

# Structural damage detection based on residual force vector and imperialist competitive algorithm

Z.H. Ding<sup>1,2</sup>, R.Z. Yao<sup>1</sup>, J.L. Huang<sup>1</sup>, M. Huang<sup>1</sup> and Z.R. Lu<sup>\*1</sup>

<sup>1</sup>School of Engineering, Sun Yat-sen University, Guangzhou, Guangdong Province, 510006, P.R. China

<sup>2</sup>Center for Infrastructural Monitoring and Protection, Curtin University, Bentley, WA6102, Australia

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**Abstract.** This paper develops a two-stage method for structural damage identification by using modal data. First, the Residual Force Vector (RFV) is introduced to detect any potentially damaged elements of structures. Second, data of the frequency domain are used to build up the objective function, and then the Imperialist Competitive Algorithm (ICA) is utilized to estimate damaged extents. ICA is a heuristic algorithm with simple structure, which is easy to be implemented and it is effective to deal with high-dimension nonlinear optimization problem. The advantages of this present method are: (1) Calculation complexity can be decreased greatly after eliminating many intact elements in the first step. (2) Robustness, ICA ensures the robustness of the proposed method. Various damaged cases and different structures are investigated in numerical simulations. From these results, anyone can point out that the present algorithm is effective and robust for structural damage identification and is also better than many other heuristic algorithms.

**Keywords:** structural damage identification; residual force vector; optimization problem; ICA; heuristic algorithm

## 1. Introduction

Due to the variety of attributes, such as impact, fatigue and corrosion, civil structures usually have to encounter damage. High-quality and reliable structural components play a crucial role in structures' security, therefore the structural damage identification (SDI) has become a major research topic over the past few decades.

Many scholars have presented variety of methods for structural damage identification by monitoring the change of structural responses, mainly including sensitivity methods (Parloo *et al.* 2003, Gomes and Silva 2008, Lu and Law 2007, Li *et al.* 2014, Li and Lo 2015), wavelet transform methods (Li *et al.* 2006, Ovanesova and Suarez 2004, Li *et al.* 2014), modal force error methods (Sheinman 1996, Zimmerman and Kaouk 1994), modal residual methods (Lim 1994), and optimization method (Hassiotis and Jeong 1993).

Besides, the Residual Force Vectors (RFV), induced from the natural frequencies and mode shapes of intact and damaged structures, has been utilized to identify structural damages as well. For example, Ricles and Kosmatka (1992) used RFV to estimate the possible damaged regions, and then assessed the destructive extents of stiffness through using a weighted sensitivity analysis based on a first-order Taylor series expansion of mode shapes and eigenvalues of a non-damped structural system. Zimmerman and Kaouk (1994) made use of a minimum rank update theory to identify structural damages and the suspect elements are at

first detected by the RFV. Li and Smith (1995) utilized RFV directly to solve the parameter changes of damaged elements, and then the frequency domain data are used to damage estimation. More recently, Liu and Yang (2006) used an original Finite Element Method (FEM) model and a subset of measured eigenvalues and eigenvectors to determine the number of damaged elements and then localized them by the particular matrix. After that, damaged extents could be easily acquired. In general, the research mentioned above fully showed that the RFV is an effective way for damage localization.

In addition, from a viewpoint of mathematics, once building up the objective function related to the damage parameters, the structural damage identification problem can be boiled down to a non-linear optimal problem, therefore many artificial intelligence methods have been introduced to solve this problem, such as artificial neural network (ANN) (Dackermann *et al.* 2014); genetic algorithm (GA) (Friswell and Penny 1998, Perry *et al.* 2006, Au *et al.* 2003); supported vector machine (SVM) (Satpal *et al.* 2016); and the particle swarm optimization (PSO) (Kang *et al.* 2012, Gökdağ 2014). However, when dealing with some large and complex structures or some structures with many elements, these mentioned algorithms may not acquire satisfactory estimated results (Kang *et al.* 2012).

To overcome the problems mentioned above and consider the merit of RFV, it is natural to combine these two type of methods and develop a two-step algorithm for damage identification. That is, in the first stage, the RFV is introduced to estimate the suspected damaged elements, and thus the number of parameters to be optimized greatly decreased. In the second step, Imperialist Competitive Algorithm (Atashpaz-Gargari 2007) is used to assess the

\*Corresponding author, Professor  
E-mail: lvzhr@mail.sysu.edu.cn

damaged extents. This algorithm (ICA) is also a kind of heuristic algorithm and has a better global optimal capacity.

Moreover, in purpose to verify the proposed method comprehensively, various damage cases and different structures are investigated in numerical examples. The subsequent content is organized as follow: in section 2, it will describe the theoretical background; in section 3, it will deliberately illustrate numerical simulations and conclusions will be given in section 4.

## 2. Theoretical background

### 2.1 The residual force vector method

Based on general finite element analysis, the characteristic equation of intact structure without considering damping can be expressed as follow

$$\mathbf{K}\varphi_j = \lambda_j \mathbf{M}\varphi_j \quad (1)$$

where  $\mathbf{K}$ ,  $\mathbf{M}$  represent the global stiffness matrix and mass matrix of the finite element model of the intact structure,  $\lambda_j$ ,  $\varphi_j$  represent the  $j$ th eigenvalue and the associated eigenvector of the intact structure, respectively.

The corresponding equation of damaged structure can be expressed in Eq. (2)

$$\mathbf{K}_d \varphi_{dj} = \lambda_{dj} \mathbf{M}_d \varphi_{dj} \quad (2)$$

where  $\mathbf{K}_d$ ,  $\mathbf{M}_d$  represent the global stiffness matrix and mass matrix of the damaged structure,  $\lambda_{dj}$ ,  $\varphi_{dj}$  represent eigenvalue and the associated eigenvector of the damaged structure, respectively.

