Structural Engineering and Mechanics, *Vol. 62, No. 5 (2017) 537-550*
DOI: https://doi.org/10.12989/sem.2017.62.5.537

Observer-Teacher-Learner-Based Optimization: An enhanced meta-heuristic for structural sizing design

Mohsen Shahrouzi*, Mahdi Aghabaglou[a] and Fataneh Rafiee[a]

Department of Engineering, Kharazmi University, 43 Shahid-Mofatteh, Tehran, Iran

(Received December 4, 2015, Revised September 10, 2016, Accepted April 11, 2017)

Abstract. Structural sizing is a rewarding task due to its non-convex constrained nature in the design space. In order to provide both global exploration and proper search refinement, a hybrid method is developed here based on outstanding features of *Evolutionary Computing* and *Teaching-Learning-Based Optimization*. The new method introduces an observer phase for memory exploitation in addition to vector-sum movements in the original teacher and learner phases. Proper integer coding is suited and applied for structural size optimization together with a fly-to-boundary technique and an elitism strategy. Performance of the proposed method is further evaluated treating a number of truss examples compared with teaching-learning-based optimization. The results show enhanced capability of the method in efficient and stable convergence toward the optimum and effective capturing of high quality solutions in discrete structural sizing problems.

Keywords: discrete optimization; constrained structural sizing, hybrid evolutionary computing

1. Introduction

As a widely interested field, structural sizing for weight minimization have been studied by several researchers (Kaveh 2014, Yang 2010, Arora 2004). It is usually addressed by non-linearity and narrowness of the feasible region in its non-convex design space (Shahrouzi and Pashaei 2013). In many applications, structural sections are limited to a practical list and should be assessed by discrete variables. Such requirements make the optimization procedure more complicated than continuous search. Meta-heuristic algorithms are interested solutions for such problems since they can search the design space by sampling it without using any gradient information or approximation of discrete variables (Kaveh and Mahdavi 2014).

The majority of meta-heuristic algorithms are bio-inspired while the others mimic some physical, chemical or natural phenomena (Fister *et al*. 2013). Performance of such population-based methods, depends on fine tuning their control parameters to achieve proper balance between exploration and exploitation (Crepinsek 2013). Therefore the less the number of control parameters, the less tuning effort is required to achieve the best performance of the corresponding algorithm.

Teaching-Learning-Based Optimization, TLBO, is a meta-heuristic introduced by Rao *et al*. (2011) with the least common parameters. It is based on stochastic movement in several directions guided by different strategies via the teacher phase and the learner phase. In the teacher-phase the search agents are focused toward the global-best solution up to the current iteration while in the learner phase social incorporation of agents is highlighted by a fitness-based competition.

Application of TLBO as a meta-heuristic optimizer is being extended in various engineering problems. Rao *et al*. (2012) studied optimization of large-scale nonlinear problems by TLBO. Togan (2012) employed this method for optimal design of planar frames. Degertekin and Hayalioglu (2013), applied TLBO to size optimization of truss structures. Makiabadi *et al*. (2013) extended its application to sizing of truss bridges. Baghlani et al. (2013) used TLBO for geometry and size optimization of truss structures with eignevalue constraints.

A number of TLBO variants have already been developed by several investigators. Rao and Patel (2012) introduced an elitist TLBO that employs a ranked portion of population by additional elite-size parameter and some extra effort for duplicate elimination in the algorithm. They applied Deb's heuristic tournament strategy (Deb 2000) to handle the limitations in the constrained problems. It requires more fitness evaluations than their unconstrained TLBO. Rajasekhar *et al*. (2012), offered an opposition-based strategy to enhance elitist TLBO. Rao and Patel (2013) improved the basic TLBO by introducing the concept of number of teachers, adaptive teaching factor, tutorial training and self motivated learning. Crepinsek *et al*. (2012) analyzed performance of some TLBO variants in a number of test functions. Pholdee and Bureerat (2014) reported that TLBO falls within medium-to-high rank among 24 meta-heuristics in structural weight minimization under frequency constraints. Vecek *et al*. (2014) reported similarities between some variants of *Differential Evolution*, DE and TLBO via a novel ranking mechanism. More rigorous survey on TLBO variants and applications is given

*Corresponding author, Ph.D.
E-mail: shahruzi@khu.ac.ir
[a]M.Sc. Student

http://www.techno-press.com/journals/sem&subpage=7
ISSN: 1225-4568 (Print), 1598-6217 (Online)

