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A simple analytical approach for thermal buckling of thick functionally graded sandwich plates

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Abstract. This study aimed to presents a simple analytical approach to investigate the thermal buckling behavior of thick functionally graded sandwich by employing both the sinusoidal shear deformation theory and stress function. The material properties of the sandwich plate faces are continuously varied within the plate thickness according to a simple power-law distribution in terms of the volume fractions of the constituents. The core layer is still homogeneous and made of an isotropic material. The thermal loads are considered as uniform, linear and non-linear temperature rises across the thickness direction. Numerical examples are presented to prove the effect of power law index, loading type and functionally graded layers thickness on the thermal buckling response of thick functionally graded sandwich.

Keywords: functionally graded materials; thermal buckling; sandwich plate; sinusoidal shear deformation theory; stress function

1. Introduction

Due to the important performance heat resistance ability and outstanding characteristics comparatively to conventional composites, Functionally Graded Materials (FGMs) which are microscopically composites and made of mixture of metal and ceramic constituents have gained considerable attention recent years. These materials are manufactured by a continuous variation of the gradient of the volume fractions of the constituents (Koizumi 1997); the FGM is hence adapted for various practical applications, such as thermal coatings of barrier for ceramic engines, gas turbines, nuclear fusions, optical thin layers, biomaterial electronics, etc. As a result, the mechanical response of FGM structures is of considerable importance in both research and industrial fields (Tounsi *et al.* 2013, Bouderba *et al.* 2013, Chakraverty and Pradhan 2014, Liang *et al.* 2014, Zidi *et al.* 2014, Ait Atmane *et al.* 2015, Ait Yahia *et al.* 2015, Akbaş 2015, Meksi *et al.* 2015, Arefi 2015a,b, Arefi and Allam 2015, Kar and Panda 2015, Zemri *et al.* 2015, Barati and Shahverdi 2016, Bounouara *et al.* 2016, Laoufi *et al.* 2016, Besseghier *et al.* 2017).

In recent years, buckling and post-buckling behaviors of functionally graded (FGM) structures under different types of loading are important for practical applications and have received considerable interest. Shariat *et al.* (2005) employed the classical plate theory (CPT) to discuss the buckling response of geometrically imperfect FG plates. Chen and Liew (2004) have considered a nonlinearly distributed in-plane edge loads and they have presented a two-dimensional elastic plane stress problem of FG plate based on the Mindlin's plate assumption for buckling response using the radial basis function. Using the principle of minimum potential energy and differential quadrature method along with first-order shear deformation theory, Liew *et al.* (2004) studied the thermal buckling and post-buckling response of FG hybrid plates. Na and Kim (2006) developed a finite element formulation to investigate the instability of clamped unsymmetric composite FG plates. In their work, temperature dependency of material properties is also incorporated. Matsunaga (2009) presented a two-dimensional global higher-order deformation theory for thermal stability of FG plates subjected to uniformly and linearly distributed temperatures. Zhao *et al.* (2009) examined the buckling response of plates made of FGMs using the element-free kp-Ritz method. Lee *et al.* (2010) have employed element-free Ritz method to investigate the post-buckling of FG plates under compressive and thermal loads. A new refined hyperbolic shear deformation theory was developed by El Meiche *et al.* (2011) by employing Navier's solution method for buckling and free vibration analysis of FG sandwich plates. Using an analytical approach, Yaghoobi and Torabi (2013) studied the stability analysis of FG plates resting on two-parameter Pasternak's foundations under thermal loads. Based on the first-order shear deformation plate theory, Yaghoobi and Yaghoobi (2013) discussed the buckling response of symmetric

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sandwich plates with FGM face sheets supported by an elastic foundation and subjected to mechanical, thermal and thermo-mechanical loads. Kettaf *et al*. (2013) developed a new hyperbolic shear deformation theory to study the thermal buckling response of FG sandwich plates. Bachir Bouiadjra *et al*. (2013) studied analytically the nonlinear thermal buckling response of FG plates using an efficient sinusoidal shear deformation theory. The stability of heated FG annular plates based on the CPT was investigated by Kiani and Eslami (2013) and an exact analytical solution was developed to compute the thermal buckling load. Tebboune *et al*. (2014) discussed the thermal stability of FG plates resting on elastic foundations. Khalfi *et al*. (2014) investigated the thermal buckling behavior of solar FG plates resting on a two-parameter Pasternak's foundations using refined and simple shear deformation theory. Ait Amar Meziane *et al*. (2014) presented an efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions. Swaminathan and Naveenkumar (2014) presented an analytical formulation for the buckling investigation of FG sandwich plates based on higher order refined computational models. Yaghoobi *et al*. (2014) presented an analytical study on post-buckling and nonlinear free vibration analysis of FG beams resting on nonlinear elastic foundation under thermo-mechanical loading. Bouderba *et al*. (2016) presented the thermal stability of functionally graded sandwich plates using a simple shear deformation theory. Bousahla *et al*. (2016) studied the thermal stability of plates with functionally graded coefficient of thermal expansion. Chikh *et al*. (2017) investigated the thermal buckling behavior of cross-ply laminated plates using a simplified HSDT. Kar *et al*. (2017) studied the effect of different temperature load on thermal post-buckling response of FG shallow curved shell panels. Recently, Klouche *et al*. (2017) proposed an original single variable shear deformation theory for buckling analysis of thick isotropic plates. Bellifa *et al*. developed a nonlocal zeroth-order shear deformation theory for nonlinear postbuckling of nanobeams. Fahsi *et al*. (2017) presented a four variable refined *n*th-order shear deformation theory for mechanical and thermal buckling analysis of FG plates.

The objective of this work is to investigate the buckling response of thick FG sandwich plates subjected to thermal loads. A sinusoidal shear deformation theory together with stress function is utilized to establish governing Eqs. taking into account geometrical nonlinearity. The thermal loads are assumed as uniform, linear and non-linear temperature rises across the thickness direction. Closed form solutions for thermal stability analysis of FG sandwich plates are determined. Numerical examples are illustrated to check the accuracy of the present formulation.

2. Theoretical formulations

2.1 Material properties of FG sandwich plate

In this work, material properties of a FG sandwich plate are assumed to vary in accordance with the rule of mixtures as (Kettaf *et al.* 2013). The plate is referred to a Cartesian coordinate system *x*, *y*, *z*, where *xy* is the mid-surface *f* the plate and z is the thickness coordinator, $-h/2 \le z \le h/2$. The length, width, and total thickness of the plate are *a*, *b* and *h*, respectively (Fig. 1).

