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# Dynamic characteristics of curved inhomogeneous nonlocal porous beams in thermal environment

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**Abstract.** This paper proposes an analytical solution method for free vibration of curved functionally graded (FG) nonlocal beam supposed to different thermal loadings, by considering porosity distribution via nonlocal elasticity theory for the first time. Material properties of curved FG beam are assumed to be temperature-dependent. Thermo-mechanical properties of porous FG curved beam are supposed to vary through the thickness direction of beam and are assumed to be temperature-dependent. Since variation of pores along the thickness direction influences the mechanical and physical properties, porosity play a key role in the mechanical response of curved FG structures. The rule of power-law is modified to consider influence of porosity according to even distribution. The governing equations of curved FG porous nanobeam under temperature field are derived via the energy method based on Timoshenko beam theory. An analytical Navier solution procedure is used to achieve the natural frequencies of porous FG curved nanobeam supposed to thermal loadings with simply supported boundary condition. The results for simpler states are confirmed with known data in the literature. The effects of various parameters such as nonlocality, porosity volume fractions, type of temperature rising, gradient index, opening angle and aspect ratio of curved FG porous nanobeam on the natural frequency are successfully discussed. It is concluded that these parameters play key roles on the dynamic behavior of porous FG curved nanobeam. Presented numerical results can serve as benchmarks for future analyses of curve FG nanobeam with porosity phases.

**Keywords:** curved FG beam; porous materials; thermo-mechanical vibration; thermal loadings; timoshenko beam theory

## 1. Introduction

Functionally graded materials (FGMs) as a new class of composite structures have drawn the attention of many researchers in the smart materials and structures by minimizing or removing stress concentrations at the interfaces of the traditional composite materials (Shen 2016). The material properties of FGMs varies continuously in one or more directions. Due to high strength and high temperature resistance of FGMs, they are increasingly utilized as structural components in modern industries such as mechanical, civil, nuclear reactors, aerospace engineering and etc. as structural components (Miyamoto *et al*. 2013). For more efficient and expand applications of nano structures, they were recently synthesized by using FGMs. Actually, functionally graded model enables the nano materials to have the best properties.

Due to extensive applications of nano-structure made of FG in advanced technology, there has been an affluence of research on the discussion of nano materials with functional graded. The classical continuum theory is aptly practical in the mechanical behavior of the macroscopic structures, but it ignores the size-dependency of nano-structures, thereupon classical continuum theory is not suitable one to micro and nano scales structures. In order to bypass this defect, Eringen's nonlocal elasticity theory is suggested to consider size effect (Eringen 2002). According to this model, the stress state at a certain point is considered as a function of strain states of all points in its area. Among of assortment of nano structures, nanobeams have huge applications in the modeling of engineering applications (Eltaher *et al*. 2013, Ebrahimi and Barati 2017a, b, c, d, e, f, g, Ehyaei and Daman 2017, Ebrahimi and Shaghaghi 2016, Ebrahimi and Dabbagh 2017, Ebrahimi and Salari 2016a, b, Ebrahimi and Shafiei 2016). Based on nonlocal Timoshenko beam theory, Vibration characteristics of size dependent FG nanobeams is reported by Rahmani and Pedram (2014). By implementation of nonlocal Eringen theory, Nazemnezhad and Hosseini-Hashemi (2014) researched nonlinear vibration of functionally graded nanobeams. Furthermore, Eltaher *et al.* (2012) presented free vibration analysis of functionally graded size-dependent nanobeams. Meanwhile, Ebrahimi and Salari provided differential transform method for flexural vibrational behavior of FG nanobeams (2015). In another survey, Ebrahimi and Barati (2016) utilized a higher order refined beam theory for dynamic analysis of magneto electro embedded FG structures.

With the rapid progression of materials technology, structures with graded porosity can be considered as one of the latest developments in FGMs. The presence of pores within the microstructures of such materials are taken into account by means of the local density of the material. An efficient way of manufacturing FGMs is by a sintering

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process in which the porosities or micro voids are created by a difference in the solidification of the material constituents (Zhu *et al*. 2001). An investigation has been carried out on porosities existing in FGMs fabricated by a multi-step sequential infiltration technique (Wattanasakulpong *et al*. 2015), according to this work, it is important to take into consideration the porosity effect when designing and analyzing FGM structures. Studies on the vibration responses of porous FGM structures, especially for beams, are still limited in number.

For porous plates, Yaha *et al*. (2015) presented a non-linear vibration analysis of FGM porous annular plates resting on elastic foundations. They reported that porosity volume fraction and type of porosity distribution have a significant impact on the vibrational response of the FGM plates. Recently, Mechab *et al*. (2016) developed a nonlocal elasticity model for free vibration of FG porous nanoplates resting on elastic foundations. Wattanasakulpong and Ungbhakorn (2014) investigated linear and non-linear vibration of porous FGM beams with elastically restrained ends. Also, Wattanasakulpong and Chaikittiratana (2015) predicted flexural vibration of porous FGM beams using Timoshenko beam theory. Ebrahimi and Zia (2015) investigated the large-amplitude nonlinear vibration of porous FGM beams by utilizing Galerkin and multiple scales methods. In addition, as it is obviously known one of the most important features of FG nano-materials is thermal insulations, hence it is essential to assume changing material properties due to thermal environment. Ebrahimi *et al*. (2016) investigated thermo-mechanical vibration response of temperature-dependent FGM beams having porosities. Based on Timoshenko and higher order beam theory, Ebrahimi and Jafari (2016a, b) examined porosity effect on vibration response of FG beams subjected to thermal loadings by using differential transform and Navier solution methods. Huge application of curve nanobeams and nanorings in the empirical experiments and dynamic molecular simulations (Wang and Duan 2008) led many researchers to study the mechanical characteristics of these structures.

In comparison with straight nanobeams, curved ones possess various advantages such as large strokes (Ebrahimi and Barati 2016b). Recently, the use of curved nanobeams has been extended in different systems as nanoswitches, nanovalves and nanofilters. Literature survey indicates that there are few researches on the vibration behavior of FGM curved nanostructures like beam, ring and arches. Yan and Jiang (2011) investigated the electromechanical response of curved piezoelectric nanobeam with the consideration of surface effects. In addition, a new numerical technique, the differential quadrature method has been developed for dynamic analysis of the nanobeams in the polar coordinate system by Kananipour *et al.* (2014). In addition, investigating surface effects on thermomechanical behavior of embedded circular curved nanosize beams has been studied by Ebrahimi and Daman (2016a). However, they (Ebrahimi and Daman 2016a) have presented the radial vibration of embedded double-curved-nanobeam-systems. As well as, Wang and Duan (2008) have surveyed the free vibration problem of nanorings/arches. In this research the problem was formulated on the framework of Eringen's nonlocal theory of elasticity according to allow for the small length scale effect. Moreover, Ansari *et al.* (2013), developed vibration of FG curved microbeams with framework of modified strain gradient elasticity model. Furthermore, Out-of-plane frequency analyze of FG circular curved beams in thermal environment has been investigated by Malekzadeh *et al.* (2010). In addition, Hosseini and Rahmani (2016) presented free vibration of shallow and deep curved functionally graded nanobeam based on nonlocal Timoshenko curved beam model.

Literature search in the area of vibration behavior of FG curve nanobeam indicates that there is no published work considering size-dependency, porosity and thermal effect on vibration characteristics of FG curve nanobeams with different temperature rising based on Timoshenko beam theory. The present research makes the first achievement to develop the thermo-mechanical vibration of curved functionally graded porous nanobeams based on nonlocal Timoshenko beam theory. Curvature rather exists in all of the real beams and nanobeams. Moreover, in previous researches in order to streamline of mathematical equations, straight beam models have been used, whilst curved beam models are more practicable than straight ones. Two kinds of temperature risings namely uniform and linear through the thickness directions are considered. The modified power-law model is exploited to describe gradual variation of thermo-mechanical properties of the porous FG curve nanobeam. The size-dependent formulation is developed for the FGM Timoshenko nanobeam. Applying Hamilton's principle, governing equations of porous FG curve nanobeam are obtained together and they are solved applying an analytical solution method. Dimensionless natural frequencies are obtained respect to the effect of various parameters such as angle of curvature, temperatures changes, mode numbers, power-law index and nonlocal parameter on vibration of curved FG porous nanobeams. Comparison between the results of present research and available data in literature reveals the accuracy of this model.

## 2. Problem formulation

Consider a curved FG porous nanobeam with even porosity distributions with length $L$ in $\theta$ direction and rectangular cross-section of width $b$ and thickness $h$

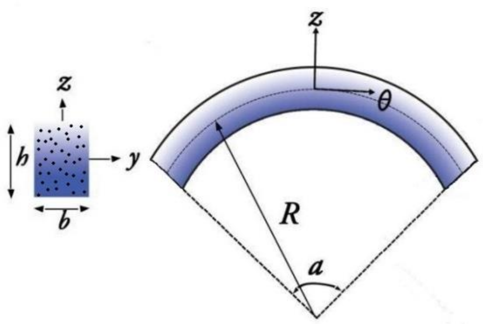



Fig. 1 geometric of curved FG porous nanobeam

Table 1 Temperature dependent coefficients of Young's modulus, thermal expansion coefficient, mass density and thermal conductivity for *SUS*304 and $Si_3N_4$

| Material | Properties | $P_0$ | $P_{-1}$ | $P_1$ | $P_2$ | $P_3$ |
|---|---|---|---|---|---|---|
| $Si_3N_4$ | $E$ (Pa) | 348.43e+9 | 0 | -3.070e-4 | 2.160e-7 | -8.946e-11 |
| | $\lambda_1$ ($K^{-1}$) | 5.8723e-6 | 0 | 9.095e-4 | 0 | 0 |
| | $\rho$ (Kg/$m^3$) | 2370 | 0 | 0 | 0 | 0 |
| | $\kappa$ (W/mk) | 13.723 | 0 | -1.032e-3 | 5.466e-7 | -7.876e-11 |
| *SUS*304 | $E(Pa)$ | 201.04e+9 | 0 | 3.079e-4 | -6.534e-7 | 0 |
| | $\lambda_1$ ($K^{-1}$) | 12.330e-6 | 0 | 8.086e-4 | 0 | 0 |
| | $\rho$ (Kg/$m^3$) | 8166 | 0 | 0 | 0 | 0 |
| | $\kappa$ (W/mk) | 15379 | 0 | -1.264e-3 | 2.092e-6 | -7.223e-10 |

according to Fig. 1. Curved FG porous nanobeam is composed of $Si_3N_4$ and *SUS*304 materials with the thermo-mechanical material properties presented in Table 1 and exposed to a thermal loading.