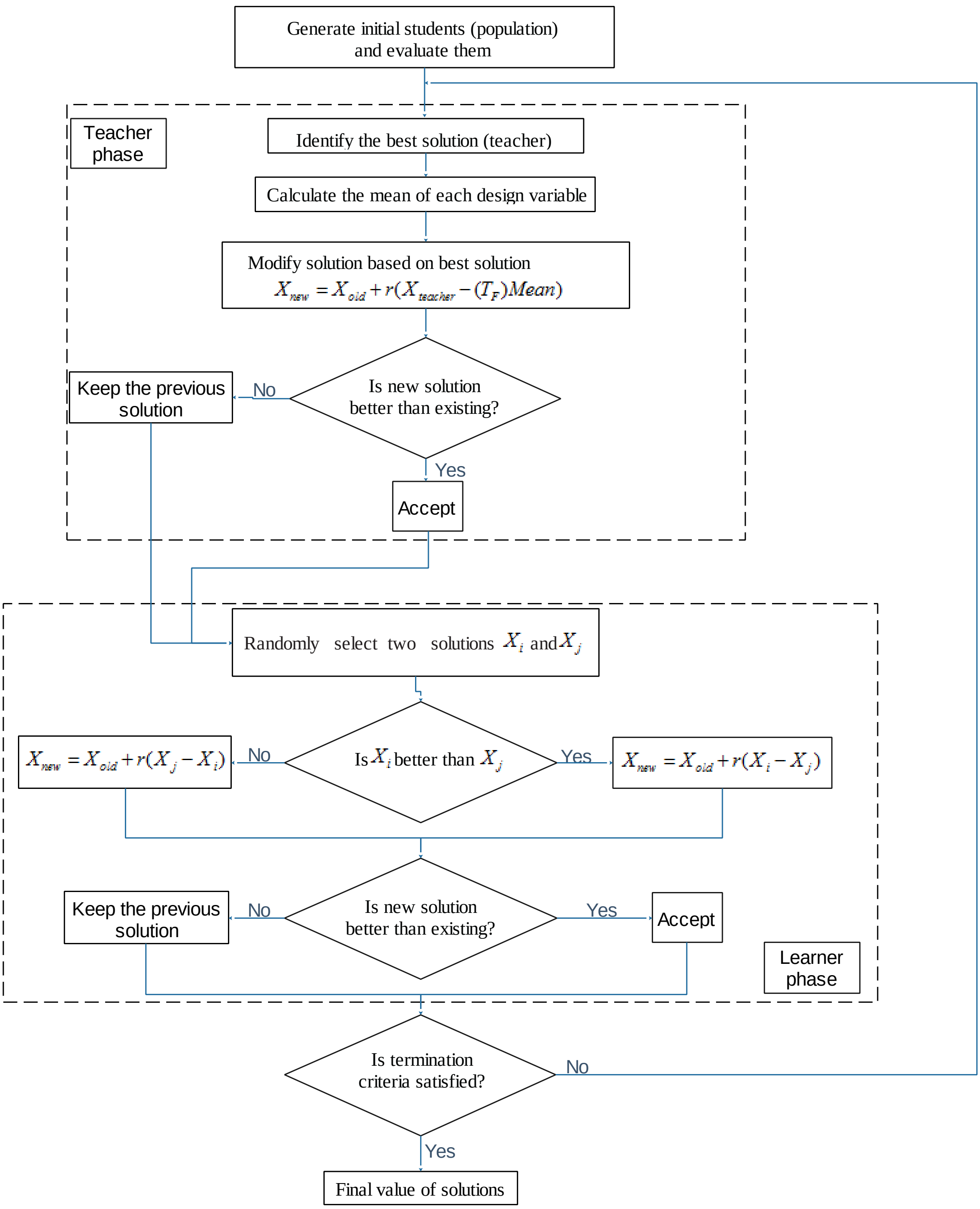


Fig. 1 Flowchart of TLBO

by Rao (2016).

The present work improves TLBO by introducing a new search agent embedded to the teaching phase. It is mainly based on memory exploitation by stochastic information exchanging between the artificial classmates. The method is utilized for a number of benchmark examples in structural sizing for weight minimization under behavioral and side constraints. For efficient constraint handling of discrete sizing problems; proper techniques are distinctly suited to treat side/behavioral constraints. Performance improvement of TLBO via the proposed strategy is evaluated treating a number of benchmark examples. Further comparison of the results with the related literature works is also provided to declare suitability of the proposed OTLBO for structural sizing problems.

