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Contact problem for a stringer plate weakened by a periodic system of variable width slots

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Abstract. We consider an elastic isotropic plate reinforced by stringers and weakened by a periodic system of rectilinear slots of variable width. The variable width of the slots is comparable with elastic deformations. We study the case when the slots faces get in contact at some area. Determination of parameters characterizing the partial closure of variable width slots is reduced to the solution of a singular integral equation. The action of the stringers is replaced with unknown equivalent concentrated forces at the points of their connection with the plate. The contact stresses and contact zone sizes are found from the solution of the singular integral equation.

Keywords: stringer plate; periodic system of variable width slots; contacting of slot faces; contact zone; contact stresses

1. Introduction

As is known (Finkel 1977, Parton and Morozov 1985), one of the effective means of inhibition of crack growth may be supporting stiffeners on the path of crack propagation. In theory of fracture a problem of the crack “healing” in body is of great value. The problem of opened crack closing is the first step in solving this issue. The reinforcing elements reduce deformation of the stretched plate in the direction perpendicular to the crack, and in this connection the stress intensity factor declines in the vicinity of the crack end.

Extensive literature (Tolkachev 1963, Dolgikh and Fil'shtinskii 1976, Vanin 1985, Broek 1982) has been devoted to deformation of unlimited plate reinforced by a regular system of ribs whose cross sections are very narrow rectangles. A considerable attention was drawn to studying fracture of plates reinforced by regular system of stringers (Cherepanov 1979, Mirsalimov 1986, Maksimenko 1988, Savruk and Kravets 1994, 1995, Kravets 1999, Mir-Salim-zadeh 2007, Mir-Salim-zada 2011). In the mentioned papers, the Griffiths crack (model), i.e., a crack with noninteracting faces was considered, and it was established that at joint action of tensile stress and stiffeners, the stress intensity factors may have negative value. This means origination of contractive stresses in the vicinity of the crack tips, where the crack faces get in contact at some area, and this reduces to appearance of contact stresses. Recently, there have been published a number of papers devoted to investigation of bodies with cracks (slots) with regard to possibility of contact of crack faces (Birinci and Cakiroglu 2003, Perel 2007, Mirsalimov 2009, Birinci 2011, Hasanov 2012, Goryacheva *et al*. 2012, Mirsalimov and Rustamov 2013, Belhouari *et al*. 2014, Mirsalimov and Mustafayev 2014, Mir-Salim-zada 2014, Mirsalimov and Mustafayev 2015a, b, Mirsalimov 2016). To other fracture mechanics problems are devoted works (Birinci *et al*. 2010, Patra *et al*. 2014, Ibraheem *et al*. 2015, Wu *et al*. 2015, Farahpour *et al*. 2015, Yaylacı 2016). The issues of partial contact of the slot faces in the reinforced plate have been poorly studied to day.

2. Formulation of the problem

Let us consider an elastic isotropic thin plate weakened by a periodic system of rectilinear slots of variable width $h(x)$ comparable with elastic deformations. At the points $z=\pm(2m+1)L\pm iky_0$ $(m=0,1,2,\ldots;\ \ k=1,2,\ldots)$ (Fig. 1), lateral stiffeners were fastened to the plate symmetrically with regard to its surface with constant step. The action of the fastened supporting ribs, in the design scheme is replaced by the unknown concentrated forces applied at the fixing points. It is assumed that at deformation the stringer’s thickness is invariable, and the stress state is uniaxial.

The following assumptions are accepted: a) stringers do not resist bending and work only to tension; b) plane stress state is realized in a thin-walled sheet structural element (plate); c) the supporting system of stringers is of truss-type, their weakening at the expense of setting of fixing points is not considered; d) the sheet element and supporting elements interact in one plane and only at fixing points; e) all fixing points are the same, their radius (the bond area) is small compared with their steps and other typical sizes. The action of the fixing point is modelled in the stringer by the action in the entire rib of the concentrated force applied at the point, corresponding to the center of the fixing point; in the plate by the action of concentrated force.

