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Analysis of cable structures through energy minimization

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Abstract. In structural mechanics, traditional analyses methods usually employ matrix operations for obtaining displacement and internal forces of the structure under the external effects, such as distributed loads, earthquake or wind excitations, and temperature changing inter alia. These matrices are derived from the well-known principle of mechanics called minimum potential energy. According to this principle, a system can be in the equilibrium state only in case when the total potential energy of system is minimum. A close examination of the expression of the well-known equilibrium condition for linear problems, $P=K\Delta$, where P is the load vector, K is the stiffness matrix and Δ is the displacement vector, it is seen that, basically this principle searches the displacement set (or deformed shape) for a system that minimizes the total potential energy of it. Instead of using mathematical operations used in the conventional methods, with a different formulation, meta-heuristic algorithms can also be used for solving this minimization problem by defining total potential energy as objective function and displacements as design variables. Based on this idea the technique called Total Potential Optimization using Meta-heuristic Algorithms (TPO/MA) is proposed. The method has been successfully applied for linear and non-linear analyses of trusses and truss-like structures, and the results have shown that the approach is much more successful than conventional methods, especially for analyses of non-linear systems. In this study, the application of TPO/MA, with Harmony Search as the selected meta-heuristic algorithm, to cables net system is presented. The results have shown that the method is robust, powerful and accurate.

Keywords: cable structures; TPO/MA method; minimum potential energy; structural analyses; geometric nonlinearity; metaheuristic algorithms; harmony search

1. Introduction

Cable net structures are systems that consist of only from cable elements. Actually belonging to group of truss-like structures, their members are either unloaded, or they are under tension. Because of this behavior, in most of the cases, pretension forces are to be applied on some or all members in order provide the system stability under different loading condition. As application, these structures are mostly used in roof systems. Studies on cable structures dates back to the beginning of the 20th century. Brief information on studies conducted at the last five decades is summarized in this section.

Saafan (1970) presented a representation of stress-strain relation of materials and the finite deflection theory analysis of a spatial system of nets using nonlinear theory. Baron and Venkatesan (1971) used the direct stiffness approach for nonlinear analysis of cable and truss structures by proposing various solution techniques and modification of existing linear approaches. Kar and Okazaki (1973) proposed an iterative method for highly nonlinear cable problems and compared the method with several previously developed methods. A method based on a discrete mathematical model was proposed for single layer cable nets by West and Kar (1973), the primary concern of the analyses being the determination of the member forces and the joint displacements resulting from live load and temperature changes. Ozdemir (1979) investigated nonlinear analyses of cable structures by a proposed finite element for static and dynamic conditions. Monforton and El-Hakim (1980) proposed a search method based on the principle of minimum potential energy for pin-ended truss and cable structures by considering geometric and material nonlinearities. Sinclair and Hodder (1981) derived analytical solutions for elastic cables systems under distributed and concentrated vertical. A small strain elastic catenary element was developed by Jayaraman and Knudson (1981) and the element was used for the members which are initially curved or curved as a result of. Lewis *et al*. (1984) investigated a dynamic relaxation technique for the analyses of nonlinear pretensioned cable networks. A procedure generating the stiffness of a nonlinear three-dimensional cable supported structures was proposed by minimizing the numerical computations (Desai *et al*. 1988). Nishino *et al.* (1989) used optimization because of different shapes of flexible cable networks and the formulations are effective than ordinary structural analyses. Eisenloffel and Adeli (1994) developed an interactive analysis program in order to solve the nonlinear problem of tensile network structures by employing an iterative Newton-Raphson

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method. Ali and Abdel-Ghaffar (1995) investigated passive control systems for seismic isolation by considering a two-dimensional model for a single-plane harp-type cable-stayed bridge. Kwan (1998) developed an approach for nonlinear static behavior of cable networks which separates the complexity of a nonlinear numerical algorithm from the basic structural principles. Liew *et al*. (2001) investigated limit state behavior of cable tensioned steel structures and demonstrated the effect of prestressing forces in improving the load bearing behavior of systems. Gasparini and Gautam (2002) studied the static behavior of cable structures by considering several issues and obtained solution of exact geometrically nonlinear equilibrium equations in non-dimensional form. By combining the analytic solution and virtual work principle Wang *et al.* (2003) developed a two-node catenary cable element with high computing precision and simple expression. Gambhir and Batchelor (1977) presented a method based on considering the prestressed cable nets a series of finite length curved elements. Kanno and Ohsaki (2005) investigated cable networks by using a minimum principle of complementary energy involving on stress component as variables with geometrical nonlinearities and nonlinear elastic materials. In order to carry out the simulation effect of net systems composed of multiple cables, a deformable catenary element was formulated by Andreu *et al.* (2006). Yang and Tsay (2007) investigated generalized displacement control method for nonlinear structures with a two node cable element. Such *et al.* (2009) dealt with three-dimensional cable structures under static loading by using a method which is a mix between nonlinear displacement method and a force density method. Kmet and Kokorudova (2006) presented a study on the non-linear closed-form static computational model of the prestressed cable trusses having unmovable, movable, or elastic yielding supports. Chen *et al.* (2010) formulated the analyses of Suspen-Dome structures by applying multi-node sliding cable elements. For solving static nonlinear cable system, Nuhoglu (2011) proposed a point based iterative method by adapting a simple convergence procedure checking the displacement increments after each iteration step. Thai and Kim (2011) developed a spatial two-node cable element for nonlinear cable structures by using an incremental-iterative solution based on the Newmark direct integration method and the Newton-Raphson method for solving the nonlinear equations. Vu *et al.* (2012) presented a spatial catenary cable element for the analysis of cable supported and tension structures.

