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Surface effects on vibration and buckling behavior of embedded nanoarches

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Abstract. The present paper deals with the free vibration and buckling problem with consideration of surface properties of circular nanobeams and nanoarches. The Gurtin-Murdach theory is used for investigating the surface effects parameters including surface tension, surface density and surface elasticity. Both linear and nonlinear elastic foundation effect are considered on the circular curved nanobeam. The analytically Navier solution is employed to solve the governing equations. It is obviously detected that the natural frequencies of a curved nanobeams is substantially influenced by the elastic foundations. Besides, it is revealed that by increasing the thickness of curved nanobeam, the influence of surface properties and elastic foundations reduce to vanished, and the natural frequency and critical buckling load turns into to the corresponding classical values.

Keywords: vibration; critical buckling load; elastic foundation; circular curved nanobeam

1. Introduction

Nano materials are attracting many researchers over the recent years due to their improvement of the quality properties (Ebrahimi and Barati 2016a, Ebrahimi and Salari 2015). Atomistic modeling and experimental researches show that, the size effect gains important when the dimensions of structures become very small. Due to this fact, the size effect plays an important role in the mechanical behavior of micro-and nanostructures (Şimşek 2014). Among various nano structures, nanobeams have more important applications (Daulton *et al.* 2010, Hu *et al.* 2010, Ebrahimi and Barati 2017a, b, c, d, e, f, g, Ebrahimi and Daman 2016a, Ebrahimi and Daman 2017, Ehyaei and Daman 2017, Ebrahimi and Shaghaghi 2016, Ebrahimi and Dabbagh 2017, Ebrahimi and Salari 2016a, b, Ebrahimi and Shafiei 2016) and Lots of studies have been performed to investigate the size-dependent response of structural systems based on Eringen's nonlocal elasticity theory (Ebrahimi and Salari 2015a, b, c, d, Ebrahimi *et al.* 2015a, 2016c, Ebrahimi and Barati 2016 a, b, c, d, e, Ebrahimi and Hosseini, 2016 a,c). A nonlocal beam theory is presented by Thai (2012), in this research, bending, buckling, and vibration of nanobeams have been investigated. However, the nonlinear vibration of the piezoelectric nanobeams based on the Timoshenko beam theory and nonlocal modeling has been investigated by Liao-Liang *et al.* (2012). In addition, Murmu and Adhikari (2010), have investigated the nonlocal transverse vibration of double-nanobeam-system. In this research, an analytical method has been developed for determining the natural frequencies of the nonlocal double-nanobeam-system. Also Eltaher *et al.* (2012), have presented free vibration analysis of functionally graded (FG) size-dependent nanobeams using finite element method. However, wave propagation in a microbeam based on the modified couple stress theory has been studied by Kocaturk and Akbas (2013). Also, Taghizadeh *et al.* (2015) have presented nonlocal integral elasticity analysis of beam bending by using finite element method. Meanwhile, nonlinear vibration analysis of a nonlocal sinusoidal shear deformation carbon nanotube using differential quadrature method has been investigated by Pour *et al.* (2015). In addition, buckling analysis of linearly tapered micro-columns based on strain gradient elasticity has been investigated by Akgoz and Civalek (2013). However, Zemri *et al.* (2015) have presented a mechanical response of functionally graded nanoscale beam by using refined nonlocal shear deformation theory beam theory. Nevertheless, comparison of various refined nonlocal beam theories for bending, vibration and buckling analysis of nanobeams has been studied by Berrabah and Tounsi (2013). Aissani *et al.* (2015) have presented a new nonlocal hyperbolic shear deformation theory for nanobeams embedded in an elastic medium. Also, bending, buckling and vibration of axially functionally graded beams based on nonlocal strain gradient theory has been presented by Li *et al.* (2017). Recently new classes of theories are also developed as functionally graded materials (Bourada *et al.* 2015, Ebrahimi and Barati 2016b). Because the nanobeams has the high proportion of the surface to volume, the surface stress effects have important role in their mechanic's behavior of these structures. Hence Gurtin and Murdach (1978) have considered surface stress effects. In this theory the surface is considered as a part of (nonphysical) the two-dimensional with zero thickness (mathematically) which has covered the total volume. This theory has used in many researches about nanobeams.

