	108.850	149.811
	$[45/-45]_s$	26.175	40.859	63.173	92.919	129.636
FF	$[0/90]_s$	23.106	35.330	59.248	97.842	144.218
	0/90/0/90	22.974	34.539	54.716	86.151	127.011
	$[90/0]_s$	22.874	33.235	50.082	73.523	104.137
	$[45/-45]_s$	22.873	32.504	47.729	67.772	92.985

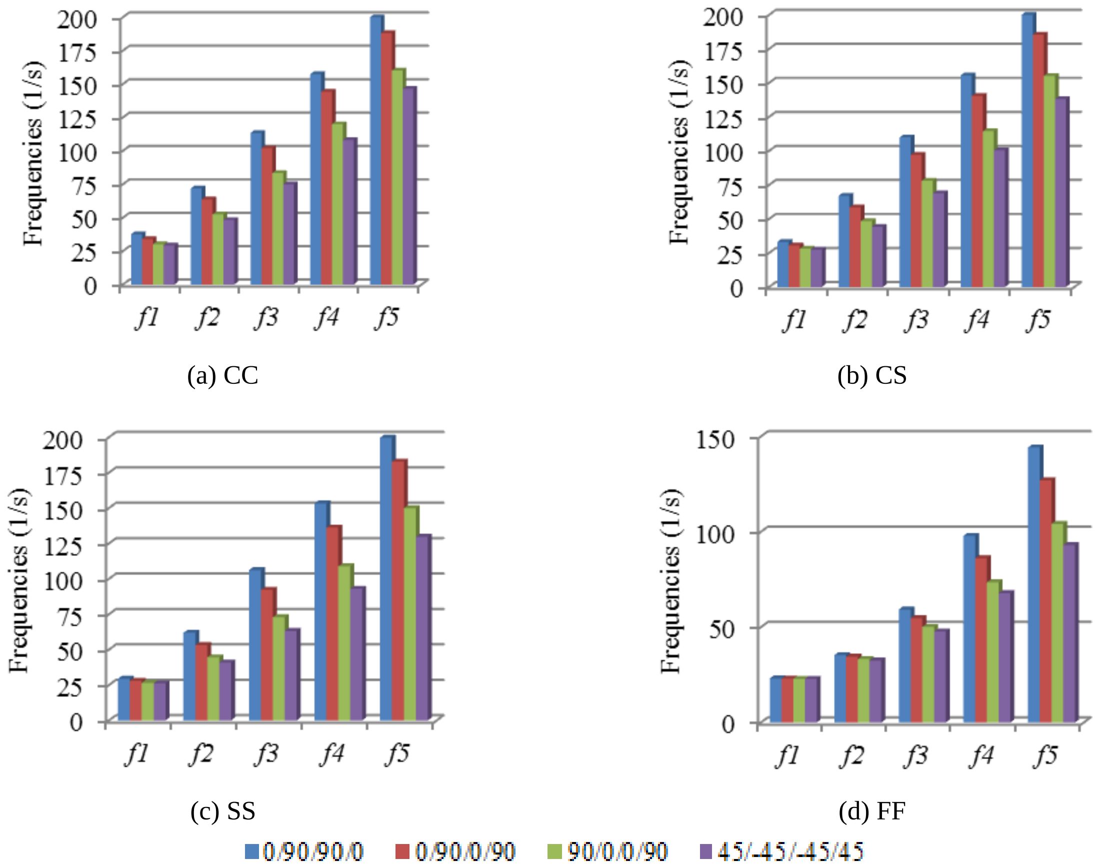


Fig. 3 Comparison of first five frequencies for various boundary conditions and lamination scheme

considered under the beam. Elasticity modulus of subsoil (E_{s1}= E_{s2}) is 68950 kN/m^2, Poisson ratio of the subsoil (ν_s) equals to 0.25 and subsoil depth (H_s) under the beam is considered as 5 m. The beam is modelled in SAP2000 using 4-node thick plate element. 20 finite elements are used for 5 m length of the beam.