The relation between length of circular curved beam ($\theta$) and the angle of curvature of beam ($\alpha$) can be written as (Setoodeh *et al*. 2016)

$$\theta = R\alpha \tag{1}$$

The effective material properties of the curved FGM porous beam change continuously in the thickness direction based on modified power-law distribution. According to this model, the effective material properties ($P_f$) of porous FG curve nanobeam by using the modified rule of mixture can be expressed by (Wattanasakulpong and Ungbhakorn 2014)

$$P_f = P_u(V_u - \frac{\vartheta}{2}) + P_l(V_l - \frac{\vartheta}{2}) \tag{2}$$

where $\vartheta$ is the volume fraction of porosity and ($P_l$, $P_u$) are the properties of materials at the lower surface and upper surface, respectively, in addition, ($V_l$, $V_u$) are the corresponding volume fractions related by

$$V_u = \left(\frac{z}{h} + \frac{1}{2}\right)^p \tag{3}$$

$$V_u + V_l = 1 \tag{4}$$

Hence, from Eqs. (2) and (3), the impressive material properties of the curved FG porous beam can be defined as

$$P(z) = (P_u - P_l)\left(\frac{z}{h} + \frac{1}{2}\right)^p + P_l - (P_u + P)\frac{\vartheta}{2} \tag{5}$$

where $p$ is the nonnegative variable parameter (power-law exponent) Power-law exponent determines the distribution profile of material through the thickness of the beam in $z$ direction. Based on this distribution, the bottom surface ($z=h/2$) is pure *SUS*304, whiles the top surface ($z=-h/2$) of curved FG porous nanobeam stands for pure $Si_3N_4$. To prognosticate the treatment of curved FG materials under high temperature more accurately, it is essential to assume the temperature dependency on material properties. The nonlinear equation of thermo-elastic material properties in function of temperature $T(K)$ can be defined as (Touloukian 1966)

$$P = P_0\left(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3\right) \tag{6}$$

where $P_0$, $P_{-1}$, $P_1$, $P_2$ and $P_3$ are the temperature dependent factors that accessible in the Table 1. Based on Timoshenko beam theory, the displacement field at any point of the curved beam model can be remarked as (Hosseini and Rahmani 2016)

$$u_\theta(\theta, z, t) = \left(1 + \frac{z}{R}\right)u(\theta, t) + z\varphi(\theta, t) \tag{7a}$$

$$u_z(\theta, z, t) = w(\theta, t) \tag{7b}$$

where $w$ and $u$ interpret the radial and tangential displacement of curved FG porous beam. In addition, $\varphi$ is total bending rotation of cross sections of curved FGP beam. The nonzero strains of Timoshenko curved beam theory are expressed as

$$\varepsilon_{\theta\theta}^0 = \frac{\partial w}{\partial \theta} - \frac{u}{R} \tag{8a}$$

$$\kappa = \frac{\partial \varphi}{\partial \theta} \tag{8b}$$

$$\gamma_{\theta z} = \frac{\partial u}{\partial \theta} - \varphi + \frac{w}{R} \tag{8-c}$$

Here $\gamma$ denotes shear strain in curved beam model.

$$\varepsilon_{\theta\theta} = \left(\varepsilon_{\theta\theta}^0 + z\kappa\right) \tag{9}$$

Through extended Hamilton's principle, the governing equation of motion can be derived by

$$\int_0^t \delta\left(U_s - T + W_{ext}\right) \tag{10}$$

where $U_s$, $T$ and $W_{ext}$ are strain energy, kinetic energy and work done by external exerted loads, respectively. The first variation of strain energy $U_s$ can be written as

$$\delta U_s = \int_V \left(\sigma_{\theta\theta}\delta\varepsilon_{\theta\theta} + \sigma_{\theta z}\delta\gamma_{\theta z}\right)dV \tag{11}$$

Substituting Eqs. (6) and (7) into Eq. (21) the first variation of strain energy yields

$$\delta U_s = \int_0^L \left(N\left(-\frac{\delta u}{R} + \frac{\partial \delta w}{\partial \theta}\right) + M\left(\frac{\partial \delta\varphi}{\partial \theta}\right) + Q\left(\frac{\partial \delta u}{\partial \theta} - \delta\varphi + \frac{\delta w}{R}\right)\right)d\theta \tag{12}$$

In which the variables at the last expression are expressed as: $M$, $N$, and $Q$ define bending moment of cross section, axial force, and shear force, respectively. These stress resultants are expressed as

$$N = \int_A \sigma_{\theta\theta} dA \;,\; M = \int_A \sigma_{\theta\theta} z dA \;,\; Q = \int_A K_{shear}\sigma_{\theta z} dA \tag{13}$$

where $K_{Shear}$ expresses the shear correction factor. The first variational of the virtual kinetic energy of present curve beam model can be written in the form as

$$T = \frac{1}{2}\int_0^L \int_A \rho(z,T)\left(\left(\frac{\partial u_\theta}{\partial t}\right)^2 + \left(\frac{\partial u_z}{\partial \theta}\right)^2\right) dA d\theta \tag{14}$$

$$\begin{aligned}\delta T = [I_0(\frac{\partial u}{\partial t}\frac{\partial \delta u}{\partial t}) + (I_0 + \frac{2I_1}{R} + \frac{I_2}{R^2})\frac{\partial w}{\partial t}\frac{\partial \delta w}{\partial t} \\ +(I_1 + \frac{I_2}{R})(\frac{\partial \varphi}{\partial t}\frac{\partial \delta w}{\partial t} + \frac{\partial w}{\partial t}\frac{\partial \delta \varphi}{\partial t}) + I_2\frac{\partial \varphi}{\partial t}\frac{\partial \delta \varphi}{\partial t}]\end{aligned} \tag{15}$$

where mass moments of inertias ($I_0$,$I_1$,$I_2$) are defined as follows

$$(I_0, I_1, I_2) = \int_A \rho(z,T)(1, z, z^2) dA \tag{16}$$

The first variation of work done by applied forces can be written in the form

$$\delta W_{ext} = \frac{1}{2}\int_0^L (N_T)\frac{\partial w}{\partial \theta}\delta\frac{\partial w}{\partial \theta} d\theta \tag{17}$$

where $N_T$ is in-plane applied load namely thermal loadings, which can be given as

$$N_T = \int_{\frac{-h}{2}}^{\frac{h}{2}} E(z,T)\lambda_1(z,T)\Delta T dz \tag{18}$$

By inserting the coefficients of $\delta u$, $\delta w$, $\delta\varphi$ and $\delta\psi$ equal to zero, following Euler-Lagrange equations of curved FG porous nanobeam subjected to thermal loading are obtained

$$\frac{N}{R} + \frac{\partial Q}{\partial \theta} = I_0\frac{\partial^2 u}{\partial t^2} \tag{19a}$$

$$\begin{aligned}&\frac{\partial N}{\partial \theta} - \frac{Q}{R} - N_T\frac{\partial^2 w}{\partial \theta^2} \\ &= \left(I_0 + \frac{2I_1}{R} + \frac{I_2}{R^2}\right)\frac{\partial^2 w}{\partial t^2} + \left(I_1 + \frac{I_2}{R}\right)\frac{\partial^2 \varphi}{\partial t^2}\end{aligned} \tag{19b}$$

$$\begin{aligned}&\frac{\partial M}{\partial \theta} + Q \\ &= \left(I_1 + \frac{I_2}{R}\right)\frac{\partial^2 w}{\partial t^2} + I_2\frac{\partial^2 \varphi}{\partial t^2}\end{aligned} \tag{19c}$$

Boundary conditions that are related to equation of motions are considered as

$$N = 0 \text{ or } w = 0 \text{ at } \theta = 0 \text{ and } \theta = L \tag{20a}$$

$$Q = 0 \text{ or } u = 0 \text{ at } \theta = 0 \text{ and } \theta = L \tag{20b}$$

$$M = 0 \text{ or } \varphi = 0 \text{ at } \theta = 0 \text{ and } \theta = L \tag{20c}$$

## 3. The nonlocal elasticity model for curved FG porous nanobeam

Despite the fundamental equations in classic elasticity theory, Eringen's nonlocal model (Eringen 2002) explains that the stress at a certain point x in a body is assumed as a function of strains of all points $x'$ in the near realm. This supposition is very good agreement with experiments of atomic model and lattice dynamics in phonon scattering in which for a nonlocal piezoelectric material. The basic equations with zero body force can be given as

$$\sigma_{ij} = \int_V \alpha(|x' - x|, \tau)\left[C_{ijkl}\varepsilon_{kl}(x')\right] dV(x') \tag{21}$$

where $\sigma_{ij}$ and $\varepsilon_{ij}$ are the stress, strain components, respectively; $\alpha(|x'-x|, \tau)$ is the nonlocal kernel function and $|x'-x|$ is the nonnegative distance. $\tau = e_0 a/l$ is given as size coefficient. However, the relations in Eq. (21) causes the elasticity problems difficult to solve, in addition to possible lack of determinism. Eringen (2002) presented in detail properties of non-local kernel $\alpha(|x'-x|)$ and evaluated that when a kernel takes a Green's function of linear differential operator

$$L\alpha(|x' - x|) = \delta(|x' - x|) \tag{22}$$

By matching the scattering curves with lattice models, Eringen (2002) supposed a nonlocal theory with the linear differential operator $L$ expressed as follow

$$L = 1 - (e_0 a)^2 \nabla^2 \tag{23}$$

where $\nabla^2$ is the Laplacian operator. Therefore, the fundamental relations given by Eq. (21) for nonlocal elasticity may be rewritten by differentiable form as

$$\sigma_{ij} - (e_0 a)^2 \nabla^2 \sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{24}$$