2. Overview of Teaching-Learning-Based Optimization

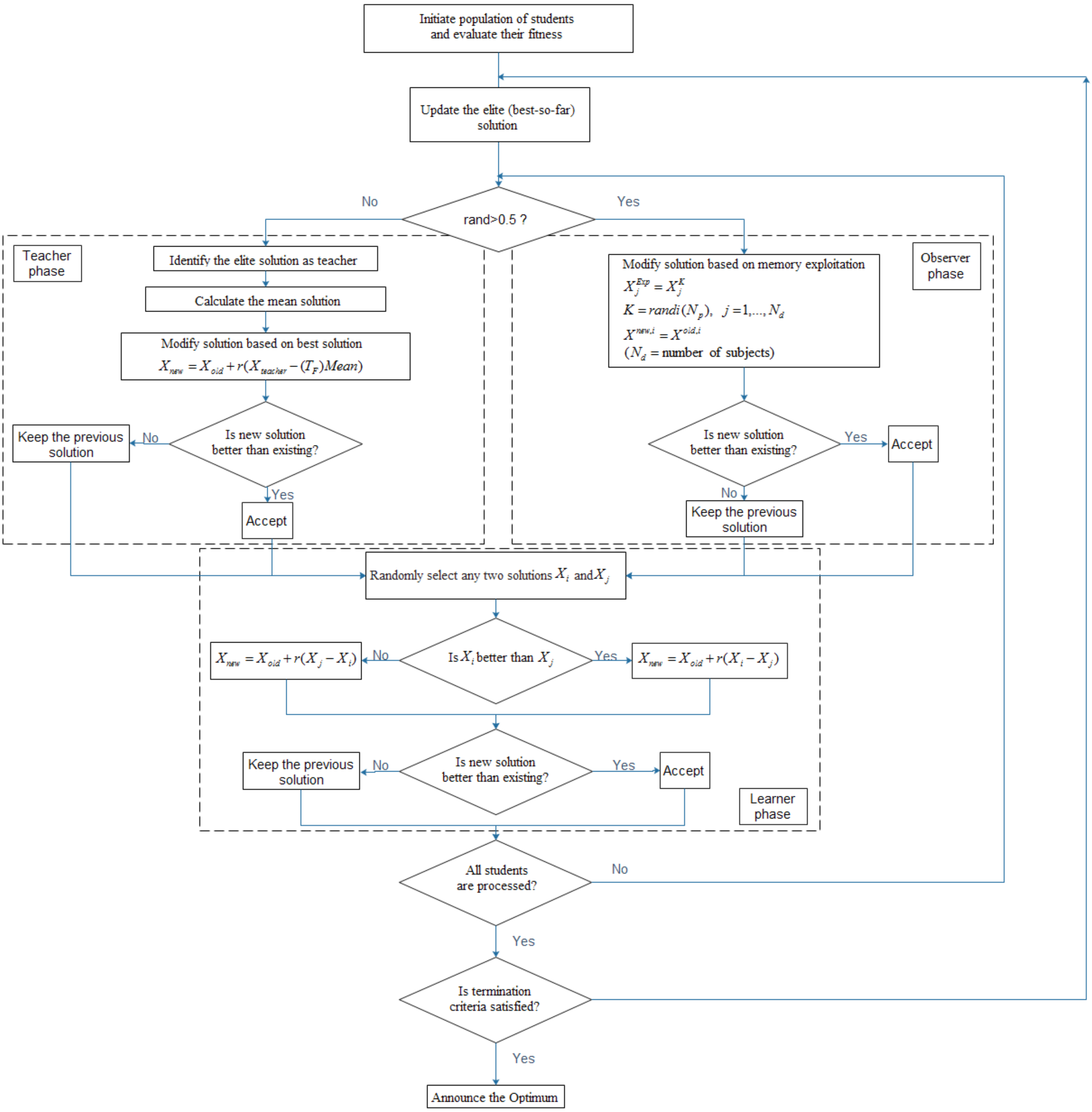


Fig. 2 Flowchart of the proposed OTLBO

Teaching-Learning-Based Optimization, TLBO is first introduced by Rao *et al*. (2011). It is a population-based method that simulates the knowledge growth process in a classroom by two distinct phases; the first is improving the mean level of the students' grades by a teacher while the second mimics learning via interaction between the students themselves. Detailed representation of TLBO variants is addressed by Rao (2016). Fig. 1 shows flowchart of the employed basic TLBO.

It is indicated that TLBO applies movement vector V from the current position of each individual X to its new position $X+rand\times V$. According to the teacher phase, any student is forced to move in the same direction like that the class mean moves toward the best found position; i.e., $V=X^{Teacher}-T_f X^{Mean}$. The teacher vector $X^{Teacher}$ can be the fittest solution of the current population or the elite solution up to the current generation (Rao and Patel 2012). The second strategy is applied in the current study by saving and updating an elite solution in an auxiliary memory next to the current population. It is somehow like movement toward the global-best position as an essential part of *Particle Swarm Optimization* (Kennedy and Eberhart 2001) provided that an amplification factor, T_f applies in TLBO that randomly switches between 1 and 2.

In the learner phase of TLBO, the movement vector of each current X^i is constructed parallel to the line connecting it to another randomly selected member of population X^j. However, based on weather X^i is fitter than X^j or not, it is decided that X^i moves toward X^j or backwards. Some other meta-heuristics like *Differential Evolution* constitutes similar movement steps to explore the search space (Vecek *et al*. 2014).

3. Observer-Teacher-Learner-Based Optimization

Both of teacher and learner phases in TLBO take advantageous of vector-sum movements. An extra way of constructing and guiding new solutions is introduced hereinafter via a new algorithm called *Observer-Teacher-Learner-Based Optimization*, OTLBO. It is based on exploitation of the current memory; i.e., the class of students in TLBO terminology. In this process, classmates' information; subject by subject is randomly taken from various students to generate a new solution called the observer. In the absence of the teacher in the class, such an observer takes its place to guide the other students. Here, the presence time is equally shared between the teacher and the observer. As this strategy introduces an extra way of search space decomposition; it is expected to improve the stochastic search capability of the algorithm (Shahrouzi 2011).

An elitist strategy is also proposed to avoid loss of best-so-far solutions via iterations of the search procedure. In the first generation, the fittest solution of the population is saved as the elite solution. It is then updated via comparison with the fittest solution of any new generation. The proposed OTLBO is described via the following algorithmic steps:

3.1 OTLBO algorithm

Step 1. Initiate *the class:* Set the iteration number to 1. Generate the population matrix: *PopMat* of N_p classmates between their lower limit $X^L{}_j$ and upper limit $X^U{}_j$ in any j^{th} subject by

$$\begin{aligned} &X^i_j = round(X^L_j + rand \times (X^U_j - X^L_j)) \\ &i \in \{1, 2, ..., N_p\}, \quad j \in \{1, 2, ..., N_d\} \end{aligned} \tag{1}$$

N_d is the number of subjects or components of any classmate vector; X^i. *rand* is a function that generates random numbers in the range [0,1]

$$PopMat = \begin{bmatrix} x^1_1 & x^1_2 & \dots & x^1_{Nd} \\ x^2_1 & x^2_2 & \dots & x^2_{Nd} \\ & & \dots & \\ x^{Np}_1 & x^{Np}_2 & \dots & x^{Np}_{Nd} \end{bmatrix} \tag{2}$$

Step 2. Decode the classmates' vectors to the design space and evaluate their fitness.

Step 3. Elitist Update:

• In the first iteration, save the fittest classmate as the elite solution.

• In any next iteration, compare previously saved elite with the fittest of the current population and select their best as the new elite solution.