Assuming structural damage only leads to the reduction of stiffness and the global stiffness matrix of the damaged structure is given as below

$$\mathbf{K}_d = \sum_{i=1}^{Nel} (1 - \alpha_i) \mathbf{k}_i \quad (3)$$

where  $\mathbf{k}_i$  represents the  $i$ th elemental stiffness matrix, and  $\alpha_i$  is the damaged extent of the  $i$ th element.  $\alpha_i=0$  means this element is intact while  $\alpha_i=1$  means this element is totally damaged. Generally speaking, the damage of mass is easily observed and thus assuming the damaged mass matrix remaining unchanged ( $\mathbf{M}=\mathbf{M}_d$ ). In this study, the RFV is used for localization of damages. As when some elements are damaged, the corresponding RFV will not equal to zero (Yang and Liu 2006), so it can find the suspected elements in this way. The RFV  $\mathbf{f}_j$  from the  $j$ th mode of the damaged structure is defined as

$$\mathbf{f}_j = \Delta \mathbf{K} \varphi_{dj} = (\mathbf{K} - \lambda_{dj} \mathbf{M}) \varphi_{dj} \quad (4a)$$

and its unfolded form can be expressed as

$$\begin{Bmatrix} f_{1j} \\ f_{2j} \\ \vdots \\ f_{nj} \end{Bmatrix} = \begin{bmatrix} \Delta \mathbf{k}_1^T \\ \Delta \mathbf{k}_2^T \\ \vdots \\ \Delta \mathbf{k}_n^T \end{bmatrix} \varphi_{dj} \quad (4b)$$

Here  $\Delta \mathbf{K}$  is the change of the global stiffness matrix. Besides,  $\Delta \mathbf{k}_i$  ( $i=1, 2, \dots, n$ ) represents the stiffness change of the  $i$ th freedom while  $n$  denotes the total number of freedoms. When some elements occur damages, it means that  $\Delta \mathbf{k}_i$  will not equal to zero. Therefore, the corresponding RFV will not equal zero as well and this is the basis to pick out potential damaged elements. On the other hand, it is usual to find that several elements share the common freedom, which means that just those elements that are next to destructive ones will become suspected elements.

In order not to miss some damaged information when a single mode is used, the normalized average value of the first  $p$  th RFV is used, i.e.,  $\bar{\mathbf{f}} = \sum_{j=1}^p \frac{|\mathbf{f}_j|}{\max |\mathbf{f}_j|} / p$  and  $p$  denotes the number of mode used to calculate.

### 2.2 Imperialist competitive algorithm

ICA (Atashpaz-Gargari 2007, Khaled and Hosseini 2015) is a new type of heuristic algorithm that is mainly based on the simulation of imperialism and its competitive process. In the algorithm, all individuals are grouped in several empires based on a series of regulations and then the strongest empire will be picked up as the optimal solution. The variety of rules are designed to make the stronger empires suppressing weak ones.

#### 2.2.1 Initial empires

Similar to other evolutionary algorithms, ICA starts with an initial populations, which are called countries. The initial size is  $N_{pop}$  and some powerful countries are chosen as empires with quantity  $N_{imp}$ . The remaining countries  $N_{col}$  become colonies that will be distributed to these empires. In terms of the empires initialization, it is based on countries' cost. To distribute colonies to each empire reasonably, the normalized cost of an empire can be defined as

$$C_n = c_n - \max_i \{c_i\} \quad (5a)$$

Where  $c_n$  is the expense of  $n$ th imperialist and  $C_n$  is its normalized cost. After normalizing the cost of each empire, the normalized power of every empire is defined by

$$P_n = \left| \frac{C_n}{\sum_{i=1}^{N_{imp}} C_i} \right| \quad (5b)$$

The normalized power of an imperialist is the portion of colonies that ought to be possessed by that imperialist. And then the number of colonies  $NC_n$  that form an empire is computed according Eq. (6)

$$NC_n = \text{round}(p_n \cdot N_{col}) \quad (6)$$

Here  $NC_n$  is the initial number of colonies of the  $n$ th empire and  $N_{col}$  is the number of all colonies.

#### 2.2.2 Movement of each colony

Since empires are usually powerful and possess much capital and thus they can attract many small countries, which is called the assimilation policy. This behavior can be

modeled in Fig. 1 in which the colony steps towards the destination by  $x_i$  units. It ought to notice that  $x_i$  is a random variable that follows uniform distribution and can be expressed as follow

$$x_i \sim U(0, \beta \times d) \quad (7a)$$

where  $\beta > 1$  causes the colonies to move closer to the imperialist, and  $d$  is the distance between the colony and its imperialist.

Besides, to make search regions around the empire, the variable  $\theta$  is introduced into the position updating. This parameter is also submitted to the uniform random distribution and is given as below

$$\theta \sim U(-\gamma, \gamma) \quad (7b)$$

where  $\gamma$  is named the assimilation coefficient ( $\gamma < 1$ ), and  $\theta$  adjusts the deviation from the original direction, which enables searching around the imperialist.

Then the final movement can be described by Eq. (7c), given below

$$x_{t+1} = x_t + \beta \cdot d \cdot \gamma \cdot U(-\theta, \theta) \quad (7c)$$

where  $x_{t+1}$  represents the place after movement. Through Eqs. 7(a)-(c), colony countries can move around an empire flexibility.

### 2.2.3 Exchanging positions and calculating the total cost of an empire

After conquering these colonies, once any empire encounters a colony where requires less cost, the imperialist will change the position with its colony and then the new forming empire will continue the new round assimilation for those colonies.

The total expense of every empire can be calculated through the cost of its imperialist plus its average colonies' cost, given as below

$$TC_n = \text{Cost}(\text{imperialist}_n) + \xi \cdot \text{mean}(\text{colonies of empire}_n) \quad (8)$$

where  $TC_n$  denotes the total cost of the  $n$ th empire and  $\xi$  is a positive number that ought to be less than 1. Anyone can point out that a smaller  $\xi$  can render the cost closer to the empires' while a large  $\xi$  can enhance the effect on the cost from those colonies.

