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# Dominant failure modes identification and structural system reliability analysis for a long-span arch bridge

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**Abstract.** Failure of a redundant long-span bridge is often described by innumerable failure modes, which make the structural system reliability analysis become a computationally intractable work. In this paper, an innovative procedure is proposed to efficiently identify the dominant failure modes and quantify the structural reliability for a long-span bridge system. The procedure is programmed by ANSYS and MATLAB. Considering the correlation between failure paths, a new branch and bound operation criteria is applied to the traditional stage critical strength branch and bound algorithm. Computational effort can be saved by ignoring the redundant failure paths as early as possible. The reliability of dominant failure mode is computed by FORM, since the limit state function of failure mode can be expressed by the final stage critical strength. PNET method and FORM for system are suggested to be the suitable calculation method for the bridge system reliability. By applying the procedure to a CFST arch bridge, the proposed method is demonstrated suitable to the system reliability analysis for long-span bridge structure.

**Keywords:** long-span arch bridge; system reliability; dominant failure mode, identification strategy; branch-and-bound

## 1. Introduction

In recent years, several catastrophic bridge collapses have strengthen the public concern about the safety of bridge structure (Chen *et al*. 2014, Cheng and Xiao 2005, Gao *et al*. 2013). The demands for systematic and efficient risk-safety assessment method for the redundant bridge systems are increasing to prevent the possible disasters subsequently (Chen and Xiao 2015, Weng and Lei 2016, Zhu *et al*. 2014a, Zhu *et al*. 2014b). Structural system reliability analysis method is usually considered a reasonable and widely accepted approach (Cho 2007, Moyo *et al*. 2004, Wang *et al*. 2016a, Zhu *et al*. 2014a). However, system reliability analysis is often computationally intractable when applied to long-span bridge structures, because the assessment of the complex structure system often leads to numerous failure modes, each of which demands structural re-analyses to account for various uncertainties and the effect of load redistribution (Lee and Song 2011, Wang *et al*. 2016b). To overcome this computational challenge, many research efforts have been made to identify the dominant failure modes which have the highest likelihood and to calculate the overall risk by system reliability analysis (Rackwitz 2001, Shao and Murotsu 1999). Those efforts can be divided into two parts: (a) Identification of failure modes; (b) Estimation of failure probabilities.

To estimate the failure probabilities, most attentions have been paid to the component (or individual mode) reliability analysis, which deals with a failure event described by a single limit state function in the space of random variables (Kim 2013, Rackwitz 2001). For the system-level reliability analysis, in which the failure event is described by multiple limit state functions, the task becomes more complex. Although several methods have already been applied to the real structure such like FORM for system (Hohenbichler and Rackwitz 1983) and PNET (Ma and Ang 1981), some challenges are still exist such as the evaluation of the multidimensional Normal integration, the expression of limit state function and the consideration of statistical dependence between failure modes.

To identify the failure mode, many research efforts have been focus on the dominant failure modes of structural systems (Kim *et al*. 2013). Those typical approaches can be broadly divided into three categories (Shao and Murotsu 1999): a) Enumeration approach; b) Plasticity-based approach; c) Simulation-based approach.

The Enumeration approach identifies the dominant failure modes of structures system by generating a failure tree including various structural failure paths (Shao and Murotsu 1999). Each failure path is formed by extending sequences of component failures step by step until finally the structural system fails The incremental loading method (Moses and Rashedi 1983) and $\beta$-unzipping method (Thoft-Christensen and Murotsu 1986) are two typical methods of this type. They are both based on the branch and bound concept, but different on the selection criterion of branch. The incremental loading method is a type of deterministic search strategy. The failure paths with random variables are generated around their means. And the utilization ratios of element strengths are used as the selection criterion to

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choose the potential failure elements at each failure stage in order to generate different failure paths. The $\beta$-unzipping is a type of probabilistic search algorithm. It enumerates failure path based on their probabilities of occurrence which is used as the branch selection criterion. At each failure stage, a failure path with a low occurrence probability will be truncated before the whole system reaches its failure, to avoid further enumeration and saving computing time.

The Plasticity-based approach is based on the plastic mechanics theory (Ditlevsen and Bjerager 1989). By assuming the structure as an ideal plastic structural system, the analytical formulations of critical plastic mechanisms can be obtained from the plastic limit analysis. Usually, the lower-bound and upper-bound theorems are applied to determine the safety margins for the structural system. This can help avoid the time consumed on the structure re-analysis at each failure step, but it still has the disadvantage on the possibility of not defining all the dominant ones if the structural system is highly redundant.

Simulation-based approach is based on the Monte Carlo simulation method (MCS). By repeating computational simulations on the randomly generated values of uncertain parameters, is the most straightforward and widely used method (Cho 2007, Lee and Song 2011, Melchers 1994). But it is also a time consuming method (Lee and Song 2012). At each failure stage, the structure reanalysis should be repeated for every sample point due to the lack of structural failure mode knowledge. Meanwhile, since the real structure usually has a very low failure probability, this method may require overwhelming computational cost. Therefore, this method is often applied together with Plasticity-based approach, such like the LP method (Corotis and Nafday 1989, Rashedi and Moses 1986).

Compared with the existing three categories, branch and bound concept based enumeration method is considered to be the most suitable method for the real bridge structure system reliability analysis, since it costs less computational effort than the simulation-based approach (Lee and Song 2012). However, some modifications are still needed when applied to the real bridge structure.

For example, the numerical models which are often used to prove the efficiency and accuracy of those typical approaches are usually very simple truss structure with few elements and fixed load position. However, a real bridge system is much more complex, which is a typical highly redundant structural system with a huge number of potential failure paths. Therefore, how to reduce the computational time has already become the biggest challenge for the application of traditional branch and bond based method. Meanwhile, the major load for a bridge is the traffic load, which is a typical moving load. How to consider the traffic load with a moving loading position is another problem needed to be solved.