In this study, the analyses of cable structures by using TPO/MA technique are presented. The applications on several examples have shown that the technique is effective, robust, and reliable on solving cable structures and that the technique can be considered as a general one applicable to all type of nonlinearities.

2. Methodology

A computer code was developed for the analyses of cable net structures by employing Total Potential Optimization using Meta-heuristic Algorithms (TPO/MA) technique (Toklu 2004, Toklu *et al*. 2013, Toklu *et al*. 2013, Toklu and Toklu 2013, Temür *et al*. 2015). The algorithm employed in the optimization is a music inspired algorithm called harmony search (HS) algorithm (Geem *et al*. 2001). HS algorithm has been successfully applied to several optimization problems in engineering with great efficiency. The methodology of the developed computer code is introduced in this section.

First, properties of the structure are defined. These properties are the number, coordinates and boundary conditions for each joint and cross-sectional and material properties of all structural members. Also, applied loads and pretension loads for each cable element are defined. Then, allowed ranges for displacement of joints are defined. The unknowns of the problem are the joint displacements within these ranges.

Then, the initial harmony memory (HM) matrix is constructed. This matrix is the combination of harmony vectors (HV) whose number is equal to harmony memory size (HMS). Each vector contains randomly generated coordinates within the defined solution range of joints. These coordinates represents the possible deformed shape of the structure under defined loading conditions. By using these generated coordinates, strain energy, work done by external loads and the total potential energy (TP) of the structure can be calculated. In order to use for comparison process, the total potential energy of system are also stored in related harmony vector.

This method can better be explained with an example. In the Fig. 1, a 2-member structure is given and the displacement range for this structure is also defined. Since joints 2 and 3 are hinged supports, the only possible joint displacements are the ones in x and y directions at joint 1. The circle around joint 1 in Fig. 1 shows the limits of the displacements of joint 1. After the application of external loads at joint 1, that joint will assume a position within that circle. In Fig. 1, five such points are marked as 1*a*, 1*b*, 1*c*, 1*d*, and 1*e*, corresponding also to five different deformed shapes of the structure. For all these shapes the total potential energy of the system can be computed easily. The method applied consists of making numerous trials, according to the rules of the algorithm, until finding a shape with a total potential energy as small as possible. Then the principle of minimum potential states that this shape with the minimum potential energy is the one corresponding to the stable equilibrium position.

The application of the method to the problem shown in

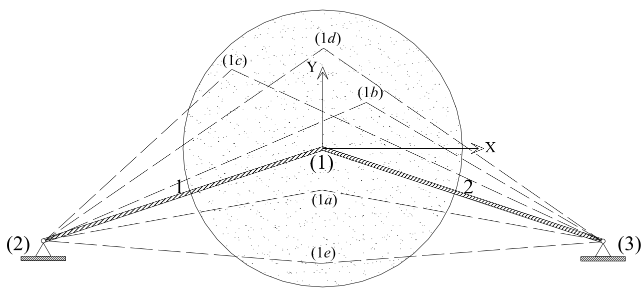


Fig. 1 An example for displacement range

Fig. 1 starts by forming HMS vectors each defining a deformed shape of the structure at hand. The set of these vectors form the initial harmony memory matrix HM

$$HM = \begin{bmatrix} \overbrace{(x_j, y_j, z_j)_1}^{HV_1} & \overbrace{(x_j, y_j, z_j)_2}^{HV_2} & \overbrace{(x_j, y_j, z_j)_3}^{HV_3} & \ldots \overbrace{(x_j, y_j, z_j)_{HMS}}^{HV_{HMS}} \\ TP_1 & TP_2 & TP_3 & \ldots TP_{HMS} \end{bmatrix}$$

The next step is the computation of TP's for each of these deformed shapes. TP for a structural member is the algebraic sum of the strain energy U in the member and the potential energy of the external forces acting on the member. The latter is equal to the negative of the work done W by the external forces during the deformation of the system. Thus the total potential can be written as

$$\Pi = U - W \tag{1}$$

For a three dimensional elastic continuum, the general expression for the strain energy is

$$U = \frac{1}{2} \int_{VOLUME} \varepsilon^T \sigma \, dV \tag{2}$$

where ε^T is the strain vector, σ is the stress vector and V is the volume of the body. The general expression for work done by external forces is

$$W = \int_{S_1} (T_x u + T_y v + T_z w) dS \tag{3}$$

In Eq. (3) u, v, and w are displacements in the x, y, and z directions. Similarly, T_x, T_y and T_z are the components in the x, y, and z directions of external forces/unit area. Substitution of Eqs. (2)-(3) into Eq. (1) yields the general expression for the total potential energy of the body at hand

$$\Pi = \frac{1}{2} \int_{VOLUME} \varepsilon^T \sigma \, dV - \int_{S_1} (T_x u + T_y v + T_z w) dS \tag{4}$$

In Fig. 2, a bar or cable element ij (undeformed member) with end coordinates (x_i; y_i; z_i) and (x_j; y_j; z_j) is given. The original length of this element (L_0) can be calculated as

$$L_0 = \left((x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2 \right)^{1/2} \tag{5}$$

and the final length (L_c) of the deformed ($i'j'$) member after end displacements (u_i; v_i; w_i) and (u_j; v_j; w_j) can be calculated as

$$L_c = \left((x_j - x_i + u_j - u_i)^2 + (y_j - y_i + v_j - v_i)^2 + (z_j - z_i + w_j - w_i)^2 \right)^{1/2} \tag{6}$$