### 2.2.4 Imperialistic competition

The most important procedure of the ICA is the intensive competition between empires. This means that

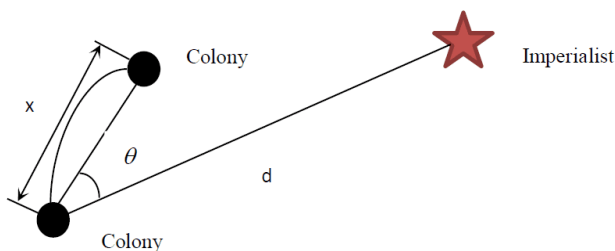


Fig. 1 Movement of a colony toward its imperialist

these stronger empires can conquer those weaker empires and occupy their colonies and make the feeble empires as colonies as well. This process will continue until only the strongest country remaining. To start with, the weakest empire is selected among these imperialists and then calculate the possession probability of each empire. The equation is given as below

$$NTC_n = TC_n - \max_i (TC_i) \quad (9a)$$

where  $TC_n$  and  $NTC_n$  denote total cost and the normalized total cost of  $n$ th empire, respectively. And  $\max_i (TC_i)$  denotes the maximum normalized cost among these empires.

After that, the possession probability  $p_{pn}$  of each empire is defined as

$$p_{pn} = \left| \frac{NTC_n}{\sum_{i=1}^{N_{imp}} NTC_i} \right| \quad (9b)$$

Obviously,  $\sum_{i=1}^{N_{imp}} p_{pi} = 1$ . And then we distribute colonies into each empire based on their possession probability, through introducing a vector  $\mathbf{P} = [p_{p1}, p_{p2}, \dots, p_{pN_{imp}}]$ . Later, another vector, with the same dimension as  $\mathbf{P}$ , is introduced and named after  $\mathbf{R}$ . This vector is filled with random numbers and is expressed as:  $\mathbf{R} = [r_1, r_2, \dots, r_{N_{imp}}]$ ,  $r_i \in (0, 1)$ . Finally, a vector  $\mathbf{A}$  is formed by subtracting  $\mathbf{R}$  from  $\mathbf{P}$ , as shown in Eq. (9c)

$$\mathbf{A} = \mathbf{P} - \mathbf{R} = [A_1, A_2, \dots, A_{pN_{imp}}] = [p_{p1} - r_1, p_{p2} - r_2, \dots, p_{pN_{imp}} - r_{N_{imp}}] \quad (9c)$$

By generating vector  $\mathbf{A}$ , the mentioned colony will be captured by the empire with the maximum index in  $\mathbf{A}$ . Actually, it is very similar to the roulette wheel in GA (Friswell and Penny 1998), but this mode can be more time-saving and efficient, since it does not require the cumulative distribute functions (Khaled and Hssein 2015).

Moreover, in the ICA, a country is defined as a feasible solution and the final empire remaining is the optimal result. And the 'cost', which is a crucial criteria to assess each country, matches the objective function value of every solution. The general structure of this algorithm is shown in Table 1.

Table 1 The structure of ICA

1: Initialize and Evaluate the empires
2: <b>while</b> stop condition is not satisfied <b>do</b>
3: move the colonies toward their relevant imperialist
4: <b>if</b> there is a colony in an empire which has a lower cost than the imperialist <b>then</b>
5: Switch the positions of that colony and of the imperialist
6: <b>end if</b>
7: Compute the total cost of all empires
8: Imperialistic competition
9: <b>If</b> there is an empire with no colony <b>then</b>
10: Eliminate this empire
11: <b>end if</b>
12: <b>end while</b>

### 2.3 Objective function

In this study, the natural frequencies and mode assurance criterion (MAC) are utilized to construct the objective function (Xu *et al.* 2016), given as follows

$$f = \sum_{i=1}^p w_{\omega_j}^2 \frac{|\omega_j^h - \omega_j^d|}{|\omega_j^d|} + w_{\varphi_j}^2 (1 - MAC_j) \quad (10)$$

$$MAC_j = \frac{(\varphi_j \cdot \varphi_{dj})^2}{\|\varphi_j\|^2 \|\varphi_{dj}\|^2} \quad (11)$$

where  $(\omega_j^h, \varphi_j)$  and  $(\omega_j^d, \varphi_{dj})$  represent the  $j$ th calculated and measured natural frequencies and mode shapes.  $w_{\omega_j}$ ,  $w_{\varphi_j}$  denote the weight factors for the natural frequencies and MACs, and in order to simplicity, all weight parameters in this paper is 1. On the other hand, Just as mentioned before, initially, there appears some discrepancy between the measured data and calculated one, and then this difference will gradually reduce when the optimal process is operated. Furthermore, it ought to be noted that the damage index  $\alpha$  is related to the natural frequencies  $\omega$  and mode shapes  $\varphi$ , therefore in the inverse problem, we can use these frequency domain parameters to assess the damage extents.

### 3. Numerical simulation

In terms of parameters setting for the ICA, according to Khaled and Hosseini (2015), the initial colony size is  $N_{pop}=100$  the number of empires is  $N_{imp}=8$ ; parameter used to calculate the mean cost  $\zeta$  is 0.1; angle parameter  $\theta$  is 0.5; assimilation coefficient  $\gamma$  is  $\pi/4$ . Furthermore, for all cases, the first three natural frequencies and three mode shapes are used to estimate the damage .