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in presence of surface properties have been studied by Gheshlaghi and Hasheminejad (2011). Nevertheless, nonlinear free vibration of functionally graded nanobeams with surface effects has been investigated by Sharabiani and Haeri-Yazdi (2013). In addition, Sahmani *et al.* have investigated Surface energy effects on the free vibration of post buckled third-order shear deformable nanobeams (2014). And they have studied Surface properties on the nonlinear forced vibration response of third-order shear deformable nanobeams (2014). In these papers they have been used to Gurtin-Murdach elasticity theory. Furthermore, the nonlinear free vibration of nanobeams with considering surface properties has been studied by Nazemnezhad *et al.* (2012). However, Hosseini-Hashemi and Nazemnezhad (2013) have presented nonlinear free vibration of functionally graded nanoscale beams with surface properties. As well as, Ansari *et al.* (2014) have investigated nonlinear forced vibration characteristics of nanobeams including surface stress effect. In this study, a new formulation of the Timoshenko beam theory has been developed through the Gurtin-Murdoch elasticity theory in which the effect of surface stress has been incorporated. Moreover, the surface and nonlocal effects on the nonlinear flexural free vibrations of elastically supported non-uniform cross section nanobeams have been investigated by Malekzadeh and Shojaee (2013) simultaneously. In addition, wave propagation in viscoelastic single-walled carbon nanotubes with surface effect under magnetic field based on nonlocal strain gradient theory has been studied by Hu and Ling (2016). Moreover, Ebrahimi and Daman (2016b) have presented investigating surface effects on thermomechanical behavior of embedded circular curved nanosize beams. However, a semi-analytical evaluation of surface and nonlocal effects on buckling and vibrational characteristics of nanotubes with various boundary conditions has been investigated by Ebrahimi *et al.* (2016). In addition, Ebrahimi and Boreiry (2015) have studied investigating various surface effects on nonlocal vibrational behavior of nanobeams. Also, surface effects on large deflection of a curved elastic nanobeam under static bending has been presented by Liu *et al.* (2016). Meanwhile, Li and Hu (2017a), have studied Torsional vibration of bi-directional functionally graded nanotubes based on nonlocal elasticity theory. In addition, Post-buckling analysis of functionally graded nanobeams incorporating nonlocal stress and microstructure-dependent strain gradient effects has been investigated by Li and Hu (2017b).

In the case of elastic foundation there are linear and nonlinear which are named as Winkler and Pasternak respectively. Elastic foundations have been employed in the size of macro and nanobeams in many recent researches as explained below.

Zhao *et al.* (2015), have investigated the axial buckling of a nanowire resting on Winkler–Pasternak substrate medium with the Timoshenko beam theory. In addition, Simple analytical expressions have been presented by Fallah and Aghdam (2011) for large amplitude free vibration and post-buckling analysis of functionally graded beams rest on nonlinear elastic foundation. Furthermore, Jang *et al.* (2011), have presented a new method of analyzing the non-linear deflection behavior of an infinite beam on a non-linear elastic foundation. Also, Niknam and Aghdam (2015) have obtained a closed form solution for both natural frequency and buckling load of nonlocal functionally graded beams resting on nonlinear elastic foundation. Moreover, the static instability of a nanobeam with geometrical imperfections with elastic foundation has been investigated by Mohammadi *et al.* (2014). In this paper, Size-dependent effect is included in the nonlinear model. Nevertheless, differential transformation method has been used to predict the buckling behavior of single walled carbon nanotube on Winkler foundation under various boundary conditions by Pradhan and Reddy (2011).

In recent years vibration of curved nanobeams and nanorings, have been worked in many empirical experiments and dynamic molecular simulations (Wang and Duan 2008). Hence some researchers are interested in studding of vibration curved nanobeams and nanorings. A unified formulation for static behavior of nonlocal curved beams has been presented by Tufekci *et al.* (2016). Yan and Jiang (2011) have investigated the electromechanical response of a curved piezoelectric nanobeam with the consideration of surface effects. In addition, a new numerical technique, the differential quadrature method has been developed for dynamic analysis of the nanobeams in the polar coordinate system by Kananipour *et al.* (2014). Moreover, Khater *et al.* (2014) have investigated the effect of surface energy and thermal loading on the static stability of nanowires. In this research, nanowires have been considered as curved fixed–fixed Euler-Bernoulli beams and has been used Gurtin-Murdoch theory to represent surface effects. The model has taken into account both von Kármán strain and axial strain. As well as, Wang and Duan (2008) have surveyed the free vibration problem of nanorings/arches. In this research the problem was formulated on the framework of nonlocal elasticity theory. Nevertheless, explicit solution has been shown for size and geometry dependent free vibration of curved nanobeams with including surface effects by Assadi and Farshi (2011). Ebrahimi and Daman (2016c) have presented an investigation of radial vibration modes of embedded double-curved-nanobeam systems. In addition, a nonlocal strain gradient refined beam model for buckling analysis of size-dependent shear-deformable curved FG nanobeams has been investigated by Ebrahimi and Barati (2017h).

To the best of the author's knowledge, there has been no record or any study regarding the vibration and buckling of circular nanoarches with surface effects and elastic foundations through any of the studies mentioned in the literature review. Therefore, there is strong scientific need to understand the vibration and buckling behavior of circular curved nanobeams with surface effects in considering the effect of elastic foundations. Actually, circular curved nanobeams or nanoarches can be applied in nano electro mechanical systems (NEMS) such as various nanosensors, nanogenerators and nanoactuators. The aim of this research is investigating the effects of Winkler and Pasternak elastic foundation on natural frequencies and critical buckling loads of nanoarches. Thus, the paper has

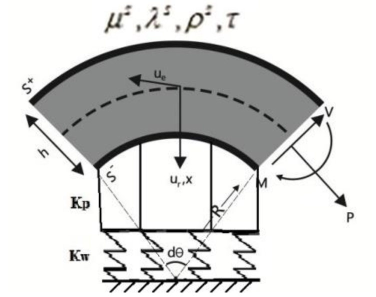

Fig. 1 Geometry of an element of a circular curved nanobeam with surface layers

presented the effects of surface density, surface elasticity and surface residual stress on natural frequencies and critical buckling loads of nanoarches.

