

# Vibration and damping behaviors of symmetric layered functional graded sandwich beams

Ersin Demir\*

Department of Mechatronics Engineering, Pamukkale University, Kinikli Campus, 20070 Denizli, Turkey

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**Abstract.** In this study, free vibration and damping behaviors of multilayered symmetric sandwich beams and single layered beams made of Functionally Graded Materials were investigated, experimentally and numerically. The beams were composed of Aluminum and Silicon Carbide powders and they were produced by powder metallurgy. Three beam models were used in the experiments. The first model was isotropic, homogeneous beams produced by using different mixing ratios. In the second model, the pure metal layers were taken in the middle of the beam and the weight fraction of the ceramic powder of each layer was increased towards to the surfaces of the beam in the thickness direction. In the third model, the pure metal layers were taken in the surfaces of the beam and the weight fraction of the ceramic powder of each layer was increased towards to middle of the beam. Then the vibration tests were performed. Consequently, the effects of stacking sequence and mixing ratio on the natural frequencies and damping responses of functionally graded beams were discussed from the results obtained. Furthermore, the results obtained from the tests were supported with a finite-element-based commercial program, and it was found to be in harmony.

**Keywords:** functionally graded; experimental investigation; dynamic analysis; frequency/modal analysis; finite element method (FEM)

## 1. Introduction

The concept of Functionally Graded Materials (FGMs) was first proposed during the heat barrier and heat shield design by a group of scientists in Japan in 1984 (Koizumi 1993). Because of the understanding of the importance of FGMs, a research project under the name "Research on the Basic Technology for the Development of Functionally Gradient Materials for Relaxation of Thermal Stress" was started by the Japan Ministry of Education and Science in 1987 (Koizumi 1997). After the project was completed in 1992, the second national project was also made entitled "Research on Energy Conversion Materials with Functionally Graded Structures". This project was completed in 1997 (Koizumi 1997). Many studies have been carried out since then to understand the behaviors of FGM. As for the topic of the vibrations of the Functionally Graded (FG) beams, many of the studies on this subject are theoretical studies. Some of them were given below.

Wu *et al.* (2005) used the semi-inverse method to solve the dynamic equation of inhomogeneous FG simply supported beams and they obtained a closed form expression for the natural frequency. Aydogdu and Taskin (2007) also studied the free vibration of simply supported FG beam. They found the governing equation by applying Hamilton's principle and used Navier type solution method to obtained frequencies in their study. They used the

classical beam theory and the different higher order shear deformation theory in the solution of the dynamic behavior of the beam. After this study, Aydogdu (2008) made an article on the vibration and buckling of axially FG simply supported beams. He achieved the analysis using the semi-inverse method. Furthermore, Sina *et al.* (2009) made another theoretical study on the free vibration of FG graded beams. They used a new beam theory in their study. They derived the equations of motion of FG beam from the first-order shear deformation plate. Ke *et al.* (2010) studied nonlinear vibration of beams made of FGMs. based on Euler-Bernoulli beam theory and von Karman geometric nonlinearity. They used direct numerical integration method and Runge-Kutta method in order to find the nonlinear vibration response of FGM beams in their analysis. Alshorbagy *et al.* (2011) made a numerical study on dynamic characteristics of functionally graded beam. They derived the equation of motion of FG beam from principle of virtual work and they employed the finite element method to discretize the model. Ke *et al.* (2010) investigated nonlinear free vibration of microbeams made of FGMs based on the modified couple stress theory and von Karman geometric nonlinearity. They employed the differential quadrature method and an iterative algorithm to determine the nonlinear vibration frequencies of the FGM microbeams. Anandrao *et al.* (2012) developed two different finite element formulations based on the theories of Euler-Bernoulli and Timoshenko. They used virtual work principles to obtain the formulations. They demonstrated the effect of transverse shear on the vibration behavior in their study. Thai and Vo (2012) developed various theories for bending and free vibration of FG beams. It was not

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\*Corresponding author, Ph.D.  
E-mail: [edemir@pau.edu.tr](mailto:edemir@pau.edu.tr)