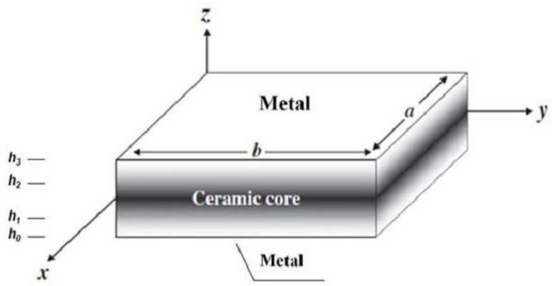

Fig. 1 Geometry of the FGM sandwich plate

The face layers of the sandwich structure are made of an isotropic material with material properties varying smoothly in the z direction only. The core layer is composed with an isotropic homogeneous material. The vertical positions of the bottom surface, the two interfaces between the core and faces layers, and the top surface are defined, respectively, by $h_0 = -h/2$, h_1, h_2 and $h_3=h/2$. The total thickness of the FG plate h is given by $h=t_C+t_F$, with $t_C=h_2-h_1$. t_C and t_F are the layer thickness of the core and all-FGM layers, respectively.

The mechanical and thermal material properties for each layer, like Young's modulus, Poisson's ratio and thermal expansion coefficient, can be expressed as (Attia *et al.* 2015, Bakora and Tounsi, 2015; Belkorissat *et al.* 2015, Mahi *et al.* 2015, Beldjelili *et al.* 2016, Boukhari *et al.* 2016, Bellifa *et al.* 2016, Benferhat *et al.* 2016, Houari *et al.* 2016, Tounsi *et al.* 2016)

$$P^{(n)}(z) = P_m + \left(P_c - P_m\right)V^{(n)} \tag{1}$$

where $P^{(n)}$ is the effective material characteristic of FGM of layer n. P_m and P_c present the property of the bottom and top faces of layer 1 ($h_0 \le z \le h_1$), respectively, and vice versa for layer 3 ($h_2 \le z \le h_3$) depending on the volume fraction $V^{(n)}$ (n=1,2,3). Note that P_m and P_c are, respectively, the corresponding properties of the metal and ceramic of the FGM sandwich plate. The volume fraction $V^{(n)}$ of the FGMs is assumed to obey a power-law function along the thickness direction (Houari *et al*. 2011, Taibi *et al*. 2015, Meksi *et al*. 2017)

$$V^{(1)} = \left(\frac{z-h_0}{h_1-h_0}\right)^k, \quad z \in [h_0, h_1] \tag{2a}$$

$$V^{(2)} = 1, \quad z \in [h_1, h_2] \tag{2b}$$

$$V^{(3)} = \left(\frac{z-h_3}{h_2-h_3}\right)^k, \quad z \in [h_2, h_3] \tag{2c}$$

where *k* is the power-law index, which takes values greater than or equals to zero. The core layer is independent of the value of *k* which is a fully ceramic layer. However, the value of *k* equal to zero represents a fully ceramic plate.

2.2 Kinematics

In this study, the sinusoidal shear deformation plate theory (Touratier 1991) is utilized. The displacement field can then be written as

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w(x,y)}{\partial x} + \Psi(z)\ \phi_x(x,y) \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w(x,y)}{\partial y} + \Psi(z)\ \phi_y(x,y) \\ w(x,y,z) &= w_0(x,z) \end{aligned} \tag{3a}$$

With

$$\Psi(z) = \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right) \tag{3b}$$

where u_0, v_0 and w_0 are generalized displacement at the mid-plane of the plate in the x, y, and z directions, respectively; ϕ_x, ϕ_y are the slope rotations in the (x, z) and (y, z) planes, respectively; and h is the plate thickness.

The non-linear von Karman strain-displacement equations are as follows

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} + z\begin{Bmatrix}k_x\\ k_y\\ k_{xy}\end{Bmatrix} + \Psi(z)\begin{Bmatrix}\eta_x\\ \eta_y\\ \eta_{xy}\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{xz}\\ \gamma_{yz}\end{Bmatrix} = \Psi'(z)\begin{Bmatrix}\gamma_{xz}^0\\ \gamma_{yz}^0\end{Bmatrix}, \tag{4}$$

Where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} = \begin{Bmatrix}u_{0,x} + (w_{0,x})^2/2\\ v_{0,x} + (w_{0,y})^2/2\\ u_{0,y} + v_{0,x} + w_{0,x}w_{0,y}\end{Bmatrix},$$

$$\begin{Bmatrix}k_x\\ k_y\\ k_{xy}\end{Bmatrix} = \begin{Bmatrix}-w_{0,xx}\\ -w_{0,yy}\\ -2w_{0,xy}\end{Bmatrix},\quad \begin{Bmatrix}\eta_x\\ \eta_y\\ \eta_{xy}\end{Bmatrix} = \begin{Bmatrix}\phi_{x,x}\\ \phi_{y,y}\\ \phi_{x,y} + \phi_{y,x}\end{Bmatrix}, \tag{5}$$

$$\begin{Bmatrix}\gamma_{xz}^0\\ \gamma_{yz}^0\end{Bmatrix} = \begin{Bmatrix}\phi_x\\ \phi_y\end{Bmatrix},$$

2.3 Constitutive equations

The linear constitutive relations of a FG plate can be expressed as

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{xz}\end{Bmatrix} = \frac{E}{1-\nu^2}\begin{bmatrix}1 & \nu & 0 & 0 & 0\\ \nu & 1 & 0 & 0 & 0\\ 0 & 0 & \frac{1-\nu}{2} & 0 & 0\\ 0 & 0 & 0 & \frac{1-\nu}{2} & 0\\ 0 & 0 & 0 & 0 & \frac{1-\nu}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_x - \alpha\,\Delta T\\ \varepsilon_y - \alpha\,\Delta T\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} \tag{6}$$

where ΔT is temperature rise from stress free initial state or temperature difference between two surfaces of the FG plate.