The parameter $e_0 a$ is the scale coefficient disclosing the nano scale effect on the responses of structures of nanoscale. The nonlocal parameter, $\mu = (e_0 a)$ is experimentally determined for different materials. For a curved FGM porous nanobeam under thermo-mechanical loading in the one-dimensional case, the nonlocal fundamental relations (24a) and (24b) can be streamlined as

$$\sigma_{\theta\theta} - \mu^2\frac{\partial^2 \sigma_{\theta\theta}}{\partial \theta^2} = E(z,T)\varepsilon_{\theta\theta} \tag{25a}$$

$$\sigma_{\theta z} - \mu^2\frac{\partial^2 \sigma_{\theta z}}{\partial \theta^2} = G(z,T)\gamma_{\theta z} \tag{25b}$$

Calculating Eq. (25) by integrating over cross-section area of the curved beam, force-strain and moment–strain of nonlocal curved FG porous Timoshenko beam model will be determined as

$$N - \mu^2\frac{\partial^2 N}{\partial \theta^2} = A\left(\frac{\partial w}{\partial \theta} - \frac{u}{R}\right) + B\frac{\partial \varphi}{\partial \theta} \tag{26a}$$

$$M - \mu^2\frac{\partial^2 M}{\partial \theta^2} = B\left(\frac{\partial w}{\partial \theta} - \frac{u}{R}\right) + D\frac{\partial \varphi}{\partial \theta} \tag{26b}$$

$$Q - \mu^2\frac{\partial^2 Q}{\partial \theta^2} = K_{Shear}C\left(\frac{w}{R} + \frac{\partial u}{\partial \theta} - \varphi\right) \tag{26c}$$

where $K_{Shear}$ defined as correction factor and assumed to be 5/6 . Consequently, coefficients are obtained as

$$\{A, B, D\} = \int_A E(z,T)\{1, z, z^2\} dA \tag{27a}$$

$$C=\int_A \frac{E(z,T)}{1+\nu(z,T)}dA \tag{27b}$$

The nonlocal governing equations of curved FG Porous Timoshenko nanobeam supposed to thermal loading in terms of displacement can be derived by inserting Eq. (27) into Eq. (19) as follows

$$\begin{aligned}&A\left(\frac{\partial w}{\partial\theta}-\frac{u}{R}\right)+B\frac{\partial\varphi}{\partial\theta}\\&+K_{Shear}CR\left(\frac{\partial w}{R\partial\theta}+\frac{\partial^2 u}{\partial\theta^2}-\frac{\partial\varphi}{\partial\theta}\right)\\&=I_0R\left(\frac{\partial^2 u}{\partial t^2}-\mu^2\frac{\partial^4 u}{\partial\theta^2\partial t^2}\right)\end{aligned} \tag{28a}$$

$$\begin{aligned}&AR\left(\frac{\partial^2 w}{\partial\theta^2}-\frac{\partial u}{R\partial\theta}\right)+BR\frac{\partial^2\varphi}{\partial\theta^2}\\&-N_TR\frac{\partial^2 w}{\partial\theta^2}+\mu^2N_TR\frac{\partial^4 w}{\partial\theta^4}\\&-K_{Shear}C\left(\frac{w}{R}+\frac{\partial u}{\partial\theta}-\varphi\right)\\&=\left(I_0+\frac{2I_1}{R}+\frac{I_2}{R^2}\right)R\frac{\partial^2 w}{\partial t^2}\\&+\left(I_1+\frac{I_2}{R}\right)R\frac{\partial^2\varphi}{\partial t^2}\\&-\mu^2\left(I_0+\frac{2I_1}{R}+\frac{I_2}{R^2}\right)R\frac{\partial^4 w}{\partial\theta^2\partial t^2}\\&-\mu^2\left(I_1+\frac{I_2}{R}\right)R\frac{\partial^4\varphi}{\partial\theta^2\partial t^2}\end{aligned} \tag{28b}$$

$$\begin{aligned}&B\left(\frac{\partial\ w}{\partial\theta^2}-\frac{\partial u}{R\partial\theta}\right)+D\frac{\partial^2\varphi}{\partial\theta^2}\\&+K_{Shear}C\left(\frac{w}{R}+\frac{\partial u}{\partial\theta}-\varphi\right)\\&=\left(I_1+\frac{I_2}{R}\right)\frac{\partial^2 w}{\partial t^2}+I_2\frac{\partial^2\varphi}{\partial t^2}\\&-\mu^2\left(I_1+\frac{I_2}{R}\right)\frac{\partial^4 w}{\partial\theta^2\partial t^2}-\mu^2 I_2\frac{\partial^4\varphi}{\partial\theta^2\partial t^2}\end{aligned} \tag{28c}$$

## 4. Solution method

In this section, analytical Navier method has been developed to solve the governing equations of curved FG in regard to find out free vibrational of a simply supported curved porous FG nanobeam. To satisfy simply supported boundary condition, the displacement quantities are presented in the following form as trigonometric functions

$$u(\theta,t)=\sum_{n=1}^{\infty}U_nCos\left(\frac{n\pi}{L}\theta\right)e^{i\omega_n t} \tag{29a}$$

$$w(\theta,t)=\sum_{n=1}^{\infty}W_nSin\left(\frac{n\pi}{L}\theta\right)e^{i\omega_n t} \tag{29b}$$

$$\varphi(\theta,t)=\sum_{n=1}^{\infty}\phi_nCos\left(\frac{n\pi}{L}\theta\right)e^{i\omega_n t} \tag{29c}$$

where $W_n$, $U_n$ and $\phi_n$ are the unknown coefficients which are obtained for each $n$ value. The boundary conditions for simply supported curved FG porous nanobeam can be given as

$$\begin{gathered}w(0)=0,\ \frac{\partial w}{\partial\theta}(L)=0,\ u(0)=u(L)=0\\\frac{\partial\varphi}{\partial\theta}(0)=\frac{\partial\varphi}{\partial\theta}(L)=0\end{gathered} \tag{30}$$

Inserting Eq. (29) into Eq. (28) respectively, leads to Eq. (31)

$$\begin{aligned}&A\left(-\frac{U_n}{R}-\left(\frac{n\pi}{L}\right)W_n\right)-B\left(\frac{n\pi}{L}\right)\phi_n\\&+K_{Shear}CR\left[-\frac{1}{R}\left(\frac{n\pi}{L}\right)W_n-\left(\frac{n\pi}{L}\right)^2U_n+\left(\frac{n\pi}{L}\right)\phi_n\right]\\&=RI_0\left(-\omega_n^2U_n-\mu^2\omega_n^2\left(\frac{n\pi}{L}\right)^2U_n\right)\end{aligned} \tag{31a}$$

$$\begin{aligned}&AR\left[-\frac{1}{R}\left(\frac{n\pi}{L}\right)U_n-\left(\frac{n\pi}{L}\right)^2W_n\right]\\&-BR\left(\frac{n\pi}{L}\right)^2\phi_n-N_TR\left(\frac{n\pi}{L}\right)^2W_n\\&-\mu^2N_TR\left(\frac{n\pi}{L}\right)^4W_n\\&-K_{Shear}C\left[\frac{1}{R}W_n+\left(\frac{n\pi}{L}\right)U_n-\phi_n\right]=\\&-\left(I_0+\frac{2I_1}{R}+\frac{I_2}{R^2}\right)R\omega_n^2W_n\\&-\left(I_1+\frac{I_2}{R}\right)R\omega_n^2\phi_n\\&-\mu^2R\left(\frac{n\pi}{L}\right)^2\left(I_0+\frac{2I_1}{R}+\frac{I_2}{R^2}\right)\omega_n^2W_n\\&-\mu^2\left(I_1+\frac{I_2}{R}\right)R\left(\frac{n\pi}{L}\right)^2\omega_n^2\phi_n\end{aligned} \tag{31b}$$

$$\begin{aligned}&BR\left[-\frac{1}{R}\left(\frac{n\pi}{L}\right)U_n+\left(\frac{n\pi}{L}\right)^2W_n\right]-RD\left(\frac{n\pi}{L}\right)^2\phi_n\\&+K_{Shear}CR\left[\frac{1}{R}W_n+\left(\frac{n\pi}{L}\right)U_n-\phi_n\right]\\&=-R\left(I_1+\frac{I_2}{R}\right)\omega_n^2W_n-RI_2\omega_n^2\phi_n\\&-R\mu^2\left(I_1+\frac{I_2}{R}\right)\left(\frac{n\pi}{L}\right)^2\omega_n^2W_n\\&-R\mu^2I_2\left(\frac{n\pi}{L}\right)^2\omega_n^2\phi_n\end{aligned} \tag{31c}$$