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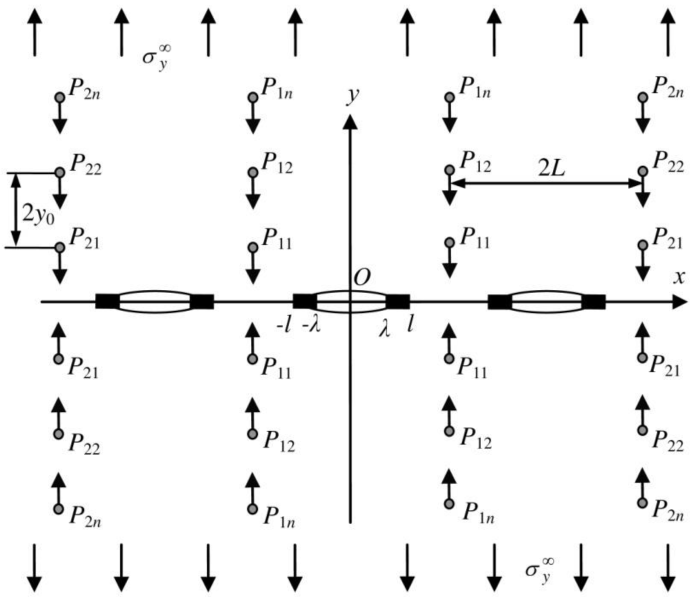

Fig. 1 Design scheme of contact problem for a stringer plate

At infinity the uniform tensile stress $\sigma_y^\infty = \sigma_0$ acts on the plate. Under the action of the external load σ_0 and concentrated forces P_{mn} in the zone of compressive stresses the slot faces will get in contact at some areas and that will contribute to emergence of contact stresses on the these areas. The contact areas are assumed to be adjacent to the slot tips; their size is not known beforehand and may be compared with the slot's length. In the end zones where the slot faces get in contact, there will arise normal $\sigma_y=q(x)$ stresses. The values of contact stresses and concentrated forces P_{mn} are unknown beforehand and should be defined in the process of solution of a boundary value problem of fracture mechanics. The parameters λ_1 and λ_2 characterizing the boundary of contact area between the slot faces also should be determined in the course of problem solution. For the problem under consideration, we can previously say that the zone between the slot faces will always begin from the end points of slots.

The problem under consideration consists of development of a mathematical model that allows to determine the contact stresses in the areas $(-l + m\omega,\ -\lambda + m\omega)$ and $(\lambda + m\omega,\ l + m\omega)$, the values of concentrated forces P_{mn} $(m,n = \pm1,\pm2,...)$, the stress-strain state outside the slot.

Outside the contact area the slots faces are load free. The boundary conditions of the problem under consideration have the form:

on noncontacting areas of slots faces for y=0, $|x - m\omega| < \lambda$

$$\sigma_y^+(x,0) = \sigma_y^-(x,0) = 0$$
$$\tau_{xy}^+(x,0) = \tau_{xy}^-(x,0) = 0 \tag{1}$$

on the contact areas $\lambda \le |x - m\omega| \le l$

$$\sigma_y^+(x,0) = \sigma_y^-(x,0) = q(x)$$
$$v^+(x,0) - v^-(x,0) = -h(x) \tag{2}$$

Here x is the affix of the points of slot contour; the quantities $\sigma_y^+(x)$, $\tau_{xy}^+(x)$, $v^+(x)$ belong to the upper faces of the slot, $\sigma_y^-(x)$, $\tau_{xy}^-(x)$, $v^-(x)$ to the lower faces of the slot. Because of symmetry of the problem under consideration $\sigma_y^+ = \sigma_y^-$; $\tau_{xy}^+ = \tau_{xy}^-$; $v^+ = -v^-$; $h(x) = h(-x)$.