2. Problem statements

In plane free vibration of curved nanobeam is considered. As it shown in Fig. 1, the radius curvature and thickness are considered R and h respectively. Additional surface effects are supposed for all the external surfaces.

The dynamic equilibrium equations for a circular curved Euler-Bernoulli beam, are given as

$$\begin{aligned}&\frac{\partial V}{\partial \theta}+P=\rho AR\frac{\partial^2 u_r}{\partial t^2}+Rb\rho^s\left(\frac{\partial^2 u_r^+}{\partial t^2}+\frac{\partial^2 u_r^-}{\partial t^2}\right)-fR\\&\frac{\partial P}{\partial \theta}-V=\rho AR\frac{\partial^2 u_\theta}{\partial t^2}+Rb\rho^s\left(\frac{\partial^2 u_\theta^+}{\partial t^2}+\frac{\partial^2 u_\theta^-}{\partial t^2}\right)-pR\\&\frac{\partial M}{\partial \theta}+RV=0\end{aligned}\tag{1}$$

where $F(\theta,t)$ is the shearing force, $P(\theta,t)$ is the tensile force, A is the cross sectional area, ρ is the mass density, ρ^s is the surface density of the nanoring and b is the width of nanoring In Eq. (1). It should be notice the displacement components of the surface property must satisfy the following relations (Assadi and Farshi 2011).

$$\begin{aligned}&\ddot{u}_r^+=\ddot{u}_r^-=\ddot{u};\\&\ddot{u}_\theta^++\ddot{u}_\theta^-=2\ddot{u}_\theta\end{aligned}\tag{2}$$

By employing Eq. (2) and substituting into Eq. (1) the equilibrium equations can be determined as follow.

$$\begin{aligned}&P-\frac{1}{R}\frac{\partial^2 M}{\partial \theta^2}=\left(\rho AR+2Rb\rho^s\right)\frac{\partial^2 u_r}{\partial t^2}-fR\\&\frac{\partial P}{\partial \theta}+\frac{1}{R}\frac{\partial M}{\partial \theta}=\left(\rho AR+2Rb\rho^s\right)\frac{\partial^2 u_\theta}{\partial t^2}-pR\end{aligned}\tag{3}$$

The normal stress resultant P from Eq. (3) should be vanished. Therefore, obtains the relation between radial displacement and bending moment such as Eq. (4)

$$\begin{aligned}&\frac{1}{R}\left(\frac{\partial^2 M}{\partial \theta^2}+\frac{\partial^4 M}{\partial \theta^4}\right)+R\left(\frac{\partial p}{\partial \theta}-\frac{\partial^2 f}{\partial \theta^2}\right)\\&=\left(\rho AR+2Rb\rho^s\right)\left(\frac{\partial^2 u_r}{\partial t^2}-\frac{\partial^4 u_r}{\partial t^2\partial \theta^2}\right)\end{aligned}\tag{4}$$

The stress components of the surface layers must satisfy the following equilibrium relations (Gurtin and Murdoch 1978)

$$\begin{aligned}&\tau_{\alpha\beta}^{\pm}=\tau\delta_{\alpha\beta}+\left(\mu^s-\tau\right)\left(u_{\alpha\beta}^{\pm}+u_{\beta\alpha}^{\pm}\right)\\&+\left(\lambda^s+\tau\right)u_{\gamma\gamma}^{\pm}\delta_{\alpha\beta}+\tau u_{\alpha\beta}^{\pm}\end{aligned}\tag{5}$$

where $\tau^{\pm}$ are residual surface tensions under unconstrained conditions, μ^s and λ^s are the surface Lamé constants for the surface material.

$$\varepsilon_{\theta\theta}=\frac{1}{R}\left[-u_r+\frac{\partial u_\theta}{\partial \theta}-\frac{x}{R}\frac{\partial}{\partial \theta}\left(u_\theta+\frac{\partial u_r}{\partial \theta}\right)\right]\tag{6}$$

Considering inextensible deformation of the curved element at x=0, it can be conclude that $u_r=\partial u_\theta/\partial\theta$. The stress-strain relation for the surface material can be determined as

$$\tau_{\theta\theta}^{\pm}=\tau\pm\frac{h\left[2\mu^s+\lambda^s\left(1-\nu\right)-\nu\tau\right]}{2R^2}\left(u_r+\frac{\partial^2 u_r}{\partial \theta^2}\right)\tag{7}$$

Bending moment resultant M due to normal stress σ_{xx} can be described by integrating the strain components on the cross section as follow as

$$M=-b\int_{-\frac{h}{2}}^{\frac{h}{2}}E\varepsilon_{\theta\theta}x\,dx+\frac{bh}{2}(\tau_{\theta\theta}^{+}-\tau_{\theta\theta}^{-})\tag{8}$$

By inserting Eq. (6) and (7) into Eq. (8), the bending moment of curved element, can be obtained as

$$M=\left\{\frac{EI}{R^2}+\frac{bh^2\left[2?^{s}+\lambda^s\left(1-\nu\right)-\nu\tau\right]}{2R^2}\right\}\left(u_r+\frac{\partial^2 u_r}{\partial \theta^2}\right)\tag{9}$$