Step 4. Perform either *Teacher phase* or *Observer phase* with the same probability:

a. Teacher phase: For any i^{th} student, try to improve mean mark of the class by help of the teacher using the following relation

$$X^{new} = q(X^i + rand \times (X^{Teacher} - T_f X^{Mean})) \tag{3}$$

$X^{Teacher}$ is the current elite solution. T_f denotes a teaching factor that randomly switches to either 1 or 2. Any component X_l^{Mean} in the above relation is determined by averaging over all classmates in their j^{th} subject. q(.) as a fly-to-boundary function is further described in the next section.

b. Observer phase: For any i^{th} student, construct an observer solution, X^{Exp} by exploiting the current memory via the following relation

$$\begin{aligned} &X^{Exp}_l = X^k_l \\ &k = randi(N_P),\ l = 1, 2, ..., N_d \end{aligned} \tag{4}$$

randi(N_p) generates a random integer between 1 and N_p. The observer vector is constructed by getting any its l^{th} subject mark from the corresponding subject of the k^{th} student. At this stage, take X^{New} as X^{Exp}.

Step 5. Evaluate fitness of X^{New}. Replace the current X^i with X^{New} if it is fitter than X^i

$$X^i = X^{new} \qquad \text{if} \quad Fit(X^{new}) > Fit(X^i) \tag{5}$$

Step 6. Learner Phase

During this phase, students interact with each other to increase their scientific level via these steps:

• Randomly select a pair of students with different numbers; *i* and $j \neq i$

• Update X^{New} by the following relations

$$\begin{aligned} &X^{new} = q(X^i + rand \times (X^j - X^i)) \qquad \text{if} \quad Fit(X^j) > Fit(X^i) \\ &X^{new} = q(X^i + rand \times (X^i - X^j)) \qquad otherwise \end{aligned} \tag{6}$$

• Evaluate fitness of X^{New}. Replace the current X^i with X^{New} if it is fitter than X^i in the same way as in Eq. (5)

Step 7. Repeat steps (4) to (6) for all classmates

Step 8. If iteration number has not reached N_I increase it by 1 and return to step (3)

Step 9. Decode and announce the final $X^{Teacher}$ as the optimal design. Flowchart of the proposed OTLBO is demonstrated in Fig. 2.

3.2 The fly-to-boundary function for side constraints

Design variables are usually limited to upper and lower bounds. The fly-to-boundary function q(.) is utilized here to avoid such out-of-bound infeasibilities. Therefore, q(.) is applied by the following relation

$$q(x_j) = \max(\ \min(\text{round}(x_j),\ x^U_j),\ \ x^L_j) \tag{7}$$

It preserves that any design vector component takes the nearest integer number between X_j^L and X_j^U as the lower and upper bounds, respectively. Due to application of vector-sum movements undesired non-discrete values may be generated during optimization. Therefore, a rounding function is included in $q(.)$ to correct such a value into the nearest discrete value.

In addition, the behavioral constraints should be specifically defined for the problem in hand. In the present study, they are described via the problem formulation.

3.3 Direct index coding

During the structural optimization, it is necessary to evaluate nodal displacements and member stresses by structural analyses to evaluate the corresponding behavioral constraints. Prior to that, the design vector should be decoded into the entire structural model.

Direct Index Coding, DIC, is shown to be very efficient for discrete structural problems (Kaveh and Shahrouzi 2005). In the present work, a DIC is utilized so that any j^{th} design variable is assigned an integer index between $N_j^{Sections}$ and 1. $N_j^{Sections}$ denotes the number of available cross sections to be selected for the j^{th} group of structural members.

Definition of the function $q(.)$ in Eq. (7) is extended so that no zero or non-integer value can be returned by it as a section index. Hence, all the lower bounds x_j^L are set to 1 for such an encoding scheme in the structural sizing problem.

Some variants of TLBO apply extra duplicate elimination in the algorithm (Crepinsek *et al*. 2012) which results in uncontrolled function calls via iterations of the search. However in the present work, a basic form of TLBO is utilized so that the number of fitness evaluations is exactly twice the product of the population size by the iteration number plus one population size for the initiation. It is worth mentioning that the proposed OTLBO applies the same number of fitness calls as TLBO; leading to true iteration-wise comparison.

4. Problem formulation for discrete structural weight minimization

Structural weight minimization is concerned here when member peroperties can only be chosen from a discrete set of practical profiles. In addition to variable bounds, there are several behavioral constraints that should be evaluated via structural analyses. It is a cumbersome numerical task, specially for large structures when no analytical method can be directly employed. In the other hand, the available structural profiles are practically limited to a specific discrete list which generally makes this discrete optimization more complicated than a continuous problem.

It is desired that all stress and deflection regulations are satisfied at the optimal design. For a given truss structure with N_m members that are grouped into N_d sizing variables, the present optimization problem is formulated as follows

$$\begin{aligned} &\textit{Minimize} \quad w(X)=\rho\sum_{j=1}^{Nd}\sum_{k=1}^{Nm(j)} l_k A_k \\ &\textit{Subject to} \\ &\qquad x_i \in \{x_i^{LB},\ldots,x_i^{UB}\}, \quad i=1,\ldots,N_d \\ &\qquad g_c(X)\le 0\ , \qquad c=1,\ldots,N_c \end{aligned} \tag{8}$$

Where ρ stands for material weight per unit volume and l_k is length of the k^{th} member in the j^{th} group. A_k denotes its cross-sectional area which is determined after decoding the design vector: $X=\{x_1,\ldots,x_{Nd}\}$.

Due to the prescribed DIC, the design vector contains integer section indices from the lower bound to the upper bound in order to be chosen for any j^{th} group with $N_m(j)$ structural members.