In this paper, focusing on those problems, some modifications are made to the traditional branch and bound method (Traditional Method). An automatically procedure of dominant failure mode identification and system reliability analysis for bridge structure is introduced. The efficient of this procedure is proved by its application to the system reliability analysis of a CFST arch bridge.

## 2. Improved stage critical strength branch and bound method for bridge structure

Stage Critical Strength Branch and Bound Method is a type of incremental load method introduced by Cong Dong (2001). This method improves the incremental load method by bringing in the stage critical strength as the branch and bound selection criterion. This improvement can save some computational effort, but the improvement is still not enough to apply it to real complex bridge structures.

From the discussion on the traditional methods, it can be concluded that: a) searching dominant failure modes of a real complex bridge structure takes too much effort, while b) most of the actually useless computation times are wasted on the searching un-dominant failure path and structure re-analyses at each failure stage. To speed up the searching progress, an improved selective search strategy is proposed based on the Traditional Method. The improved method considers the correlation between paths in the branching operation to prevent highly correlated paths. This improved searching strategy contained the following steps:

*Step0. Set up parametric bridge structure FE model.*

Usually, a real bridge structure is a typical highly redundant structural system, which makes the bridge FE model contains hundreds or thousands elements. However, the bridge FE model is needed to be rebuilt and reanalyzed to ignore the already failed element, which will cost most of the computation effort. Modern professional software like ANSYS helps a lot to solve this problem, through the ANSYS Parametric Design Language (APDL), the FE mode can be set up by parametric method and reanalyzed in an explicit mode.

*Step1. Calculate the load factor $a_{r_k}^{(i)}$.*

At this part, all the random variables of components are fixed at their mean values, and a deterministic structural analysis is performed to identify a failure path. Since the FEM is already rebuilt with the removal of the failed elements, the load redistribution can be considered. $a_{r_k}^{(i)}$ is the load effect of component $r_k$, which is caused by the standard unit external load at failure stage $i$.

In traditional Method, the standard external load is in the form of unit concentrated load which cannot reflect the uncertainty characters of traffic load. To make the traditional method suitable to bridge structures, traffic load model from bridge code is applied as the standard external load, which contains two parts: a uniformly distributed load and a concentrated load. The uniformly distributed load represents the normal traffic load while the concentrated load represents the heavy truck which sometimes may be overloaded. Since the traffic load is a kind of moving load, in order to reflect this uncertainty, influence line method is applied when calculates the $a_{r_k}^{(1)}$ by FE program at the first failure stage.

*Step2. Determine the component residual resistance $R_{r_k}^{(k)}$*

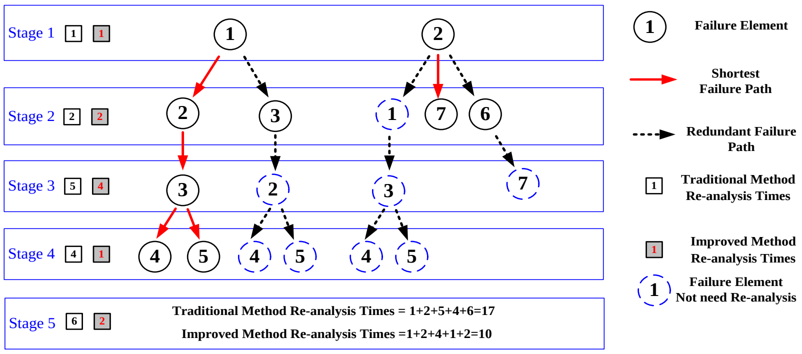


Fig. 1 Comparison of traditional and improved operation parameter

At stage $k$ for a failure path, define $\Delta F_{r_i}^{(i)}$ as the external load increment at stage $i$. The $R_{r_k}^{(k)}$ is calculated as Eq. (1).

$$\begin{cases} R_{r_k}^{(k)} = R_{r_k}^{I_{r_k}} - G_{r_k}^{(k)} - I_{r_k} \times \sum_{i=1}^{k-1} a_{r_i}^{(i)} \Delta F_{r_i}^{(i)} m_{r_i} \\ \Delta F_{r_i}^{(i)} = \dfrac{R_{r_i}^{(i)}}{a_{r_i}^{(i)}} \\ I_{r_k} = sign[a_{r_k}^{(k)}] \end{cases} \tag{1}$$

Where $R_{S,r_k}^{I_{r_k}}$ is the initial resistance for component $r_k$. $m_{r_i}$ is the component failure type indicator. For example, if the component failure type is brittle $m_{r_i}$=1, else if it is ductile $m_{r_i}$=0. $G_{r_k}^{(k)}$ is the load effect caused by dead load.

*Step3. Branch and bound operation.*

Define $R_{S,r_k}^{(k)}$ as the stage critical strength of bridge system at stage $k$ supposing the failure component number is $r_k$, the branch and bound operation is summarized as a optimization problem (Eq. (2)).

$$\begin{aligned} &\text{Find} \quad r_k \\ &\text{s.t.} \quad R_{S,r_k}^{(k)} \le c_k R_{S(\min)}^{(k)} \\ &\qquad\; 0 \le c_k < \infty \\ &\qquad\; r_k \left[ r_k \in (1,2,\cdots,n), r_k \notin (r_1, r_2, \cdots, r_{k-1}) \right] \\ &where \begin{cases} R_{S,r_k}^{(k)} = \Delta F_{r_k}^{(k)} + \sum_{i=1}^{k-1} \Delta F_{r_i}^{(i)} m_{r_i} = \mathbf{m}^{(k)} \mathbf{\Delta F}^{(k)} \\ \qquad\;\; = \sum_{i=1}^{k} \beta_{R_i}^{(k)} R_{r_i}^{I_{r_i}} - \beta_G^{(k)} g \\ \mathbf{m}^{(k)} = \left[ m_{r_1}, m_{r_2}, \cdots, m_{r_{k-1}}, 1 \right] \\ \mathbf{\Delta F}^{(k)} = \left[ \Delta F_{r_1}^{(1)}, \Delta F_{r_2}^{(2)}, \cdots, \Delta F_{r_{k-1}}^{(k-1)}, \Delta F_{r_k}^{(k)} \right]^T \end{cases} \end{aligned} \tag{2}$$

$c_k$ is the bounding parameter, with a chosen value based on the required degree of accuracy. Traditionally, when $c_k \ge 1.2$, the branches of failure path contributes little to the system reliability (Dong 2001).