necessary to use the shear correction factor due to the theory that they developed. Li *et al.* (2013) derived characteristic equations in closed form for the free vibration of axially in homogeneous beams. Then, they reduced the characteristic equations to the classical equations of Euler-Bernoulli beams. Pradhan and Chakraverty (2013) used the classical and first order shear deformation beam theories to make the vibration analysis of FG beams in their study. They obtained the governing equation of system via Rayleigh-Ritz method in their study and found the effects of constituent volume fractions, slenderness ratios and the beam theories on the natural frequencies. Demir *et al.* (2013a) investigated the free vibration behavior of a sandwich FG beam resting on Winkler elastic foundation. The width of the beam were taken as variable in that study. The theoretical results obtained were compared with a finite element based program. Vo *et al.* (2014) made a finite element model for vibration and buckling of FG sandwich beams. The model used was based on a refined shear deformation theory. They used the Hamilton principle to obtain the governing equations of motion. They took the core of the sandwich beam model as fully metal of ceramic and the other surfaces were composed of a FGM across the depth. Bambill *et al.* (2015) overcame the free vibrations of axially FG beams. They considered the variation of the material properties and the geometry of the beams as stepwise change. They used the differential quadrature method with domain decomposition technique in their analysis and they used Timoshenko beam theory to obtain the governing equations of motion. Consequently, they presented the effects of stepped changes in materials properties and geometry on the free vibrations of axially FG beams. Nyugen *et al.* (2015) proposed a new higher order shear deformation theory in order to solve buckling and free vibration of isotropic and FG sandwich beams. They derived the equation of motion from Lagrange's equations. They compared their results with those obtained by higher and first order shear deformation beam theories. Wang *et al.* (2016) investigated free vibration of a FG beam which has variable material properties along the beam length and thickness. They derived the characteristic equation in closed form and reduced the governing equation to the classical form of Euler-Bernoulli beam. Jing *et al.* (2016) developed a new approach based on combination of cell-center finite volume method and Timoshenko beam theory to analyze static and free vibration of FG beams. They derived and simplified the equation of motion from Hamilton's principle. They considered three different kinds of shear corrections

factors and found the optimal shear correction factor. Wang *et al.* (2008) studied the beam model formulation for FGMs beams. Apart from these theoretical studies, there are also some studies on the vibrations of damaged beams (Ke *et al.* 2009, Wei *et al.* 2012, Aydin 2013, Cunedioğlu 2015).

It can be seen from the literature survey that many of the studies are theoretical work and there are not enough experimental study in the open literature. Some of the experimental studies were given as follows. Kapuria *et al.* (2008) overcame solution of static and free vibration of layered FG beams. They used third order zigzag theory with the modified rule of mixtures for solution. They fabricated the FG beams using powder metallurgy and thermal spraying techniques and they did experiments to validate the results obtained. Wattanasakulpong *et al.* (2012) employed an improved third order shear deformation theory to formulate the governing equation and used the Ritz method to solve the governing equation. They fabricated the layered FG beams using multi-step sequential infiltration technique and discussed the results obtained.

According to the literature survey, it was not found an experimental study on the vibration and damping behavior of layered FG sandwich beams in the open literature. In this work, vibration and damping behaviors of FG symmetric layered sandwich beams and single layered beams were investigated, experimentally and numerically. Three different beam models were considered in the study and FG beam specimens were fabricated for each model. Free vibration experiments were performed after fabrication. The results obtained from experiments were compared with those obtained from the finite-element-based commercial program SolidWorks (SolidWorks Corp., USA). The results were found to be in good agreement.

As a result of the study performed, the effects of the mixing ratio of the powders constituting the FG beam and stacking sequence of the layers of the FG sandwich beams on the free vibration and damping behavior were determined.

## 2. Design of specimens

Three beams model were used in the experiment. In the first model, isotropic and homogeneous single layered beams were produced from different mixtures of Aluminum (Al) and Silicon Carbide (SiC) powders in order to see the effect of mixing ratio of powders and calculate the mechanical properties of the sandwich beams. The beam

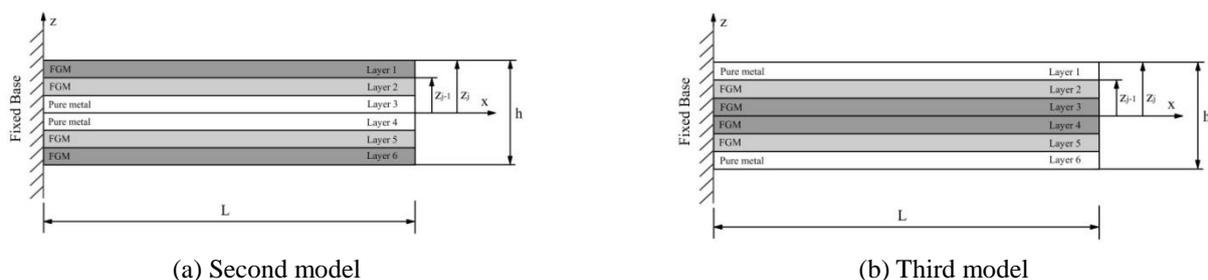


Fig. 1 Schematic representation of second and third models

Table 1 The weight ratios of SiC powder in the beam specimens

Model no	Specimen code	Weight ratio of SiC powders [%]					
		Layer1	Layer2	Layer3	Layer4	Layer5	Layer6
1	B1			0			
	B2			5			
	B3			10			
	B4			15			
	B5			20			
	B6			25			
	B7			30			
	B8			35			
	B9			40			
2	SB1-1	10	5	0	0	5	10
	SB1-2	20	10	0	0	10	20
	SB1-3	30	15	0	0	15	30
	SB1-4	40	20	0	0	20	40
3	SB2-1	0	5	10	10	5	0
	SB2-2	0	10	20	20	10	0
	SB2-3	0	15	30	30	15	0
	SB2-4	0	20	40	40	20	0