#### 3.1 61-bar truss structure

In order to verify the proposed algorithm and make comparison with other methods in the reference (Ding *et al.* 2016), A 61-bar truss is regarded as the first numerical example. The geometrical parameters of the truss are shown in Fig. 2. This structure has 26 nodes and 61 elements. Besides, the structure's Young's modulus is  $E=70$  Gpa and the density is  $\rho=2.7 \times 10^3$  kg/m<sup>3</sup> while a Poisson's ratio is  $\mu=0.33$ . In terms of the boundary condition, it has been modeled by three springs with large stiffness, i.e.,  $K_{1,1}=2 \times 10^{10}$  N/m;  $K_{1,2}=2 \times 10^{10}$  N/m; and  $K_{25,1}=2 \times 10^{10}$  N/m. Meanwhile, the corresponding degrees of freedoms (DOFs) are deleted in modeling process, therefore, the total DOFs is 49. On the other hand, in order to obtain the natural frequencies and mode shapes are utilized to calculate, there appears a sinusoidal excitation  $F(t)=1500\sin(20\pi t)$  acting at the vertical direction of the 5<sup>th</sup> node and lasting 10 s.

#### Damage case 1

The first damage case assumes the 3<sup>rd</sup> element has 15% reduction in Young's modulus, i.e.,  $\alpha_3=0.15$ . Fig. 3 exhibits the RFV value of each freedom in this case. Anyone can

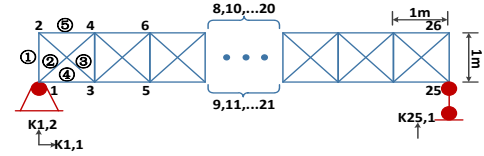


Fig. 2 A 61 bar truss structure (1,2...26 denote node number, ①,②...denote element) (Ding *et al.* 2015)

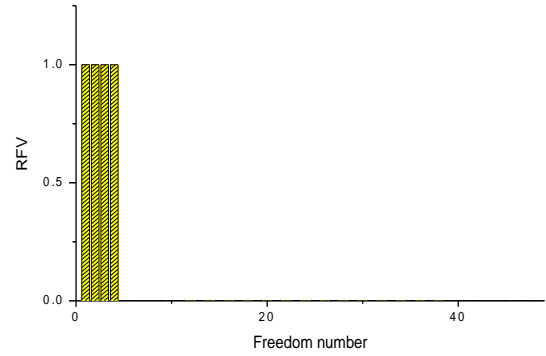


Fig. 3 RFV value of each freedom of case 1

point out that the values of the first four DOFs are 1, which fully illustrate the proposed modified formula used to calculate RFV is extremely sensitive to locate structures with single damage. Meanwhile, it means that the 1<sup>st</sup>, 3<sup>rd</sup>, 4<sup>th</sup>, 5<sup>th</sup>, 6<sup>th</sup>, 7<sup>th</sup>, 9<sup>th</sup> are the suspected damage elements, because these elements are related the mentioned DOFs. After that, Fig. 4(a) shows the evolutionary process of the objective function value. Anyone can find that after several number of iterations, the present ICA converges to the neighborhood of zero. Just as mentioned in section 2.3, with iteration, when the discrepancy between the calculate data and the measured one (the objective function value) is small enough, it means that the identified result  $\alpha$  will very close to the preset values. To be more detail, Fig. 4(b) reveals the iteration process of the damage factor  $\alpha_3$ . It can be found that  $\alpha_3$  converges to the preset value only after 20 cycles, which fully illustrates the calculation efficiency and accuracy of this method.

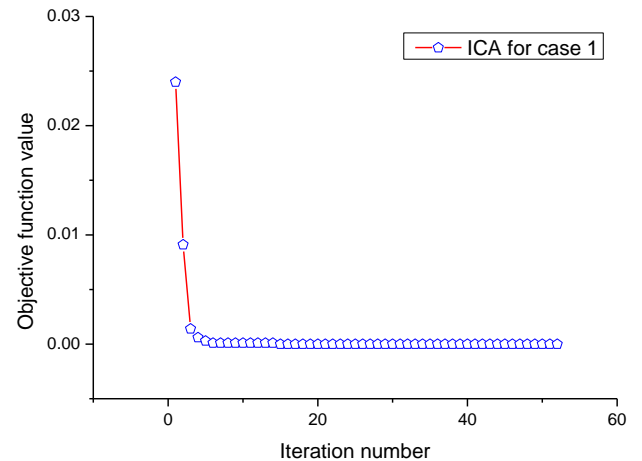


Fig. 4(a) Iteration of the objective function value of case 1

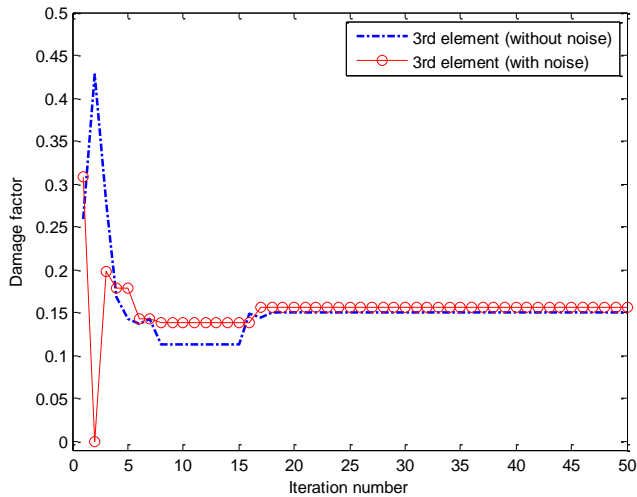


Fig. 4(b) Iteration of the damage factor of case 1

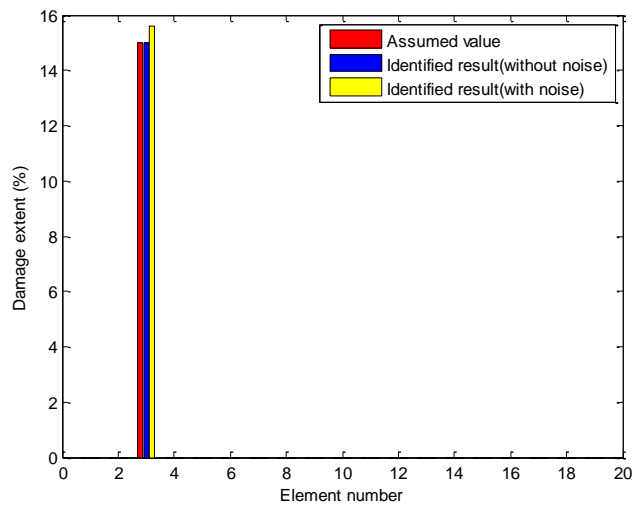


Fig. 5 Identified results of case 1 (those elements not shown are intact)