Side constraints on the number of available sections are satisfied by the function $q(.)$. Behavioral constraints for the resulting stress, deflection and stability measures are defined as follows

$$g_\sigma^k(X)=\frac{\sigma_k}{(\sigma_k)_{allowable}}-1\le 0,\quad k=1,\ldots,N_m \tag{9}$$

$$g_d^n(X)=\frac{d_{n,h}}{(d_{n,\mathrm{h}})_{allowable}}-1\le 0,\quad n=1,\ldots,N_n \tag{10}$$

$$g_\lambda^k(X)=\frac{\lambda_k}{(\lambda_k)_{allowable}}-1\le 0,\quad k=1,\ldots,N_m \tag{11}$$

$d_{n,h}$ is the n^{th} node's displacement at the h^{th} degree of freedom. σ_k and λ_k denote the combined stress for the k^{th} member and its slenderness ratio, respectively. N_m is the number of members while N_n stands for the number of nodes.

For implementation of meta-heuristic search, the aforementioned constrianed formulation is changed to the unconstrained form by the following penalty function

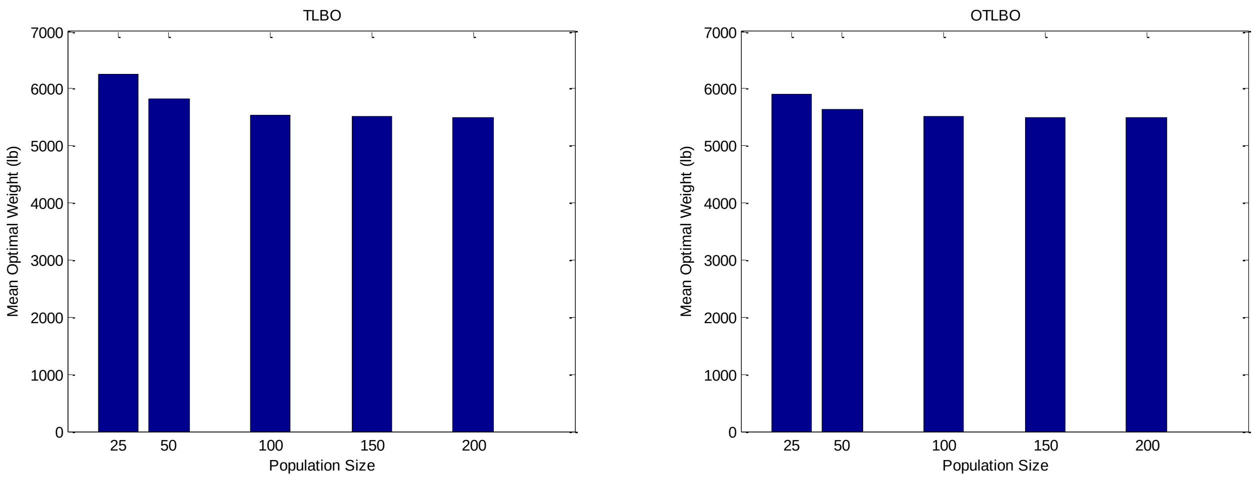
$$\textit{Maximize } Fit(X)=-w(X)\times(1+\kappa_p\sum_{c=1}^{N_C}\max(0,g_c(\underline{X}))\ ) \tag{12}$$

in which κ_p stands for the prescribed penalty factor and w is the strucural weight as a raw cost function.

5. Numerical simulation

Structural optimization has been a classic rewarding task for investigators since early 1910's due to its wide application in the real world problems. Relatively narrow feasible region with respect to the entire search space as well as non-analytical derivation of objective function has led it to be classified in the non-convex complex optimization category. It is dealt here with OTLBO and TLBO and consequent statistical results are derived in each example using several independent runs.

In order to better study behavior of the algorithms a diversity measure is utilized. In this article the following



(a) (b)

Fig. 4 Mean optimal weight vs. population size for (a) TLBO and (b) OTLBO in the 10-bar truss design

Table 1 Optimal design comparison for the 10-bar truss

Element Group	Section Variable	Optimal cross section area (in²)					Present Work	
		Li *et al.* 2009	Kripka 2004	Camp *et al.* 2004	Turkkan 2003	Sonmez 2011	TLBO	OTLBO
1	A1	30.00	33.50	33.50	33.50	33.50	33.50	33.50
2	A2	1.62	1.62	1.62	1.62	1.62	1.62	1.62
3	A3	22.90	22.90	22.90	22.90	22.90	22.90	22.90
4	A4	13.50	14.20	14.20	14.20	14.20	14.20	14.20
5	A5	1.62	1.62	1.62	1.62	1.62	1.62	1.62
6	A6	1.62	1.62	1.62	1.62	1.62	1.62	1.62
7	A7	7.97	7.97	7.94	7.97	7.97	7.97	7.97
8	A8	26.50	22.90	22.90	22.90	22.90	22.90	22.9
9	A9	22.00	22.00	22.00	22.00	22.00	22.00	22.00
10	A10	1.80	1.62	1.62	1.62	1.62	1.62	1.62
Best (lb)		5531.98	5490.74	5490.74	5490.74	5490.74	5490.74	5490.74
Average (lb)		—	—	—	—	5510.35	5515.63	5501.49
Worst (lb)		—	—	—	—	5734.38	5734.29	5545.79
Standard deviation (lb)		—	—	—	—	—	49.25	17.67

definition of diversity index, DI, is applied as given by Kaveh and Zolghadr (2014)

$$DI(t) = \frac{1}{N_p}\sum_{i=1}^{N_p} norm\left(\frac{X^{Elite}(t) - X^{i}(t)}{X^{U} - X^{L}}\right) \tag{13}$$

where $X^{Elite}(t)$ denotes the elitist individual up to the corresponding iteration, t and N_p is the population size.