Table 2 Comparison of dimensionless natural frequencies of S-S curved FG nanobeams for different amounts of slenderness, mode number and nonlocality where $p$=0 and $\alpha=\pi/3$

| $L/h$ | $\omega_n$ | $\mu^2$=0 | | $\mu^2$=1 | | $\mu^2$=2 | | $\mu^2$=3 | | $\mu^2$=4 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Hosseini and Rahmani (2016) | Present | Hosseini and Rahmani (2016) | Present | Hosseini and Rahmani (2016) | Present | Hosseini and Rahmani (2016) | Present | Hosseini and Rahmani (2016) | Present |
| 10 | $n$=1 | 8.1991 | 6.9425 | 6.9425 | 7.2020 | 7.2020 | 7.4929 | 7.4929 | 7.8222 | 7.8222 | 8.1991 |
| | $n$=2 | 35.7451 | 22.2576 | 22.2576 | 24.1855 | 24.1855 | 26.7204 | 26.7204 | 30.2666 | 30.2666 | 35.7451 |
| | $n$=3 | 77.3993 | 36.2732 | 36.2732 | 40.4308 | 40.4308 | 46.4500 | 46.4500 | 56.3256 | 56.3256 | 77.3993 |
| 20 | $n$=1 | 8.2912 | 7.0205 | 7.0205 | 7.2829 | 7.2829 | 7.5771 | 7.5771 | 7.9101 | 7.9101 | 8.2912 |
| | $n$=2 | 37.2875 | 23.2180 | 23.2180 | 25.2291 | 25.2291 | 27.8733 | 27.8733 | 31.5725 | 31.5725 | 37.2875 |
| | $n$=3 | 84.3180 | 39.5156 | 39.5156 | 44.0449 | 44.0449 | 50.6022 | 50.6022 | 61.3605 | 61.3605 | 84.3180 |
| 30 | $n$=1 | 8.3177 | 7.0429 | 7.0429 | 7.3061 | 7.3061 | 7.6012 | 7.6012 | 7.9353 | 7.9353 | 8.3177 |
| | $n$=2 | 37.7658 | 23.5159 | 23.5159 | 25.5527 | 25.5527 | 28.2309 | 28.2309 | 31.9776 | 31.9776 | 37.7658 |
| | $n$=3 | 86.7084 | 40.6359 | 40.6359 | 45.2935 | 45.2935 | 52.0367 | 52.0367 | 63.1000 | 63.1000 | 86.7084 |

Table 3 Comparison of dimensionless natural frequency of S-S curved FG nanobeams for different amounts of slenderness, mode number and nonlocality where $p$=1 and $\alpha=\pi/2$

| $L/h$ | $\omega_n$ | $\mu^2$=0 | | $\mu^2$=1 | | $\mu^2$=2 | | $\mu^2$=3 | | $\mu^2$=4 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Hosseini and Rahmani (2016) | Present | Hosseini and Rahmani (2016) | Present | Hosseini and Rahmani (2016) | Present | Hosseini and Rahmani (2016) | Present | Hosseini and Rahmani (2016) | Present |
| 10 | $n$=1 | 4.5601 | 4.5601 | 4.3504 | 4.3504 | 4.1673 | 4.1673 | 4.0055 | 4.0055 | 3.8612 | 3.8612 |
| | $n$=2 | 23.7375 | 23.7375 | 20.0993 | 20.0993 | 17.7444 | 17.7444 | 16.0611 | 16.0611 | 14.7808 | 14.7808 |
| | $n$=3 | 53.2817 | 53.2817 | 38.7745 | 38.7745 | 31.9762 | 31.9762 | 27.8325 | 27.8325 | 24.9704 | 24.9704 |
| 20 | $n$=1 | 4.6675 | 4.6675 | 4.4530 | 4.4530 | 4.2655 | 4.2655 | 4.0999 | 4.0999 | 3.9522 | 3.9522 |
| | $n$=2 | 25.0039 | 25.0039 | 21.1716 | 21.1716 | 18.6911 | 18.6911 | 16.9179 | 16.9179 | 15.5694 | 15.5694 |
| | $n$=3 | 58.3285 | 58.3285 | 42.4472 | 42.4472 | 35.0050 | 35.0050 | 30.4689 | 30.4689 | 27.3356 | 27.3356 |
| 30 | $n$=1 | 4.7208 | 4.7208 | 4.5038 | 4.5038 | 4.3142 | 4.3142 | 4.1466 | 4.1466 | 3.9972 | 3.9972 |
| | $n$=2 | 25.5362 | 25.5362 | 21.6223 | 21.6223 | 19.0889 | 19.0889 | 17.2780 | 17.2780 | 15.9008 | 15.9008 |
| | $n$=3 | 60.4005 | 60.4005 | 43.9551 | 43.9551 | 36.9551 | 36.9551 | 31.5512 | 31.5512 | 28.3067 | 28.3067 |

By finding determinant of the coefficient matrix of the following equations and setting this multinomial to zero, we can find natural frequencies $\omega_n$.

$$\left\{[K]-\omega_n^2[M]\right\}\begin{Bmatrix} U_n \\ W_n \\ \phi_n \\ \psi_n \end{Bmatrix}=0 \qquad (32)$$

where [$K$] and [$M$] are stiffness and mass matrixes.

## 5. Numerical results and discussion

Through this section, validation of the proposed model is confirmed by comparing the obtained results with those of perfect curve FG nanobeam regardless of temperature risings presented by Hosseini and Rahmani (2016) in Table 2 and Table 3 for different power-law index and opening angles. It is observed that the present results agree very well with the given by Hosseini and Rahmani (2016). Then, the influence of nonlocality, porosity volume fractions, type of temperature rising, gradient index, Then, the influence of nonlocality parameter, porosity volume fraction, FG material graduation, different types of thermal loadings opening angle and aspect ratio on the natural frequencies of porous FG curve nanobeam under thermal loadings explored. The non-dimensional natural frequency ($\lambda$) can be calculated by the relation in Eq. (33) as

$$\lambda=\omega\frac{L^2}{h}\sqrt{\frac{\rho_u}{E_u}} \qquad (33)$$

The amounts of nonlocality for the curved FG porous nanobeams is considered as constant in the numerical results. Increasing the nonlocality parameter tends to decrease the natural frequency. The reason is that the presence of the nonlocal effect tends to decrease the stiffness of the nanostructures and hence decrease the values of natural frequencies.

In Table 4 frequency results of porous FG curved nanobeam under thermal environment are presented for various temperature changings (($\Delta T$=30, 60, 90)), opening angle ($\alpha=\pi/2$, $\pi/3$, $\pi/4$), power-law indexes ($p$ = 0.2, 0.5, 1), porosity volume fractions ($\vartheta$=0,0.1,0.2) and at constant value of aspect ratio ($L/h$=30) and nonlocal parameter ($\mu^2$=2). It can be observed from Table 4 that, by increasing the material gradient index, the dimensionless natural frequencies decrease. This is due to the increment in flexibility of the curved FG nanobeams, while the

Table 4 Temperature and gradient index effect on fundamental frequency of a S-S curved FG nanobeam with various porosity, curvature and thermal loading $L/h$=30, $\mu^2$=2

| | | | $\Delta T$=30 [$K$] | | | $\Delta T$=60 [$K$] | | | $\Delta T$=90 [$K$] | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\vartheta$ | Load type | Power-law Exponent | | | Power-law Exponent | | | Power-law Exponent | | |
| | | | 0.2 | 0.5 | 1 | 0.2 | 0.5 | 1 | 0.2 | 0.5 | 1 |
| α=π/4 | 0 | UTR | 6.5399 | 5.5380 | 4.8444 | 6.4502 | 5.4505 | 4.7589 | 6.3603 | 5.3568 | 4.6666 |
| | | LTR | 6.5703 | 5.5685 | 4.8743 | 6.5275 | 5.5276 | 4.8345 | 6.4848 | 5.4835 | 4.7911 |
| | 0.1 | UTR | 6.8107 | 5.6266 | 4.8418 | 6.7297 | 5.5482 | 4.7655 | 6.6436 | 5.4642 | 4.6831 |
| | | LTR | 6.8385 | 5.6544 | 4.8689 | 6.8000 | 5.6181 | 4.8337 | 6.7591 | 5.5787 | 4.7951 |
| | 0.2 | UTR | 7.1850 | 5.7388 | 4.8351 | 7.1153 | 5.6694 | 4.7680 | 7.0354 | 5.5950 | 4.6954 |
| | | LTR | 7.2102 | 5.7638 | 4.8595 | 7.1760 | 5.7320 | 4.8289 | 7.1397 | 5.6975 | 4.7950 |
| α=π/3 | 0 | UTR | 5.9831 | 5.0550 | 4.4132 | 5.8217 | 4.8901 | 4.2513 | 5.6444 | 4.7113 | 4.0747 |
| | | LTR | 6.0390 | 5.1109 | 4.4679 | 5.9590 | 5.0339 | 4.3925 | 5.8793 | 4.9513 | 4.3111 |
| | 0.1 | UTR | 6.2414 | 5.1446 | 4.4183 | 6.0899 | 4.9971 | 4.2741 | 5.9275 | 4.8378 | 4.1172 |
| | | LTR | 6.2924 | 5.1953 | 4.4678 | 6.2206 | 5.1269 | 4.4009 | 6.1445 | 5.0535 | 4.3287 |
| | 0.2 | UTR | 6.5953 | 5.2559 | 4.4196 | 6.4600 | 5.1257 | 4.2929 | 6.3157 | 4.9855 | 4.1554 |
| | | LTR | 6.6415 | 5.3015 | 4.4638 | 6.5779 | 5.2416 | 4.4055 | 6.5106 | 5.1774 | 4.3424 |
| α=π/2 | 0 | UTR | 4.4773 | 3.7430 | 3.2362 | 4.0290 | 3.2754 | 2.7688 | 3.4896 | 2.7039 | 2.1776 |
| | | LTR | 4.6242 | 3.8902 | 3.3810 | 4.4108 | 3.6824 | 3.1752 | 4.1919 | 3.4521 | 2.9448 |
| | 0.1 | UTR | 4.7116 | 3.8440 | 3.2702 | 4.2967 | 3.4326 | 2.8607 | 3.8170 | 2.9434 | 2.3593 |
| | | LTR | 4.8445 | 3.9765 | 3.3998 | 4.6547 | 3.7932 | 3.2186 | 4.4483 | 3.5916 | 3.0173 |
| | 0.2 | UTR | 5.0212 | 3.9619 | 3.3008 | 4.6564 | 3.6042 | 2.9466 | 4.2434 | 3.1890 | 2.5248 |
| | | LTR | 5.1408 | 4.0800 | 3.4155 | 4.9741 | 3.9208 | 3.2583 | 4.7942 | 3.7470 | 3.0850 |