• *Vibration equation*

Using Eq. (9) in Eq. (4) yields the governing equation for vibration of the curved nanobeam as

$$\begin{aligned}&\frac{\lambda}{R}\left(\frac{\partial^6 u_r}{\partial \theta^6}+2\frac{\partial^4 u_r}{\partial \theta^4}+\frac{\partial^2 u_r}{\partial \theta^2}\right)+R\left(\frac{\partial p}{\partial \theta}-\frac{\partial^2 f}{\partial \theta^2}\right)\\&=\left(\rho AR+2Rb\rho^s\right)\left(\frac{\partial^2 u_r}{\partial t^2}-\frac{\partial^4 u_r}{\partial \theta^2\partial t^2}\right)\end{aligned}\tag{10}$$

where f and λ, defined as follow

$$f=-K_w u+K_p\nabla^2 u\tag{11}$$

$$\lambda=\frac{(EI)+0.5bh^2\left(2\mu^s+\lambda^s\left(1-\upsilon\right)-\upsilon\tau\right)}{R^2}\tag{12}$$

• *Critical buckling load equation*

$$\begin{aligned}&-\frac{\lambda}{R}\left(\frac{\partial^5 u_r}{\partial \theta^5}+2\frac{\partial^3 u_r}{\partial \theta^3}+\frac{\partial u_r}{\partial \theta}\right)\\&-K_w R\frac{\partial u_r}{\partial \theta}+\frac{K_p}{R}\frac{\partial^3 u_r}{\partial \theta^3}-\frac{N}{R}\frac{\partial^3 u_r}{\partial \theta^3}=0\end{aligned}\tag{13}$$

where N is the critical buckling load

3. Solution method

For free vibration of circular curved nanobeam or nanoring, analytical solution is considered. The radial displacement can be considered as

$$u_r(\theta,t)=\bar{u}_r(\theta)e^{i\omega_n t} \tag{14}$$

In which ω_n is the natural frequency of the nanoring. Navier solution is employed for simply-simply supported of the circular curved nanobeam. Therefore Eq. (14) can be rewritten as follow

$$u_r(\theta,t)=\sin\left(\frac{n\pi}{\alpha}\theta\right)e^{i\omega_n t} \tag{15}$$

where $\sin\left(\frac{n\pi}{\alpha}\theta\right)$ is the corresponding deformation shape of the circular curved nanobeam and nanoring, and i is the conventional imaginary number $\sqrt{-1}$. Substituting Eq. (15) into Eqs. (10) and (13), dimensionless natural frequencies of the circular curved nanobeam and nanoring with surface properties and elastic foundation, can be obtained as

$$\Omega_n^2=\frac{\frac{\lambda}{R}\left(\lambda_n^6-2\lambda_n^4+\lambda_n^2\right)+R\left(K_w\left(\lambda_n^2\right)+\frac{K_p}{R^2}\left(\lambda_n^4\right)\right)}{\left(\rho AR+2Rb\rho^s\right)\left(1+\lambda_n^2\right)} \times\frac{(R\alpha)^4\rho A}{EI};\quad \lambda_n=\frac{n\pi}{\alpha} \tag{16}$$

The dimensionless natural frequency of curved beam without surface effects and elastic foundation can be written as

$$\Omega_{n0}^2=\frac{\mathrm{EI}\left(\lambda_n^6-2\lambda_n^4+\lambda_n^2\right)}{\left(\rho AR^4\right)\left(1+\lambda_n^2\right)}\frac{(R\alpha)^4\rho A}{EI}, \quad \lambda_n=\frac{n\pi}{\alpha} \tag{17}$$

The dimensionless critical buckling load with surface effects and elastic foundations is determined as follow

$$N_{cr}^0=\frac{-\frac{\lambda}{R}\left(\lambda_n^5-2\lambda_n^3+\lambda_n\right)-K_wR\lambda_n-\frac{K_p}{R}\lambda_n^3}{-\frac{1}{R}\lambda_n^3}\frac{(R\alpha)^2}{EI}, \quad \lambda_n=\frac{n\pi}{\alpha} \tag{18}$$

4. Numerical results

In this section, The bulk elastic properties are E=177.3 Gpa, ρ=7000 Kg/m^3, ν=0.27 and b=5 nm. The surface elastic properties are λ^s=−8 N/m, μ^s=2.5 N/m, τ=1.7 N/m and ρ^s=7×10^{-6} Kg/m^2 (Gurtin and Murdoch 1978). To validating results, elastic foundations are eliminated and Simply-Simply supported boundary conditions are considered. Therefore, the plot of frequency ratio, with and without surface properties, versus thickness of curved nanobeam illustrated in Fig 2 and Fig. 3.