The elongation of the element (ΔL) is equal to length differences between deformed (L_c) and undeformed length (L_0) of the element and the uniform strain in a member is

$$\varepsilon = \frac{\Delta L}{L_0} \tag{7}$$

For a given strain (ε), stress (σ) can be calculated using

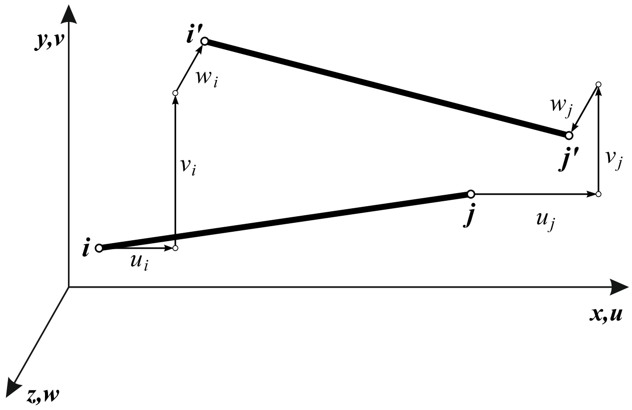


Fig. 2 3D - Deformation of truss element ij

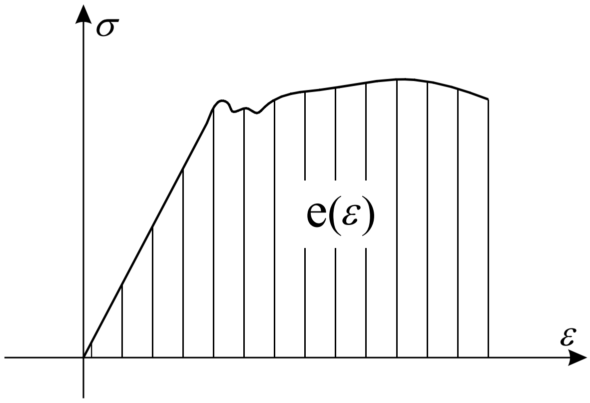


Fig. 3 Stress-strain diagram for an example structural material

the material stress-strain diagram (Fig. 3). By the help of these values, it is possible to calculate the strain energy as

$$U = \int_{VOLUME} e(\varepsilon) \, dV \tag{8}$$

where $e(\varepsilon)$ is strain energy density (area that cover strain-stress diagram) and can be written as

$$e(\varepsilon) = \int_0^{\varepsilon} \sigma(\varepsilon) d\varepsilon \tag{9}$$

V is the volume of the body and $\sigma(\varepsilon)$ represent the relationship between σ and ε, which can be linear or not, and is assumed to be known and integrable. For instance, for a truss member made of linear elastic material stress-strain relation is described as

$$\sigma = E\varepsilon \tag{10}$$

where E represents modulus of elasticity and the strain energy density can be written as

$$e = \frac{1}{2} E \varepsilon^2 \tag{11a}$$

If the material is nonlinear, then the strain energy density in the member should be computed by performing the integral given in Eq. (9). In general, a stress strain diagram as in Fig. 3 is given either by segments of lines or by a continuous function. In both cases, the integration can be performed with no great difficulty (Toklu 2004). For instance, for a cable with modulus of elasticity in compression and E in tension, the strain energy density would become

$$e = \begin{cases} \frac{1}{2}E\varepsilon^2, & \varepsilon > 0 \\ 0 \quad , & \varepsilon < 0 \end{cases} \tag{11b}$$

For a truss-like structure that consists of N_m prismatic members and N_P loads, total potential energy is written as

$$\Pi = \sum_{j=1}^{N_m} e_j A_j L_j - \sum_{i=1}^{N_p} P_i u_i \tag{12}$$

In Eq. (12), A_j represents cross-sectional area of the j^{th} member and A_jL_j is volume of that member, u_i's are generalized deflections coupled with the generalized loads P_i.

According to HS algorithm rules, after process of generation of initial HM matrix and computation of TP's for each vector, a new vector must be generated (Geem *et al*. 2001). This new vector can be generated from an existing vector (from one of the vectors stored in HM matrix) or it can be generated randomly from whole range (as the same process with previous generated vectors). The new vector is generated from an existing vector with the possibility of HMCR. If this is the case, the new vector will be in the neighborhood of the existing vector within a small range. In HS algorithm, this range is defined as multiples of the previous range, the factor between being called the pitch adjacent rate (PAR). Then by using the coordinates of new vector, the total potential energy of the new vector is calculated and that value is compared with those corresponding to the vectors already in HM matrix. If the newly generated vector is not the worst, it is replaced with the worst existing value. Then the process continues from the generation of new vector. This iterative process is repeated until the iteration number reaches to a predefined value or another finishing criteria pointing out to a convergence case becomes satisfied (Geem *et al*. 2001, Toklu *et al*. 2013).

Once the deflected shape is found at the end of cycles of the algorithm, then all other unknowns of the problem, like member forces and support reactions of the system can be found by simple static equilibrium conditions.