As another issue, CPU time consumption is compared between TLBO and OTLBO provided that the same platform is used in each example. For this purpose, a time ratio is defined and calculated as

$$TR = \frac{Elapsed\ time\ by\ OTLBO}{Elapsed\ time\ by\ \ TLBO} \tag{14}$$

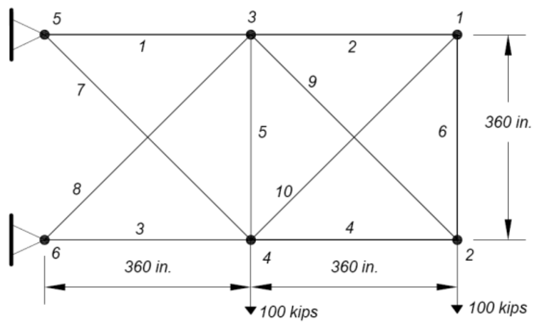
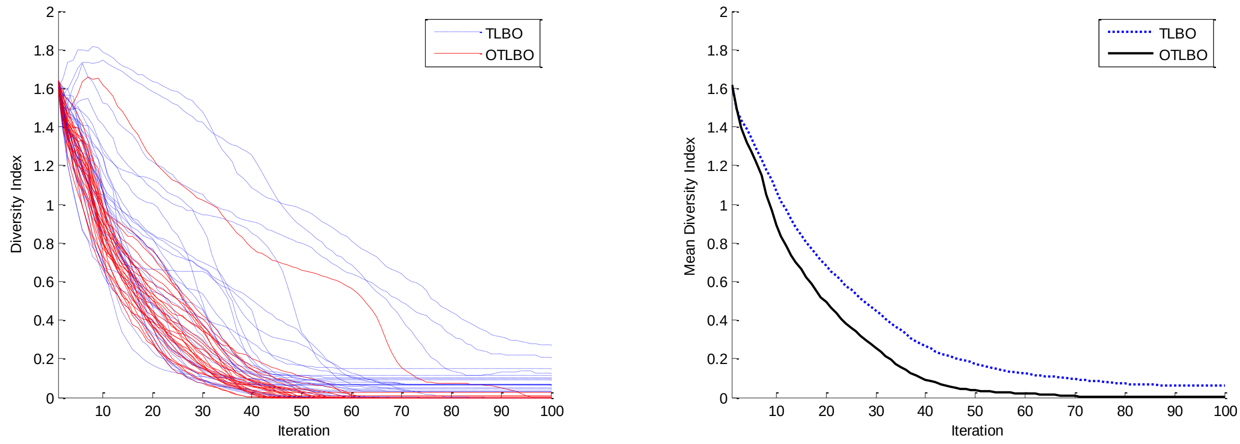


Fig. 3 The 10-bar truss

Every example is solved with both TLBO and the proposed OTLBO. For the sake of true comparison in each



(a) (b)

Fig. 5 DI in (a) independent runs of TLBO and OTLBO, (b) its mean for the 10-bar truss design

of the independent runs, identical initial population is employed by both the methods. Consequently, the convergence histories are drawn and statistical measures are derived. The results are further compared with those reported in literature for the corresponding problems.

5.1 The 10-bar planar truss design

The cantilever truss of Fig. 3 is considered as the first example with 10 design variables.

Several research works have already addressed optimal design of this benchmark using FPGA (Turkkan 2003), SA (Kripka 2004), ACO (Camp and Bichon 2004), HPSO (Li *et al.* 2009) and ABC (Sonmez 2011).

Design parameters include modulus of elasticity, $E = 68.971 \times 10^3 \text{MPa}$, density $\rho = 2768 \text{ kg/m}^3$, allowable stress, $F_y = \pm 172.25 \text{ MPa}$ and allowable displacements, $\pm 50.8 \text{ mm}$. A set of 42 discrete values of cross section areas are available including: {10.45, 11.61, 12.84, 13.74, 15.35, 16.90, 16.97, 18.58, 18.90, 19.93, 20.19, 21.81, 22.39, 22.90, 23.42, 24.77, 24.97, 25.03, 26.97, 27.23, 28.97, 29.61, 30.97, 32.06, 33.03, 37.03, 46.58, 51.42, 74.19, 87.10, 89.68, 91.61, 100, 103.23, 109.03, 121.29, 128.39, 141.93, 147.74, 170.97, 193.55, 216.13} (cm^2) or {1.62, 1.80, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80, 4.97, 5.12, 5.74, 7.22, 7.97, 11.5, 13.5, 13.9, 14.2, 15.5, 16.0, 16.9, 18.8, 19.9, 22.0, 22.9, 26.5, 30.0, 33.5} (in^2). After a number of trials, the penalty factor κ_p is adjusted to 30 in order to insure feasibility of final designs.

As both TLBO and OTLBO are parameterless algorithms; they are majorly controlled by the population size; N_p. In this regard a sensitivity analysis is performed using 30 independent runs. Initial population of both TLBO and OTLBO are taken identical at each run but different from the other run while the maximum iteration number is fixed to 100. It is observed that variation of mean fitness is less than 1% for the N_p of 100 and greater (Fig. 4). Consequently, a population size of 150 is selected to obtain further results in this example.