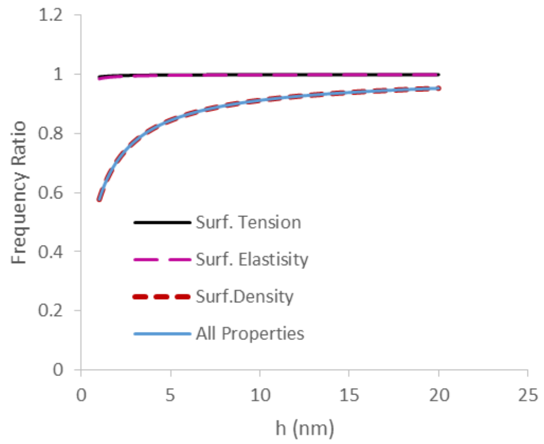

Fig. 2 Comparison of effect various surface properties on natural frequency respect to thickness *h* (nm)

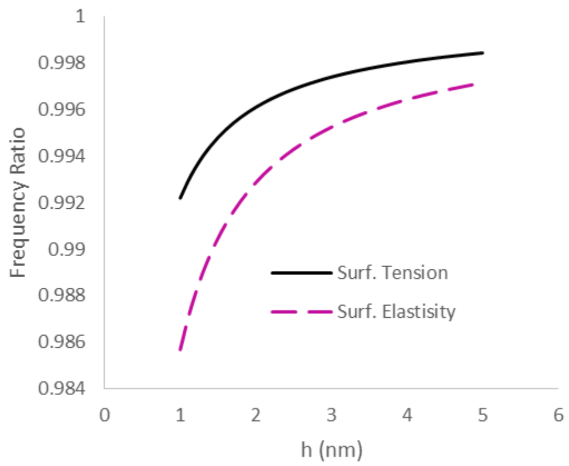

Fig. 3 Comparison of effect surface tension and elasticity, on natural frequency respect to thickness *h* (nm)

We reached to reasonable results in this survey that can represent the validity of our research as it shown in Fig. 2 and Fig. 3 and It is detected, the results are very good agreement with reference (Assadi and Farshi 2011)

4.1 Effect of thickness on frequency ratio with different radius of curvature

In this subsection, the effect of the thickness (h) with various curvature radiuses on frequency ratio with and without surface effects is examined. The same material and geometric parameters are selected is used for the results by the present model in Fig. 2. In addition the Winkler and Pasternak elastic foundation for this case, are 10^{10} N/m^2 and 10^{-6} N respectively and opening angle is $\alpha=\pi/2$. To highlight the elastic foundations properties effect, on the natural frequencies of the curved nanobeams with surface effects, the dispersion curves are presented in Fig. 4. It is clearly seen that, at the low values of thickness h, the greater values of curved nanobeams with elastic foundations effects. Hence, it is shown that by increasing thickness h, the elastic foundations effects tend to vanished. However, the Fig. 4 reveals that, the elastic foundations

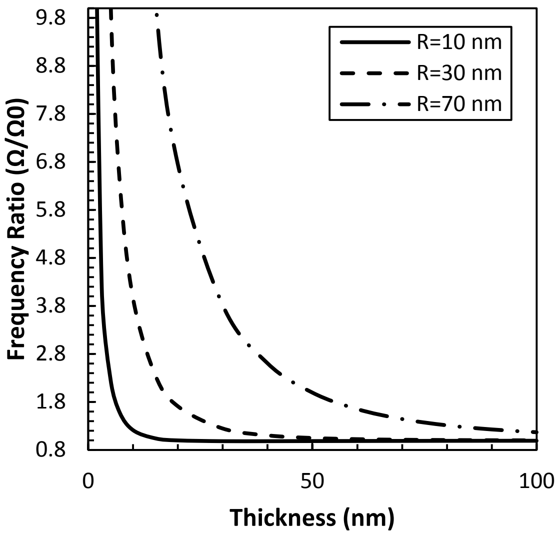

Fig. 4 Frequency ratio with and without elastic foundations effects versus thickness *h* for different radius of curvature

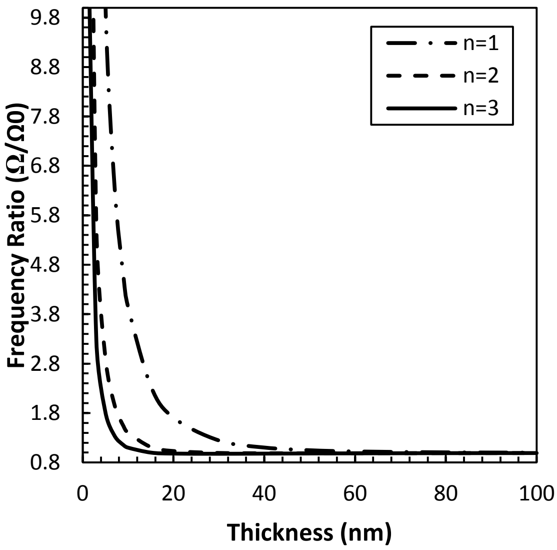

Fig. 5 First three frequency ratios of nanoarches with and without elastic foundations effects versus thickness *h*

effects play important role in higher curvature radiuses.

4.2 Analysis of higher modes on frequency ratio of curved beam with and without surface effects and elastic foundation

The frequency ratio with different modes number, with and without elastic foundations has been illustrated in Fig. 5. In this case, the following parameters are selected R=30 nm, K_w=10^{10} N/m^2, K_p=10^{-6} N.