To include the uncertainty of the measured data, 1% uniform noise is added to the natural frequencies and 10% uniform noise is added to the mode shapes (Kang *et al.* 2012). Fig. 4(b) also records the evolutionary process of  $\alpha_3$  in this scenario. Even with data polluted by noise, the present method can still identify the damage successfully with the maximum error of 0.58%, less than 1.11% obtained by Ding *et al.* (2015). The final identified results are presented in Fig. 5.

### Damage case 2

In the second damage case, it assumes that 1<sup>st</sup>, 2<sup>nd</sup>, and 61<sup>st</sup> element has 10% reduction in Young's modulus, respectively. This means  $\alpha_1=0.1$ ,  $\alpha_2=0.1$  and  $\alpha_{61}=0.1$ . Similar with case 1, Fig. 6 shows the normalized RFV value for each DOF of this scenario, which can be found that there appear big values in the 1<sup>st</sup>, 5<sup>th</sup>, 6<sup>th</sup> and 49<sup>th</sup> DOFs. Therefore it can be induced that the 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup>, 5<sup>th</sup>, 6<sup>th</sup>, 8<sup>th</sup>, 9<sup>th</sup>, 10<sup>th</sup>, 11<sup>th</sup>, 12<sup>th</sup>, 14<sup>th</sup>, 57<sup>th</sup>, 60<sup>th</sup>, 61<sup>st</sup> are the suspected damage elements. After that, Fig. 7 exhibits the iteration process of the objective function value in this case. Roughly

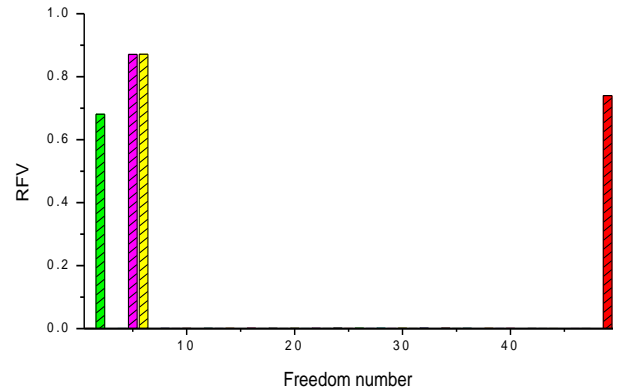


Fig. 6 RFV value of each freedom of case 2

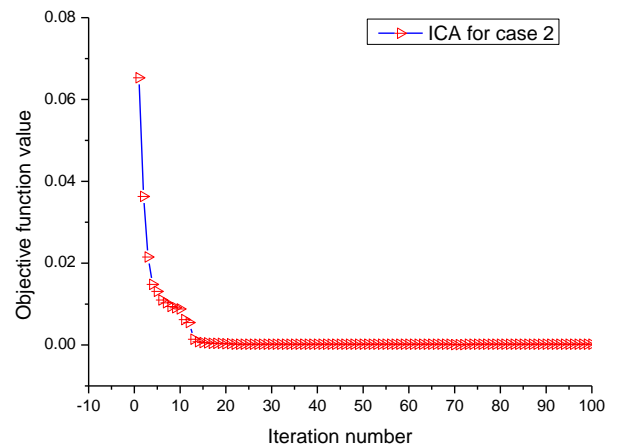


Fig. 7 Iteration of the objective function value of case 2

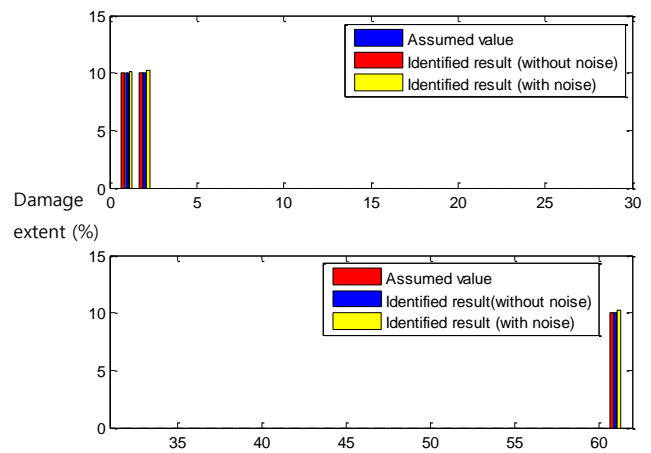


Fig. 8 Identified results of case 2

the same as Fig. 4, only a few number of iteration is required to get the objective function values convergent to the neighbor of zero, which is also indicated that the estimated results will extremely close to assumed values. Finally, from observing Fig. 8, one can find that when the input data is without noise pollution, the proposed method can obtain the identification results correctly. Furthermore, even though the 1% uniform noise is added into the natural frequencies and 10% noise is put in the mode shapes, the

present algorithm still has capacity to acquire satisfactory estimated results. And the maximum error is only 0.24%, occurring in the 61<sup>st</sup> element.