The stress and strain state in an infinite plate in the conditions of the plane problem with the cuts along the axis Ox is described by two analytic functions $\Phi(z)$ and $\Omega(z)$ (Muskhelishvili 2010)

$$\sigma_x + i\tau_{xy} = \Phi(z) + \overline{\Phi(z)} - \overline{\Omega(z)} - (z - \bar{z})\overline{\Phi'(z)}$$
$$\sigma_y - i\tau_{xy} = \Phi(z) + \overline{\Phi(z)} + \overline{\Omega(z)} + (z - \bar{z})\overline{\Phi'(z)}$$
$$2\mu\left(\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}\right) = \kappa\Phi(z) - \overline{\Phi(z)} - \overline{\Omega(z)} - (z - \bar{z})\Phi'(z) \tag{3}$$
$$\Omega(z) = z\Phi'(z) + \Psi(z)$$

where v is the Poisson ratio of the plate material; $\kappa = (3 - v)/(1 + v)$ is the Muskhelishvili elastic constant.

For determining the functions $\Phi(z)$ and $\Omega(z)$, we have the boundary value problem

for y = 0, $|x - m\omega| \le \lambda$

$$\Phi(z) + \overline{\Phi(z)} + \Omega(z) = 0$$

for y = 0, $\lambda < |x - m\omega| \le l$ (4)

$$\Phi(z) + \overline{\Phi(z)} + \Omega(z) = q(x)$$

3. Solution of boundary value problem

We look for the solution of boundary value problem (4) in the form

$$\begin{aligned}\Phi(z)=\Phi_0(z)+\Phi_1(z)\\ \Omega(z)=\Omega_0(z)+\Omega_1(z)\end{aligned} \qquad (5)$$

$$\Phi_0(z)=\frac{1}{4}\sigma_0-\frac{i}{2\pi(1+\kappa)h_*}\times$$

$$\times\sum_{m=-\infty}^{\infty}{}'\sum_{n=-\infty}^{\infty}{}'P_{mn}\left[\frac{1}{z+iy_0n-m_*L}-\frac{1}{z-iy_0n-m_*L}\right]$$

$$\Omega_0(z)=z\Phi_0'(z)+\Psi_0(z)$$

$$\Psi_0(z)=\frac{1}{2}\sigma_0-\frac{i\kappa}{2\pi(1+\kappa)h_*}\times \qquad (6)$$

$$\times\sum_{m=-\infty}^{\infty}{}'\sum_{n=-\infty}^{\infty}{}'P_{mn}\left[\frac{1}{z+iy_0n-m_*L}-\frac{1}{z-iy_0n-m_*L}\right]+$$

$$+\frac{i}{2\pi(1+\kappa)h_*}\sum_{m=-\infty}^{\infty}{}'\sum_{n=-\infty}^{\infty}{}'P_{mn}\times$$

$$\times\left[\frac{m_*L-iy_0n}{(z-iy_0n-m_*L)^2}-\frac{m_*L+iy_0n}{(z+iy_0n-m_*L)^2}\right]$$

Here h_* is the plate thickness; $m_*=2m+1$; the prime at the sign of sum means that at summation the index $n=m=0$ is excluded.

For determining the analytic functions $\Phi_1(z)$ and $\Omega_1(z)$, we have the boundary value problem:

For determining the functions $\Phi(z)$ and $\Omega(z)$, we have the boundary value problem

for $y=0$, $|x-m\omega|\le\lambda$

$$\Phi_1(z)+\overline{\Phi_1(z)}+\Omega_1(z)=f(x)$$

for $y=0$, $\lambda<|x-m\omega|\le l$ (7)

$$\Phi_1(z)+\overline{\Phi_1(z)}+\Omega_1(z)=f(x)+q(x)$$

where

$$\Omega_1(z)=z\Phi_1'(z)+\Psi_1(z)$$

$$f(x)=-\left[\Phi_0(x)+\overline{\Phi_0(x)}+x\Phi_0'(x)+\Psi_0(x)\right]$$

By the conditions of symmetry with respect to the axis x, the function $f(x)$ is real, therefore based on (7) on all the real axis there will be $\operatorname{Im}\Omega_1(z)=0$. Consequently, taking into account the conditions at infinity, we get $\Omega_1(z)=0$. For the function $\Phi_1(z)$ we get the Dirichlet problem