Traditional Method uses $c_k$ as the only bounding operation parameter, which only considers the locally most-likely-to-fail elements. Two types of redundant failure paths will be retained which can be instead by the other failure path. For example, the failure path 2-1-3-4 can be replaced by 1-2-3-4 while the failure path 2-6-7 can be replaced by 2-7 (Fig. 1).

If those redundant failure paths can be selected in an early stage, much computation cost can be saved. Therefore, a new constraint condition is added to the traditional branch and bound operation based on the concept of shortest failure path. Totally, there are two types of shortest failure path. At each failure stage, the failure path with lowest stage critical strength is defined as the stage shortest failure. While if the stage shortest failure path has caused the structural system failure already, it will be defined as the dominant failure mode. Therefore, if a local failure path has a dominant failure mode as its subset, this path will be a redundant failure path which can be ignored. The comparison of the re-analysis numbers is shown in Fig. 1. The traditional method needs 17 FEM re-analysis while the improved method only uses 10 FEM re-analysis. The improved Branch and bound operation criteria is shown as Eq. (3).

$$\begin{cases} \text{Find} \quad r_k \\ \text{s.t.} \quad R_{S,r_k}^{(k)} \le c_k R_{S(\min)}^{(k)} \\ \qquad\;\; r_k \left[ r_k \in (1,2,\cdots,n), r_k \notin (r_1, r_2, \cdots, r_{k-1}) \right] \\ \qquad\;\; 0 \le c_k < \infty \\ \qquad\;\; r_1 \to r_2 \to \cdots \to r_k \notin \text{Redundant Failure Paths} \end{cases} \tag{3}$$

*Step4. Repeat Step0 to Step3 until all the dominant failure modes are identified.*

Finally, if at stage $k$, the structure system is failed with failure mode $r_1 \to r_2 \to \cdots \to r_k$, the bridge structural system limit state equation can be expressed as Eq. (4). $F_{traffic}$ a is the design traffic load, $\beta_i^{(k)}$ and $\beta_G^{(k)}$ are the parameters calculated by $a_{r_k}^{(i)}$ and $m_{r_i}$.

$$\begin{aligned} Z_{r_1 \to r_2 \to \cdots \to r_k} &= R_{S,r_k}^{(k)} \times F_{traffic} - F_{traffic} \\ &= \left( \sum_{i=1}^{k} \beta_{R_i}^{(k)} R_{r_i}^{I_{r_i}} - \beta_G^{(k)} g \right) \times F_{traffic} - F_{traffic} \\ &= 0 \end{aligned} \tag{4}$$

Eq. (4) shows that the limit state function of dominate failure mode can be expressed by the final stage critical strength of bridge system, which help the calculation of the reliability index and correlation coefficient of failure mode becomes easier than the other methods.

## 3. Reliability evaluation for bridge system based on FORM and PNET Method

Usually, the bridge system failure probability $P_{f,system}$ can be expressed as the union probability of the failure modes (Eq. (5))

$$P_{f,system} = P\left[\bigcup_{\text{all possible failure modes}} E_{i,mode}\right] \tag{5}$$

The number of failure modes will be enormous. However, in most cases, only a small fraction of failure modes (dominant failure modes) contributes significantly to the overall failure probability of the system. In that case, the failure probability of the system can be described as Eq. (6)

$$\begin{aligned} P_{f,system} &= P\left[\bigcup_{\text{all possible failure modes}} E_{i,mode}\right] \\ &\approx P\left[\bigcup_{\text{dominant failure modes}} E_{j,mode}\right] \\ &= P\left[\bigcup_{k=1} P_{f,mode}\left(G_k(\mathbf{u}) \le 0\right)\right] \end{aligned} \tag{6}$$

$G_k(\mathbf{u})$ is the limit state equation of $k$th dominant failure mode.

For the past few decades, many research efforts have been made to estimate the failure probabilities of component that deals with a failure event described by a single limit state function. In this paper, the First Order Reliability method (FORM) is suggested as the calculation method for $P_{f,mode}$, since the limit state equation $G_k(\mathbf{u}) = 0$ can be obtained by Eq. (4).

For the calculation of $P_{f,system}$ two types of traditional methods are suggested to fit for different condition. One method is the FORM for system (Hohenbichler and Rackwitz 1983); the other is the Probabilistic Network Evaluation Technique (PNET) (Ma and Ang 1981).

*(1) FORM for system*

The traditional FORM approach can be easily extended to system reliability problems by looking at the complementary events and using the symmetry property of the standard Normal space (Hohenbichler and Rackwitz 1983). As a series system reliability analysis, the bridge system failure probability $P_{f,system}$ can be calculated as Eq. (7)

$$P_{f,system} = P\left[\bigcup_{i=1}^{m} G_i(\mathbf{u}) \le 0\right] \approx 1 - \Phi_m(\boldsymbol{\beta}, \mathbf{R}) \tag{7}$$

$m$ is the number of failure modes. $\boldsymbol{\beta} = [\beta_1, \beta_2, \cdots \beta_k]$ is the vector of reliability index of dominant failure modes, $\mathbf{R} = [\rho_{ij}]$ is the matrix of correlation coefficients between each dominant failure modes. $\Phi_m(\cdot)$ is the joint cumulative distribution function of multivariate normal distribution.