The trends of Fig. 5 are similar to Fig. 4. It is noted that with an increase of thickness in curved nanobeam *h* in Fig. 5, the frequency ratios tend to one at three natural frequency mode numbers. It is revealed that in high values of thickness the influences of elastic foundations effects have been diminished in all mode numbers.

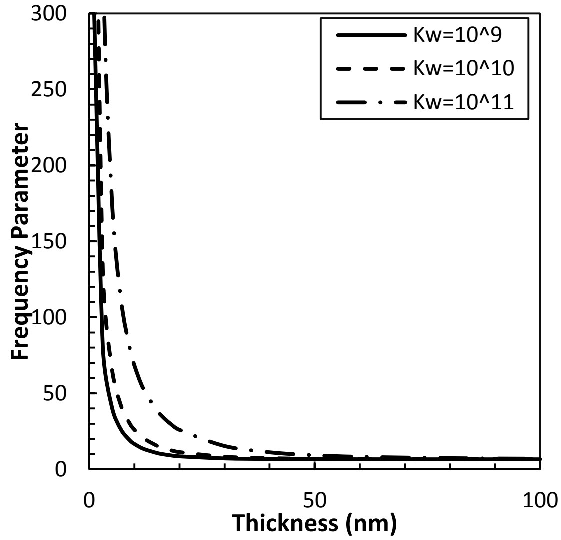

Fig. 6 Dimensionless natural frequency of nanoarches respect to thickness *h* for various Winkler elastic foundations

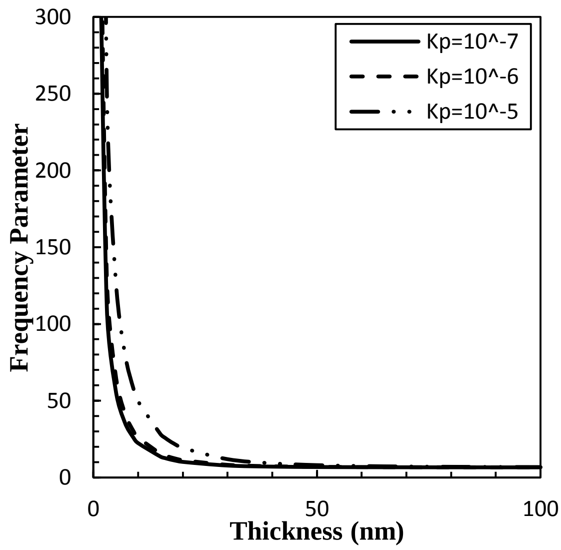

Fig. 7 Dimensionless natural frequency respect to thickness *h* for various Pasternak elastic foundations

4.3 Effect of Winkler foundation on frequency parameter

In this subsection, the effect of the Winkler elastic foundations of curved nanobeams with surface effects on the vibration frequencies is investigated respect to thickness of curved nanobeam. For this aim, the variation of fundamental dimensionless natural frequency respect to thickness with various Winkler elastic foundations is considered as shown in Fig. 6. In the case, the Pasternak elastic foundation assume constant and it is equal to 10^{-6} N.

From Fig. 6, it is seen that the Winkler elastic foundation can significantly influence the vibration of curved nanobeam with surface effects. It is also observed that as thickness of curved nanobeam heightens, the fundamental frequencies decrease, which indicates that the

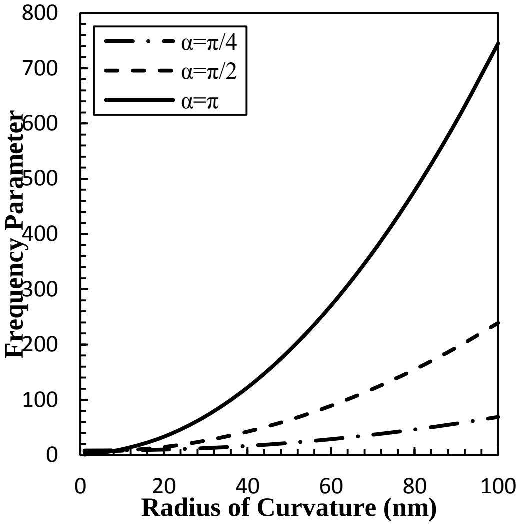

Fig. 8 Dimensionless natural frequency respect to radius of curvature for various curvature angles

Winkler elastic foundation has an important role in dimensionless frequencies. As it shown in Fig. 6, as Winkler values increase, the dimensionless natural frequencies also increase.

4.4 Effect of Pasternak foundation on frequency parameter of curved nanobeam

To evaluate the influence of the Pasternak elastic foundation on vibration curved nanobeam with surface effects, Fig. 7 presents the natural frequency of the Euler-Bernoulli model with respect to different values of Pasternak elastic foundation. For this aim, the variation of fundamental dimensionless natural frequency respect to thickness with various Pasternak elastic foundations is considered as shown in Fig. 7. In the case, the Winkler elastic foundation assume constant and it is equal to 10^{10} N/m^2.

It can be seen from Fig. 7 that the dimensionless frequency is more sensitive to low thicknesses. As the thickness of curved nanobeam increases, the dimensionless frequency decreases. However, it is observed from that as Pasternak values increase, the dimensionless natural frequencies also increase.