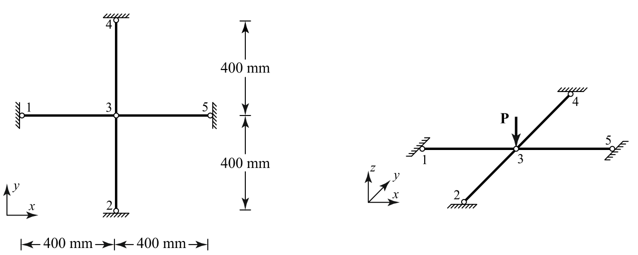


Fig. 4 Flat cable net 1×1 (Lewis 1989)

3. Numerical examples

The proposed method is applied on six numerical examples taken from existing literature. Comparisons are done as to the deflections and the total potential energies of the deflected shapes. Maximum iteration numbers are set as 5000, 50000, 10^4, 10^6, 10^5, and 5×10^6 for Example 1, 2, 3, 4, 5, and 6, respectively.

3.1 Example 1: Flat cable net 1×1

The first example is a one free and four fully constrained system that consists of four cables (Fig. 4) (Lewis 1989, Halvordson 2007). The cross-sectional area of each cable is 0.785 mm^2 and the modulus of elasticity is 124800 N/mm^2. The pretension force at each cable is 200 N, there is a downward load on the node number 3 with intensity of 15 N. The displacement of the node 3 and the total potential energy values of the system are presented in Table 1 obtained by two studies in the literature and by TPO/MA proposed in this study.

3.2 Example 2: Flat cable net 2×1

In Fig. 5, a flat system with two free nodes and having seven cables is given (Lewis 1989, Buchholdt 1999, Halvordson 2007). The cross-sectional area of each cable and modulus of elasticity of the material are 2 mm^2 and 11000 N/mm^2, respectively. Two point loads in downward direction with 200 N intensity are applied at nodes 1 and 2.

Table 1 TP energies (Nmm) and displacements (mm) for example 1

Node	Lewis (1989)			Halvordson (2007)			Presented method		
	δ_x	δ_y	δ_z	δ_x	δ_y	δ_z	δ_x	δ_y	δ_z
3	0	0	6.97	0	0	6.98	0	0	6.97
Energy	271.8088			271.8087			271.8088		

Table 2 TP energies (Nmm) and displacements (mm) for example 2

Node	Lewis (1989)			Halvordson (2007)			Presented method		
	δ_x	δ_y	δ_z	δ_x	δ_y	δ_z	δ_x	δ_y	δ_z
4	0	-3.3	199.7	0	-3.3	199.7	0	-3.3	199.7
5	0	3.3	199.7	0	3.3	199.7	0	3.3	199.7
Energy	-37448.49			-37448.49			-37448.49		

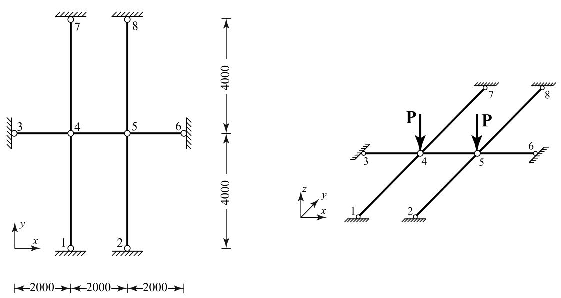


Fig. 5 Flat cable net 2×1 (Lewis 1989)

Table 3 TP energies (Nmm) and displacements (mm) example 3

Node	Lewis (1989)			Kwan (1998)			Halvordson (2007)			Present method		
	δ_x	δ_y	δ_z	δ_x	δ_y	δ_z	δ_x	δ_y	δ_z	δ_x	δ_y	δ_z
4	-0.1	-0.1	-12.2	-0.08	-0.08	-12.17	-0.07	-0.07	-12.2	-0.07	-0.07	-12.17
5	0.0	-0.1	-11.2	0.04	-0.08	-11.18	0.04	-0.08	-11.2	0.04	-0.08	-11.18
8	-0.1	0.0	-11.2	-0.08	0.05	-11.18	-0.08	0.04	-11.2	-0.08	0.04	-11.18
9	0.0	0.0	-5.6	-0.04	-0.04	-5.59	-0.04	-0.04	-5.59	-0.04	-0.04	-5.59
Energy	706.9226			704.8925			704.8477			704.8458		