for $y=0$, $|x-m\omega|\le\lambda$

$$\operatorname{Re}\Phi_1(z)=\frac{1}{2}f(x)$$

for $y=0$, $\lambda<|x-m\omega|\le l$ (8)

$$\operatorname{Re}\Phi_1(z)=\frac{1}{2}\left(f(x)+q(x)\right)$$

as $z\to\infty$ $\quad\Phi_1(z)\to 0$

By means of transformation $w=\sin(\pi z/\omega)$ we pass from the plane z to the parametric plane of complex variable w. Herewith, the exterior of the periodic system of slots of the plane z passes to infinitely sheeted Riemann surface with the slot $(-l_0,l_0)$, where $l_0=\sin(\pi l/\omega)$.

The sought-for solution of the problem (8) should be found in the class of everywhere bounded functions. We write the sought-for solution of the problem in the form

$$\Phi_1(z)=\frac{X(z)}{2i\omega}\int_{-l_0}^{l_0}F_*(x)\frac{\cos(\pi x/\omega)}{\sin(\pi x/\omega)-\sin(\pi z/\omega)}dx \qquad (9)$$

Here

$$F_*(x)=\frac{f(x)}{X(x)}\quad\text{for}\quad |x-m\omega|<\lambda$$

$$F_*(x)=\frac{q(x)+f(x)}{X(x)}\quad\text{for}\quad \lambda\le|x-m\omega|<l$$

$$X(z)=\sqrt{\sin^2(\pi z/\omega)-\sin^2(\pi l/\omega)}$$

Under the function $X(z)$ we mean the branch that for large $|z|\to\infty$ has the form $\sin(\pi z/\omega)$.

Subject to behavior of the function $\Phi_1(z)$ at infinity, the solvability condition of the boundary value problem is represented in the form

$$\int_{-l_0}^{l_0}\frac{[f(x)+q(x)]\cos(\pi x/\omega)}{X_*(x)}dx=0 \qquad (10)$$

where $X_*(x)=\sqrt{\sin^2(\pi l/\omega)-\sin^2(\pi x/\omega)}$.

This relation is used to determine the size of the end contact zone.

For terminal determination of the potential $\Phi_1(z)$ it is necessary to find the contact stresses $q(x)$ on the areas of contact between the slot faces, i.e., for $\lambda\le|x-m\omega|\le l$, and also the values of concentrated forces P_{mn}.

4. Definition of contact stresses

Using the Kolosov-Muskhelishvili relations and the boundary values of the function $\Phi_1(z)$, on the sections $|x-m\omega|\le l$ we get the following equality

$$\Phi_1^+(x) - \Phi_1^-(x) = \frac{2\mu i}{1+\kappa}\frac{\partial}{\partial x}\left[v^+ - v^-\right]$$

Using the Sokhotskii-Plemelj formulas (Muskhelishvili 2008), with regard to formula (9), we find

$$\frac{2\mu}{1+\kappa}\frac{d}{dx}\left(v^+ - v^-\right) = -\frac{X_*(x)}{\omega}\int_{-l_0}^{l_0}\frac{\left[f(x)+q(x)\right]\cos(\pi t/\omega)}{X_*(x)\left(\sin(\pi t/\omega)-\sin(\pi x/\omega)\right)}dt \tag{11}$$

Taking into account conditions (2), we get the singular integral equation

$$\int_{L_1+L_2}\frac{\left[f(x)+q(x)\right](\pi/\omega)\cos(\pi t/\omega)}{X_*(x)\left(\sin(\pi t/\omega)-\sin(\pi x/\omega)\right)}dt = \frac{2\pi\mu}{1+\kappa}\frac{h'(x)}{X_*(x)} \tag{12}$$

where $L_1 = \left[-l_0, -\lambda_0\right]$, $L_2 = \left[\lambda_0, l_0\right]$, $\lambda_0 = \sin(\pi\lambda/\omega)$.