The evaluation of the multidimensional Normal integration is certainly a very important task and approximate methods have been developed over the years. If the correlation coefficients between each dominant failure modes are close to each other, the above multidimensional Normal integration can be approximately calculated as Eq. (8). $\varphi(\cdot)$ is the PDF of standard normal distribution. $\Phi(\cdot)$ is the CDF of standard normal distribution.

$$\begin{cases} P_{f,system} = \Phi_m(\boldsymbol{\beta}, \mathbf{R}) \approx \int_{-\infty}^{\infty} \varphi(t) \prod_{i=1}^{m} \Phi\left(\dfrac{\beta_i - \sqrt{\bar{\rho}}t}{\sqrt{1-\bar{\rho}}}\right) dt \\ \bar{\rho} = \dfrac{1}{m(m-1)} \sum_{i \ne j}^{m} \rho_{ij} \end{cases} \tag{8}$$

*(2) PNET*

The PNET method assumes that the failure modes are highly correlated with $\rho_{ij} \ge \rho_{up}$, while those with $\rho_{ij} \ge \rho_{down}$ are statistically independent. $\rho_{up}$ and $\rho_{down}$ are the demarcating correlations. Their value depends on the failure probabilities of the single failure modes.

For those highly correlated failure modes, for example failure mode $i$ with failure probability $P_i$ having a correlation coefficient $\rho_{ij}$ with another mode $j$ with failure probability $P_j$ will be ignored, if $\rho_{ij} \ge \rho_{up}$ and $P_i \le P_j$. The remaining modes are defined as the representative modes and the failure probability of the system can be approximated by Eq. (9).

$$P_{f,system} = 1 - \prod_{i=1}^{n}\left(1 - P_{f,i}\right) \tag{9}$$

$P_{f,i}$ is the failure probability of representative failure mode.

The above two reliability evaluation methods are fit for different situations. FORM for system is the suitable method when the correlation coefficients between each dominant failure modes are very near. PNET method will be a proper method when the dominant failure modes can be divided into two groups separately with high or low correlation coefficient. Correlation coefficients between dominant failure modes are the key parameters for both two reliability evaluation methods. The proposed method can get the limit state functions of dominate failure modes, so this will not be a problem.

## 4. Procedure of dominant failure mode identification and system reliability analysis for bridge structure

A bridge may collapse in different failure modes, depending upon the combination of applied loads and the strengths of various elements. Although focus on the dominant failure modes has already simplified the problem greatly, identification of the dominant failure modes is still a very complex and time-consuming process.

In order to quickly and automatically identify the dominant failure modes in order to analyze the system reliability of bridge structure, a procedure is proposed. This

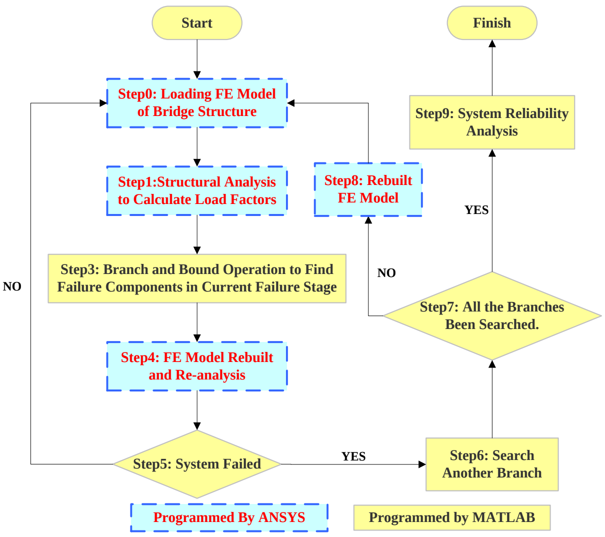


Fig. 2 Flow chart of dominant failure modes identification and structural reliability analysis

procedure is actualized by calling the ANSYS program in MATLAB program. The ANSYS program is used to build up the bridge FE model in ANSYS Parametric Design Language (APDL), so that the MATLAB can run it at backstage, which can save almost half computation time.

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The MATLAB is used to program the dominant failure mode identification scheme and reliability analysis procedure. The procedure is designed as an automatically program. There is no need for manual operation during the dominant failure identification and reliability analysis process. The flowchart of this procedure is shown as Fig. 2.

## 5. Application to a CFST arch bridge

### *5.1 Dominant failure modes identification for the arch rib*

The safety assessment of existing Concrete filled steel

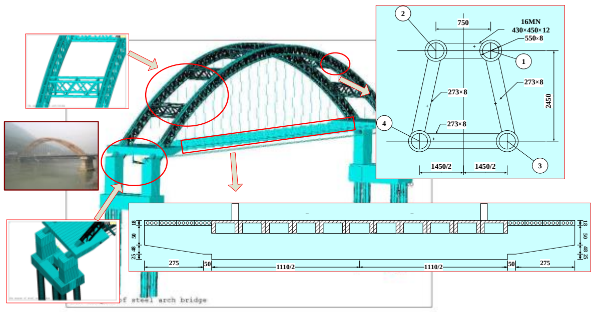


Fig. 3 FEM of the CFST arch bridge

tubular (CFST) arch bridge has become a research focus in China. In this section, a CFST arch bridge is illustrated as an application example of the proposed procedure of dominant failure mode identification and system reliability analysis for bridge structures.

The CFST arch bridge is located in Sichuan Province with 2 lanes and 13 m wide and 138 m long in span. The bridge is numerically modeled by ANSYS program as shown in Fig. 3, which has 58 link elements for suspenders and tie bar and 6482 beam elements for all the other components. The details about the FEM model are listed in Table 1.