Fig. 2 Planetary ball mill

specimens were obtained by increasing the weight ratio of SiC powders from 0% to 40% in 5% increments. As for the second and the third model, the schematic representations of them were shown in Figs. 1(a)-(b).

It can be seen from the figure that in the second model, multilayered symmetric sandwich beams with different stacking sequences were produced. The stacking sequences of the sandwich beams were changed along the thickness direction and each layer were distributed isotropically and homogeneously. In this model, the pure metal was located in the middle layers of the beam and the weight ratio of SiC powders was increased toward the upper and lower surfaces of the beam as shown in Fig. 1(a). In the third model, contrary to second model, the pure metal was located in the top and bottom layers of the beam and the weight ratio of SiC powders was increased toward the middle of the beam, as shown in Fig. 1(b). The last two models were composed of 6 layers. The design of the three model described above

were also summarized in Table 1. The dimensions of the entire models discussed were taken as 10 mm (width)×9 mm (thickness)×50 mm (length).

### 3. Production of the beam specimen

#### 3.1 Preparation of powder mixture

Firstly, powder mixture was carefully prepared for the production of beam specimens. The beam specimens were fabricated from Al and SiC powders as mentioned in the above section. In addition, Zinc Stearate powder was also added in a weight ratio of 2% to the powder mixture as mold lubricant. Some properties of powders used in the mixture were described below.

Al powder used in the mixture was supplied from Atlantic Equipment Engineers Industry, USA. The priority of Al powder is 99,8% and particle size of Al powder is 100 mesh. Its density is 2.699 g/cm<sup>3</sup>. As for the SiC powder, it was supplied from Interabrasive Industry, TURKEY. The priority of it SiC 99,9% and particle size of it is <1500 Grit. Its density is 3.217 g/cm<sup>3</sup>. Furthermore, the particle size of Zinc Stearate powder was 325 Mesh and the density of it was 1.095 g/cm<sup>3</sup>.

These powders were weighed in the weight ratio given in Table 1 by a precision weighing, PRECISA XB 220A (manufactured by Precisa Gravimetrics AG, SWITZERLAND). The powders were weighed by a precision 0.1 mg. After the powders were weighed in appropriate proportions, they were mixed by Bench-Top Planetary Ball Mill (SFM-1 QM - 3SP2) manufactured by MTI Corporation, USA. The Planetary Ball Mill has four



Fig. 3 Cold pressing process



Fig. 4 Beam specimens sintered in the tube oven

ceramic jars installed on one turning plate as shown in Fig. 2.

The direction of rotation of the turning plate was opposite to that of jars. Moreover,  $ZrO_2$  ceramic balls of 12 mm diameter were put into jars to ensure good mixing. The amount of balls and powders were not exceeded the three quarter of total volume of each jar. The Planetary Ball Mill was set to change the direction of rotation after each half an hour with an interval of 1 min. The powders were mixed for three hours in the Planetary Ball Mill.

### 3.2 Cold pressing of the powder mixture

The powder mixture prepared was pressed by a hydraulic press. Just before pressing, the walls of the mold space and upper and lower punch were lubricated with Aluminum mold lubricants (Sumidera 460). The powder mixture was then added to the space of the mold. The compression was performed for 60 seconds at a pressure of about 450 MPa. The compression process was shown in Fig. 3.

The powders were mechanically consolidated as beam specimens after compression process. However, the strength of the beam specimens obtained by the compression process was fairly low. Therefore, the sintering process was carried out. This process was given below.

### 3.3 Sintering

The sintering process was performed to increase the strength of the beam specimens. Porous structure of the beam specimens became homogeneous structure by sintering process. Furthermore, the bonds between powder particles were also strengthened by sintering process. The beam specimens were sintered in a tube oven under flowing argon gas as shown in Fig. 4.

The sintering program was started at room temperature. The beam specimens were heated from room temperature to 400°C with a heating up rate of 10 degree/minute and the temperature was kept at that degree for 60 minute. Then, the temperature was increased to 600°C with a same heating up rate and the temperature was kept at that degree for 180 minute.

In order to prevent oxidation of the Al powders, air was evacuated from the oven by a vacuum pump and the Argon gas was pumped into the oven at a constant flow rate.