By using the virtual work principle to minimize the functional of total potential energy function result in the expressions for the nonlinear equilibrium equations of a plate as

$$N_{x,x} + N_{xy,y} = 0 \tag{7a}$$

$$N_{xy,x} + N_{y,y} = 0 \tag{7b}$$

$$(M_{x,xx} + 2M_{xy,xy} + M_{y,yy}) + N_x w_{,xx} + 2\ N_{xy} w_{,xy} + N_y w_{,yy} = 0 \tag{7c}$$

$$S_{x,x} + S_{xy,y} - Q_x = 0 \tag{7d}$$

$$S_{xy,x} + S_{y,y} - Q_y = 0 \tag{7e}$$

The force and moment resultants (N, Q, S and M) of the FG sandwich plate are obtained by

$$(N_i,\ M_i,\ S_i) = \int_{-h/2}^{h/2} \sigma_i\,(1, z, \Psi(z))\,dz,\quad (i = x, y, xy) \tag{8a}$$

$$Q_i = \int_{-h/2}^{h/2} \sigma_j\ \Psi'(z)\,dz,\quad (i = x, y);\ \ (j = xz, yz) \tag{8b}$$

Substitution of Eqs. (4) and (6) into Eq. (8) yields the constitutive relations as

$$\begin{aligned}(N_x, M_x, S_x) = \frac{1}{1-\nu^2}[&(E_1, E_2, E_3)(\varepsilon_x^0 + \nu\,\varepsilon_y^0) + (E_2, E_4, E_5)(k_x + \nu\,k_y)\\ &+ (E_3, E_5, E_7)(\eta_x + \nu\,\eta_y) - (1+\nu)(\Phi_1, \Phi_2, \Phi_3)]\end{aligned} \tag{9a}$$

$$\begin{aligned}(N_y, M_y, S_y) = \frac{1}{1-\nu^2}[&(E_1, E_2, E_3)(\varepsilon_y^o + \nu\,\varepsilon_x^0) + (E_2, E_4, E_5)(k_y + \nu\,k_x)\\ &+ (E_3, E_5, E_7)(\eta_y + \nu\,\eta_x) - (1+\nu)(\Phi_1, \Phi_2, \Phi_3)]\end{aligned} \tag{9b}$$

$$\begin{aligned}(N_{xy}, M_{xy}, S_{xy}) = \frac{1}{2(1+\nu)}[&(E_1, E_2, E_3)\gamma_{xy}^0 + (E_2, E_4, E_5)k_{xy}\\ &+ (E_3, E_5, E_7)\eta_{xy}]\end{aligned} \tag{9c}$$

$$(Q_x, Q_y) = \frac{1}{2(1+\nu)}E_8\left(\gamma_{xz}^0, \gamma_{yz}^0\right) \tag{9d}$$

Where

$$\begin{aligned}\left(E_1, E_4, E_5, E_7\right) &= \int_{-h/2}^{h/2}\left(1, z^2, z\,\Psi(z), \Psi(z)^2\right)E(z)dz,\\ \left(E_2, E_3\right) &= \int_{-h/2}^{h/2}\left(z, \Psi(z)\right)E(z)dz = (0,0),\\ E_8 &= \int_{-h/2}^{h/2}\left(\Psi'(z)\right)^2\ E(z)\ dz\end{aligned} \tag{10a}$$

$$(\Phi_1, \Phi_2, \Phi_3) = \int_{-h/2}^{h/2}(1, z, \Psi)E(z)\,\alpha(z)\,\Delta T(z)\,dz \tag{10b}$$

The last three Eq. (7) may be rewritten into two Eqs. in terms of variables w_0 and $\phi_{x,x}$+$\phi_{y,y}$ by substituting Eqs. (5) and (9) into Eqs. (7c)-(7e). Subsequently, elimination of the variable $\phi_{x,x}$+$\phi_{y,y}$ from two the resulting equations leads to the following system of equilibrium equations

$$N_{x,x} + N_{xy,y} = 0 \tag{11a}$$

$$N_{xy,x} + N_{y,y} = 0 \tag{11b}$$

$$(D_1D_3 - D_2^2)\nabla^6 w - D_1D_4\ \nabla^4 w - D_3\nabla^2(N_x w_{,xx} + 2N_{xy}w_{,xy} + N_y w_{,yy}) + D_4(N_x w_{,xx} + 2N_{xy}w_{,xy} + N_y w_{,yy}) = 0 \tag{11c}$$

Where

$$D_1 = \frac{E_4}{(1-\nu^2)},\quad D_2 = \frac{E_5}{(1-\nu^2)},\quad D_3 = \frac{E_7}{(1-\nu^2)},\quad D_4 = \frac{E_8}{2(1+\nu)}. \tag{12}$$

For a FG sandwich plate, Eq. (11) are modified into form as

$$\left(D_2^2 - D_1D_3\right)\nabla^6 w + D_1D_4\nabla^4 w + D_3\nabla^2\left[f_{,yy}\left(w_{,xx}\right) - 2f_{,xy}\left(w_{,xy}\right) + f_{,xx}\left(w_{,yy}\right)\right] - D_4\left[f_{,yy}\left(w_{,xx}\right) - 2f_{,xy}\left(w_{,xy}\right) + f_{,xx}\left(w_{,yy}\right)\right] = 0 \tag{13}$$

where $f(x,y)$ is stress function defined by

$$N_x = f_{,yy}\ ,\ N_y = f_{,xx}\ ,\ N_{xy} = -f_{,xy} \tag{14}$$

The geometrical compatibility equation for a sandwich plate is written as

$$\varepsilon^0_{x,yy} + \varepsilon^0_{y,xx} - \gamma^0_{xy,xy} = w^2_{,xy} - w_{,xx}w_{,yy} \tag{15}$$

From the constitutive relations (9) and Eq. (14) one can write

$$(\varepsilon^0_x, \varepsilon^0_y) = \frac{1}{E_1}[(f_{,yy}, f_{,xx}) - \nu(f_{,xx}, f_{,yy}) + \Phi_1(1{,}1)]$$
$$\gamma^0_{xy} = -\frac{1}{E_1}[\ 2(1+\nu)\, f_{,xy}] \tag{16}$$