Table 4 The displacements of downward direction (mm) for example 4

Node	Experiment (Lewis *et al*. 1984)	Stiffness matrix (Krishna 1978)	Dynamic relaxation (Lewis *et al*. 1984)	Minimum Energy (Sufian and Templeman 1992)	Dynamic relaxation (Kwan 1998)	Approximation by series (Kwan 1998)	Thai and Kim (2011)	Vu (2012)	Andreu (2006)	Present method
5	19.5	19.6 (0.51)	19.3 (1.03)	19.3 (1.03)	19.38 (0.62)	19.52 (0.10)	19.56 (0.31)	19.38 (0.63)	19.51 (0.05)	19.48 (0.10)
6	25.3	25.9 (2.37)	25.3 (0.00)	25.5 (0.79)	25.62 (1.26)	25.35 (0.20)	25.70 (1.58)	25.39 (0.36)	25.65 (1.38)	25.59 (1.15)
7	22.8	23.7 (3.95)	23.0 (0.88)	23.1 (1.32)	22.95 (0.66)	23.31 (2.24)	23.37 (2.50)	23.09 (1.27)	23.37 (2.5)	23.17 (1.62)
10	25.4	25.3 (0.39)	25.9 (1.97)	25.8 (1.52)	25.57 (0.67)	25.86 (1.81)	25.91 (2.01)	25.65 (1.00)	25.87 (1.85)	25.75 (1.38)
11	33.6	33.0 (1.79)	33.8 (0.60)	34.0 (1.19)	33.79 (0.57)	34.05 (1.34)	34.16 (1.67)	33.72 (0.36)	34.14 (1.61)	33.86 (0.77)
12	28.8	28.2 (2.08)	29.4 (2.08)	29.4 (2.08)	29.32 (1.81)	29.49 (2.40)	29.60 (2.78)	29.25 (1.57)	29.65 (2.95)	29.27 (1.63)
15	25.2	25.8 (2.38)	26.4 (4.76)	25.7 (1.98)	25.43 (0.91)	25.79 (2.34)	25.86 (2.62)	25.41 (0.84)	25.86 (2.62)	25.65 (1.79)
16	30.6	31.3 (2.29)	31.7 (3.59)	31.2 (1.96)	31.11 (1.67)	31.31 (2.32)	31.43 (2.71)	30.74 (0.45)	31.47 (2.84)	30.96 (1.18)
17	21.0	21.4 (1.90)	21.9 (4.29)	21.1 (0.48)	21.28 (1.33)	21.42 (2.00)	21.56 (2.67)	21.01 (0.07)	21.57 (2.71)	21.03 (0.14)
20	21.0	22.0 (4.76)	21.9 (4.29)	21.1 (0.48)	21.16 (0.76)	21.48 (2.29)	21.57 (2.71)	20.61 (1.87)	21.62 (2.95)	21.33 (1.57)
21	19.8	21.1 (6.57)	20.5 (3.54)	19.9 (0.51)	19.79 (0.05)	20.00 (1.01)	20.14 (1.72)	18.88 (4.67)	20.15 (1.77)	19.67 (0.66)
22	14.2	15.7 (10.56)	14.8 (4.23)	14.3 (0.70)	14.29 (0.63)	14.40 (1.41)	14.55 (2.46)	13.54 (4.62)	14.55 (2.46)	14.04 (1.13)
Total Diff.		39.56	31.24	14.09	10.94	19.46	25.74	17.71	25.69	13.12
% Max. Diff.		10.56	4.76	2.08	1.81	2.40	2.78	4.67	2.95	1.79

Numbers in parentheses are the percentage deviations from experimental values.

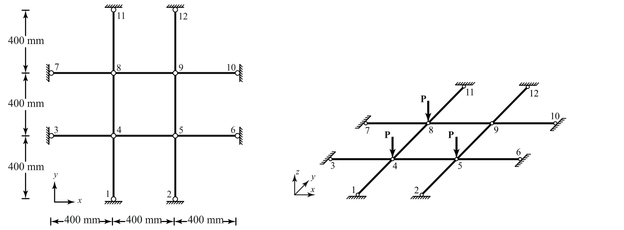


Fig. 6 Flat cable net 2×2

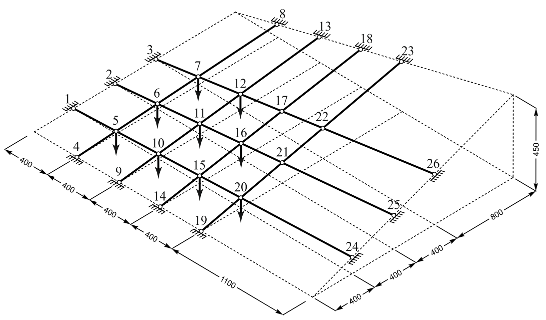


Fig. 7 Hyperbolic Paraboloid net system (Krishna 1978)

The pretension in the cables is 500 N. The results can be seen in Table 2.

3.3 Example 3: Flat cable net 2×2

The third example is a 3×3 square grid with cell side lengths of 400 mm (Lewis 1989, Kwan 1998, Halvordson 2007). System consists of 12 cables with four free and eight fully constrained joints (Fig. 6). Cross-sectional area (A) and the modulus of elasticity (E) multiplication of each member is 97.97 kN. The pretension force at each cable is 200 N, and loads on the nodes can be seen in Fig. 6. The displacement and the total potential energy values of the system are given in Table 3.

3.4 Example 4: Hyperbolic paraboloid net

The fourth example is a hyperbolic paraboloid cable network system with 31 cables as shown in Fig. 7. The cross-sectional area of each cable is 0.785 mm^2 and the modulus of elasticity of material is 128.3 kN/mm^2. The pretension force at each cable is 200 N. Concentrated loads with intensity of 15.7 N are applied at all free joints (see Fig. 7). The system is examined experimentally by Lewis (1984) and numerically by several authors. Comparisons are given in Table 4.

Table 5 TP energies (Nmm) and displacements (mm) for spatial cable network

Node	z-coord	Lewis (1989)			Thai, Kim (2011)			Present method		
		δ_x	δ_y	δ_z	δ_x	δ_y	δ_z	δ_x	δ_y	δ_z
1	1000.0	0	0	0	0	0	0	0	0	0
2	2000.0	0	0	0	0	0	0	0	0	0
3	3000.0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0
7	819.5	-5.14	0.42	30.41	-5.03	0.41	29.86	-5.03	0.4	29.46
8	1409.6	-2.26	0.47	17.70	-2.23	0.46	17.29	-2.22	0.39	17.08
9	1676.9	0	-2.27	-3.62	0	-2.31	-3.61	0	-3.12	-3.19
13	0	0	0	0	0	0	0	0	0	0
14	687.0	-4.98	0	43.49	-4.92	0	42.85	-4.92	0	42.84
15	1147.8	-2.55	0	44.47	-2.55	0	44.26	-2.55	0	44.27
16	1317.6	0	0	41.65	0	0	42.08	0	0.00	42.08
Energy		6505782			6505527			6505019		