Since the arch rib is the key elements for the CFST arch bridge and its failure can be considered as the failure of the whole bridge system, the analysis in this paper is focused on the failure of arch rib. The arch rib is treated as an ideal truss structure. The failure type of chord members and web members of arch rib are considered to be ductile.

According to the design documents, the design traffic load of this bridge is Road Class II in the Chinese bridge design code. The load pattern is offset load at the upper lane and the concentrated load is placed at the position of each hanger crossbeam (Fig. 4). Since this bridge is a symmetric structure, only half span is analyzed. The searching results of dominant failure components at first failure stage with critical strength is found at load location 8 which is near 1/3 span.

The failure sequences of dominant failure modes in Table 2 are shown in Fig. 5. It's obvious that all the top chords near the loading location are failed. For the failure mode at location 01 to location 09, the bottom chords at arch springing are failed, while at location 10~13 the failed parts are near the joint of first wind brace.

### *5.2 System reliability analysis of the arch rib*

Table 1 Material properties and parameters for major component of the bridge FEM model

| Component | Material | Element Type | Elastic Modulus(E)(GPa) | Density(kg/m3) | Poisson ratio |
|---|---|---|---|---|---|
| arch rib | 16Mn | Beam 44 | 206 | 7800 | 0.3 |
| concrete filled | C40 | Beam 44 | 32.5 | 2600 | 0.1667 |
| beam | C40 | Beam 44 | 32.5 | 2600 | 0.1667 |
| suspender | Steel strand | Link10 | 190 | 7800 | 0.3 |
| cushion cap | C30 | Beam 44 | 30.0 | 2600 | 0.1667 |
| pile | C30 | Beam 44 | 30.0 | 2600 | 0.1667 |
| bar | steel | Beam 44 | 206 | 7800 | 0.3 |