Substituting Eq. (16) into Eq. (15), the compatibility equations of a FG sandwich plate becomes

$$\nabla^4 f\ - E_1(w^2_{0,xy} - w_{0,xx}\, w_{0,yy}) = 0 \tag{17}$$

We are here concerned with the exact solution of Eqs. (13) and (18) for a simply supported FG sandwich plate. In this case, the proposed solutions of w and f respecting boundary conditions are considered to be (Librescu and Lin 1997, Lin and Librescu 1998)

$$w = W\sin(\lambda_m x)\sin(\delta_n y) \tag{18a}$$

$$f = A_1\cos(2\lambda_m x) + A_2\cos(2\delta_n y) + A_3\sin(\lambda_m x)\sin(\delta_n y) + \frac{1}{2}N_{x0}y^2 + \frac{1}{2}N_{y0}x^2 \tag{18b}$$

where $\lambda_m = m\pi/a$, $\delta_n = n\pi/b$, m, n are odd numbers and W is amplitude of the deflection. The coefficients A_i (i=1, 2, 3) are obtained by substitution of Eqs. (19a, b) into Eq. (18) as

$$A_1 = \frac{E_1\delta_n^2}{32\lambda_m^2}W^2\ ,\ A_2 = \frac{E_1\lambda_m^2}{32\delta_n^2}W^2\ ,\ A_3 = 0 \tag{19}$$

Then, setting Eqs. (19a, b) into Eq. (13) and using the Galerkin method for the resulting Eq. yield

$$\left(\left(D_1D_3 - D_2^2\right)\left(\lambda_m^2 + \delta_n^2\right)^3 + D_1D_4\left(\lambda_m^2 + \delta_n^2\right)^2\right)W + \frac{E_1}{16}\left(D_3\left(5\left(\lambda_m^4\delta_n^2 + \lambda_m^2\delta_n^4\right) + \lambda_m^6 + \delta_n^6\right) + D_4\left(\lambda_m^4 + \delta_n^4\right)\right)W^3 + \left(D_3\left(\lambda_m^2 + \delta_n^2\right) + D_4\right)\times\left(N_{x0}\lambda_m^2 + N_{y0}\delta_n^2\right)W = 0 \tag{20}$$

3. Thermal buckling solution

A simply supported FG sandwich plate with all immovable edges is considered here. The in-plane condition on immovability at all edges, i.e., u_0=0 at x=0, a and v_0=0 at y=0, b, is given in an average sense as (Tung and Duc 2010)

$$\int_0^b\int_0^a \frac{\partial u_0}{\partial x}dxdy = 0\ ,\ \int_0^a\int_0^b \frac{\partial v_0}{\partial x}dydx = 0 \tag{21}$$

From Eqs. (5) and (8) one can determine the following expressions in which Eq. (14) has been introduced

$$\frac{\partial u_0}{\partial x} = \frac{1}{E_1}(f_{,yy} - \nu f_{,xx}) - \frac{1}{2}w^2_{,x} + \frac{\Phi_1}{E_1} \tag{22}$$

$$\frac{\partial v_0}{\partial y} = \frac{1}{E_1}(f_{,xx} - \nu f_{,yy}) - \frac{1}{2}w^2_{,y} + \frac{\Phi_1}{E_1} \tag{23}$$

Introduction of Eq. (19) into Eq. (23) and then the result into Eq. (22) give

$$N_{x0} = -\frac{\Phi_1}{1-\nu} + \frac{E_1}{8(1-\nu^2)}(\lambda_m^2 + \nu\,\delta_n^2)\,W^2 \tag{24}$$

$$N_{y0} = -\frac{\Phi_1}{1-\nu} + \frac{E_1}{8(1-\nu^2)}(\nu\lambda_m^2 + \delta_n^2)\,W^2 \tag{25}$$

When the deflection dependence of fictitious edge loads is ignored, i.e., W=0, Eq. (24) becomes

$$N_{x0} = N_{y0} = -\frac{\Phi_1}{1-\nu} \tag{26}$$

Substituting Eqs. (24) into Eq. (21) yields the expression of thermal parameter as

$$\frac{\Phi_1}{1-\nu} = \left[\frac{\left(D_1D_3 - D_2^2\right)\left(\lambda_m^2 + \delta_n^2\right)^2 + D_1D_4\left(\lambda_m^2 + \delta_n^2\right)}{D_3\left(\lambda_m^2 + \delta_n^2\right) + D_4}\right] + \left[\frac{E_1\left[D_3\left(5\left(\lambda_m^4\delta_n^2 + \lambda_m^2\delta_n^4\right) + \lambda_m^6 + \delta_n^6\right) + D_4\left(\lambda_m^4 + \delta_n^4\right)\right]}{16\left[D_3(\lambda_m^2 + \delta_n^2) + D_4\right]\left(\lambda_m^2 + \delta_n^2\right)} + \frac{E_1\left[\left(\lambda_m^4 + 2\,\upsilon\,\lambda_m^2\delta_n^2 + \delta_n^4\right)\right]}{8\left(1-\nu^2\right)\left(\lambda_m^2 + \delta_n^2\right)}\right]W^2 \tag{27}$$

3.1 Uniform temperature rise

The FG sandwich plate is subjected to temperature environments uniformly raised from stress free initial state T_i to final value T_f, and temperature change $\Delta T = T_f T_i$ is assumed to be independent from thickness variable. The thermal parameter Φ_1 is obtained from Eqs. (10b), and substitution of the result into Eq. (26) yields

$$\Delta T_{cr} = \frac{(1-\nu)}{L\,[D_3(\lambda_m^2 + \delta_n^2) + D_4]}\times\left[(D_1D_3 - D_2^2)(\lambda_m^2 + \delta_n^2)^2 + D_1D_4(\lambda_m^2 + \delta_n^2)\right] \tag{28}$$

where

$$L = \int_{-h/2}^{h/2}\alpha(z)E(z)dz \tag{29}$$

Table 1 Material properties used in the FG sandwich plate

Properties	Metal: Ti-6A1-4V	Ceramic: ZrO_2
E (GPa)	66.2	244.27
ν	0.3	0.3
α (10^{-6}/K)	10.3	12.766

Table 2 Minimum critical temperature parameter (αT_{cr}) of the simply supported isotropic plate (a/b=1, t_C/h, E=1.0×10^6 N/m^2, ν=0.3)

a/h	Present theory	Kettaf *et al*. (2013)	Matsunaga (2005)
10	0.1198×10^{-1}	0.1198×10^{-1}	0.1183×10^{-1}
20	0.3119×10^{-2}	0.3119×10^{-2}	0.3109×10^{-2}
100	0.1265×10^{-3}	0.1265×10^{-3}	0.1264×10^{-3}