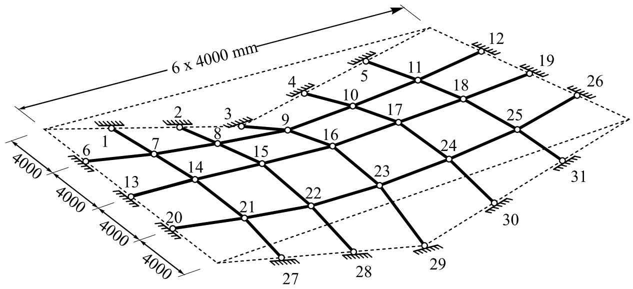


Fig. 8 Spatial cable network system (Lewis 1989)

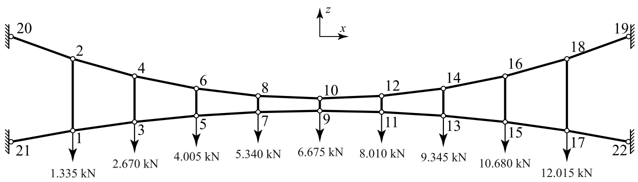


Fig. 9 Dual cable system under load case 1 (Thornton and Birnstiel 1967)

Table 6 Nodal coordinates and pretension forces for dual cable system

Coordinates [mm]						Pretension forces [kN]					
Node	X	Z	Node	X	Z	Element	Forces	Element	Forces	Element	Forces
1	-12194	-4633	7	-3048	-3719	20-2	23.65	21-1	45.22	1-2	1.78
2	-12194	-1097	8	-3048	-2926	2-4	23.10	1-3	44.94	3-4	1.78
3	-9144	-4206	9	0	-3658	4-6	22.69	3-5	44.72	5-6	1.78
4	-9144	-1951	10	0	-3048	6-8	22.41	5-7	44.58	7-8	1.78
5	-6096	-3901	20	-15240	0	8-10	22.27	7-9	44.51	9-10	1.78
6	-6096	-2560	21	-15240	-5182						

3.5 Example 5: Spatial cable network

Example 5 is a spatial cable network, consisting of 38 cables, with planar dimensions 24 m×16 m, as shown in Fig. 8. The structure has symmetry with respect to both x and y axes; the z-coordinates for a quarter of the structure are given in Table 5. The pretension forces in x-direction and in y-direction are 90 kN and 30 kN, respectively. The cross-sectional area of each cable is 350 mm^2 in x-direction and 120 mm^2 in y-direction. Elasticity modulus of all cables is 160 kN/mm^2. In all internal nodes, structure is subjected to downward concentrated loads of 6.8 kN. The results can be seen in Table 5.

3.6 Example 6: Dual cable

In the sixth example, the counter stressed dual cable structure is studied for different load cases (Thornton and Birnstiel 1967, Nishino *et al*. 1989). Before applying the external loads, system is designed under pretension forces of 44.50 kN for the parabolic shaped tie-down (stabilizing) and 22.25 kN for load (bearing) cables, respectively (Table 6). Then, system is loaded with vertical concentrated loads varying linearly from 1.335 kN a joint 1 to 12.015 kN at joint 17 (load case 1, Fig. 9). Finally, the system is analyzed for a single concentrated load with 50 kN in the y-direction

Table 7 TP energies (Nmm) and displacements (mm) for dual cable (load case 1)

Node	Thornton and Birnstiel (1967)		Nuhoglu (2011)		Present method	
	δ_x	δ_z	δ_x	δ_z	δ_x	δ_z
1	-21.3	-110.6	-21.9	-112.8	-21.62	-111.77
2	39.3	-111.3	40.2	-114.3	39.70	-112.54
3	-25.9	-146.6	-26.8	-151.2	-26.36	-148.49
4	50.3	-146.0	51.8	-150.8	50.88	-148.18
5	-23.2	-125.3	-23.7	-130.1	-23.66	-126.74
6	46.9	-123.4	48.4	-128.6	47.64	-125.56
7	-19.8	-66.8	-20.4	-69.5	-20.12	-67.55
8	40.5	-63.1	42.0	-67.6	41.18	-66.67
9	-20.4	8.8	-19.2	9.7	-18.99	10.39
10	37.8	13.7	39.0	12.2	38.35	12.66
11	-20.4	84.4	-21.3	88.4	-20.97	87.42
12	41.1	87.8	42.3	90.2	41.77	89.29
13	-23.8	140.8	-24.7	146.0	-24.37	144.30
14	48.5	142.3	49.9	146.9	49.30	145.00
15	-25.3	158.2	-25.9	162.1	-25.53	160.28
16	53.0	157.9	53.0	162.1	53.59	159.53
17	-19.2	118.0	-19.8	120.7	-19.52	118.81
18	42.3	115.2	43.3	117.6	42.50	115.68
Energy	-1994844		-2021280		-2170278	

Table 8 TP energies (Nmm) and displacements (mm) for dual cable (load case 2)

Node	Nuhoglu (2011)			Present method		
	δ_x	δ_y	δ_z	δ_x	δ_y	δ_z
1	-4.9	150.6	46.3	-4.95	150.45	46.38
2	14.0	191.4	49.7	13.94	191.07	49.67
3	-7.0	299.9	85.6	-7.05	299.84	85.83
4	20.1	385.0	86.3	20.17	384.61	86.36
5	-6.4	444.4	116.4	-6.45	444.35	116.25
6	20.4	586.7	109.7	20.31	586.04	109.78
7	-3.9	576.1	144.8	-3.99	575.84	144.77
8	14.0	810.5	109.4	14.18	810.28	109.74
9	0.0	677.0	194.8	0	676.74	195.26
10	0.0	1086.6	38.7	0	1087.10	38.68
Energy	-39504210			-39516190		

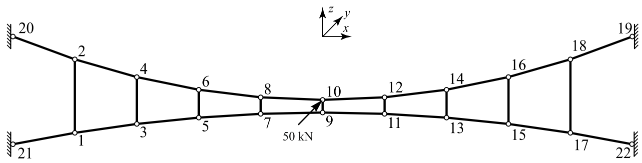


Fig. 10 Dual cable system under load case 2

at joint 10 (load case 1, Fig. 10). The modulus of elasticity is 165.54 GN/m^2 and the cross-sectional areas are 64.5 mm^2 for hanger cables, 645 mm^2 for stabilizing cables and 1290 mm^2 for bearing cables. The results of analyses of the system can be seen in Tables 7-8.