Table 5 Nonlocality and gradient index effect on fundamental frequency of a S-S curved FG nanobeam with various porosity and thermal loading $L/h$=30, $\alpha=\pi/2$

| $\vartheta$ | Load type | $\mu^2$=1 | | | $\mu^2$=2 | | | $\mu^2$=3 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Power-law Exponent | | | Power-law Exponent | | | Power-law Exponent | | |
| | | 0.2 | 0.5 | 1 | 0.2 | 0.5 | 1 | 0.2 | 0.5 | 1 |
| $\Delta T$=30 [$K$] | | | | | | | | | | |
| 0 | UTR | 4.7107 | 3.9436 | 3.4142 | 4.4773 | 3.7430 | 3.2362 | 4.2696 | 3.5640 | 3.0772 |
| | LTR | 4.8505 | 4.0837 | 3.5518 | 4.6242 | 3.8902 | 3.3810 | 4.4233 | 3.7183 | 3.2291 |
| 0.1 | UTR | 4.9514 | 4.0451 | 3.4457 | 4.7116 | 3.8440 | 3.2702 | 4.4983 | 3.6649 | 3.1137 |
| | LTR | 5.0780 | 4.1713 | 3.5689 | 4.8445 | 3.9765 | 3.3998 | 4.6373 | 3.8036 | 3.2496 |
| 0.2 | UTR | 5.2709 | 4.1642 | 3.4736 | 5.0212 | 3.9619 | 3.3008 | 4.7994 | 3.7819 | 3.1469 |
| | LTR | 5.3849 | 4.2768 | 3.5828 | 5.1408 | 4.0800 | 3.4155 | 4.9244 | 3.9055 | 3.2669 |
| $\Delta T$=60 [$K$] | | | | | | | | | | |
| 0 | UTR | 4.2869 | 3.5032 | 2.9751 | 4.0290 | 3.2754 | 2.7688 | 3.7760 | 3.0692 | 2.5809 |
| | LTR | 4.6475 | 3.8863 | 3.3566 | 4.4108 | 3.6824 | 3.1752 | 4.1890 | 3.5002 | 3.0128 |
| 0.1 | UTR | 4.5584 | 3.6565 | 3.0599 | 4.2967 | 3.4326 | 2.8607 | 4.0616 | 3.2306 | 2.6802 |
| | LTR | 4.8974 | 3.9970 | 3.3969 | 4.6547 | 3.7932 | 3.2186 | 4.4387 | 3.6114 | 3.0593 |
| 0.2 | UTR | 4.9247 | 3.8257 | 3.1392 | 4.6564 | 3.6042 | 2.9466 | 4.4163 | 3.4052 | 2.7729 |
| | LTR | 5.2261 | 4.1253 | 3.4334 | 4.9741 | 3.9208 | 3.2583 | 4.7501 | 3.7387 | 3.1021 |
| $\Delta T$=90 [$K$] | | | | | | | | | | |
| 0 | UTR | 3.7845 | 2.9758 | 2.4347 | 3.4896 | 2.7039 | 2.1776 | 3.1818 | 2.4498 | 1.9330 |
| | LTR | 4.4404 | 3.6689 | 3.1535 | 4.1919 | 3.4521 | 2.9448 | 3.9531 | 3.2570 | 2.7690 |
| 0.1 | UTR | 4.1095 | 3.2018 | 2.5975 | 3.8170 | 2.9434 | 2.3593 | 3.5502 | 2.7050 | 2.1368 |
| | LTR | 4.7016 | 3.8063 | 3.2069 | 4.4483 | 3.5916 | 3.0173 | 4.2217 | 3.3991 | 2.8467 |
| 0.2 | UTR | 4.5362 | 3.4374 | 2.7473 | 4.2434 | 3.1890 | 2.5248 | 3.9783 | 2.9621 | 2.3196 |
| | LTR | 5.0551 | 3.9605 | 3.2695 | 4.7942 | 3.7470 | 3.0850 | 4.5612 | 3.5560 | 2.9195 |

percentage of metal phase increases when power-law index increases. Also, ascending the temperature changes decrease in the dimensionless natural frequency and it may be observed that these parameters have a significant effect on the dimensionless frequency parameter. By perusing the result of Table 4, it may be observed that the variations of dimensionless frequencies of porous FG curved nanobeam depends on porosity parameter and gradient index. For example, while $0 \text{ p } p \text{ p } 1$, increasing of porosity parameter leads to increment of non-dimensional natural frequency for

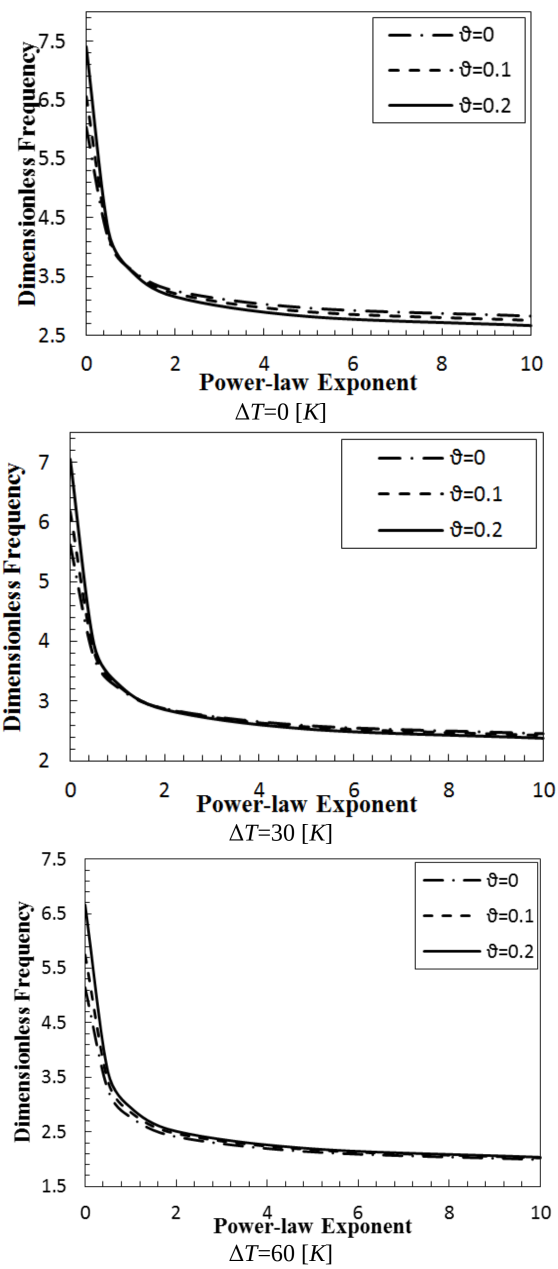



Fig. 2 The variation of the fundamental dimensionless frequency of S-S curved FG porous nanobeam respect to gradient index and porosity for different uniform temperature changes ($\alpha=\pi/2$, $\mu^2=2$, $L/h=30$)

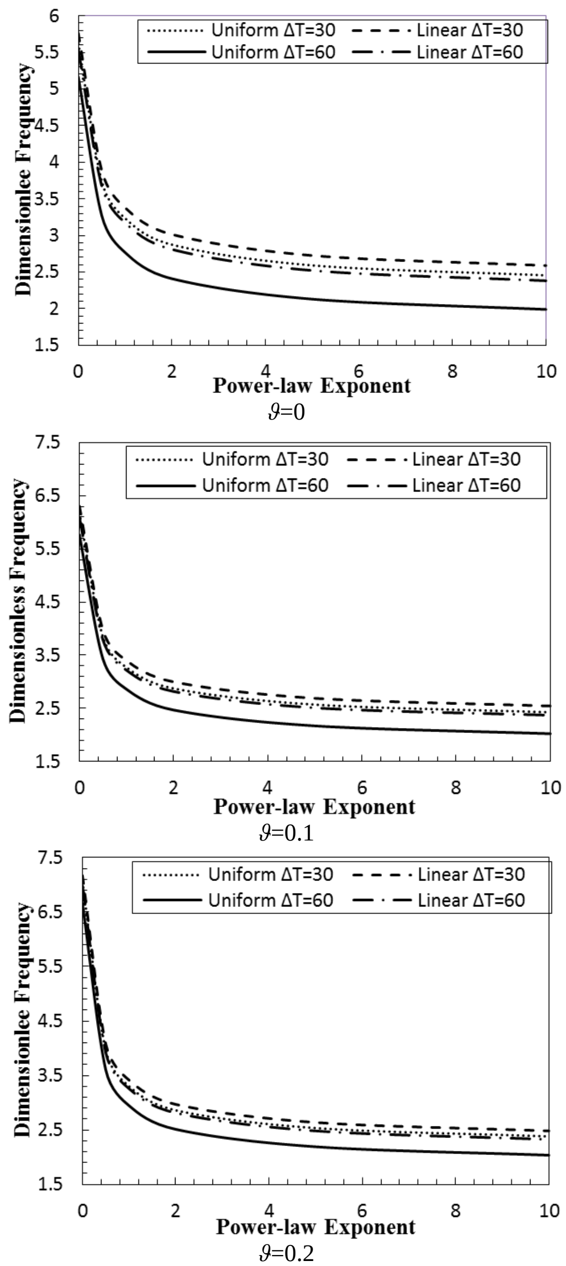



Fig. 3 The variation of the dimensionless frequency of S-S curved FG porous nanobeam respect to gradient index and porosity for different uniform and linear temperature changes ($\alpha=\pi/2$, $\mu^2=2$, $L/h=30$)

all of opening angles and temperature risings.

The tendency of dimensionless frequency changes is opposite for 1p $p$. However, this tendency is different to increasing the temperature changes. While the temperature changes increase, the dimensionless natural frequency decreases. It also can be seen that with the increase the opening angle from $\pi/4$ to $\pi/2$ the natural frequency decreases significantly.