(a) Intact specimen

(b) Cut specimen

Fig. 5 Beam specimens obtained after the second pressing

### 3.4 Hot pressing after sintering

The specimens were subjected to hot-pressing at a pressure of about 450 MPa for 60 second to improve the adhesion of the particles of the powders immediately after the sintering.

An example of the specimens obtained after the second pressing were shown in Fig. 5(a). Sharp corners of the specimens in the vertical direction were rounded in order to remove easily from the mold after compressing, as also shown in Fig. 5(a). As for the Fig. 5(b), the rounded portion of one end of the each specimen was cut to calculate the natural frequencies of the beam specimens easily.

## 4. Determination of the effective material properties of the sandwich beams

In order to compare the vibration results obtained from experiments and numerical studies, the material properties of the sandwich beams were calculated utilizing the material properties of single layered beams. The elasticity moduli of the single layered beams were determined from

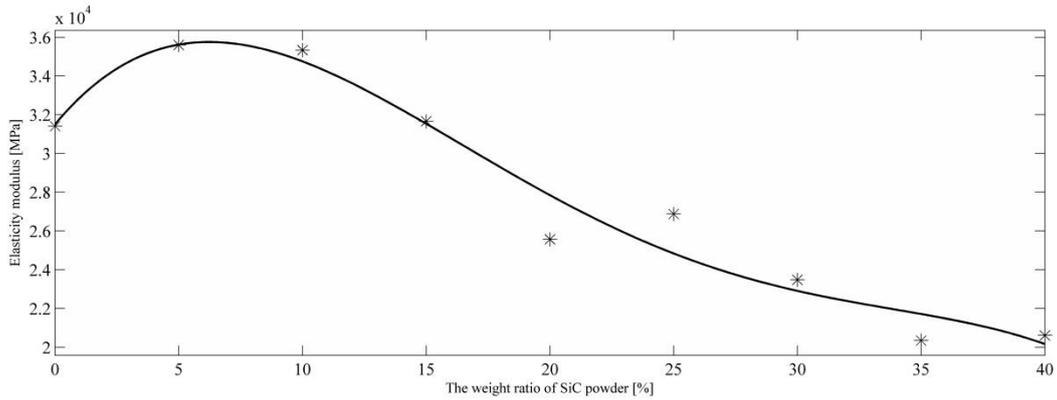


Fig. 6 Variation of the elasticity modulus with the weighting ratio of SiC powders

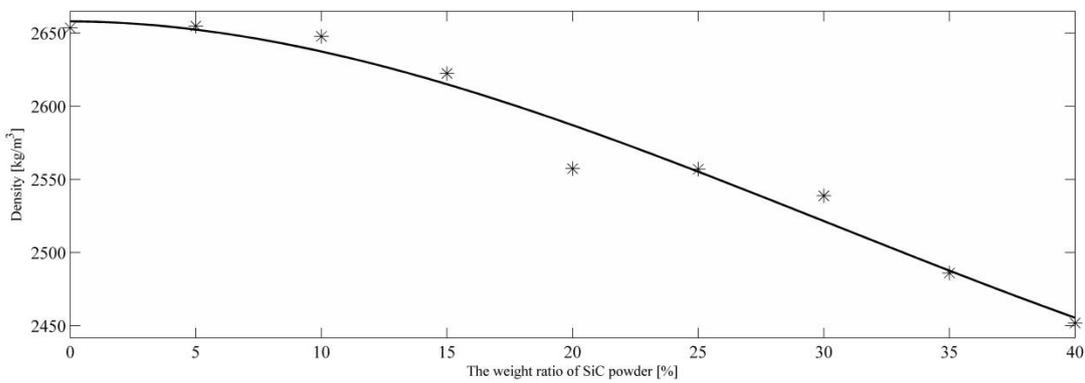


Fig. 7 Variation of the mass density with the weighting ratio of SiC powders

the vibration tests and the mass densities of the single layered beams were determined by above-mentioned precision weighing, PRECISA XB 220A. The variation of the elasticity modulus and the mass density of the single layered beams due to the change in the weighting ratio of SiC powders were given in Figs. 6 and 7, respectively.

Besides, the polynomial trend lines were added to the figures as well. The conclusions drawn from these graphs were discussed in the Results and Discussion section. After the material properties of the single layered beams determined, the above-mentioned calculation based on the theory of laminated composite beams (Gibson 1994) was given as follow.