3.2 Buckling of FGM plates subjected to graded temperature change across the thickness

We consider that the temperature of the top surface is T_t and the temperature varies from T_t, according to the power law variation through-the-thickness, to the bottom surface temperature T_b in which the plate buckles. In this case, the temperature through-the-thickness is given by

$$T(z) = \Delta T\left(\frac{z}{h}+\frac{1}{2}\right)^{\gamma} + T_t \tag{30}$$

where the buckling critical temperature difference $\Delta T=T_b-T_t$ and γ is the temperature exponent ($0<\gamma<\infty$). Note that the value of γ equal to unity represents a linear temperature distribution within the thickness. While the value of γ excluding unity represents a non-linear temperature variation through-the-thickness. Similar to the previous loading case, the buckling temperature change ΔT_{cr} can be determined, for the present theory, as

$$\Delta T_{Cr} = e_1^2 - \frac{T_m L}{H} \quad \text{and}$$
$$e_1^2 = \frac{(1-\nu)}{L\,[D_3(\lambda_m^2+\delta_n^2)+D_4]} \times \begin{bmatrix} (D_1D_3-D_2^2)(\lambda_m^2+\delta_n^2)^2 + \\ D_1D_4(\lambda_m^2+\delta_n^2) \end{bmatrix} \tag{31}$$

where

$$H = \int_{-h/2}^{h/2} \alpha(z)\,E(z)\left(\frac{z}{h}+\frac{1}{2}\right)^{\gamma} dz \tag{32}$$

4. Numerical results

The general formulation presented in the previous sections for the thermal stability analysis of the FG sandwich plates subjected to the uniform, linear and non-linear temperature rises through-the-thickness is examined here. The shear correction factor for the first shear deformation theory (FSDT) is set equal to 5/6. For the linear and non-linear temperature rises through-the-thickness, T_t=25°C. FG plates made of the combination of Titanium and Zirconia are considered in this work. The Young's modulus and the coefficient of thermal expansion for Titanium and Zirconia are given in Table 1.

Table 3 Critical buckling temperature ($10^3\alpha_0\Delta T_{cr}$) of a homogeneous isotropic plate under uniform temperature rise

b/a	Theory	a/h=5	a/h=10	a/h=15	a/h=25	a/h=50
0.5	Present	81.18684	27.73637	13.23204	4.94986	1.25824
	Kettaf *et al*. (2013)	81.15170	27.73347	13.23144	4.94979	1.25825
	CPT	126.53339	31.63335	14.05927	5.06134	1.26533
1	Present	41.33313	11.97926	5.48643	2.00646	0.50500
	Kettaf *et al*. (2013)	41.32613	11.97877	5.48633	2.00644	0.50500
	CPT	50.61336	12.65334	5.62371	2.02453	0.50613
2	Present	27.73637	7.63958	3.46069	1.25824	0.31588
	Kettaf *et al*. (2013)	27.73347	7.63938	3.46065	1.25824	0.31589
	CPT	31.63335	7.90834	3.51482	1.26533	0.31633
5	Present	23.56351	6.39261	2.88676	1.04785	0.26288
	Kettaf *et al*. (2013)	23.56145	6.39248	2.88674	1.04785	0.26288
	CPT	26.31895	6.57974	2.92433	1.05276	0.26319

In order to verify the validity of the proposed method in predicting the thermal buckling behaviors of plates, a comparison has been performed with the results reported by Kettaf *et al*. (2013) and Matsunaga (2005) for homogeneous isotropic plates under uniform temperature rise. The critical buckling temperature difference has been given in Table 2.

The examination of this table demonstrates that the results provided by the present analytical method are in a good agreement with those reported by Kettaf *et al*. (2013) and Matsunaga (2005).

To check also the validity of the proposed formulation, comparisons are carried out between the thermal stability results obtained from the present method and those determined by other plate theories. For a homogeneous isotropic plate k=0, $E(z)=E_0$, $\alpha(z)=\alpha_0$, ν=0.3. Critical stability temperature change ($10^3\alpha_0\Delta T_{cr}$) for different values of the thickness ratio a/h and aspect ratio b/a of a homogeneous plate is presented in Table 3.

With the increase of the thickness ratio a/h, severe decrement for critical stability temperature can be clearly seen. Also, it can be remarked that the critical stability temperature for the homogeneous plate diminishes gradually as the plate aspect ratio b/a increases. The difference between the shear deformation plate theories and the CPT decreases as the ratios a/h or b/a increase because the plate becomes thin or long.

Tables 4-6 provide the critical stability temperature difference ($10^{-3}\Delta T_{cr}$) for FG sandwich plates subjected to the uniform, linear and nonlinear temperature distribution through the thickness, respectively.

The comparison between the present formulation and the method developed by Kettaf *et al*. (2013) as well as classical plate theory (CPT) is established. From the results presented in Tables 4-6, it can be seen that there is a very good agreement between the present formulation and the plate theory proposed by Kettaf *et al*. (2013). Tables 4-6 demonstrate also the effect of the layer thickness of the core

Table 4 Critical buckling temperature of FG sandwich square plates under uniform temperature rise versus gradient index k and t_C/h (a/h=5)

t_C/h	Theory	k						
		0	0.2	0.5	1	2	5	10
0	Present	3.23775	3.07197	2.87277	2.69065	2.63460	2.94205	3.31226
	Kettaf *et al.* (2013)	3.23720	3.07138	2.87207	2.68975	2.63325	2.93978	3.30959
	CPT	3.96470	3.66606	3.34559	3.06734	2.96200	3.32950	3.82441
0.2	Present	3.23775	3.05598	2.83194	2.59458	2.39953	2.35401	2.42818
	Kettaf *et al.* (2013)	3.23720	3.05543	2.83135	2.59388	2.39856	2.35252	2.42641
	CPT	3.96470	3.64978	3.30066	2.95538	2.68016	2.59922	2.68195
0.4	Present	3.23775	3.05956	2.84318	2.60545	2.37450	2.19992	2.17714
	Kettaf *et al.* (2013)	3.23720	3.05915	2.84285	2.60512	2.37406	2.19921	2.17624
	CPT	3.96470	3.66567	3.33354	2.99117	2.67295	2.43609	2.39804
0.5	Present	3.23775	3,07014	2,86992	2,64976	2,42900	2,24005	2,17784
	Kettaf *et al.* (2013)	3.23720	3,06980	2,86974	2,64965	2,42885	2,23972	2,17737
	CPT	3.96470	3,68764	3,38155	3,06366	2,75801	2,50252	2,41816
0.6	Present	3.23775	3.08741	2.91146	2.71909	2.52297	2.34310	2.27458
	Kettaf *et al.* (2013)	3.23720	3.08713	2.91139	2.71917	2.52309	2.34313	2.27452
	CPT	3.96470	3.71993	3.45164	3.17226	2.89771	2.65182	2.55878
0.8	Present	3.23775	3.14474	3.04107	2.93038	2.81650	2.74092	2.65609
	Kettaf *et al.* (2013)	3.23720	3.14445	3.04101	2.93052	2.81681	2.74134	2.65659
	CPT	3.96470	3.81800	3.66058	3.49712	3.33246	3.21552	3.10423
1	Present	3.23775	3.23775	3.23775	3.23775	3.23775	3.23775	3.23775
	Kettaf *et al.* (2013)	3.23720	3.23720	3.23720	3.23720	3.23720	3.23720	3.23720
	CPT	3.96470	3.96470	3.96470	3.96470	3.96470	3.96470	3.96470