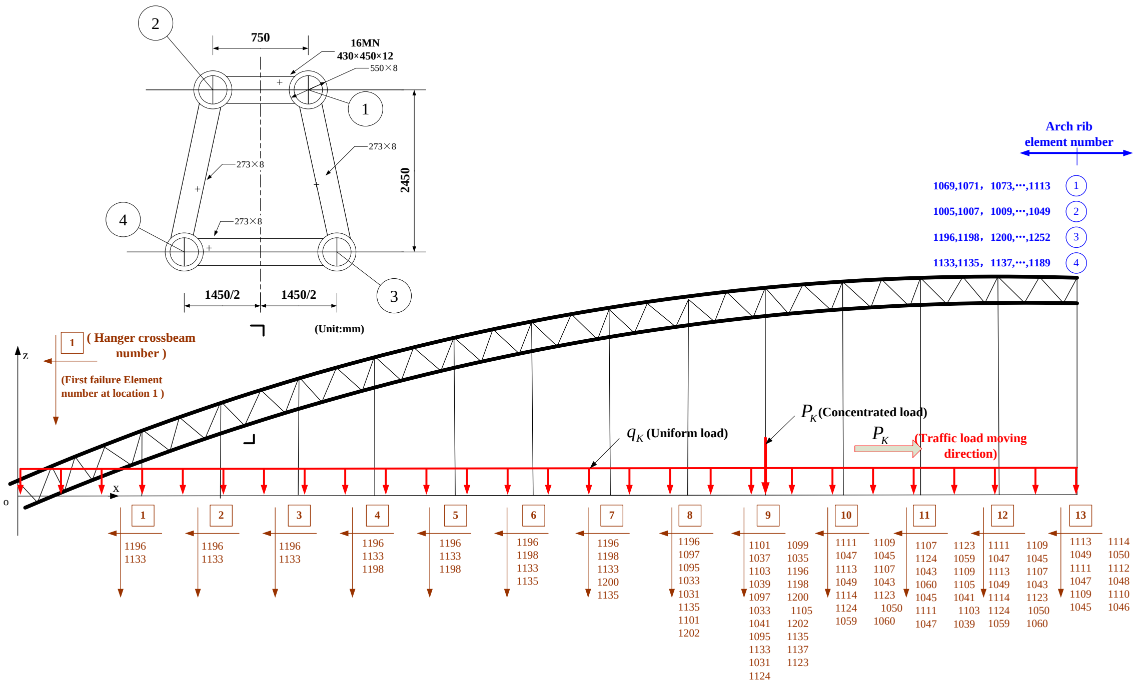



Fig. 4 Dominant failure components at first failure stage

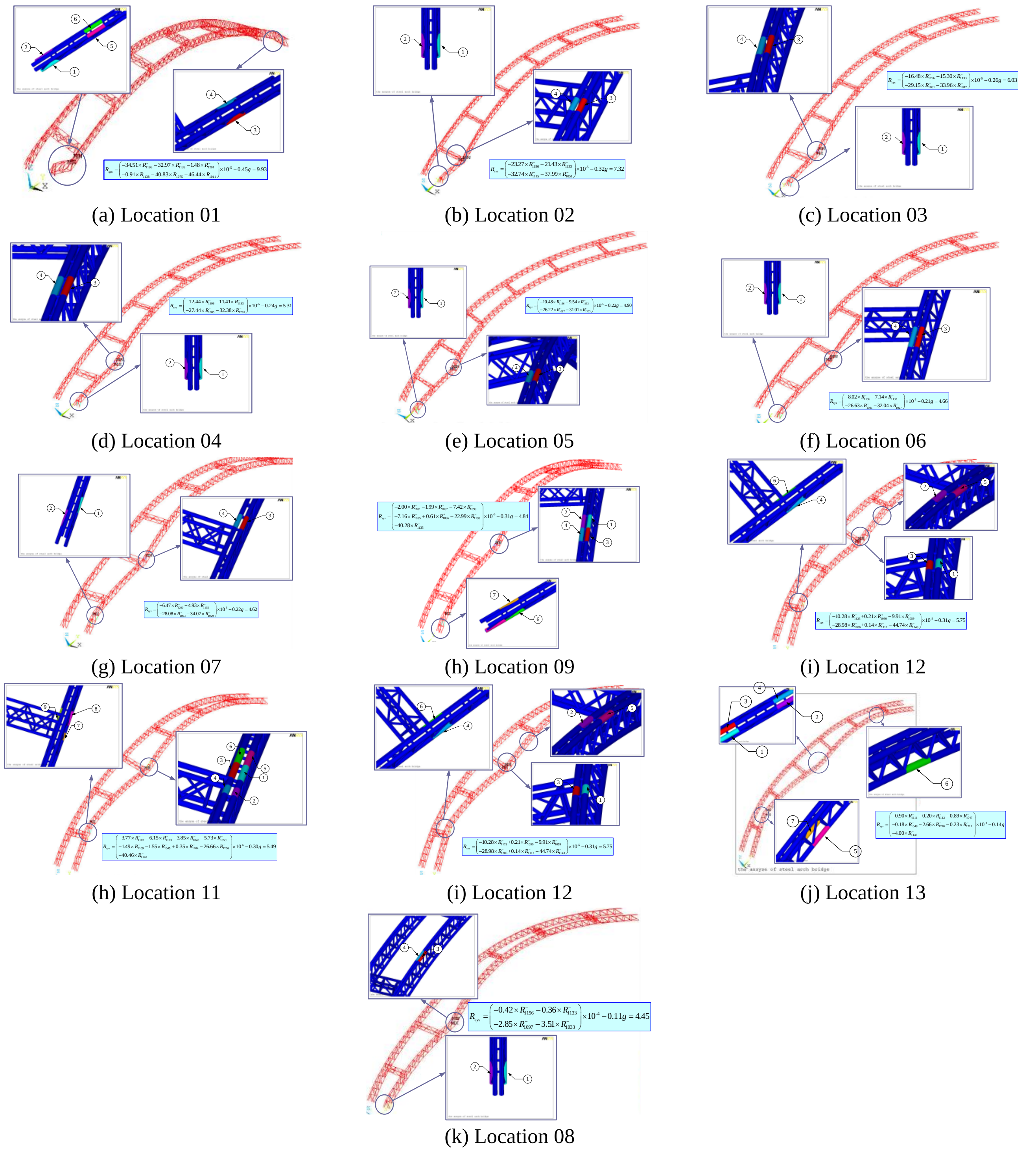
Table 2 Expressions for system final critical strength at different load location

| Location | Expression for system final critical strength $\times F_{traffic}$ |
|---|---|
| 1 | $R_{sys}=\left(-3.45\times R_{1196}^{-}-3.30\times R_{1133}^{-}-0.15\times R_{1201}^{-}-0.09\times R_{1138}^{-}-4.08\times R_{1075}^{-}-4.64\times R_{1011}^{-}\right)\times 10^{-4}-0.23g=9.93$ |
| 2 | $R_{sys}=\left(-2.33\times R_{1196}^{-}-2.14\times R_{1133}^{-}-3.27\times R_{1115}^{-}-3.80\times R_{1051}^{-}\right)\times 10^{-4}-0.17g=7.32$ |
| 3 | $R_{sys}=\left(-1.65\times R_{1196}^{-}-1.53\times R_{1133}^{-}-2.92\times R_{1081}^{-}-3.40\times R_{1017}^{-}\right)\times 10^{-4}-0.14g=6.03$ |
| 4 | $R_{sys}=\left(-1.24\times R_{1196}^{-}-1.14\times R_{1133}^{-}-2.74\times R_{1085}^{-}-3.24\times R_{1201}^{-}\right)\times 10^{-4}-0.12g=5.31$ |
| 5 | $R_{sys}=\left(-1.05\times R_{1196}^{-}-0.95\times R_{1133}^{-}-2.62\times R_{1087}^{-}-3.10\times R_{1203}^{-}\right)\times 10^{-4}-0.11g=4.90$ |
| 6 | $R_{sys}=\left(-0.80\times R_{1196}^{-}-0.71\times R_{1133}^{-}-2.66R_{1091}^{-}-3.20\times R_{1027}^{-}\right)\times 10^{-4}-0.11g=4.66$ |
| 7 | $R_{sys}=\left(-0.65\times R_{1200}^{-}-0.49\times R_{1135}^{-}-2.81\times R_{1093}^{-}-3.41\times R_{1029}^{-}\right)\times 10^{-4}-0.11g=4.62$ |
| 8 | $R_{sys}=\left(-0.42\times R_{1196}^{-}-0.36\times R_{1133}^{-}-2.85\times R_{1097}^{-}-3.51\times R_{1033}^{-}\right)\times 10^{-4}-0.11g=4.45$ |
| 9 | $R_{sys}=\begin{pmatrix}-0.20\times R_{1101}^{-}-0.20\times R_{1037}^{-}-0.74\times R_{1099}^{-}-0.72\times R_{1035}^{-}\\+0.06\times R_{1096}^{+}-2.30\times R_{1198}^{-}-4.03\times R_{1135}^{-}\end{pmatrix}\times 10^{-4}-0.16g=4.84$ |
| 10 | $R_{sys}=\left(-1.03\times R_{1123}^{-}+0.02\times R_{1050}^{+}-0.99\times R_{1059}^{-}-2.90\times R_{1206}^{-}+0.02\times R_{1112}^{+}-4.48\times R_{1143}^{-}\right)\times 10^{-4}-0.16g=5.75$ |
| 11 | $R_{sys}=\begin{pmatrix}-0.38\times R_{1107}^{-}-0.62\times R_{1123}^{-}-0.39\times R_{1043}^{-}-0.57\times R_{1059}^{-}\\-0.15\times R_{1109}^{-}-0.16\times R_{1045}^{-}+0.04\times R_{1204}^{+}-2.67\times R_{1206}^{-}-4.05\times R_{1143}^{-}\end{pmatrix}\times 10^{-4}-0.15g=5.49$ |
| 12 | $R_{sys}=\left(-1.03\times R_{1123}^{-}+0.02\times R_{1050}^{+}-0.99\times R_{1059}^{-}-2.90\times R_{1206}^{-}+0.02\times R_{1112}^{-}-4.48\times R_{1143}^{-}\right)\times 10^{-4}-0.16g=5.75$ |
| 13 | $R_{sys}=\begin{pmatrix}-0.90\times R_{1111}^{-}-0.20\times R_{1112}^{-}-0.89\times R_{1047}^{-}-0.18\times R_{1048}^{-}\\-2.66\times R_{1210}^{-}-0.23\times R_{1211}^{-}-4.00\times R_{1147}^{-}\end{pmatrix}\times 10^{-4}-0.14g=5.61$ |