### 3.2 A two-span continuous beam

In this section, the two-span continuous beam, shown in Fig. 9, is investigated to verify the effective of the proposed algorithm. The physical parameters of the beam are: mass density  $\rho=2800 \text{ kg/m}^3$ , Young's modulus  $E=34 \text{ Gpa}$ , total length  $L=30 \text{ m}$ , width  $b=0.5 \text{ m}$  and height  $h=1.0 \text{ m}$ . The FEM of the beam consists of 20 Euler-Bernoulli beam elements with two DOFs at each node. Since a large stiffness is added to the restrained DOF when FEM modeling, the total quantity of DOFs of this beam is 42. To obtain the vibration data, there appears an impact load, acting at the position 7.5 m from the left end of the beam (shown in Fig. 9). Besides, the formula of this impact is given as below

$$F(t) = \begin{cases} 3.0 \times 10^4 \text{ N} & (0.05 \text{ s} \leq t \leq 0.1 \text{ s}) \\ 0 & (0 \leq t < 0.05 \text{ s or } t > 0.1 \text{ s}) \end{cases} \quad (12)$$

#### Damage case 3

The third case supposes that the 6<sup>th</sup>, 11<sup>th</sup>, and 18<sup>th</sup> element has 2%, 5% and 8% reduction of Young's module, respectively. That means that  $\alpha_6=0.02$ ,  $\alpha_{11}=0.05$  and  $\alpha_{18}=0.08$ . Likewise, The RFV in this case is exhibited in Fig. 10. From observing it, anyone can find some distinct values in 11<sup>th</sup> to 14<sup>th</sup> DOF, 21<sup>st</sup> to 24<sup>th</sup> DOF, and 35<sup>th</sup> to 38<sup>th</sup> DOF. Afterwards, based on the match result between Fig. 9 and Fig. 10, it can be referred that the 5<sup>th</sup>, 6<sup>th</sup>, 7<sup>th</sup>, 10<sup>th</sup>, 11<sup>th</sup>, 12<sup>th</sup>, 17<sup>th</sup>, 18<sup>th</sup> and 19<sup>th</sup> are the potential damage elements. Then, we use the ICA to calculate the damaged extents. Fig.

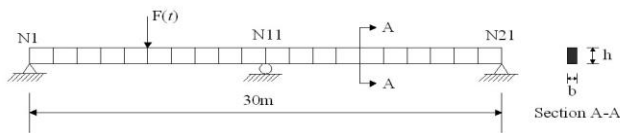


Fig. 9 A two-span continuous beam

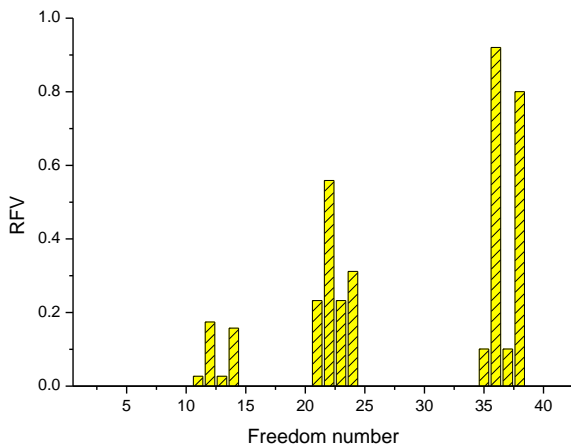


Fig. 10 RFV value of each freedom of case 3

11 presents the iteration process of the three damage parameters, and any person can observe that it take 60 cycles, 62 cycles, 75 cycles for  $\alpha_{18}$ ,  $\alpha_{11}$ , and  $\alpha_6$  to converge to the preset value correctly. Moreover, the Final identified results are shown in Fig. 12. Even under the same noise level as mentioned above, the proposed algorithm can still acquire a satisfied identified result with small errors. The corresponding identification error are 0.16% at the 6<sup>th</sup> element, 0.17% at the 11<sup>th</sup> element and 0.01% at the 18<sup>th</sup> element, respectively.

#### Damage case 4

The fourth scenario assumes that 3<sup>rd</sup> element has 5% damage of Young's Modulus while 14<sup>th</sup> element has 50% damage of Young's Modulus. That means the damage index are  $\alpha_3=0.05$  and  $\alpha_{14}=0.5$ . In the first stage, the RFV of this case is calculated and shown in Fig. 13. It is clear that there appear some large values on the 5<sup>th</sup> to 8<sup>th</sup> DOFs and 27<sup>th</sup> to 30<sup>th</sup> DOFs. This implies that 2<sup>nd</sup> to 4<sup>th</sup>, 13<sup>th</sup> to 15<sup>th</sup> elements are the suspected elements. And then the ICA is carried out and Fig. 14 showcases the iteration detail of the damage factors. It only spends 55 cycles and 70 cycles for the  $\alpha_{14}$  and  $\alpha_3$  to converge the assuming values correctly, which