t_C (ceramic layer) on the thermal stability response of the FG sandwich plates. As can be observed from Tables 4 and 5, the thermal stability temperatures are reduced with the increase in gradient index k. Consequently, the increase in thermal stability temperature of FG sandwich plate could be attributed to the ceramic property. Indeed, this remark is also confirmed when a small gradient index is considered ($k\leq 2$) for all values of t_C. A small gradient index k shows that the ceramic is the dominant constituent in FG sandwich plates. However, Table 6 demonstrates that the thermal stability temperatures increase with the increase in gradient index k when the plate is subjected to a non-linear temperature variation with γ=5. It can be seen that the thermal stability temperature decreases with increasing the thickness of the core layer (t_C) for all considered gradient index.

Finally, it should be noted from the comparison carried out within Tables 2 to 6 that the present formulation involves only two Eqs. (13) and (18) contrary to the theory

Table 5 Critical buckling temperature of FG sandwich square plates under linear temperature rise versus gradient index k and t_C/h (a/h=5)

t_C/h	Theory	k						
		0	0.2	0.5	1	2	5	10
0	Present	6.42550	6.09396	5.69554	5.33130	5.21920	5.83411	6.57458
	Kettaf *et al.* (2013)	6.42441	6.09275	5.69414	5.32949	5.21651	5.82957	6.56918
	CPT	7.87940	7.28211	6.64118	6.08468	5.87400	6.60901	7.59882
0.2	Present	6.42550	6.06196	5.61388	5.13917	4.74907	4.65803	4.80632
	Kettaf *et al.* (2013)	6.42441	6.06087	5.61271	5.13775	4.74712	4.65504	4.80264
	CPT	7.87940	7.24955	6.55131	5.86076	5.31032	5.14843	5.31369
0.4	Present	6.42550	6.06912	5.63636	5.16089	4.69900	4.34984	4.24818
	Kettaf *et al.* (2013)	6.42441	6.06830	5.63571	5.16024	4.69812	4.34842	4.26735
	CPT	7.87940	7.28133	6.61708	5.93233	5.29588	4.82217	4.70737
0.5	Present	6,42550	6,09028	6,68985	5,24951	4,80800	4,43010	4,30568
	Kettaf *et al.* (2013)	6,42441	6,08961	5,68948	5,24929	4,80770	4,42943	4,30474
	CPT	7,87940	7,32529	6,71310	6,07732	5,46601	4,95505	4,78633
0.6	Present	6.42550	6.12481	5.77291	5.38818	4.99595	4.63609	4.84881
	Kettaf *et al.* (2013)	6.42441	6.12425	5.77278	5.38833	4.99619	4.63616	4.49905
	CPT	7.87940	7.38985	6.85328	6.29453	5.74542	5.25352	5.06756
0.8	Present	6.42550	6.23948	6.03215	5.81076	5.58301	5.35903	5.26229
	Kettaf *et al.* (2013)	6.42441	6.23889	6.03202	5.81104	5.58362	5.35987	5.26317
	CPT	7.87940	7.58600	7.27115	6.94424	6.61492	6.29563	6.15846
1	Present	6.42550	6.42550	6.42550	6.42550	6.42550	6.42550	6.42550
	Kettaf *et al.* (2013)	6.42441	6.42441	6.42441	6.42441	6.42441	6.42441	6.42441
	CPT	7.87940	7.87940	7.87940	7.87940	7.87940	7.87940	7.87940

Table 6 Critical buckling temperature of FG sandwich square plates under non-linear temperature rise versus gradient index k and t_C/h (a/h=5, γ=5)

t_C/h	Theory	k						
		0	0.2	0.5	1	2	5	10
0	Present	19.27655	20.57527	21.62878	22.43462	23.06831	23.77153	24.07624
	Kettaf *et al.* (2013)	19.27322	20.57122	21.62347	22.42701	23.05643	23.75304	24.05661
	CPT	23.63820	24.58692	25.21986	25.60494	25.96247	26.92893	27.82720
0.2	Present	19.27655	20.43384	21.35073	22.00140	22.38252	22.66383	22.87336
	Kettaf *et al.* (2013)	19.27322	20.43016	21.34626	21.99533	22.37338	22.64929	22.85562
	CPT	23.63820	24.43703	24.91598	25.09061	25.02775	25.04991	25.28770
0.4	Present	19.27655	20.24827	21.00993	21.54426	21.82347	21.87961	21.88456
	Kettaf *et al.* (2013)	19.27322	20.24553	21.00745	21.54152	21.81937	21.87237	21.87534
	CPT	23.63820	24.29255	24.66557	24.76464	24.59556	24.25535	24.13098
0.5	Present	19,27655	20,13433	20,80528	21,28377	21,54917	21,60383	21,58327
	Kettaf *et al.* (2013)	19,27322	20,13209	20,80394	21,28287	21,54783	21,60059	21,57856
	CPT	23,63820	24,29255	24,66557	24,76464	24,59556	24,25535	24,13098