4.5 Effect of radius of curvature with different curvature angle on frequency parameter

To understand the influence of the radius change R on the first dimensionless natural frequency of curved nanobeam with surface effects Fig. 8. Present the natural frequencies of curved nano beam with respect to curvature radius for different angles of curvatures. Effects of the curvature radius change on the natural frequencies of curved nanobeams are shown in Fig. 8. In this case, the following parameters are selected: h=10 nm, K_w=10^{10} N/m^2, K_p=10^{-6}.

In Fig. 8, it is noted that with an increase of radius of curvature R, the dimensionless natural frequency increase.

Table 1 Radius of curvatures and opening angle effects on first three dimensionless frequency of a S-S curved nanobeam embedded in elastic medium with surface effects (h=10 nm)

R (nm)	n=1			n=2			n=3		
	Opening angle			Opening angle			Opening angle		
	$\pi/4$	$\pi/2$	π	$\pi/4$	$\pi/2$	π	$\pi/4$	$\pi/2$	π
10	8.5860	7.9899	10.4824	35.5131	34.3442	31.9595	80.4543	79.1681	75.1266
20	9.9584	14.5663	33.1482	36.7241	39.8337	58.2651	81.6298	84.1989	98.9823
30	12.5597	26.0609	70.3180	38.8763	50.2389	104.243	83.6543	93.4159	143.790
40	16.5435	42.3610	122.245	42.1267	66.1739	169.444	86.6159	107.609	210.267
50	21.9300	63.3998	188.974	46.6265	87.7200	253.599	90.6203	127.320	297.848
60	28.6895	89.1502	270.519	52.4893	114.758	356.601	95.7771	152.774	406.031
70	36.7900	119.601	366.884	59.7808	147.160	478.403	102.187	183.985	534.516
80	46.2081	154.746	478.071	68.5258	184.833	618.985	109.934	220.873	683.140
90	56.9286	194.584	604.080	78.7233	227.714	778.334	119.078	263.334	851.811
100	68.9415	239.111	744.914	90.3584	275.766	956.445	129.660	311.279	1040.47

Meanwhile it is also found that at the same curvature radius, the frequency at higher angle of curvature is greater than other frequencies.

According to Table 1, it is obviously can be seen, the dimensionless frequency increase with increasing radius of curvatures. It is interesting to say that natural frequencies also increase with increase opening angles. The results in Table 1 can be used for design of curved nanobeams and nanorings in future.

4.6 Critical buckling loads of circular curved nanobeam embedded in elastic medium

In this section, critical buckling loads of circular curved nanobeam embedded in elastic medium is discussed. For this purpose, the variation of critical buckling loads respect to radius and angle of curvatures with and without surface properties are presented in Table 2. In the case, the Winkler and Pasternak elastic foundation assume constant and it is equal to 10^9 and 10^{-5} respectively.

According to Table 2, it can be seen, the dimensionless critical buckling load increase with increasing radius of curvatures. Also should be noted that, critical buckling loads, decrease with increase opening angles. These results can be used for design of curved nanobeams and nanorings in future.

The critical buckling load ratio with and without elastic foundations has been detected in Fig. 9. In this case, the following parameters are selected R=30 nm, K_w=10^9 N/m^2, K_p=10^{-5} N, $\alpha = \frac{\pi}{2}$.

To highlight the elastic foundations effect, on the critical buckling loads of the circular curved nanobeams, the dispersion curves are presented in Fig. 9. It is obviously seen that, at the low values of thickness h, the greater values of curved nanobeams with elastic foundations effects. Hence, it is shown that by increasing thickness h, the elastic foundations effects tend to vanished. However, the Fig. 9, reveals that, the elastic foundations effects play important role in higher curvature radiuses. To evaluate the effect of

Table 2 Radius of curvatures and opening angle effects on critical buckling load of a S-S curved nanobeam embedded in elastic medium with and without surface effects (h =10 nm)

R (nm)	$\alpha = \pi/4$		$\alpha = \pi/2$		$\alpha=\pi$	
	With. S*	Without. S	With. S	Without. S	With. S	Without. S
10	9.0538	9.0920	7.1973	7.2217	6.6806	6.6806
20	10.3064	10.3445	12.2079	12.2323	26.7304	26.7304
30	12.3940	12.4321	20.5605	20.5849	60.1735	60.1735
40	15.3169	15.3551	32.2577	32.2821	107.0500	107.0500
50	19.0754	19.1135	47.3029	47.3273	167.4159	167.4159
60	23.6696	23.7077	65.7008	65.7252	241.3434	241.3434
70	29.0999	29.1380	87.4567	87.4811	328.9207	328.9207
80	35.3668	35.4049	112.5772	112.6016	430.2520	430.2520
90	42.4706	42.5087	141.0698	141.0942	545.4575	545.4575
100	50.4120	50.4501	172.9431	172.9675	674.6735	674.6735