The limit state function is determined by arch rib's axial strength and external axial force.

Based on Eq. (4), the limit state function of each dominant failure mode can be expressed as Eq. (10). $\alpha_{R_i}$, $\alpha_G$ and $\alpha_{traffic}$ are random variables represent the uncertainties of the resistance, dead load and traffic load.

$$
\begin{aligned}
&Z_{r_1\to r_2\to\cdots\to r_k}\\
&=\left(\sum_{i=1}^{k}\alpha_{R_i}\beta_{R_i}^{(k)}R_{r_i}^{I_{r_i}}-\alpha_G\beta_G^{(k)}g\right)\times F_{traffic}-\alpha_{traffic}F_{traffic} \qquad (10)\\
&=0
\end{aligned}
$$



(a) Location 01 (b) Location 02 (c) Location 03
(d) Location 04 (e) Location 05 (f) Location 06
(g) Location 07 (h) Location 09 (i) Location 12
(h) Location 11 (i) Location 12 (j) Location 13
(k) Location 08

Fig. 5 Failure sequences for dominant failure modes at different load location

The probabilistic distribution and statistical parameters obtained from the literature survey and assumptions are shown in Table 3. All these random variables are assumed to be statistical independent.

Based on the limit state equations of all the dominant failure modes, the reliability index of each failure mode is calculated by FORM. The results are shown in Fig. 6. The minimum reliability index is 6.933 with the loading position at location 08.

The correlation coefficients between each failure mode are shown in Fig. 7.

Table 3 Probabilistic properties of random variables

| Variable | Type | Parameter | Reference |
|---|---|---|---|
| $\alpha_{R_i}$ | Normal | $\mu_{\alpha_{R_i}}=1.05, \sigma_{\alpha_{R_i}}=0.10$ | (Nowak and Cho 2007) |
| $\alpha_G$ | | $\mu_{\alpha_{R_i}}=1.0212, \sigma_{\alpha_{R_i}}=0.0462$ | (GB/T50283-1999 1999) |
| $\alpha_{traffic}$ | | $\mu_{\alpha_{R_i}}=0.6684, \sigma_{\alpha_{R_i}}=0.1994$ | |

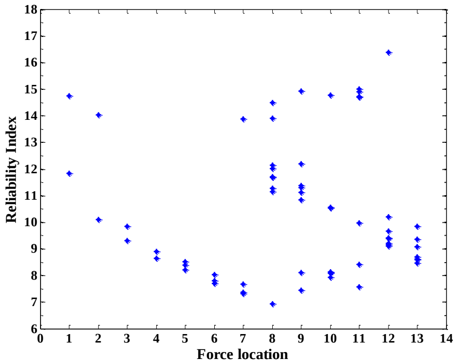



Fig. 6 Reliability indexes of all the 70 dominant failure modes

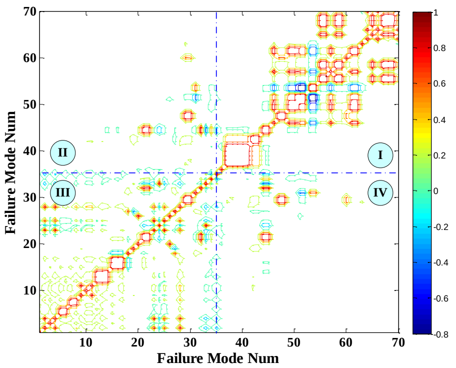



Fig. 7 Contour plot for correlation coefficients of failure modes

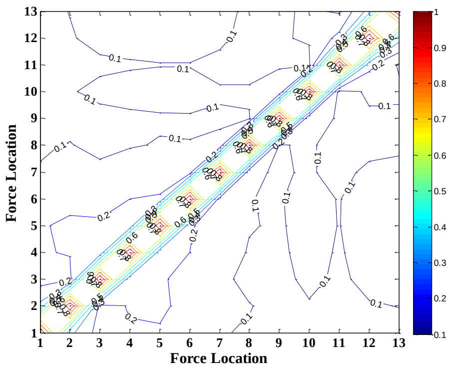



Fig. 8 Contour plot for correlation coefficients at different force locations

Most failure modes have a positive correlation between each other, while only a few have a negative correlation.

The correlation coefficients between each failure mode at different location are shown in Fig. 8. The result shows that the correlation coefficients will decrease when the distance between loading location increases.

The statistical results for the correlation coefficients between each failure mode are shown in Fig. 9.