Table 5 illustrates the variation of fundamental natural frequencies with changing of the nonlocality parameter at constant aspect ratio ($L/h=30$) and opening angle ($\alpha=\pi/2$) of curved FG porous nanobeam with simply supported boundary condition at both ends and different material distribution under uniform and linear temperature rises. It can be observed that from results of Table 5, the non-dimensional fundamental frequencies of curved FG porous nanobeam decrease with the increase of temperature for both thermal loading types. Also, it may be observed that, size effect parameter, has an effect on vibration of nonlocal curved FG porous beam.

So, that by increasing nonlocal parameter, dimensionless natural frequency decreases for all porosity parameter and thermal loading types. In addition, Table 5 reveals that, the dimensionless frequency of the curved FG porous nanobeam subjected to LTR is greater than subjected to UTR. Also, the discrepancy between non-dimensional frequencies of different temperature changes (UTR and LTR) becomes greater by increasing the temperature changes. The reason is that the stiffness of the curved FG porous nanobeam for linear temperature rise is the larger than uniform temperature rise.

In order to clearly understand the porosity effect on the vibration of S-S FG curve nanobeam subjected to UTR, Fig. 2 demonstrate variations of the natural frequency of this model with material graduation, temperature changings and porosity parameter at a constant value of aspect ratio ($L/h$=30), nonlocal parameter ($\mu^2$=2) and opening angle ($\alpha=\pi/2$). It is observed from the results of Fig. 2 that porosity effect according to the even distribution depends on the value of power-law index and temperature changings. For example, at ($\Delta T$=0) by increasing the porosity parameter the natural frequency of curved FG porous nanobeam first increases at lower gradient indexes, however, an opposite behavior is observed from a certain value of the power index. In other words, from a certain value of power-law index, increasing porosity volume fraction leads to lower non-dimensional frequencies. Moreover, it is observed that this certain value of the power-law index is dependent on temperature changing value. In addition, it is found that temperature changing indicates reducing impacts on the natural frequency of porous FG curved nanobeam when their values change from zero to positive one which highlights the saliency of the role of temperature environment. Also, it is beheld that dimensional frequency of curved FG nanobeam decrease as the power-law exponent increases for all values of temperature changings and porosity parameters. While the power exponent is in the range of 0 to 2, the natural frequencies decrease with high pace compared to those when the power-law index is in the realm of between 2 and 10.

The variations of fundamental natural frequencies of FG porous curved nanobeam subjected to UTR and LTR against power-law index for different temperature changings and are illustrated in Fig. 3 at constant values of ($L/h$=30, $\mu^2$=2, $\alpha=\pi/2$). Both type of thermal loading including uniform and linear temperature changes are considered with different temperature changes $\Delta T$=30, 60. It obviously can be understood from the results of Fig. 3 that the fundamental natural frequencies of curved FG porous nanobeam decrease with the increase of power-law exponent for both type of thermal loadings. However, it is seen that the variation of frequency with respect to power-law exponent according to the linear temperature rise is less sensible than uniform temperature rising. In addition, the effect of uniform and linear temperature changes is clearly can be seen in Fig. 3. The dimensionless natural frequencies decreased by rising temperature change for all values of power-law exponents and porosity parameter. It is worth mentioning, the difference between frequency results according to different values of temperature changings increases with uniform temperature rise. Thus, both types of temperature rises have a prominent effect on the natural frequency of the curved FG porous nanobeams.

To indicate the influences of opening angle on the non-dimensional frequency of S-S FG porous curved nanobeam

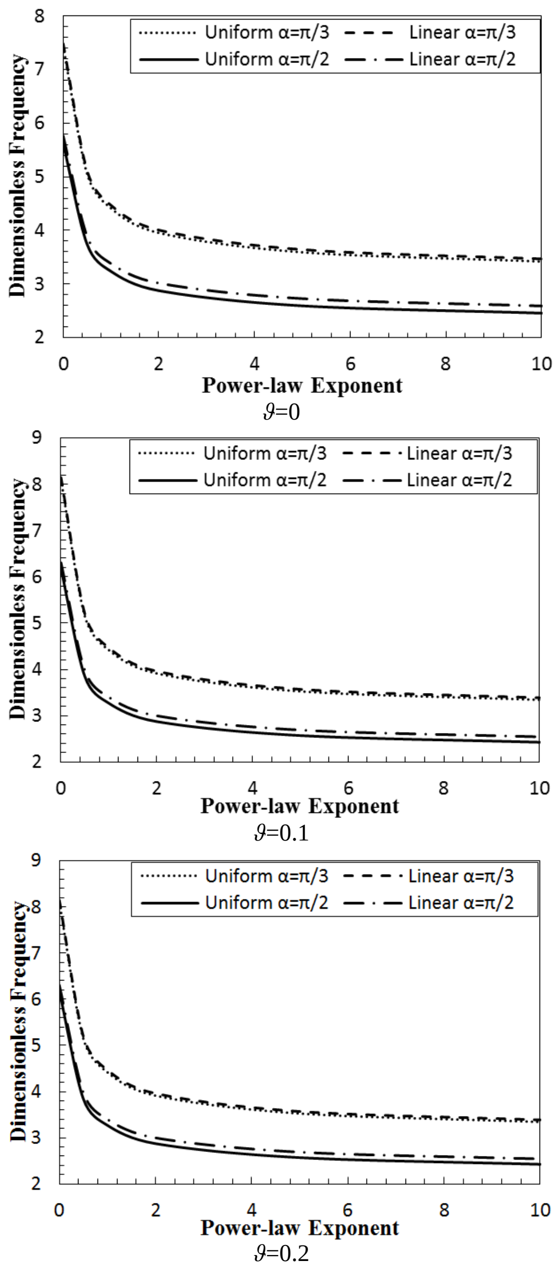



Fig. 4 The variation of the frequency of S-S curved FG porous nanobeam respect to gradient index, opening angles and porosity parameter for different uniform and linear temperature changes ($\Delta T$=30, $\mu^2$=2, $L/h$=30)

for different thermal loadings, Fig. 4 is presented. Fig. 4 demonstrates the frequency results versus gradient index for perfect ($\alpha$=0) and porous ($\alpha$=0.1, 0.2) FG nanobeam at ($\Delta T$=30, $L/h$=30, $\mu^2$=2). It is seen from Fig. 4, an increase in opening angle, gives rise to a decrease in the fundamental dimensionless frequency for all porosity parameter and gradient index. In addition, it can be seen that, with increasing the opening angles, discrepancy of uniform and linear temperature rises becomes greater.

Fig. 5 shows the influence of nonlocal parameter on the vibration behavior of curved FG porous nanobeam with respect to different porosity parameter and power-law exponent at $L/h$=30, $\alpha=\pi/2$ and $\Delta T$=30. It is revealed that

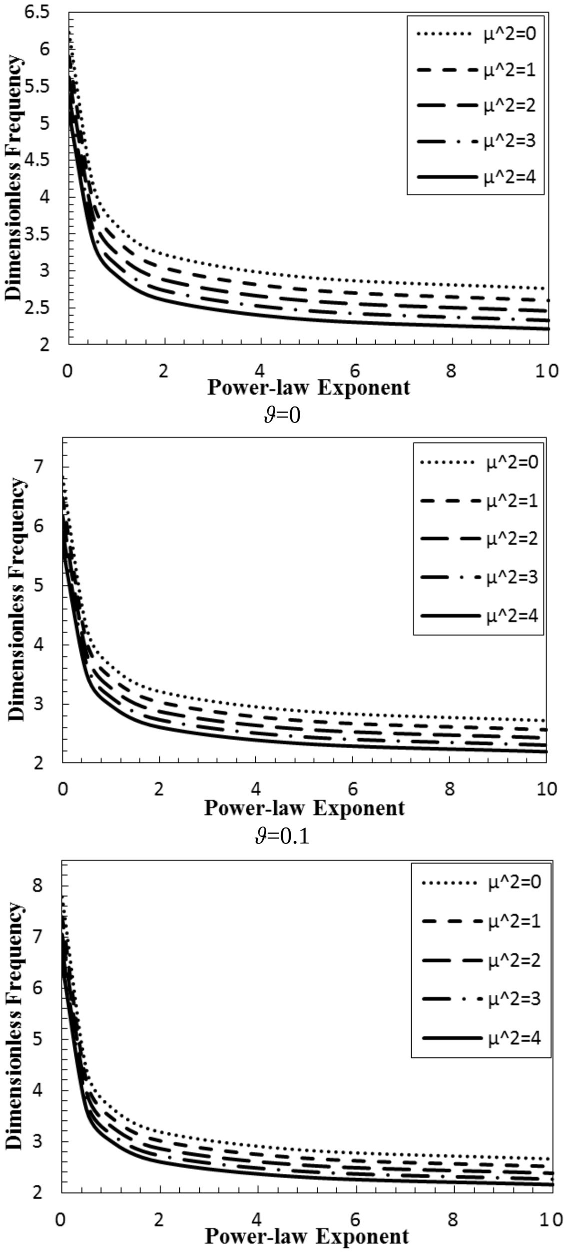



Fig. 5 The variation of the frequency of S-S curved FG porous nanobeam respect to gradient index and nonlocality and porosity parameter for different nonlocal parameter ($\Delta T$=30, $\alpha=\pi/2$, $L/h$=30)