The bending moment of a symmetric laminated FG sandwich beam was written as that of laminated composite beam by utilizing this theory as follow (Demir *et al.* 2013a)

$$M = \frac{2b}{3r} \sum_{j=1}^{m/2} E_j (z_j^3 - z_{j-1}^3) \quad (1)$$

where  $M$ ,  $b$ ,  $r$ ,  $m$  and  $E_j$  were bending moment, width, radius of curvature, number of layers and elasticity modulus of  $j$ th layer of the sandwich beam, respectively. As for the  $z_j$ , it was the distance between the outer surface of the  $j$ th layer and the neutral plane as shown in Fig. 1. The most commonly known form of the bending moment was also given as follow

$$M = \frac{E_{ef} I_{yy}}{r} \quad (2)$$

where  $E_{ef}$  and  $I_{yy}$  were the effective elasticity modulus and the cross-sectional inertia moment about the neutral axis of the sandwich beam, respectively. Effective elasticity modulus was obtained by substituting Eq. (1) into Eq. (2) as follow

$$E_{ef} = \frac{8}{h^3} \sum_{i=1}^{m/2} E_j (z_j^3 - z_{j-1}^3) \quad (3)$$

Furthermore, the effective mass density of the sandwich beam was written in the form similar to Eq. (3) as follow

$$\rho_{ef} = \frac{8}{h^3} \sum_{i=1}^{m/2} \rho_j (z_j^3 - z_{j-1}^3) \quad (4)$$

The usage of the effective material properties in the vibration analysis of the symmetric FG sandwich beams were discussed in the previous studies (Demir *et al.* 2013a, b).

### 5. Experimental setup and tests

Free vibration measurements were performed with

experimental setup as shown in Fig. 8.

Boundary conditions of the beam specimens were considered to be clamped-free (C-F) and the specimens were clamped vertically by a mechanical clamp as shown in the figure. Free vibration was initiated by a single external mechanical pulse. The vibration signals were acquired by an accelerometer (3035A1G, Dytran Instruments, USA). The accelerometer was attached very close to the clamped end of the beam specimen to reduce the influence of the weight of the accelerometer (2.3 g). The signals acquired were amplified using a single-channel current source power unit (4102C, Dytran Instruments, USA). Afterward, the signals were collected, digitalized and sent to a computer via a data acquisition card (NI USB 6008, National Instruments, USA). A Matlab code (MathWorks, USA) was developed to convert the signal from the time domain to the frequency domain by using Fast Fourier Transform Method.

In the test, 17 different beams with different mixing ratio were fabricated and used. The first 9 of the 17 beams were the variations of model 1. The next 4 were the variations of Model 2 and the next 4 were the variations of Model 3. As mentioned earlier, the material contents of these beams were given in Table 1. Moreover, in order to obtain accurate data, 5 specimens were used for each of the 17 beam in the test and the test was repeated 3 times for each test specimens. The arithmetic mean of the 3 values obtained for each specimen was taken as the final value. An example of frequency domain response of a specimen (specimen code: SB1-3) was shown in Fig. 9. The peak value of the curve corresponds to the natural frequency of the specimen.

The damping ratios of the beams were also calculated from the peak values of the free vibration curves in the time domain. Details of this calculation were described in the next section.

## 6. Calculating the damping ratios

The damping test was also repeated three times for each specimen as vibration test. Final results were found from the arithmetic mean of the values obtained from each tests.



Fig. 8 Experimental setup

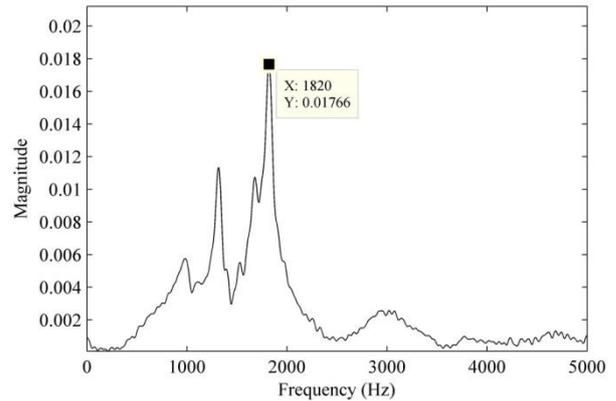


Fig. 9 Frequency domain response of the specimen SB1-3

The damping ratios of the beam specimens were obtained from the time history graphs of the free vibrations of the specimens. The decay time of the free vibration was short because of the length and stiffness of the beam specimens. Therefore two methods were used to evaluate and verify the data obtained from damping tests, i.e., Curve Fitting Technique and Logarithmic Decrement Method. In the first method, the envelope curves of free vibration response were obtained by considering seven peak values from the upper half of the time history graph. The exponential equations of the envelope curves were acquired by using the curve fitting technique with Curve Fitting Module of MATLAB (MathWorks, USA). The well-known equation of the time-dependent displacement for damped vibrations was written as follows (Krodkiewski 2008)

$$x(t) = C e^{-\xi \omega_n t} \sin(\omega_d t + \phi) \quad (5)$$

where  $C$ ,  $\xi$ ,  $t$ ,  $\phi$  were the amplitude, the damping ratio, the time and the phase shift, respectively. The phase shift was calculated from the initial conditions.  $\omega_d$  was damped natural frequency and it was equal to  $\omega_n \sqrt{1 - \xi^2}$ . The undamped natural frequency was represented by  $\omega_n$ . Because the damping rate is quite small, the damped and undamped natural frequencies were almost the same. Therefore, the exponentially decay term  $-\xi \omega_n$  was obtained from the exponential equations of the envelope curve.