It shows that above 93% failure modes have a correlation coefficient less than 0.3 and above 4.2% failure modes have a correlation coefficient more than 0.8. The reason for this result is that most of the dominant failure modes have very few similar failure elements with each

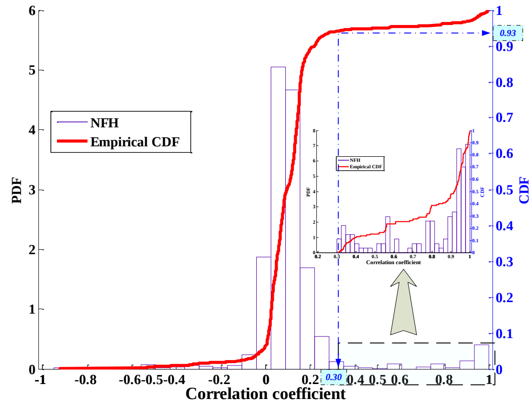



Fig. 9 Static analysis for correlation coefficients of failure modes

Table 4 Comparison of structural re-analysis times for different methods

| Failure Mode | Structural re-analysis times required to identify the failure modes | | |
|---|---|---|---|
| | MCS Method | Traditional Method | Proposed Method |
| load location 8 | 23420 | 116 | 105 |

other. The correlation coefficients between each dominant failure modes are apparently divided into two groups. As a result, the PNET might be a proper method to calculate the bridge system failure probability. Finally, the reliability index of the bridge system is calculated by PNET method, which is equal to 6.72.

### *5.3 Comparison with MCS method and traditional method*

In order to examine the efficiency of the proposed method in the dominate failure mode identification process, the Monte Carlo simulation method (MCS) is carried out for comparison. The MCS method directly generates the original random variables in Table 3 from their distributions, and the structural analysis is then performed to check the failures of the components as well as the progressive failure of the whole bridge structure due to the load re-distribution. Since the number of simulations for MCS will be greatly increased with failure probability decrease, only the failure modes on load location 8 with the largest failure probability are compared. In order to reduce the computation time of MCS to an acceptable level, the standard deviations of all the variables in Table 3 will be expanded to five times of the original ones. This change will influence the accuracy of failure probability results calculated by MCS. So, here we only check the efficiency of the proposed method. For the proposed method, the accuracy of the failure probability depends on the calculation method such like FORM, which will not be compared in this paper. The MC simulations will be terminated when all the dominant failure modes by the proposed method is obtained. For a fair comparison, the MCS result is the total number of structural re-analysis until

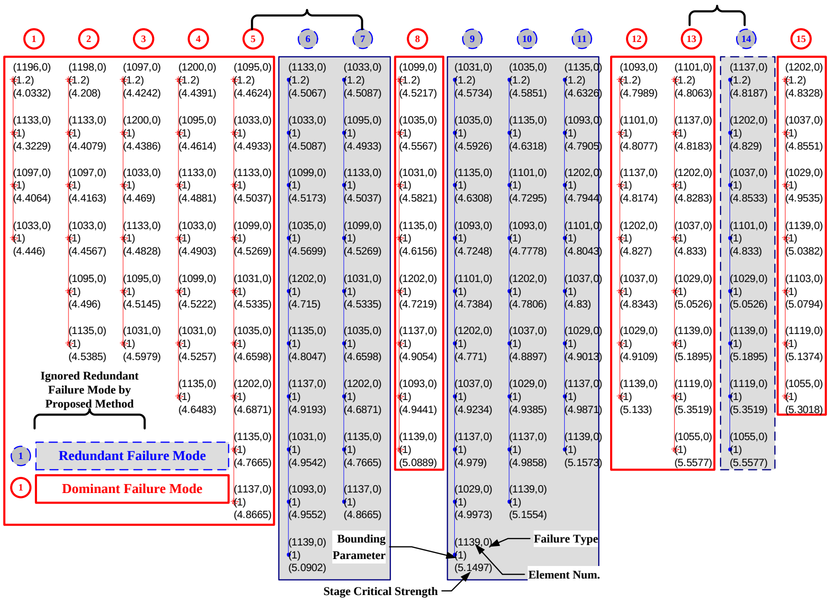



Fig. 10 Failure modes identified at load location 8

the MCS method identifies the same dominant failure modes as those identified by the proposed searching method.

Fig. 10 shows the dominant failure modes identified by proposed method and traditional method at load location 8. The comparison results are listed in Table 4. The MCS result is the number of structural re-analysis required to identify the same failure modes in Fig. 10.

The number of re-analysis required by Traditional Method and the proposed searching method is much less than the MCS method. That is because MCS method needs to repeat the structure reanalysis for every sample point due to the lack of structural failure mode knowledge at each failure stage. Compared with the Traditional Method, the proposed searching method can chose out those redundant failure paths earlier (No.7 and No.14 in Fig. 10). As a result, structural re-analysis of proposed method is 11 less than the traditional in load location 8. For all the load locations, the proposed method uses only 679 calculation times of structural reanalysis to identify the total 70 dominant failure modes. Save almost 1/3 computation effort compared with the traditional method which takes 1034 calculation times.

## 6. Conclusions

This paper develops an efficient method to identify the dominant failure modes and estimate the system reliability for bridge structures. In the proposed method, dominant failure modes are effectively identified by a selective searching technique based on the improved Stage Critical Strength Branch and Bound Method. The bridge system failure probability is calculated by PNET method and FORM for system. The advantages of the proposed approach are as follows:

- By taking advantage of the correlation between paths in the branching operation, the improved method can select those redundant failure paths earlier than the traditional method. Since the redundant failure paths contribute little to the failure probability of the system, computation cost can be saved by ignoring them.
- Two reliability evaluation methods, the PNET method and FORM for system, are fit for different situations depending on the condition of correlation coefficients between each dominant failure mode. For the limit state functions of dominate failure modes can be expressed easily by the final stage critical strength of bridge system, the PNET method and FORM for system are suggested to be the suitable calculation method for the bridge system reliability in the proposed method.
- Compared with the MCS method and traditional Stage Critical Strength Branch and Bound Method, the proposed method can save a lot of computational effort. By applying the procedure to a CFST arch bridge, the algorithm is demonstrated to be suitable for long-span bridge structural systems reliability analysis.

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