The example is solved for centrally concentrated static load as considering the ratio of L/h=20 and soil parameters are tabulated in Table 3. As seen from the table, vertical deformation parameter (γ) and subgrade reaction modulus (C_k) increase from [0/90]$_s$ lamination to [45/-45]$_s$ lamination while shear parameter (C_{2t}) decreases in the same cases. Similarly, vertical deformation parameter and subgrade reaction modulus decrease from CC boundary condition to FF boundary condition while shear parameter increases in the same cases.

First five frequencies of laminated orthotropic beams resting on three-parameter elastic foundation for various boundary conditions and lamination scheme are presented in Table 4 and Fig. 3. Soil parameters presented in Table 3 are used in the frequency analysis.

Frequencies are listed from the smallest to the largest as CC, CS, SS and FF when viewed in terms of boundary conditions. If examined in terms of lamination scheme, frequencies is listed from the smallest to the largest as [45/-45]$_s$, [90/0]$_s$, [0/90/0/90]$_s$ and [0/90]$_s$. [45/-45]$_s$ lamination scheme for FF boundary condition gives the smallest frequencies in comparison to the other lamination scheme

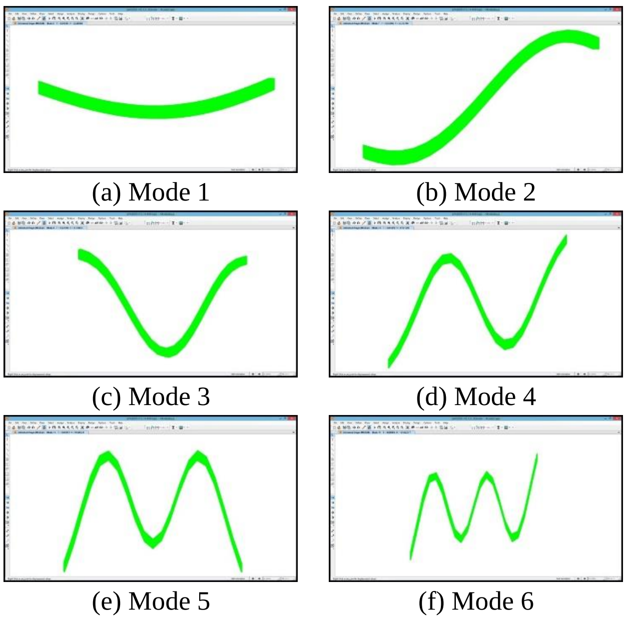


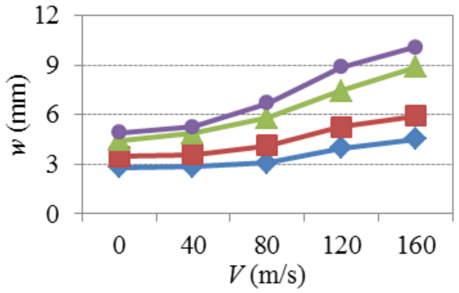
Fig. 4 First six mode shapes for [45/-45]$_s$ laminated orthotropic beam on elastic foundation

and boundary conditions. The difference between frequencies for various lamination scheme and boundary conditions is greater for higher modes.

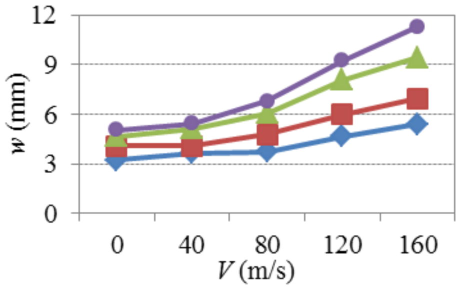
In this study, mode shapes of the beam are also obtained for all parameters considered but since the presentation of

Table 5 Displacements (mm) of laminated orthotropic beams for various support conditions, velocity of moving load and lamination scheme