Table 1 gives the best design achieved by OTLBO, a truss weighing 5490.74 lb, which is the same as the global optimum reported by TLBO (Rao et al. 2011), FPGA (Turkkan, 2003), SA (Kripka, 2004), ACO (Camp and Bichon, 2004), HPSO (Li et al. 2009) and ABC (Sonmez, 2011). The best, worst and average results for a number of independent runs by TLBO and OTLBO are distinctly reported in Table 1. In this example that the best results are

Table 2 Available sections for the 52-bar truss design

No.	in²	No.	in²	No.	in²
1	0.111	23	2.620	45	7.970
2	0.141	24	2.630	46	8.530
3	0.196	25	2.880	47	9.300
4	0.250	26	2.930	48	10.850
5	0.307	27	3.090	49	11.500
6	0.391	28	1.130	50	13.500
7	0.442	29	3.380	51	13.900
8	0.563	30	3.470	52	14.200
9	0.602	31	3.550	53	15.500
10	0.766	32	3.630	54	16.000
11	0.785	33	3.840	55	16.900
12	0.994	34	3.870	56	18.800
13	1.000	35	3.880	57	19.900
14	1.228	36	4.180	58	22.000
15	1.266	37	4.220	59	22.900
16	1.457	38	4.490	60	24.500
17	1.563	39	4.590	61	26.500
18	1.620	40	4.800	62	28.000
19	1.800	41	4.970	63	30.000
20	1.990	42	5.120	64	33.500
21	2.130	43	5.740		
22	2.380	44	7.220		

the same, OTLBO has been superior in the mean or the worst results than ABC (Sonmez 2011) which itself has outperformed the employed TLBO.

Both TLBO and OTLBO have captured the global optimum in less than 40 iterations. However, OTLBO has obtained its results with lower standard deviation than TLBO. Numerical simulations show that OTLBO has captured the global optimum of this example by 10350 structural analyses; that is 15% lower than 12150 analyses required by TLBO.

The trace of diversity index over independent runs of TLBO and OTLBO is given in Fig. 5. It can be realized that both the methods obey from a decreasing trend of DI variation from higher values in early iterations toward lower ones near the end. However, OTLBO exhibits more stable convergence about its mean over different runs with respect to TLBO. Mean DI among iterations is lower for OTLBO than TLBO, specially in final iterations after it has found the global optimum. Such a DI trace, also confirms less standard deviation about the final solution leading to more success rate of OTLBO. That means the proposed hybridization has successfully improved TLBO features via OTLBO algorithm.

5.2 The 52-bar truss design

As the second benchmark example, the 52-bar planar truss of Fig. 6 is treated here. It has already been studied implementing GA (Wu and Chow 1995), HS (Lee and Geem 2004), HPSO (Li *et al*. 2009), DHPSACO (Kaveh and Talatahari 2009) and SDE (Kaveh and Hosseini 2014). All members have elasticity modulus of $E = 2.07 \times 10^5$ MPa, material density of $\rho = 7860\,\mathrm{kg/m^3}$ and allowable stress of $\sigma_{\max} = 180.5$ MPa. The uppermost nodes are imposed to the loads $P_x = 100$ kN and $P_y = 200$kN in *x* and *y* directions, respectively.

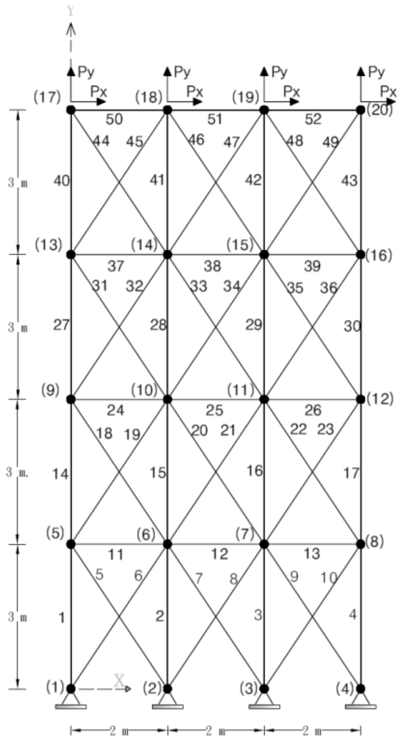


Fig. 6 The 52-bar planar truss

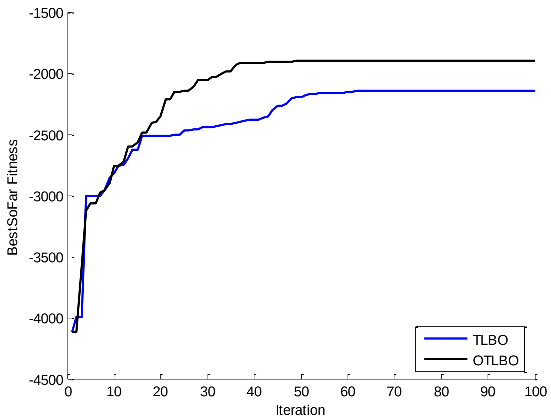


Fig. 7 Convergence trace in the 52-bar truss design

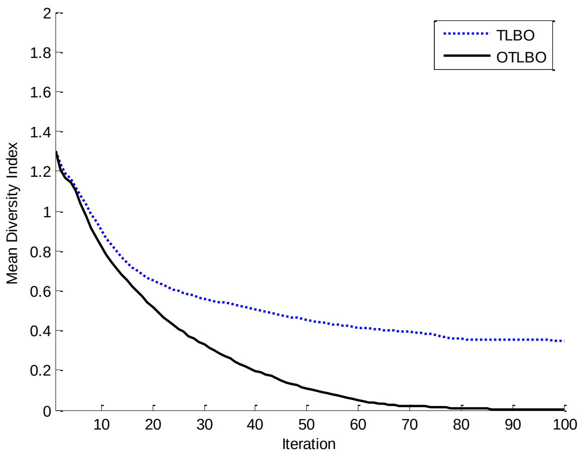


Fig. 8 Mean DI for the 52-bar truss design

Table 2 gives the available sections for optimization. The structural members are associated with 12 groups: (1) $A_1 - A_4$, (2) $A_5 - A_{10}$, (3) $A_{11} - A_{13}$, (4) $A_{14} - A_{17}$, (5) $A_{18} - A_{23}$, (6) $A_{24} - A_{26}$, (7) $A_{27} - A_{30}$, (8) $A_{31} - A_{36}$, (9) $A_{37} - A_{39}$, (10) $A_{40} - A_{43}$, (11) $A_{44} - A_{49}$, (12) $A_{50} - A_{52}$. In this example κ_p is taken 50. The population size and total number of iterations are taken the same as previous example.