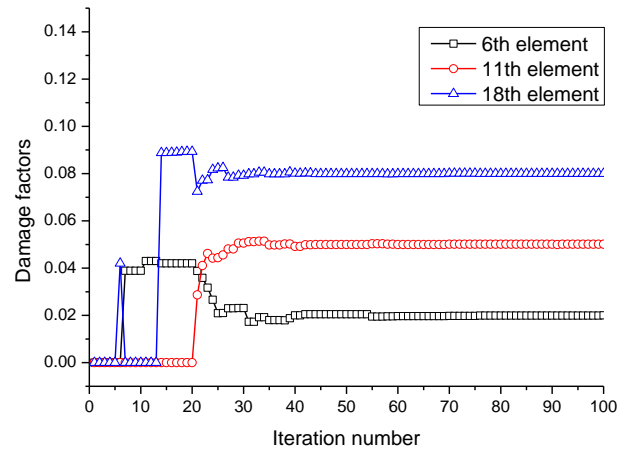


Fig. 11 Iteration of the damage factors of case 3

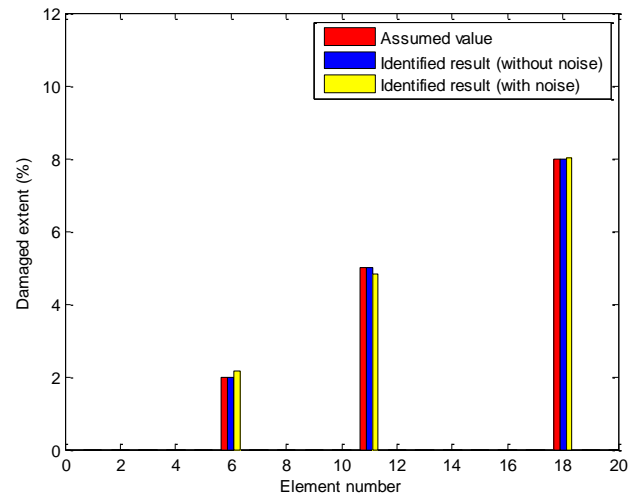


Fig. 12 Identified results of case 3



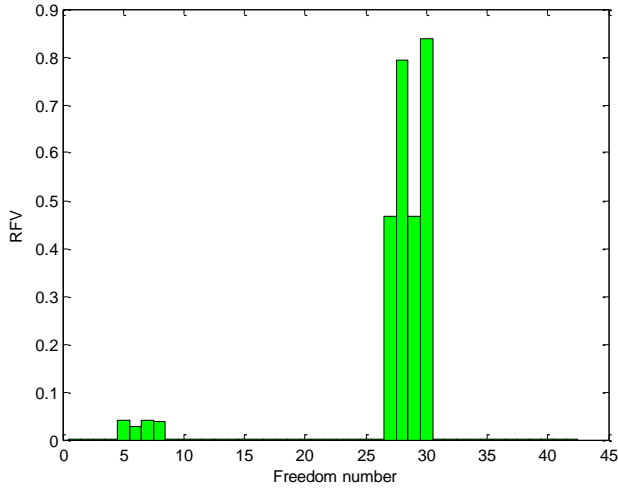


Fig. 13 RVF value of each freedom of case 4

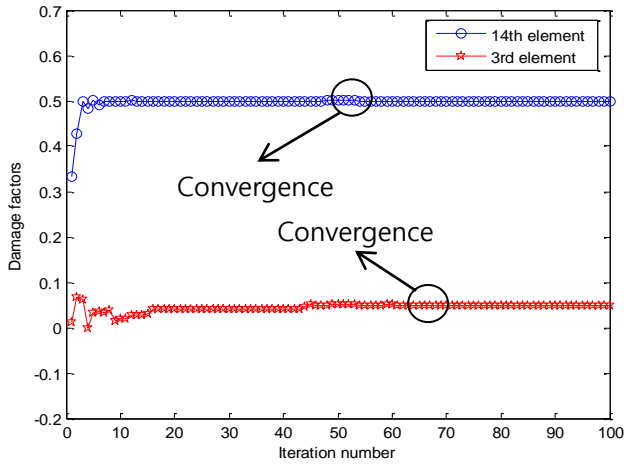


Fig. 14 Iteration of the damage factors of case 4

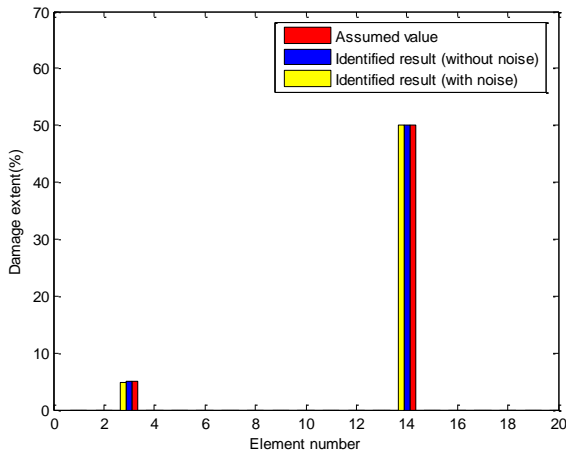


Fig. 15 Identified results of case 4

also demonstrate the effectiveness of the proposed algorithm further. Likewise, when the input is polluted with the same level noise, the method can obtain a relative accurate outcomes as well. (The 3<sup>rd</sup> and 14<sup>th</sup> elements' absolute errors are 0.0888% and 0.0304%, respectively). Fig. 15 records the final result for this case.

### 3.3 A cantilevered plate

In this part, a cantilevered plate is investigated to verify the present method further. The plate has a fixed end at the left with the dimensions of 5000 mm×2500 mm×50 mm as shown in Fig. 16. The physical material properties of the plate are: Young's modulus  $E=25$  GPa, density  $\rho=2800$  kg/m<sup>3</sup> and Poisson ratio  $\mu=0.2$ . When FEM modeling, the plate unit was discretized into fifty 4-node Reissner-Mindlin plate elements. Besides, the left end is completely fixed and thus the relevant constrained DOFs are deleted in modeling, therefore the total number of the DOFs is 180. In addition, to acquire the frequency domain data, there is an impact load, acting at 24<sup>th</sup> node. Its direction is parallel to the z-axis negative direction. The related formula is given as below,

$$F(t) = \begin{cases} 10^5 (t - 0.02) \text{ N} & (0.02 \text{ s} \leq t \leq 0.04 \text{ s}) \\ 10^5 (0.06 - t) \text{ N} & (0.04 \text{ s} < t \leq 0.06 \text{ s}) \\ 0 & (t < 0.02 \text{ s or } t > 0.06 \text{ s}) \end{cases} \quad (13)$$