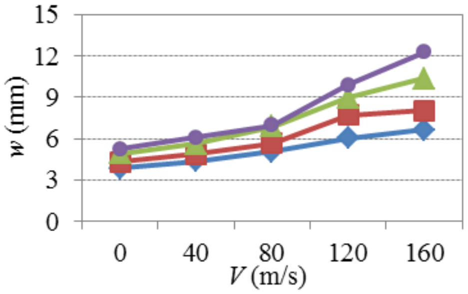
Boundary Conditions		Velocity (m/s)					
		Without soil	0	40	80	120	160
CC	[0/90]$_s$	5.24	2.78	2.83	3.08	3.95	4.51
	[0/90/0/90]	8.53	3.45	3.56	4.12	5.23	5.89
	[90/0]$_s$	19.88	4.44	4.85	5.80	7.44	8.85
	[45/-45]$_s$	35.33	4.90	5.26	6.69	8.87	10.06
CS	[0/90]$_s$	7.96	3.23	3.66	3.73	4.66	5.40
	[0/90/0/90]	13.50	3.85	4.07	4.77	5.98	6.94
	[90/0]$_s$	34.14	4.68	5.14	6.08	8.08	9.45
	[45/-45]$_s$	65.10	5.06	5.45	6.82	9.25	11.31
SS	[0/90]$_s$	15.10	3.84	4.35	5.05	6.03	6.64
	[0/90/0/90]	25.22	4.30	4.88	5.66	7.71	8.03
	[90/0]$_s$	74.13	4.94	5.64	6.88	8.97	10.38
	[45/-45]$_s$	161.39	5.24	6.07	6.97	9.91	12.27
FF	[0/90]$_s$	–	4.45	10.07	10.39	8.55	11.40
	[0/90/0/90]	–	4.50	10.44	9.96	9.30	12.64
	[90/0]$_s$	–	5.24	8.51	8.89	10.52	15.84
	[45/-45]$_s$	–	5.50	8.99	9.31	11.29	18.46



(a) CC

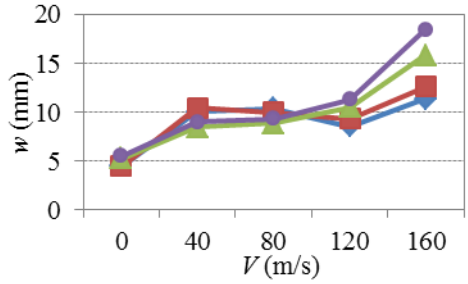


(b) CS



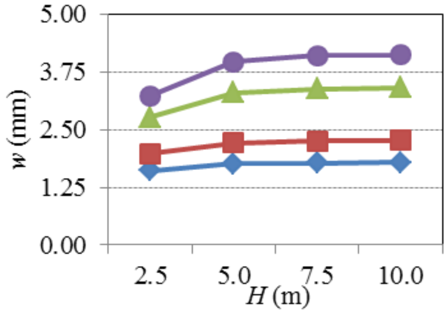
(c)

Fig. 5 The effect of the load velocity and lamination scheme on the response of laminated orthotropic beams

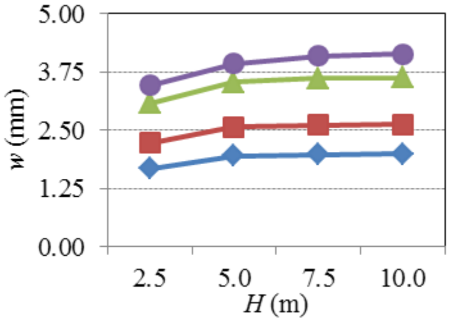


(d) FF

Fig. 5 Continued



(a) for centrally concentrated load



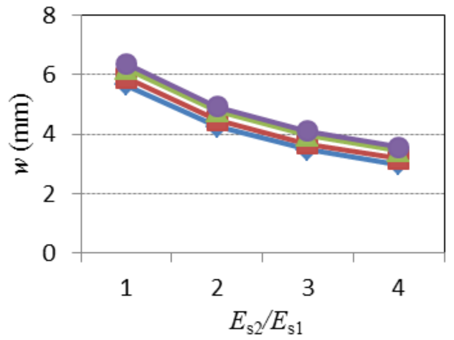
(b) for moving load

Fig. 6 The effect of the subsoil depth on the central displacement of laminated orthotropic beams for SS boundary conditions