Solving the appropriate Riemann problem (Muskhelishvili 2008), we can get the solution of singular integral Eq. (12).

Solving Eq. (12), with regard to boundedness of the contact stresses at the ends of the contact zone, we get formulas for the normal stresses $q(x)$

$$q(x) = \frac{\sqrt{\sin^2(\pi l/\omega) - \sin^2(\pi x/\omega)}}{\pi^2}\times X_1^+(x)\int_{L_1+L_2}\frac{f_*(t)(\pi/\omega)\sin(\pi t/\omega)}{X_*(t)(\sin(\pi t/\omega)-\sin(\pi x/\omega))}dt \tag{13}$$

Here

$$f_*(x_*) = \frac{2\mu\pi}{1+\kappa}\frac{h'(x)}{X_*(x)} - \int_{-l_0}^{l_0}\frac{f(x)(\pi/\omega)\cos(\pi t/\omega)}{X_*(x)\left(\sin(\pi t/\omega)-\sin(\pi x/\omega)\right)}dt$$

$$X_1^+(x) = \sqrt{(\cos(\pi t/\omega)+\sin(\pi l/\omega))(\cos(\pi t/\omega)+\sin(\pi\lambda/\omega))}\times\sqrt{(\cos(\pi x/\omega)-\sin(\pi\lambda/\omega))(\sin(\pi l/\omega)-\sin(\pi x/\omega))}$$

For calculating the contact stresses, it is necessary to find the values of concentrated forces P_{mn} ($m,n=\pm1,\pm2,\ldots$). Because of symmetry and periodicity of the problem $P_{mn}=P_{m1}$ ($m=1,2\ldots$).

5. Definition of values of concentrated forces

For determining the values of concentrated forces P_{mn} ($m,n=\pm1,\pm2,\ldots$), we use the Hooke law and the method of joining of two asymptotics of the desired solution. According to this law, the desired value of the concentrated force P_{mn} acting on each fixing point from the side of the rigidity rib equals

$$P_{mn} = \frac{E_S F_m}{2y_0 n}\Delta v_{mn} \tag{14}$$

where $m=0,1,2,\ldots$; $n=1,2,\ldots$; E_S is the Young modulus of the stringer material; F_m is the area of the cross section of the stringer (without loss of generality we will assume that $F_m = F$); $2y_0 n$ is the distance between the fixing points; Δv_{mn} is mutual displacement of the considered fixing points that equals the elongation of the appropriate area of the stringer.

Denote by a_0 the radius of the fixing point (adhesion area). Accept a natural assumption that mutual elastic displacement of the points a_0 in the considered problem of elasticity theory equals the above mentioned mutual displacement of the fixing points $z = m_*L + i(ny_0 - a_0)$ and $z = m_*L - i(ny_0 - a_0)$. This additional condition of compatibility of displacements permits to find effectively the solution of the stated problem. By complex potentials (5), (6), (9) and the Kolosov-Muskhelishvili formula (Muskhelishvili 2010) for displacements, after fulfilling elementary, although some bulky calculations, we find mutual displacement of the fixing points Δv_{mn}

$$\Delta v_{kr} = \Delta v_{kr}^{(0)} + \Delta v_{kr}^{(1)} + \Delta v_{kr}^{(2)}$$

$$\Delta v_{kr}^{(0)} = \frac{1}{2\pi\mu(1+\kappa)h}\sum_{m=-\infty}^{\infty}{}'\sum_{n=-\infty}^{\infty}{}' P_{mn}\times\left\{\kappa\ln\frac{C_1}{C_2} + \frac{2by_0C_3\left[2k(k-m_*)L^2 + C_3a_0\right]}{C_2C_1}\right\}$$