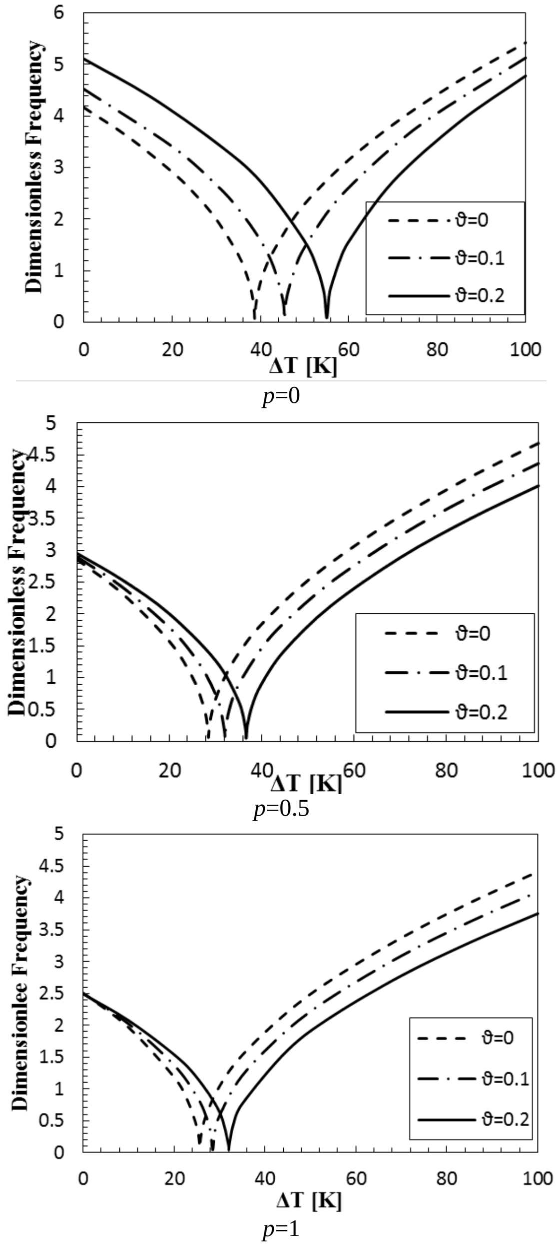
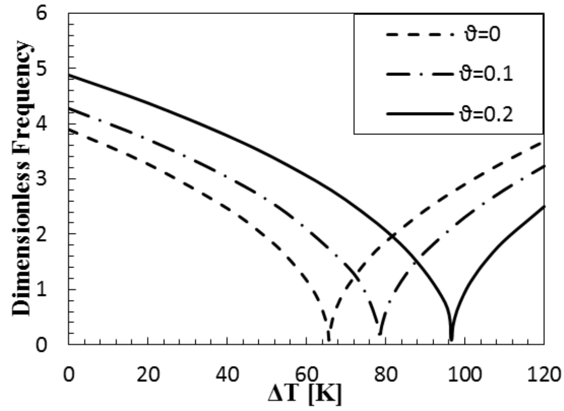



Fig. 6 Variations of the first dimensionless natural frequency of the S-S curved FG porous nanobeam with respect to uniform temperature change for different values of porosity parameter and gradient indexes ($L/h$=40, $\alpha=2\pi/3$, $\mu^2$=2)

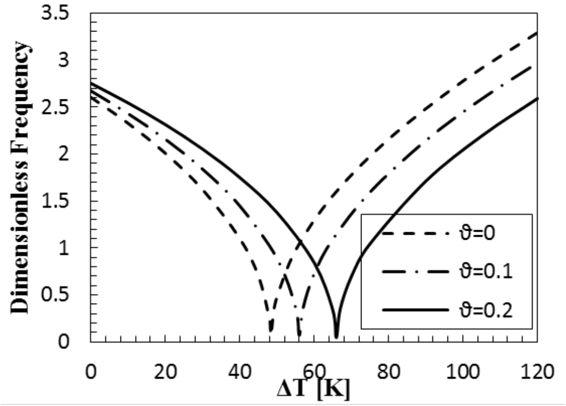
frequencies predicted by the nonlocal parameter is smaller than the local frequency due to the size effects. For this reason, the size effect plays prominent role on vibration treatment of curved FG porous nanobeams. Also, it is evident that increasing of porosity volume fraction leads to decreasing in natural frequency of porous FG curved nanobeam. Furthermore, it can be pointed that the effect of power-law exponent on the natural frequency of porous FG curved nanobeam porosity is similar previous conclusions.

The non-dimensional frequency of porous FG curved nanobeam supposed to UTR and LTR as a function of temperature changings and porosity volume fractions for different power-law indexes ($p$=0, $p$=0.5, $p$=1) at $L/h$=40, $\mu^2$=2 and $\alpha=\pi/2$ is depicted respectively in Figs. 6 and 7.

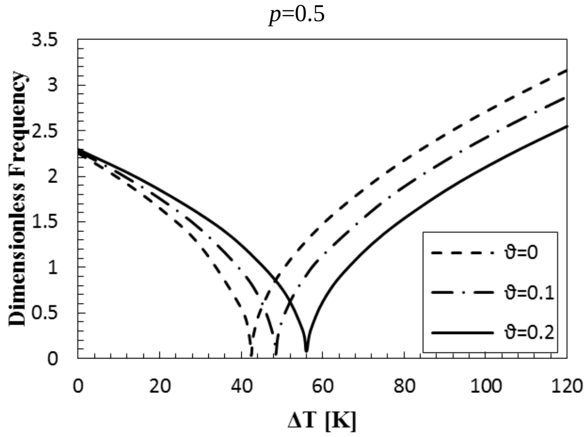
It can be observed that the frequencies of curved FG porous nano beam decrease with the increase of temperature change until reaches to zero at the critical temperature point. The reason of this phenomena is reduction in total stiffness of the beam, since stiffness of curved FG porous nanobeam decreases when temperature change increase. The important seeing within the realm of temperature before the critical temperature, is that the curved FG porous nanobeams with greater values of porosity parameter usually provide higher values of the dimensionless frequency results. Furthermore, this treatment is vice versa in the realm of temperature after the critical temperature.



$p$=0



$p$=0.5



$p$=1

Fig. 7 Variations of the first dimensionless natural frequency of the S-S curved FG porous nanobeam with respect to linear temperature change for different values of porosity parameter and gradient indexes ($L/h$=40, $\alpha$=2$\pi$/3, $\mu^2$=2)

Also, it can be observed that uniform temperature change has a softening effect on curved FG porous nanobeam at pre-buckling realm and a rise in temperature increases this effect. It is also objective that the ramified point of the curved FG porous nanobeam is postponed by assumption of the lower porosity parameter due to the fact that the lower porosity parameter results in the decrease of stiffness of the curved FG porous nanobeam. Also, it can be noted that the buckling temperatures decrease depending on an increasing in power-law index and porosity parameter.

## 6. Conclusions

In the present perusal, Thermo-mechanical vibration of porous FG curved nanobeams supposed to thermal loadings with different opening angles is studied within the framework of a first order shear deformation beam theory in which shear deformation effect is involved with a shear correction factors. Even type of porosity distributions is considered. Thermo-Mechanical properties of porous FG curved nanobeams are gradually change in the thickness direction based on modified rule of mixture. Hamilton's principle is employed to derive the governing equations and related boundary conditions. An analytically exact solution is used to solve governing partial differential equations for simply-supported boundary condition. It is indicated that the thermo-mechanical vibration characteristics of curved FG porous nanobeam significantly affected by various parameters such as porosity parameter, uniform and linear temperature rising, nonlocality, angle of curvature and gradient index. Numerical results show that:

√ By increasing the power-law index value and nonlocal parameter, the non-dimensional frequencies of porous FG curve nanobeams are found to decrease regardless of opening angle and porosity values.

√ Effect of porosity volume fraction on natural frequency of porous FG curve nanobeams depends on material graduation index value and temperature changing.

√ For porous FG curve nanobeams, increasing the volume fraction of porosity first yields an increase in natural frequency, then this trend becomes opposite for upper values of gradient index. In other words, increasing of porosity volume fraction decreases the non-dimensional frequency from a certain value of power-law index which this certain value depends on temperature changings.

√ For both temperature rises, increasing temperature changing yields reduction of natural frequency of porous FG curve nanobeams supposed thermal loadings. However, after critical temperature this behavior is vice versa.

√ Effect of temperature changings on frequencies with supposed to uniform temperature rising is more prominent than linear temperature rising.

## References


Aissani, K., Bouiadjra, M.B., Ahouel, M. and Tounsi, A. (2015), “A new nonlocal hyperbolic shear deformation theory for nanobeams embedded in an elastic medium”, *Struct. Eng. Mech.*, **55**(4), 743-763.

Ansari, R., Gholami, R. and Sahmani, S. (2013), “Size-dependent vibration of functionally graded curved microbeams based on the modified strain gradient elasticity theory”, *Arch. Appl. Mech.*, **83**, 1439-1449.

Akgoz, B. and Civalek, O. (2013), “Buckling analysis of linearly tapered micro-columns based on strain gradient elasticity”, *Struct. Eng. Mech.*, **48**(2), 195-205.

Berrabah, H. M., Tounsi, A., Semmah, A. and Adda, B. (2013), “Comparison of various refined nonlocal beam theories for bending, vibration and buckling analysis of nanobeams”, *Struct. Eng. Mech.*, **48**(3), 351-365.

Bouderba, B., Houari, M. S. A., Tounsi, A. and Mahmoud, S. R.

(2016), "Thermal stability of functionally graded sandwich plates using a simple shear deformation theory", *Struct. Eng. Mech.*, **58**(3), 397-422.

Bousahla, A. A., Benyoucef, S., Tounsi, A. and Mahmoud, S. R. (2016), "On thermal stability of plates with functionally graded coefficient of thermal expansion", *Struct. Eng. Mech.*, **60**(2), 313-335.

Eringen, A.C. (2002), *Nonlocal Continuum Field Theories*, Springer Science & Business Media.

Eltaher, M., Alshorbagy, A.E. and Mahmoud, F. (2013), "Vibration analysis of Euler-Bernoulli nanobeams by using finite element method", *Appl. Math. Model.*, **37**, 4787-4797.

Eltaher, M., Emam, S.A. and Mahmoud, F. (2012), "Free vibration analysis of functionally graded size-dependent nanobeams", *Appl. Math. Model.*, **218**, 7406-7420.