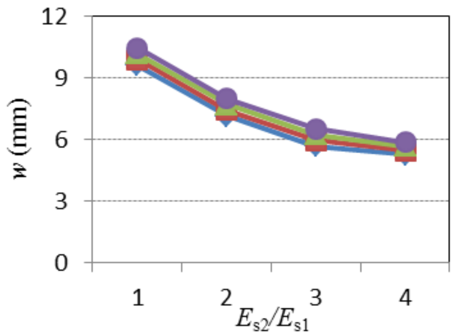
all mode shapes would take up excessive space, only mode shapes corresponding to six lowest frequency for [45/-45]$_s$ and FF boundary condition are presented. These mode shapes are given in Fig. 4.

Effects of soil-structure interaction and load velocity on the responses of laminated orthotropic beam for various boundary conditions and lamination scheme are investigated using soil parameters presented in Table 3. At first the beam is considered without and with subsoil effect for centrally concentrated load, and then the analysis is performed for beam subjected to moving load with various velocities such as 40, 80, 120 and 160 m/s. Maximum displacements obtained from the analysis are presented in Table 5 and Fig. 5.

As seen from Table 5 and Fig. 5, maximum



(a) for concentrated static load



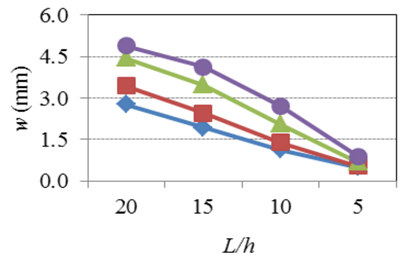
(b) for moving load

Fig. 7 The effect of subsoil elasticity modulus on the central displacement of laminated orthotropic beams for FF boundary conditions

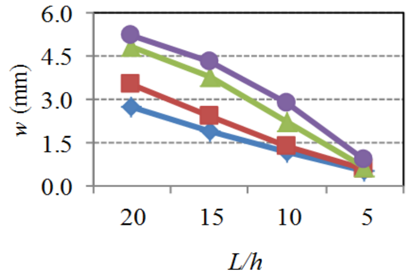
displacement of the beam for all boundary conditions tends to increase as the velocity of moving load increases. Effect of lamination scheme on displacement of beam is seen more clearly as the load velocity increases. Maximum displacements increase from $[0/90]_s$ lamination to $[45/-45]_s$ lamination except for FF boundary conditions while maximum displacements increase from CC to FF for each load velocity and lamination scheme. It has also been seen that the effects of lamination scheme reduce when the subsoil is taken into account.

Simply supported beam on elastic foundation is chosen to show the effects of subsoil depth on the responses. Calculations are performed for four depth of the subsoil such as 2.5, 5.0, 7.5 and 10.0 m. Central displacements of beam under centrally concentrated load and moving load separately are plotted in Fig. 6. The slenderness ratio (L/h) is considered as 10 and velocity of the load is 40 m/s. As seen from the figure, central displacement of beam increases as subsoil depth increases but curves for displacement are becoming horizontal as subsoil depth increases. This means that subsoil depth does not affect the results considerably after a certain value of subsoil depth (nearly 5 m for this example).