$$\Delta v_{kr}^{(1)} = \frac{\sigma_0}{\mu}\left\{\frac{1}{4}(3-\kappa)\sin\frac{\pi C_3}{\omega} + \frac{1+\kappa}{2\sqrt{2}}\sqrt{A-B} - \frac{1}{\sqrt{2}A}\sin\frac{\pi C_3}{\omega}\left[\sin\frac{\pi L}{\omega}\sqrt{A+B} + \sin\frac{\pi C_3}{\omega}\sqrt{A-B}\right]\right\}$$

$$\Delta v_{kr}^{(2)} = \frac{1}{2\pi\mu}\int_{\lambda_0}^{l_0} f_1(t,l)q(t)dt - \frac{d_1}{\pi\mu}\int_{\lambda_0}^{l_0} X_*(t) f_2(t,l)q(t)dt + \frac{1}{2\pi\mu}\int_0^{l_0} f_1(t,l)f(t)dt - \frac{d_1}{\pi\mu}\int_0^{l_0} X_*(t) f_2(t,l)f(t)dt$$

Here

$$B = 1 - \sin^2\frac{\pi l}{\omega} - \sin^2\frac{\pi C_3}{\omega}$$

$$C_1 = (k-m_*)^2L^2 + a_0^2, \qquad \lambda_0 = \sin\frac{\pi\lambda}{\omega},$$

$$C_2 = (k-m_*)^2L^2 + C_3^2, \qquad C_3 = by_0 - a_0,$$

$$b = r - n, \qquad d_1 = 2\sin\frac{\pi C_3}{\omega}$$

$$d = \sin^2 \frac{\pi t}{\omega} - 1 + d_1, \qquad C = \frac{\pi}{\omega} \cos \frac{\pi t}{\omega}$$

$$D^2 = A, \qquad A = \sqrt{B^2 + d_1^2}$$

$$f_1(t,l) = \frac{1}{2}(1+\kappa) C \ln \frac{D^2 \cos^2 \varphi + \left(D \sin \varphi - \mathrm{X}_*(t)\right)^2}{D^2 \cos^2 \varphi + \left(D \sin \varphi + \mathrm{X}_*(t)\right)^2}$$

$$f_2(t,l) = \frac{C}{D(d^2 + d_1^2)} \times$$

$$\times \left[d \cos \varphi + d_1 \sin \varphi + \frac{1}{2} d_1 (d \sin \varphi - d_1 \sin \varphi) \right]$$

$$\varphi = \frac{1}{2} \arctan \frac{d_1}{B}$$

Thus, the sought-for values of concentrated forces are determined from the solution of infinite system of Eq. (14). The obtained system of Eq. (14) and relations (10), (13) are coupled and should be solved jointly.

6. Analysis of results

Analysis of partial closure of the system of variable width slots is reduced to parametric investigation of geometric parameters and also mechanical constants of the material according to formulas (10), (13), (14) at different distributions of stresses in the plane. Immediately, by means of calculations from the obtained formulas we can determine the normal stresses in contact areas, and also the size of the contact zone.

When calculating the contact stresses, all integration intervals were reduced to one interval [-1,1], and then the integrals we replaced by finite sums by means of Gauss-type quadrature formulas. Because of unknown sizes of contact zones, the obtained algebraic system of equations turned out to be nonlinear. For solving it, the successive approximations method was used (Mirsalimov 1987).

The calculations were carried out for the following values of free parameters: $\nu = 0.3$; $\varepsilon_1 = a_0/L = 0.01$; $\varepsilon = y_0/L = 0.15$; 0.25; 0.5; $E = 7.1 \cdot 10^4 MPa$ (V95 alloy): $E_s = 11.5 \cdot 10^4 MPa$ (Al-steel composite); $F/y_0 h = 1$. The amount of stringers and fixing points accepted to be equal to 6; 10; 14. The dimensionless coordinates

$$x = \frac{l+\lambda}{2} + \frac{l-\lambda}{2} x'$$

were used at calculations. The dependence of distribution of contact stresses along the right contact zone for the slot whose width changes according to the parabolic law for $\varepsilon = 0.25$ was established (Fig. 2). Curve 1 corresponds to the dimensionless length of the slots $l_* = l/L = 0.50$; curve 2 $l_* = 0.25$:

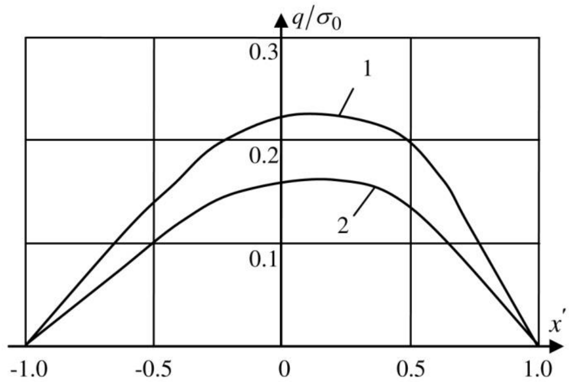

Fig. 2 Distribution of contact stresses along the right contact zone for the slot

7. Conclusions

The effective calculation scheme of partial closure of the system of variable width slots in the stringer plate is suggested. The obtained relations permit to solve the inverse problem, i.e., to determine the characteristics of strengthening elements and the stress state of the stringer plate at which the given area of contact of slot faces is achieved.

References

Belhouari, M., Amiri, A., Mehidi, A., Madani, K. and Bel Abbes, B.B. (2014), "Elastic-plastic analysis of interaction between an interface and crack in bi-materials", *Int. J. Damage Mech.*, **23**(3), 299-326.

Birinci, A. (2011), "Crack-contact problem for an elastic layer with rigid stamps", *Struct. Eng. Mech.*, **37**(3), 285-296.

Birinci, A. and Cakiroglu, F.L. (2003), "Partial closure of a crack located in an infinite elastic layer", *Eur. J. Mech. A/Solids*, **22**(4), 583-590.

Birinci, A., Birinci, F., Cakiroglu, F.L. and Erdol, R. (2010), "An internal crack problem for an infinite elastic layer", *Arch. Appl. Mech.*, **80**(9), 997-1005.

Broek, D. (1982), *Elementary engineering fracture mechanics*, Martinus Nijhoff Publishers, Hague, Netherlands.

Cherepanov, G.P. (1979), *Mechanics of brittle fracture*, McGraw-Hill, New York, USA.

Dolgikh, V.N. and Fil'shtinskii, L.A. (1976), "On a model for regular piecewise homogeneous medium", *Izv. Akad. Nauk SSSR. Mekh. Tverd. Tela*, **2**, 158-164.

Farahpour, P., Babaghasabha, V. and Khadem, M. (2015), "Stress intensity factor calculation for semi-elliptical cracks on functionally graded material coated cylinders", *Struct. Eng. Mech.*, **55**(6), 1087-1097.

Finkel, V.M. (1977), *Physical basis of fracture retardation*, Metallurgiya, Moscow, Russia.

Goryacheva, I.G., Malanchuk, N.I. and Martynyak, R.M. (2012), "Contact interaction of bodies with a periodic relief during partial slip", *J. Appl. Math. Mech.*, **76**(5), 621-630.

Hasanov, S.H. (2012), "Cohesive crack with partially contacting faces in section of the road covering", *Mech. Machines, Mechanism. Mater.*, **2**, 58-64.

Ibraheem, O.F., Abu Bakar, B.H. and Johari, I. (2015), "Behavior and crack development of fiber-reinforced concrete spandrel beams under combined loading: an experimental study", *Struct. Eng. Mech.*, **54**(1), 1-17.