4. Discussion of the results

The aim of this paper is to investigate the efficiency of the method called Total Potential Optimization using Metaheuristic Algorithms (TPO/MA) by applications on cable systems. The proposed method was performed on six numerical examples such as flat, hyperbolic-parabolic and spatial systems and results were compared with other findings in the literature. As it is seen from the results, the potential energy values of the presented method is equal or slightly better than other methods.

In order to investigate the reliability and robustness of the method, the standard deviation values are obtained for 100 independent runs. In Figs. 11-12 and Tables 9-10, the normalized standard deviations in 100 independent runs are given for nodal displacement and member forces of Examples 5 and 6.

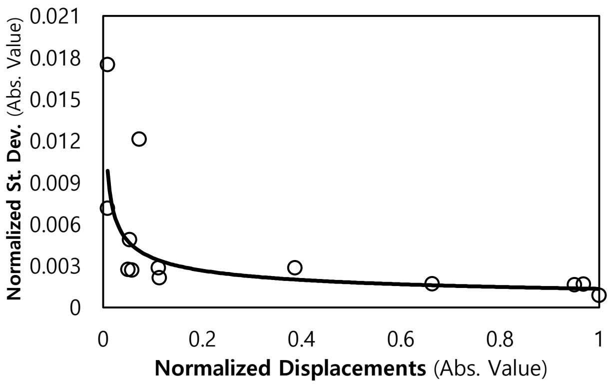
The first observation as to these 100 independent runs starting with different random seeds is that the method was able to find solutions for all the runs. This is an indication of the reliability of the method. The second observation is the closeness of the results at each run, as shown in the comparisons shown in Tables 9 and 10. For instance, in Table 9, the first line indicates that for this case, in 100 runs *x*-displacement for node 1 is found between -21.62 mm and -21.54 mm, the average being -21.59 mm, and the standard deviation being 0.014 mm. Then the normalized standard deviation, or the coefficient of variation for this case is 0.014/21.59=0.648×10^{-3} which is an unexpectedly good result for a stochastic method like this. This indicates to the result-robustness of the method proposed. Further observations on the results show that coefficient of variation gets smaller for displacements and member forces as these values get bigger. This fact, shown in Figs. 11-12 is quite understandable is one considers the dynamics of the

Table 9 Displacements evaluation of 100 independent solutions of Example 6 (load case 1) using TPO/MA

Displacements	Max [mm]	Min [mm]	Average	Diff. [%]	St. Dev.
u1	-21.54	-21.62	-21.59	0.37	0.014
w1	111.77	111.39	111.60	0.34	0.066
u2	39.70	39.58	39.65	0.30	0.022
w2	112.54	112.17	112.38	0.33	0.067
u3	-26.27	-26.36	-26.33	0.34	0.016
w3	148.49	147.92	148.30	0.38	0.094
u4	50.88	50.71	50.82	0.33	0.028
w4	148.18	147.63	148.00	0.37	0.092
u5	-23.59	-23.66	-23.63	0.30	0.015
w5	126.75	126.33	126.58	0.33	0.099
u6	47.64	47.50	47.58	0.29	0.028
w6	125.57	125.16	125.40	0.33	0.096
u7	-20.06	-20.12	-20.09	0.30	0.012
w7	67.64	67.25	67.45	0.58	0.074
u8	41.18	41.06	41.13	0.29	0.024
w8	65.76	65.38	65.57	0.58	0.072
u9	-18.94	-19.00	-18.97	0.32	0.011
w9	-10.3	-10.58	-10.43	2.69	0.053
u10	38.35	38.24	38.30	0.29	0.021
w10	-12.57	-12.85	-12.70	2.21	0.053
u11	-20.91	-20.97	-20.94	0.29	0.011
w11	-87.14	-87.47	-87.31	0.38	0.064
u12	41.77	41.66	41.72	0.26	0.021
w12	-89.00	-89.33	-89.17	0.37	0.065
u13	-24.31	-24.37	-24.34	0.25	0.012
w13	-143.84	-144.15	-144.01	0.22	0.062
u14	49.3	49.18	49.25	0.24	0.024
w14	-144.70	-145.01	-144.88	0.21	0.064
u15	-25.45	-25.53	-25.49	0.31	0.012
w15	-159.84	-160.28	-160.09	0.27	0.074
u16	53.59	53.43	53.52	0.30	0.029
w16	-159.08	-159.53	-159.33	0.28	0.077
u17	-19.47	-19.52	-19.51	0.26	0.014
w17	-118.54	-118.86	-118.75	0.27	0.089
u18	42.52	42.40	42.48	0.28	0.037
w18	-115.43	-115.74	-115.63	0.27	0.091
Maximum Values :				*2.69*	