In the second method, logarithmic decrement of damping was used to determine the damping ratio. The method was based on the principle of taking the natural logarithm of the ratio between successive peak values. It was defined as follows (Krodkiewski 2008)

$$\delta = \ln \frac{x(t)}{x(t+T_d)} = \ln \frac{C e^{-\xi \omega_n t} \sin(\omega_d t + \phi)}{C e^{-\xi \omega_n (t+T_d)} \sin(\omega_d (t+T_d) + \phi)} = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \quad (6)$$

and the damping ratio  $\xi$  was defined as,

$$\xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \quad (7)$$

In the study, seven successive peak values were considered in order to achieve more accurate results. As a

result of the operations, six damping ratios were obtained. The damping ratio of beam specimen was obtained by taking the arithmetic mean of the six values.

## 7. Numerical analysis

The numerical results obtained using from a finite element based commercial program SolidWorks (SolidWorks Corp., USA) were compared with the experimental results. This program was capable of both modelling and vibration analysis. The beam specimens were modelled in the Part Drawing Interface of SolidWorks. After three dimensional modelling process was completed, the Frequency Simulation module of the program was run. The material properties of the each beam model were

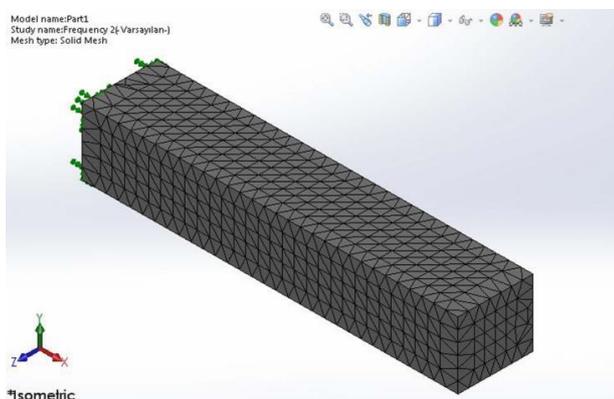


Fig. 10 Finite element model of FG beam after meshing

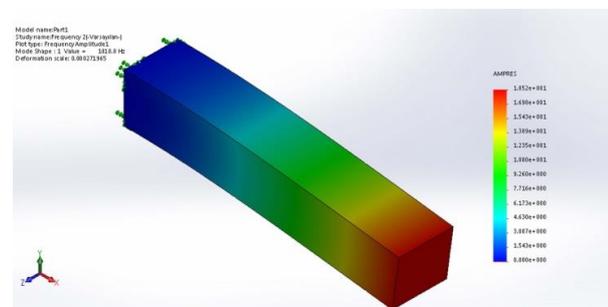


Fig. 11 Finite element model of FG beam after solution

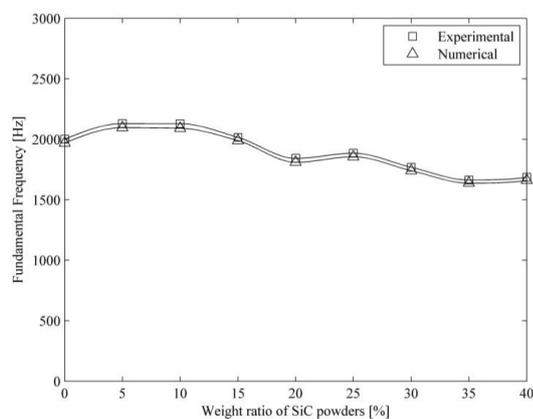


Fig. 12 Free vibration response of the single layered beam (Model no:1)

calculated from the formulas (Eqs. (3)-(4)) and added to the material library of the SolidWorks according to Table 1 and they were assigned to the each model. Then, the C-F boundary conditions were applied to the models. Finally, the models were meshed and the analysis program was run. The total node numbers and elements of FEM model were 10145 and 6450, respectively. The beam middle-meshed and the latest state of the beam (specimen code: SB1-3) after analysis were shown in Figs. 10 and 11, respectively. The data obtained from SolidWorks were compared with those from experiments and the results were found to be acceptable.