Ebrahimi, F. and Salari, E. (2015), "Size-dependent free flexural vibrational behavior of functionally graded nanobeams using semi-analytical differential transform method", *Compos. Part B: Eng.*, **79**, 156-169.

Ebrahimi, F. and Barati, M.R. (2016), "A nonlocal higher-order refined magneto-electro-viscoelastic beam model for dynamic analysis of smart nanostructures", *Int. J. Eng. Sci.*, **107**, 183-196.

Ebrahimi, F. and Barati, M.R. (2017a), "Vibration analysis of piezoelectrically actuated curved nanosize FG beams via a nonlocal strain-electric field gradient theory", *Mech. Adv. Mater. Struct.*, 1-10.

Ebrahimi, F. and Barati, M.R. (2017b), "Vibration analysis of embedded size dependent FG nanobeams based on third-order shear deformation beam theory", *Struct. Eng. Mech.*, **61**(6), 721-736.

Ebrahimi, F. and Barati, M.R. (2017c), "Porosity-dependent vibration analysis of piezo-magnetically actuated heterogeneous nanobeams", *Mech. Syst. Signal Pr.*, **93**, 445-459.

Ebrahimi, F. and Barati, M.R. (2017d), "Small-scale effects on hygro-thermo-mechanical vibration of temperature-dependent nonhomogeneous nanoscale beams", *Mech. Adv. Mater. Struct.*, **24**(11), 924-936.

Ebrahimi, F. and Barati, M.R. (2017e), "A modified nonlocal couple stress based beam model for vibration analysis of higher-order FG nanobeams", *Mech. Adv. Mater. Struct.* (accepted)

Ebrahimi, F. and Barati, M.R. (2017f), "Scale-dependent effects on wave propagation in magnetically affected single/double-layered compositionally graded nanosize beams", *Wave. Random Complex Media*, 1-17.

Ebrahimi, F. and Barati, M.R. (2017g), "Buckling analysis of nonlocal strain gradient axially functionally graded nanobeams resting on variable elastic medium", *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 0954406217713518.

Ebrahimi, F. and Zia, M. (2015), "Large amplitude nonlinear vibration analysis of functionally graded Timoshenko beams with porosities", *Acta Astronautica*, **116**, 117-125.

Ebrahimi, F., Ghasemi, F. and Salari, E. (2016), "Investigating thermal effects on vibration behavior of temperature-dependent compositionally graded Euler beams with porosities", *Meccanica*, **51**, 223-249.

Ebrahimi, F. and Jafari, A. (2016a), "Thermo-mechanical vibration analysis of temperature-dependent porous FG beams based on Timoshenko beam theory", *Struct. Eng. Mech.*, **59**, 343-371.

Ebrahimi, F. and Jafari, A. (2016b), "A higher-order thermomechanical vibration analysis of temperature-dependent FGM beams with porosities", *J. Eng.*, **2016**, Article ID 9561504, 20.

Ebrahimi, F. and Daman, M. (2016), "Investigating surface effects on thermomechanical behavior of embedded circular curved nanosize beams", *J. Eng.*, **2016**, Article ID 9848343, 11.

Ebrahimi, F. and Daman, M. (2016), "An investigation of radial vibration modes of embedded double-curved-nanobeam-systems", *Cankaya Univ. J. Sci. Eng.*, **13**, 58-79.

Ebrahimi, F., Ghadiri, M., Salari, E., Hoseini, S.A.H., Shaghaghi, G.R. (2015), "Application of the differential transformation method for nonlocal vibration analysis of functionally graded nanobeams", *J. Mech. Sci. Technol.*, **29**, 1207-1215.

Ebrahimi, F. and Barati, M.R. (2016b), "Magneto-electro-elastic buckling analysis of nonlocal curved nanobeams", *Euro. Phys. J. Plus*, **131**, 346.

Ebrahimi, F. and Shaghaghi, G.R. (2016), "Thermal effects on nonlocal vibrational characteristics of nanobeams with non-ideal boundary conditions", *Smart Struct. Syst.*, **18**(6), 1087-1109.

Ebrahimi, F. and Dabbagh, A. (2017), "Wave propagation analysis of smart rotating porous heterogeneous piezo-electric nanobeams", *Euro. Phys. J. Plus*, **132**(4), 153.

Ebrahimi, F. and Salari, E. (2016a), "Analytical modeling of dynamic behavior of piezo-thermo-electrically affected sigmoid and power-law graded nanoscale beams", *Appl. Phys. A*, **122**(9), 793.

Ebrahimi, F. and Salari, E. (2016b), "Thermal loading effects on electro-mechanical vibration behavior of piezoelectrically actuated inhomogeneous size-dependent Timoshenko nanobeams", *Adv. Nano Res.*, **4**(3), 197-228.

Ebrahimi, F. and Shafiei, N. (2016), "Application of Eringens nonlocal elasticity theory for vibration analysis of rotating functionally graded nanobeams", *Smart Struct. Syst.*, **17**(5), 837-857.

Ehyaei, J. and Daman, M. (2017), "Free vibration analysis of double walled carbon nanotubes embedded in an elastic medium with initial imperfection", *Adv. Nano Res.*, **5**(2), 179-192.

Hosseini, S. and Rahmani, O. (2016), "Free vibration of shallow and deep curved FG nanobeam via nonlocal Timoshenko curved beam model", *Appl. Phys. A*, **122**, 1-11.

Kocaturk, T. and Akbas, S.D. (2013), "Wave propagation in a microbeam based on the modified couple stress theory", *Struct. Eng. Mech.*, **46**(3), 417-431.

Kananipour, H., Ahmadi, M. and Chavoshi, H. "Application of nonlocal elasticity and DQM to dynamic analysis of curved nanobeams", *Latin Am. J. Solid. Struct.*, **11**, 848-853, 2014.

Malekzadeh, P., Haghighi, M.G. and Atashi, M. (2010), "Out-of-plane free vibration of functionally graded circular curved beams in thermal environment", *Compos. Struct.*, **92**, 541-552.

Mechab, I., Mechab, B., Benaissa, S., Serier, B. and Bouiadjra, B.B. (2010), "Free vibration analysis of FGM nanoplate with porosities resting on Winkler Pasternak elastic foundations based on two-variable refined plate theories", *J. Brazil. Soc. Mech. Sci. Eng.*, **38**(8), 2193-2211.

Miyamoto, Y., Kaysser, W.A., Rabin, B.H., Kawasaki, A. and Ford, R.G. (2013), *Functionally Graded Materials: Design, Processing and Applications*, Springer Science & Business Media.

Nazemnezhad, R. and Hosseini-Hashemi, S. (2014), "Nonlocal nonlinear free vibration of functionally graded nanobeams", *Compos. Struct.*, **110**, 192-199.

Pour, H.R., Vossough, H., Heydari, M.M., Beygipoor, G. and Azimzadeh, A. (2015), "Nonlinear vibration analysis of a nonlocal sinusoidal shear deformation carbon nanotube using differential quadrature method", *Struct. Eng. Mech.*, **54**(6), 1061-1073.

Rahmani, O. and Pedram, O. (2014), "Analysis and modeling the size effect on vibration of functionally graded nanobeams based on nonlocal Timoshenko beam theory", *Int. J. Eng. Sci.*, **77**, 55-70.

Setoodeh, A., Derahaki, M. and Bavi, N. (2015), "DQ thermal buckling analysis of embedded curved carbon nanotubes based on nonlocal elasticity theory", *Latin Am. J. Solid. Struct.*, **12**, 1901-1917.

Shen, H.S. (2016), Functionally Graded Materials: Nonlinear Analysis of Plates and Shells, CRC Press.

Tounsi, A., Houari, M.S.A. and Bessaim, A. (2016), "A new 3-unknowns non-polynomial plate theory for buckling and vibration of functionally graded sandwich plate", *Struct. Eng. Mech.*, **60**(4), 547-565.

Taghizadeh, M., Ovesy, H.R. and Ghannadpour, S.A.M. (2015), "Nonlocal integral elasticity analysis of beam bending by using finite element method", *Struct. Eng. Mech.*, **54**(4), 755-769.

Touloukian, Y.S. (1966), "Thermophysical properties of high temperature solid materials, Volume 6, intermetallics, cermets, polymers, and composite systems, Part II. cermets, polymers, composite systems", Dtic Document.

Wang, C.M. and Duan, W. (2008), "Free vibration of nanorings/arches based on nonlocal elasticity", *J. Appl. Phys.*, **104**, 014303.

Wattanasakulpong, N., Prusty, B. G., Kelly, D. W. and Hoffman, M. "Free vibration analysis of layered functionally graded beams with experimental validatio", *Mater. Des.*, 36, 182-190.

Wattanasakulpong, N. and Ungbhakorn, V. (2014), "Linear and nonlinear vibration analysis of elastically restrained ends FGM beams with porosities", *Aerosp. Sci. Technol.*, **3**2, 111-120.

Wattanasakulpong, N. and Chaikittiratana, A. (2015), "Flexural vibration of imperfect functionally graded beams based on Timoshenko beam theory", *Chebyshev Coll. Meth., Meccanica*, **50**, 1331-1342.

Yahia, S.A., Atmane, H.A., Houari, M.S.A. and Tounsi, A. (2015), "Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories", *Struct. Eng. Mech.*, **53**, 1143-1165.

Yan, Z. and Jiang, L. (2011), "Electromechanical response of a curved piezoelectric nanobeam with the consideration of surface effects", *J. Phys. D: Appl. Phys.*, **44**, 365301.

Zemri, A., Houari, M.S.A., Bousahla, A.A. and Tounsi, A. (2015), "A mechanical response of functionally graded nanoscale beam: an assessment of a refined nonlocal shear deformation theory beam theory", *Struct. Eng. Mech.*, **54**(4), 693-710.

Zhu, J., Lai, Z., Yin, Z., Jeon, J. and Lee, S. (2001), "Fabrication of ZrO 2-NiCr functionally graded material by powder metallurgy", *Mater. Chem. Phys.*, **68**, 130-135.

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