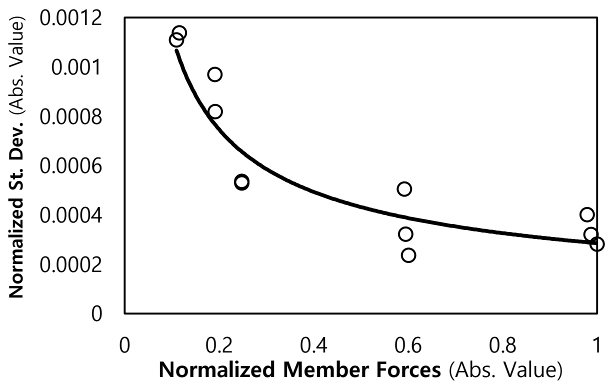
Table 10 Member forces evaluation of 100 independent solutions of Example 6 (load case 2) using TPO/MA

Members	Max [kN]	Min [kN]	Average	Diff. [%]	St. Dev.
1	67.54	67.06	67.30	0.72	0.102
2	66.58	66.10	66.35	0.71	0.092
3	65.84	65.35	65.56	0.74	0.094
4	65.17	64.69	64.88	0.75	0.104
5	64.39	63.98	64.06	0.64	0.131
6	63.56	63.02	63.32	0.85	0.116
7	62.75	62.27	62.56	0.76	0.080
8	62.22	61.81	62.04	0.66	0.086
9	61.90	61.50	61.68	0.65	0.085
10	61.75	61.25	61.54	0.82	0.116
11	117.47	116.85	117.13	0.52	0.142
12	116.12	115.36	115.68	0.65	0.138
13	114.73	114.25	114.48	0.42	0.107
14	114.64	113.64	113.86	0.88	0.138
15	114.11	113.28	113.87	0.73	0.134
16	115.31	114.42	114.62	0.77	0.110
17	116.51	115.65	116.24	0.74	0.109
18	119.14	118.22	118.70	0.78	0.124
19	122.36	121.54	122.07	0.67	0.117
20	126.90	126.24	126.38	0.52	0.149
21	5.74	5.67	5.71	1.21	0.020
22	6.70	6.55	6.60	2.39	0.033
23	7.53	7.41	7.46	1.61	0.027
24	8.43	8.27	8.33	1.90	0.031
25	9.30	9.13	9.20	1.90	0.047
26	10.16	10.00	10.08	1.61	0.037
27	11.03	10.87	10.97	1.41	0.034
28	11.94	11.83	11.90	0.93	0.024
29	12.87	12.81	12.84	0.53	0.024
Maximum Values :				*2.39*	



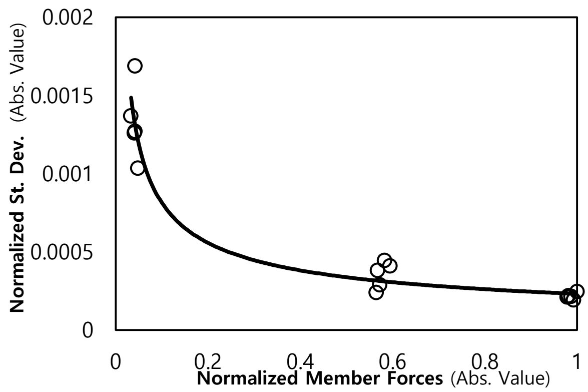
(a) Joint displacements

Fig. 11 Normalized standard deviations in 100 independent runs for the spatial cable network (Example 5)

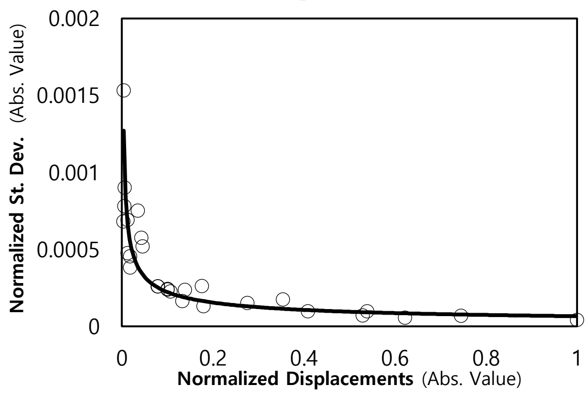


(b) Member forces

Fig. 11 Continued



(a) Joint displacements



(b) Member forces

Fig. 12 Normalized standard deviations in 100 independent runs for the dual cable system (Example 6) under load case 2

method. Indeed, greater member forces and greater nodal displacements affect the TP of the system much more than the smaller member forces and smaller displacements. Therefore, when it comes to minimizing the TP, the method becomes more effective as to these values. On the contrary, a slightly loaded member, or a node with no external loads and far from the loaded zones, have negligible effect on TP, and thus the method becomes less accurate for the values corresponding to them.

5. Conclusions

Cable net structures have been investigated as to their mechanical behavior since the beginning of 20th century. But because of their highly non-linear characteristics, there has not been a method generally accepted and used for their analyses. The literature analysis show that there are almost as many methods as the studies made on them. The method proposed in this study, TPO/MA, on the contrary, has proved itself to be a candidate for being a method general enough for being used for the static analyses of cable net structures under all conditions. It is shown that the method is robust, accurate and reliable. The meta-heuristic algorithm chosen in this study is the one known as harmony search. The study has also shown that harmony search is quite successful in finding the deflected shapes of cable net structures with minimum total potential energy. Finally, the applications have shown that the use of minimum energy principle per se is very successful in solving cable net analysis problems without recourse to simplifications of any sort.

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