#### Damage case 5

This case assumes that there appear damages in the 1<sup>st</sup>, 5<sup>th</sup>, and 14<sup>th</sup> elements with 10%, 10% and 8% reduction of the Young's modulus, independently. That means  $\alpha_1 = \alpha_5 = 0.1$ ,  $\alpha_{14} = 0.08$ . In the first step, the RFV values of each DOF are bared in Fig.17. There appear peak values at 1<sup>st</sup> to 6<sup>th</sup> DOFs, 13<sup>th</sup> to 18<sup>th</sup> DOFs, 28<sup>th</sup> to 33<sup>rd</sup> DOFs, and 37<sup>th</sup> to 42<sup>nd</sup> DOFs, which indicates that the 1<sup>st</sup> to 7<sup>th</sup>, 10<sup>th</sup>, 11<sup>th</sup>, 13<sup>th</sup> to 15<sup>th</sup> elements are the suspected damage elements. Afterwards, similar with previous cases, the ICA is conducted. Fig.18 shows the evolutionary process of damage parameters of this case. Once again, the present algorithm acquires the identified results without error when the data used is without contaminant. On the other hand, even 1% uniform noise is added to the natural frequencies and 10% uniform noise is added to the mode shapes, the method can still obtain the identified results with good accuracy as shown in Fig.18. The corresponding identification errors are 0.2% at the 1<sup>st</sup> element, 0.41% at the 5<sup>th</sup> element, and 0.27% at the 14<sup>th</sup> element, respectively.

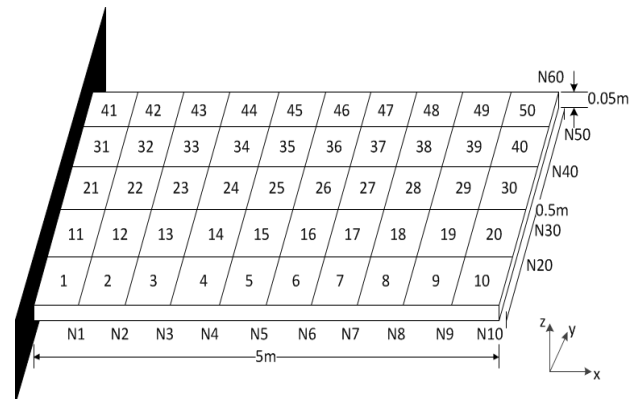


Fig. 16 A cantilevered plate

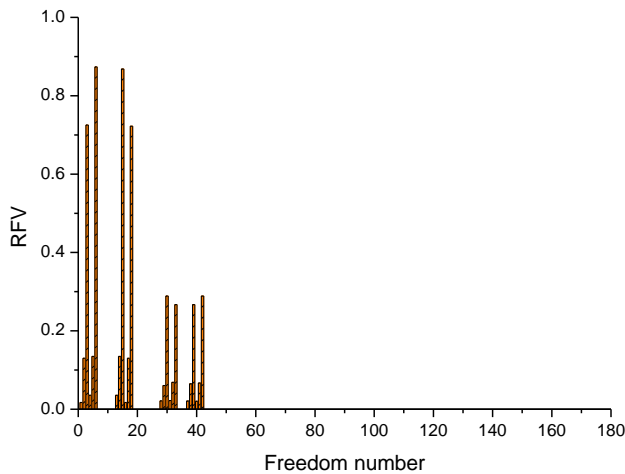


Fig. 17 RLV value of each freedom of case 5

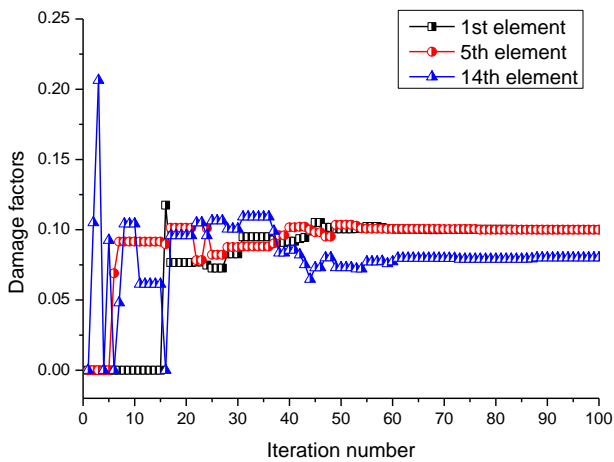


Fig. 18 Iteration of the damage factors of case 5

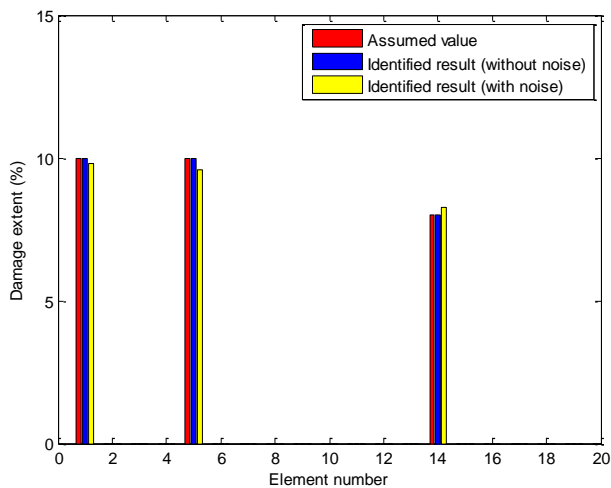


Fig. 19 Identified results of case 4 (those elements not shown are intact)

#### 4. Conclusions

Structural damage identification based on RFV and ICA algorithm is investigated in this work. In the initial stage,

the RFV is introduced to identify the possible damage location, thus the suspected damage elements can be found. This operation can effectively avoid dimensional disaster. In the second step, the imperialist competitive algorithm (ICA) is adopted to estimate damaged extents with the designed frequency object function. The performance of the present algorithm is illustrated by three numerical examples. The following conclusions can be drawn.

- The proposed RFV is extremely sensitive to localize the potential damaged elements, especially when dealing with single damage cases.
- After eliminating many intact elements, the ICA become quite effective and efficient to quantify damage extents. In particular when the input data is not polluted by artificial noise, the proposed method can get correct estimated outcomes.
- In terms of the robustness, the present algorithm can obtain satisfactory results even the noise level is relatively high.

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