Table 6 Continued

	Present	19.27655	20.00359	20.57123	20.98563	21.23853	21.32522	21.31766
0.6	Kettaf *et al.* (2013)	19.27322	20.00176	20.57076	20.98623	21.23955	21.32555	21.31715
	CPT	23.63820	24.13520	24.42100	24.51562	24.42463	24.16529	24.01085
	Present	19.27655	19.68398	19.99828	20.23774	20.41159	20.52420	20.55608
0.8	Kettaf *et al.* (2013)	19.27322	19.68210	19.99784	20.23872	20.41383	20.52740	20.55953
	CPT	23.63820	23.93190	24.10594	24.18546	24.18431	24.11121	24.05679
	Present	19.27655	19.27655	19.27655	19.27655	19.27655	19.27655	19.27655
1	Kettaf *et al.* (2013)	19.27322	19.27322	19.27322	19.27322	19.27322	19.27322	19.27322
	CPT	23.63820	23.63820	23.63820	23.63820	23.63820	23.63820	23.63820

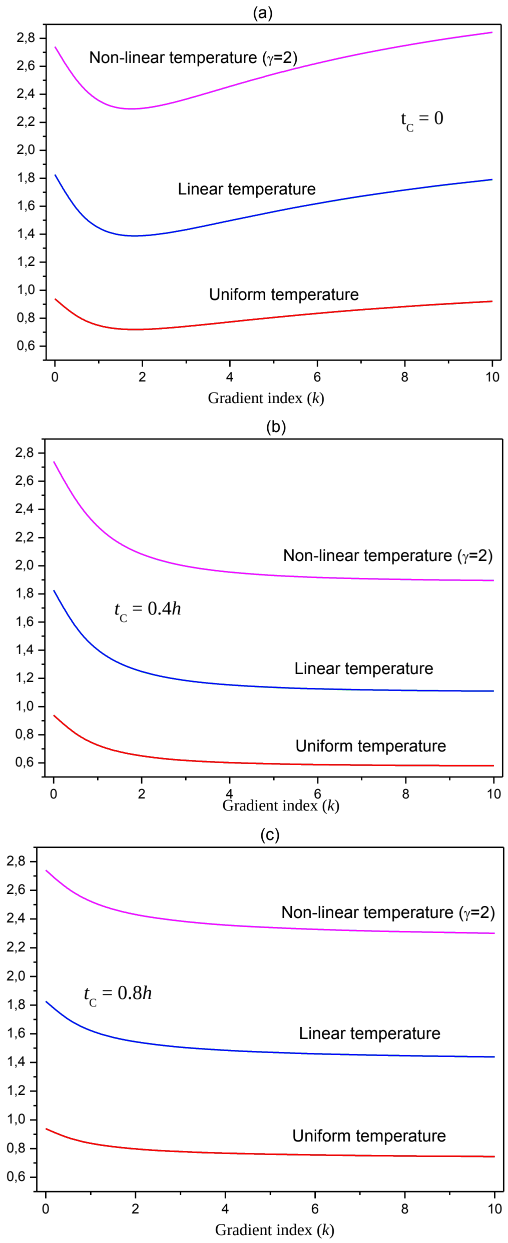

Fig. 2 Critical stability temperature difference T_{cr} versus the gradient index k for FG sandwich square plates with: (a) a/h=10, (b) t_C=0, (c) t_C=0.8h

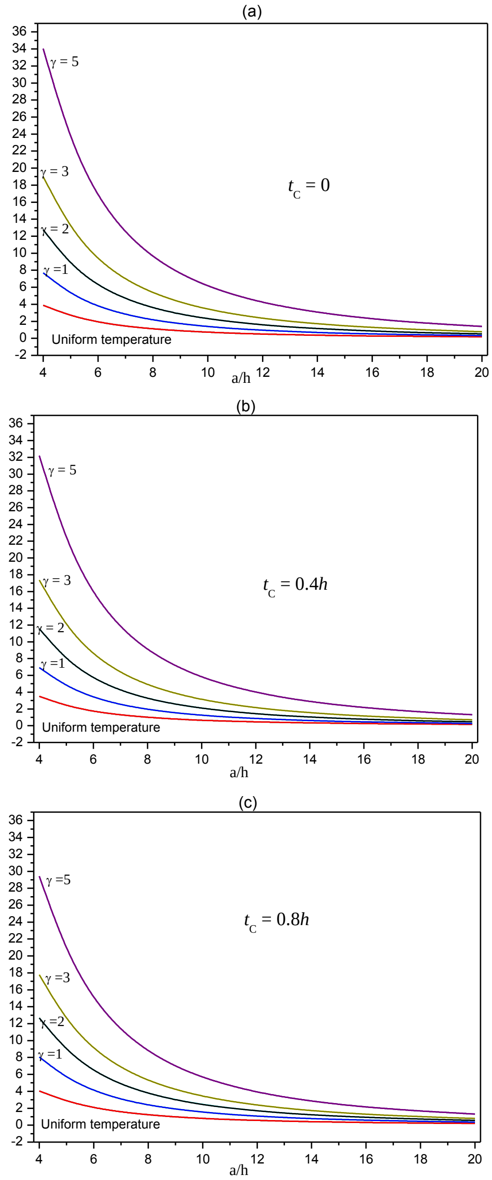

Fig. 3 Critical stability temperature difference T_{cr} versus the thickness ratio *a/h* for FG sandwich square plate (k=2): (a) t_C=0, (b) t_C=0.4h, (c) t_C=0.8h

proposed by Kettaf *et al.* (2013) where four governing equations are needed and the theory proposed by Matsunaga (2005) involving a higher number of governing equations.

Fig. 2 demonstrates the influence of the gradient index k on the critical stability temperature T_{cr} for different thickness of the core t_C of FG sandwich plates under uniform, linear and non-linear temperature change through-the-thickness using the present formulation.

It can be shown that the non-linear temperature loading produces the highest critical stability temperature T_{cr}, while T_{cr} for the plates under linear temperature loading is intermediate between the non-linear and uniform the non-linear temperature loading.