## 8. Results and discussion

In order to find vibration and the damping characteristics of the single layered and the symmetric 6-layered FG sandwich beams, experimental and numerical studies were done in this study. In the vibration tests, 5 different specimens were used for each type of the beams whose codes were given in Table 1 and the each test was repeated three times for each sample. Firstly, the vibration tests of single layered homogenous beams were performed. According to the test results obtained the variation of the fundamental frequencies by changing the weight ratio of SiC powders were shown in Fig. 12.

It was seen from Fig. 12 that the fundamental frequency of the single layered beam reached the highest value when the weight ratio of SiC powders was in the range of 5-10%. It was also seen that the fundamental frequency decreased gradually after the value 10%. Elasticity moduli of the single layered beams were obtained from the values of the fundamental frequencies. So, referring to Fig. 6, the curve characteristic was very similar to the one given in Fig. 12. It was concluded here that up to 10% increase in the ratio of SiC powder caused the improvement of the single layered beam rigidity. However, further increase in the ratio of SiC powder caused the rigidity reduction. It was estimated that more than 10% increase in the ratio of SiC powder led to notch effect.

As for the mass density of the single layered beam, it was seen in Fig. 7 that it was decreased gradually by increasing the ratio of the SiC powder. It was also estimated that increase in the ratio of SiC powder caused the occurrence of more gaps in the whole structure. Because the particle size of SiC powder was larger than that of Al.

The free vibration response of the multilayered symmetric sandwich beams were given in Figs. 13 and 14. As mentioned before, the difference between the two models is the location of pure Al layer.

The middle layers of the beam shown in Fig. 13 were composed of pure Al. However, the upper and lower layers of the beam shown in Fig. 14 were composed of pure Al. Other layers were composed of different mixtures of SiC and Al (see Table 1).

As it was seen in all two figures, the fundamental frequencies were decreased by increasing in the mixture ratio of SiC powder as expected. Because, as previously shown in Fig. 12 that increase in the ratio of SiC powder caused a decrease in the natural frequency. The highest

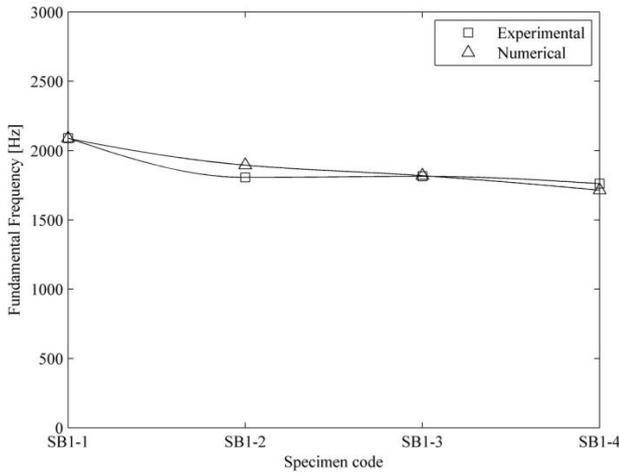


Fig. 13 Free vibration response of the multilayered sandwich beam (Model no:2)

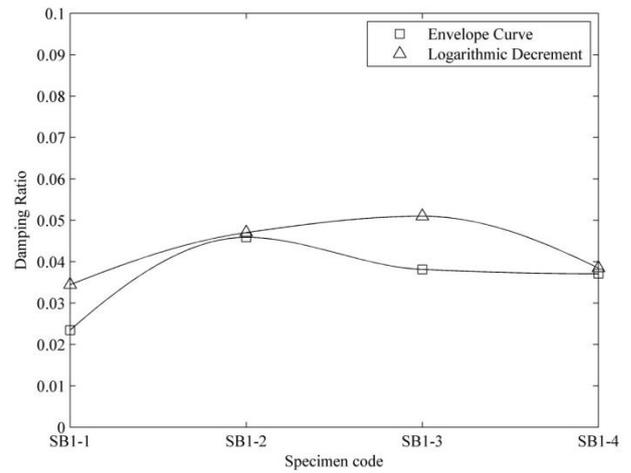


Fig. 16 Variation of damping ratio of the multilayered sandwich beam (Model no:2)

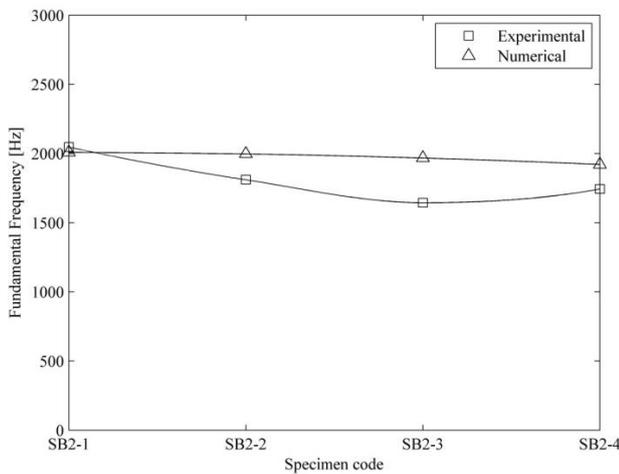


Fig. 14 Free vibration response of the multilayered sandwich beam (Model no:3)