*S: Surface Effects

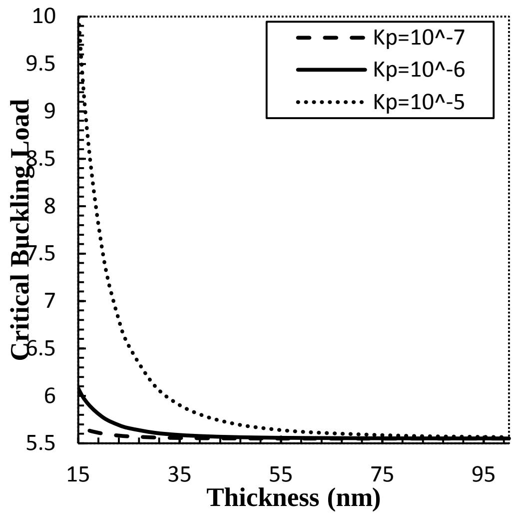

Fig. 10 Dimensionless critical buckling load respect to thickness h for various Pasternak elastic foundations

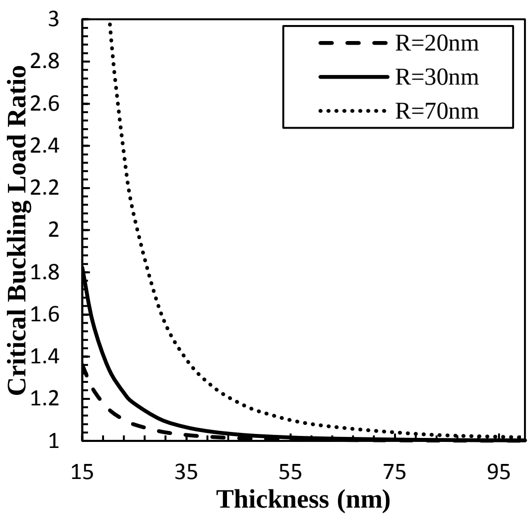

Fig. 9 Critical buckling load ratio with and without elastic foundations effects versus thickness h for different radius of curvatures

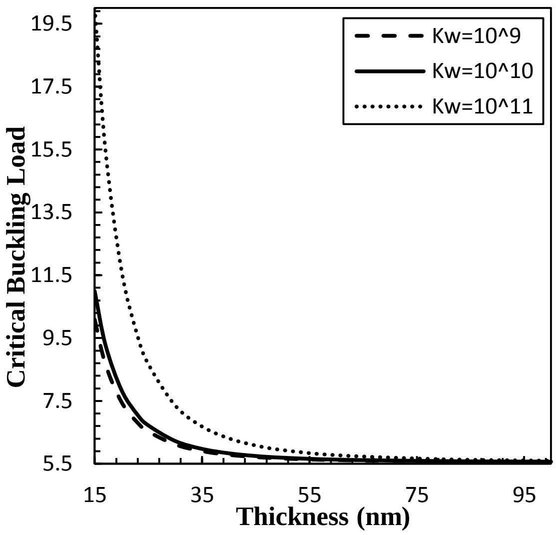

Fig. 11 Dimensionless critical buckling load respect to thickness h for various Winkler elastic foundations

the Pasternak elastic foundation on critical buckling load of circular curved nanobeam with surface effects, Fig. 10 presents the critical buckling load with respect to different values of Pasternak elastic foundations. For this aim, the variation of dimensionless critical buckling load respect to thickness with various Pasternak elastic foundations is considered as shown in Fig. 10. In the case, the Winkler elastic foundation assume constant and it is equal to 10^9 N/m^2.

It is seen from Fig. 10 that the dimensionless critical buckling load is more sensitive to low thicknesses. As the thickness of curved nanobeam increases, the dimensionless critical buckling load decreases. However, it is observed from that as Pasternak values increase, the dimensionless critical buckling load also increase.

Therefore, the effect of the Winkler elastic foundations of curved nanobeams with surface effects on the critical buckling load is investigated respect to thickness of curved nanobeam.

For this purpose, the variation of fundamental dimensionless critical buckling load respect to thickness with various Winkler elastic foundations is considered as shown in Fig. 11. In the case, the Pasternak elastic foundation assume constant and it is equal to 10^{-5} N.

From Fig. 11, it is seen that the Winkler elastic foundation can significantly influence the buckling of curved nanobeam with surface effects. It is also observed that as thickness of curved nanobeam heightens, the critical buckling loads decrease, which indicates that the Winkler elastic foundation has an important role in dimensionless critical buckling loads. As it shown in Fig. 11, as Winkler values increase, the dimensionless critical buckling loads also increase.

5. Conclusions

Derived herein are the governing equations for the free vibration and critical buckling load of circular curved nanobeam including surface elasticity, surface density and surface tension. The Winkler and Pasternak elastic foundations were considered on vibration and buckling behavior of the circular curved nanobeam. In addition, the simply-simply boundary conditions were assumed for this case. Hence the Navier method was employed to solve the governing equations. The effects of the thickness of circular curved nanobeam, Winkler and Pasternak elastic foundations, opening angle and radius of curvature, were investigated on the frequency and critical buckling load parameters of the circular curved nanobeams. It is observed that by increasing thickness *h*, the elastic foundations effects tend to vanished. Furthermore, it is shown that the elastic foundations and surface effects play an important role in vibration and buckling behavior of circular curved nanobeams.

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