Fig. 3 presents the variation of critical stability temperature T_{cr} versus the thickness ratio T_{cr} for FG sandwich square plates under various thermal loading types. It is observed that the critical temperature difference

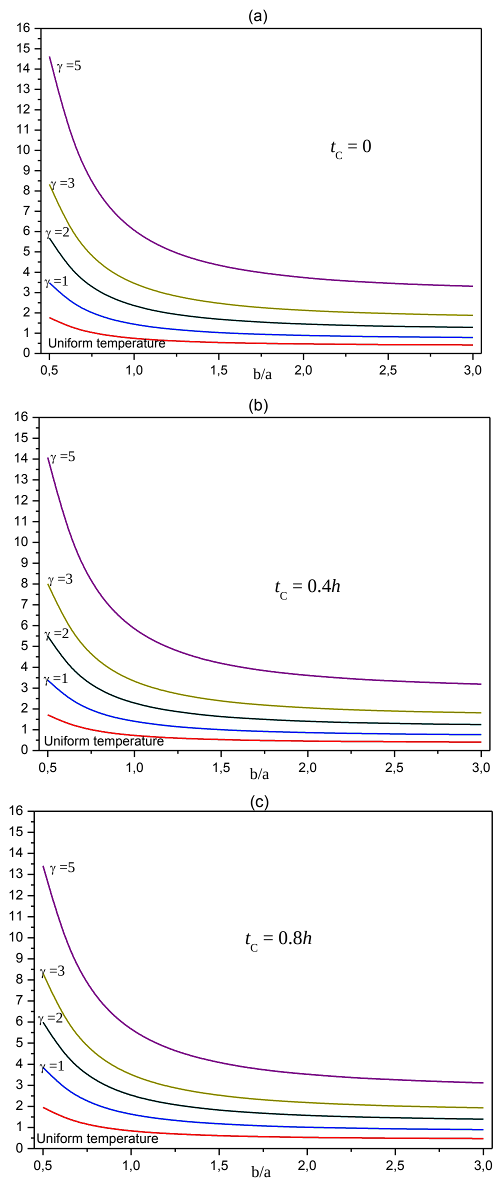

Fig. 4 Critical stability temperature difference T_{cr} versus the aspect ratio b/a for FG sandwich square plates (k=1, a/h=10): (a) t_C=0, (b) t_C=0.4h, (c) t_C=0.8h

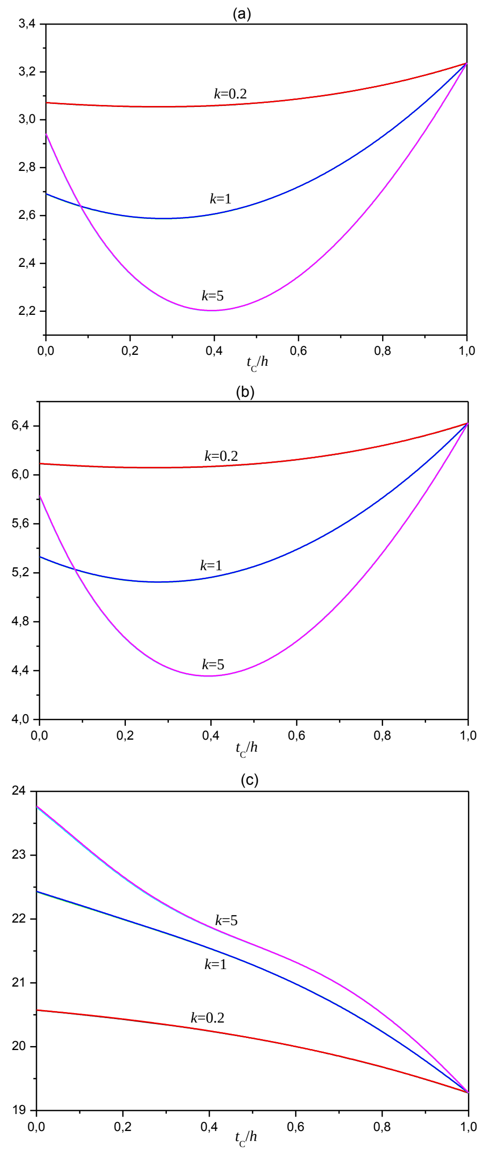

Fig. 5 Critical stability temperature change T_{cr} of FG sandwich square plates versus k and t_C/h: (a) uniform temperature; (b) linear temperature; (c) non-linear temperature (γ=5)

decreases monotonically when the plate becomes thin. It is also shown that the uniform temperature loading produces the small critical temperatures T_{cr} but the highest ones is found in the case of non-linear temperature loading. In addition, it is noticed that T_{cr} increases as the nonlinearity parameter γ increases.

Fig. 4 indicates the influence of the aspect ratio b/a on the critical stability temperature change T_{cr} of FG sandwich plates subjected to various thermal loading types.

It is observed that, regardless of the values of t_C, the critical stability T_{cr} decreases gradually with the increase of the plate aspect ratio b/a wherever the loading type is. It is also seen from Fig. 4 that the T_{cr} increases with the increase of the non-linearity parameter γ.

Fig. 5 demonstrates the effect of the core thickness t_C on the thermal stability behavior of the FG sandwich plates subjected to the uniform, linear and nonlinear cases of temperature distribution through the thickness, respectively. As can be observed from Figs. 5(a) and (b) (uniform and linear temperature), the thermal stability temperatures diminishes with increasing the gradient index. A small gradient index γ indicates that the ceramic is the dominant constituent in the FG plate. Also, it is found that the thermal stability temperatures increase for $t_C \geq 0.4$ which means that the ceramic is also the dominant constituent in the FG plate. Thus, the increase in thermal stability temperature of an FG sandwich plate could be attributed to the ceramic property. As expected, the thermal stability temperature will be maximum for the pure-ceramic plate (t_C=h) in the cases of uniform and linear temperature loading. However, in case of nonlinear temperature loading (Fig. 5(c)), the thermal stability temperature will be minimum for the pure-ceramic plate and the thermal stability temperatures increase with the increase in gradient index.

5. Conclusions

The buckling behaviors of FG sandwich plates subjected to thermal loads are presented analytically by using the sinusoidal shear deformation and stress function concept, with the assumption of power law composition for the constituent materials of FGM layers. The stability analysis of FG sandwich plates under different types of thermal loadings is presented. In conclusion, it can be said that the proposed formulation is accurate and simple in solving the thermal stability behaviors of FG sandwich plates. An improvement of present formulation will be considered in the future work to account for the thickness stretching effect by using quasi-3D shear deformation models (Bessaim *et al.* 2013, Bousahla *et al.* 2014, Belabed *et al.* 2014, Fekrar *et al.* 2014, Hebali *et al.* 2014, Meradjah *et al.* 2015, Hamidi *et al*. 2015, Bennai *et al.* 2015, Larbi Chaht *et al.* 2016, Bourada *et al.* 2015, Bennoun *et al.* 2016, Draiche *et al.* 2016,Bouafia *et al.* 2017, Benahmed *et al.* 2017).

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