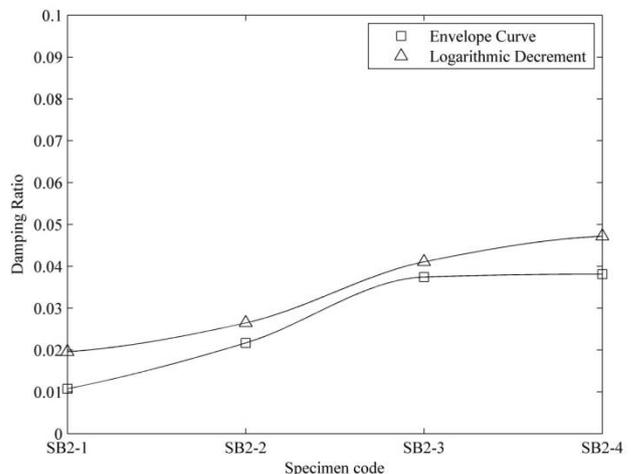


Fig. 17 Variation of damping ratio of the multilayered sandwich beam (Model no:3)

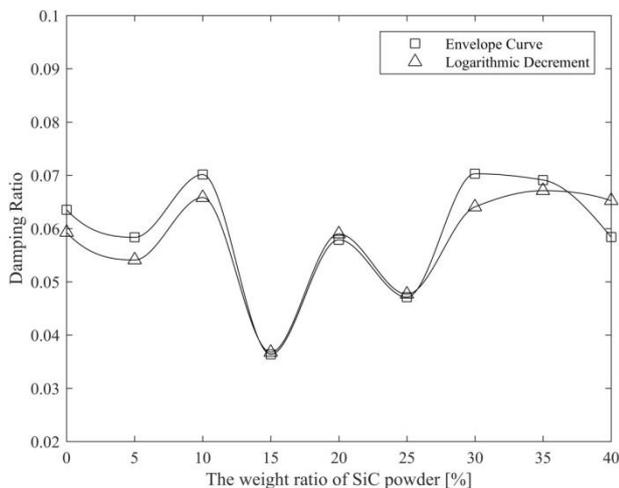


Fig. 15 Variation of damping ratio of the single layered beam (Model no:1)

natural frequencies were obtained in SB1-1 and SB2-1 curve in each graph was almost the same according to the experimental data. It was also observed that the results

obtained from tests were agreed with the results obtained from Finite Element Analysis.

Figs. 15, 16 and 17 were related to damping rates. The variation in the damping ratio of the single layered beam due to the increase in the weight ratio of SiC was shown in Fig. 15.

When the figure was analyzed, it was observed that the damping ratio was decreased suddenly in the case where the weight ratio of SiC was 15% and 25%. The maximum value of damping ratio was obtained at the value which the weight ratio of SiC was 10% and 30%. Besides, it was also seen that the damping ratio varied between about 0.035 and 0.070. When the graphic examined generally, the curve decreased to the weight ratio of 15% and then increased except for 10%.

The variation the damping ratio of the multilayered symmetric sandwich beams that pure Al layers are in the middle of the beam was shown in Fig. 16.

It was observed from the figure that the damping rate was increased first and then the decreased by increasing the ratio of SiC powder in the layers. It was also seen from the figure that the SB1-3 specimen has the highest damping

ratio according to the Envelope Curve Technique. Actually, difference between the two methods was small, as can be seen from the Fig. 16.

In contrast to the curves in Fig 16, the curves in Fig. 17 were increased gradually. Difference between the two methods was also small in here. The highest damping ratio was obtained in SB2-4 specimen, as shown in above figure.

## 9. Conclusions

It was obtained following results according to especially experiment results.

- The natural frequency of the single layered beam increased up to the weight ratio of SiC powders was 10%. Then, it decreased gradually. So, the weight ratio of SiC powders should not exceed 10% of the mixture.
- Model 2 should be preferred when designing the layered sandwich FG beams. Because the highest natural frequency obtained by Model 2 was higher than that obtained by Model 3. Moreover, the ratio of the reduction in the natural frequency obtained by using Model 2 was less than the other.
- Model 2 should be also preferred if desired high damping ratio. The damping ratios of Model2 specimens was higher than those of Model 3 specimens.
- It was also observed that the average damping ratios of the multilayered sandwich beams were higher than those of the single layered beams.

## Acknowledgments

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