Kravets, V.S. (1999), "Effect of breaks in riveted stringers on the

elastic and limiting equilibrium of a cracked plate", *Mater. Sci.*, **35**(3), 339-348.
Maksimenko, V.N. (1988), "Influence of riveted stiffeners on crack development around a hole", *J. Appl. Mech. Tech. Phys.*, **29**(2), 287-293.
Mirsalimov, V.M., (1986), "Some problems of structural arrest of cracks", *Mater. Sci.*, **22**(1), 81-85.
Mirsalimov, V.M. (1987), *Non-one-dimensional elastoplastic problems*, Nauka, Moscow, Russia.
Mirsalimov, V.M. (2009), "Simulation of bridged crack closure in a contact pair bushing", *Mech. Solid.*, **44**(2), 232-243.
Mirsalimov, V.M. (2016), "Simulation of partial closure of a variable width slot with interfacial bonds in end zones in an isotropic medium", *Int. J. Damage Mech.*, **25**(2), 266-279.
Mirsalimov, V.M. and Mustafayev, A.B. (2014), "Exact solution of contact problem for partial interaction of width variable slit faces at temperature field action", *J. Mech. Eng.*, **3**, 33-37.
Mirsalimov, V.M. and Mustafayev, A.B. (2015a), "A contact problem on partial interaction of faces of a variable thickness slot under the influence of temperature field", *Mechanika*, **21**(1), 19-22.
Mirsalimov, V.M. and Mustafayev, A.B. (2015b), "Solution of the problem of partial contact between the faces of a slot of variable width under the action of temperature fields", *Mater. Sci.*, **51**(1), 96-103.
Mirsalimov, V.M. and Rustamov, B.E. (2013), "Simulation of partial closure of a crack-like cavity with cohesion between the faces in an isotropic medium", *J. Appl. Mech. Tech. Phys.*, **54**(6), 1021-1029.
Mir-Salim-zadeh, M.V. (2007), "Fracture of an isotropic medium strengthened with a regular system of stringers", *Mech. Compos. Mater.*, **43**(1), 41-50.
Mir-Salim-zada, M.V. (2011), "Fracture of a perforated stringer plate", *Mech. Machines, Mechanism. Mater.*, **4**, 59-62.
Mir-Salim-zada, M.V. (2014), "Cracks with interfacial bonds in a perforated stringer plate", *Struct. Mech. Eng. Constr. Build.*, **5**, 34-43.
Muskhelishvili, N.I. (2010), *Some Basic Problem of Mathematical Theory of Elasticity*, Springer, Dordrecht, Netherlands.
Muskhelishvili, N.I. (2008), *Singular integral equations*, Dover publications, New-York, USA.
Parton, V.Z. and Morozov, E.M. (1985), *Mechanics of elastic-plastic fracture*, Nauka, Moscow, Russia.
Patra, P., Barik, S.P., Kundu, M. and Chaudhuri, P.K. (2014), "Plane elastostatic solution in an infinite Functionally graded layer weakened by a crack lying in the middle of layer", *Int. J. Comput. Math.*, ID: 358617.
Perel V.Y. (2007), "A new approach for finite element analysis of delaminated composite beam, allowing for fast and simple change of geometric characteristics of the delaminated area", *Struct. Eng. Mech.*, **25**(5), 501-518.
Savruk, M.P. and Kravets, V.S. (1994), "Reinforcement of a thin cracked plate by a system of parallel stringers", *Mater. Sci.*, **30**(1), 95-104.
Savruk, M.P. and Kravets, V.S. (1995), "Two-dimensional problems of the theory of elasticity for reinforced cracked plates", *Mater. Sci.*, **31**(3), 350-362.
Tolkachev, V.M. (1963), "Transmission of a load from a stringer of finite length to an infinite and semiinfinite plate", *Dokl. Akad. Nauk SSSR*, **154**(4), 86-88.
Vanin, G.A. (1985), *Micromechanics of composite materials*, Naukova dumka; Kiev, Ukraine.
Wu, X.G., Zou, R., Zhao, X. and Yu, Q. (2015), "Theoretical study of UHPCC composite column behaviors under axial compression", *Struct. Eng. Mech.*, **55**(5), 931-951.
Yaylacı, M. (2016), "The investigation crack problem through numerical analysis", *Struct. Eng. Mech.*, **57**(6), 1143-1156.

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