Further, effects of changes of subsoil elasticity modulus through the depth on responses of beam are investigated for FF boundary condition, L/h=15, H_s=10 m and V=40 m/s. Subsoil elasticity modulus at the top is kept as E_{s1}=68950 kN/m^2 and subsoil elasticity modulus at the bottom, E_{s2}, is changed depending on ratio of E_{s2}/E_{s1}=1, 2, 3 and 4. The variation between these values is linear. Fig. 7 depicts changes of central displacement of beam with various ratios of E_{s2}/E_{s1} for centrally concentrated load and moving load separately. Results show that the central displacements decrease with increasing subsoil elasticity modulus at the bottom. This is expected because the foundation becomes more rigid as the ratio of E_{s2}/E_{s1} increases.



(a) for concentrated static load



(b) for moving load

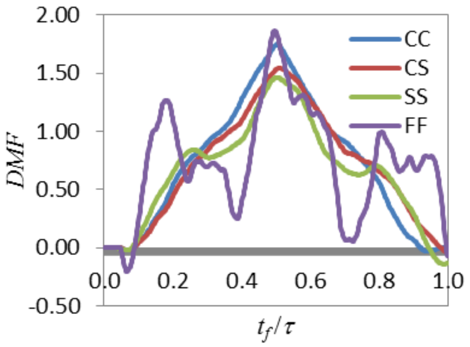
Fig. 8 The effect of beam thickness on the central displacement of laminated orthotropic beams for CC boundary conditions

Hereafter, changes of central displacement of beam with various slenderness ratio (L/h) are given for CC boundary condition in Fig. 8. In this analysis, subsoil depth is set to 5 m. The load moves along the centerline, parallel to x axis with constant amplitude, P=250 kN, and constant velocity, V=40 m/s. Elasticity modulus of subsoil through the depth is kept as 68950 kN/m^2. As seen in figures, displacements decrease as the beam thickness increases. Furthermore, the effect of lamination scheme on the responses decreases and lost its importantance with increasing beam thickness.

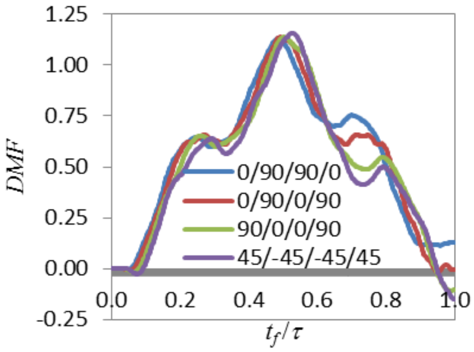
In Fig. 9, variation of dynamic magnification factors (DMF) of the beam with non-dimensional time ($t_f\ /\tau$) is given. Here DMF is defined as the ratio of maximum dynamic displacement (w_d) at the midpoint of the beam to static one (w_s). Non-dimensional time is the ratio of the time at which the load travels on the beam (t_f) to fundamental period of beam (τ). The velocity of moving load is considered as 40 m/s. Subsoil elasticity modulus through the depth is kept constant as 68950 kN/m^2. Ratio of beam length to beam thickness equals 20.

5. Conclusions

In this study, a computer tool is developed to investigate the behavior of laminated orthotropic beams subjected to



(a) for 90/0/0/90 schemes



(b) for SS boundary condition

Fig. 9 Variation of mid-point displacement of beam with load motion

moving load. Modified Vlasov Model is considered to describe the soil medium under the beam. After verifying the accuracy of the proposed model, the effects of velocity of moving load, subsoil depth, elasticity modulus of the subsoil, lamination scheme, slenderness ratio and boundary conditions on the responses are discussed. The conclusions drawn from the study are summarized below.

Comparison of the results obtained with the results in the literature display that the Consistent FEM-Vlasov Model can be used reliably. Frequencies are listed from the smallest to the largest as [45/-45]s, [90/0]s, [0/90/0/90]s and [0/90]s in terms of lamination scheme but the difference between frequencies for various lamination scheme and boundary conditions is greater for higher modes. Taking the subsoil into account reduces the effects of lamination scheme on the results but the effect of lamination scheme emerges more clearly as the load velocity increases. Subsoil depth and subsoil elasticity modulus do not affect the results after a certain value of subsoil depth. Lamination scheme lost its influence on the results with increasing beam thickness.

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