Table 3 declares that OTLBO has outperformed TLBO not only in the best result but also regarding the mean and the worst designs with 40% lower standard deviation. According to Table 3, OTLBO has also captured better final design than those reported by GA, HS, HPSO, DHPSACO and SDE (Li *et al*. 2009, Wu and Chow 1995, Lee and Geem 2004, Kaveh and Talatahari 2009, Kaveh and Hosseini 2014).

According to Fig. 7, OTLBO has considerably superior convergence rate over TLBO. It is confirmed by reporting the required number of fitness evaluations to capture the optimum which is 15450 for OTLBO; i.e., 18% less than 18750 for TLBO.

Table 3 Optimal design comparison for the 52-bar planar truss

Element Group	Section Variable	Optimal cross section area (mm^2) Wu & Chow 1995	Lee & Geem 2004	Li et al. 2009	Kaveh & Talatahari 2009	Kaveh & Hosseini 2014	Present work TLBO	OTLBO
1	A1 ~ A4	4658.055	4658.055	4658.055	4658.055	4658.055	4658.055	4658.055
2	A5 ~ A10	1161.288	1161.288	1161.288	1161.288	1161.288	1161.288	1161.288
3	A11 ~ A13	645.16	506.451	363.225	494.193	363.225	388.386	494.193
4	A14 ~ A17	3303.219	3303.219	3303.219	3303.219	3303.219	3303.219	3303.219
5	A18 ~ A23	1045.159	939.998	939.998	1008.385	939.998	939.998	939.998
6	A24 ~ A26	494.193	494.193	494.193	285.161	641.289	729.031	494.193
7	A27 ~ A30	2477.414	2290.318	2238.705	2290.318	2238.705	2238.705	2238.705
8	A31 ~ A36	1045.159	1008.385	1008.385	1008.385	1008.385	1008.385	1008.385
9	A37 ~ A39	285.161	2290.318	388.386	388.386	363.225	363.225	494.193
10	A40 ~ A43	1696.771	1535.481	1283.868	1283.868	1283.868	1283.868	1283.868
11	A44 ~ A49	1045.159	1045.159	1161.288	1161.288	1161.288	1161.288	1161.288
12	A50 ~ A52	641.289	506.451	792.256	506.451	641.289	506.451	494.193
Best (kg)		1970.142	1906.760	1905.49	1904.830	1904.126	1903.092	1902.606
Average(kg)		—	—	—	—	—	1972.846	1930.662
Worst (kg)		—	—	—	—	—	2383.310	2158.994
Standard deviation (kg)		—	—	—	—	—	108.016	63.941

The overall trend of mean DI decrease in Fig.8 is similar to the previous example except that it exhibits more difference between TLBO and OTLBO as the search progresses to the final iterations. According to Fig.8 and Table 3, OTLBO has exhibited better capability of intensification than TLBO in capturing the optimum.

5.3 The 72-bar truss design

The 72-bar truss of Fig. 9 is treated here to verify performance of the algorithms in optimal design of such a spatial structure. This example is solved by Wu and Chow (1995), Lee and Geem (2004), Li *et al*. (2009). Material density is $0.1\ lb/in^3$ and the modulus of elasticity is $10000\ ksi$. The allowable tension/compression stresses are $\pm 25\ ksi$ while nodal displacement in each direction is limited to $\pm 0.25\ in$. Loading applied to the structure consists of $5\ kip$, $5\ kip$ and $-5\ kip$ in the *x*, *y* and *z* directions, respectively. Structural members are subdivided into 16 groups for sizing: (1)$A_1 - A_4$, (2) $A_5 - A_{12}$, (3) $A_{13} - A_{16}$, (4) $A_{17} - A_{18}$, (5) $A_{19} - A_{22}$, (6) $A_{23} - A_{30}$, (7) $A_{31} - A_{34}$, (8) $A_{35} - A_{36}$, (9) $A_{37} - A_{40}$, (10) $A_{41} - A_{48}$, (11) $A_{49} - A_{52}$, (12) $A_{53} - A_{54}$, (13) $A_{55} - A_{58}$, (14) $A_{59} - A_{66}$, (15) $A_{67} - A_{70}$, (16) $A_{71} - A_{72}$. List of available cross section areas is given as: D={0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.2, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 3.0, 3.1, 3.2}$(in.^2)$.

The propsed algorithms in this example are run with a population of 100 classmates for 100 iterations where κ_p is taken 30. According to Table 4, the reported best result of GA, HS, HPSO, DHPSACO and SDE varies from

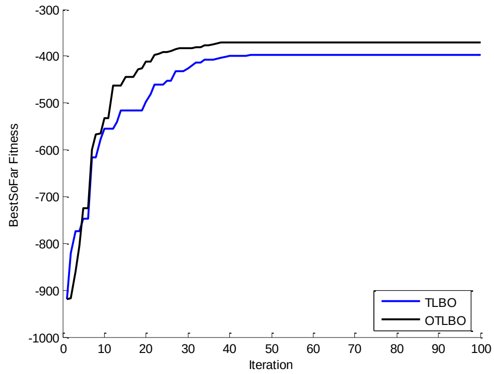
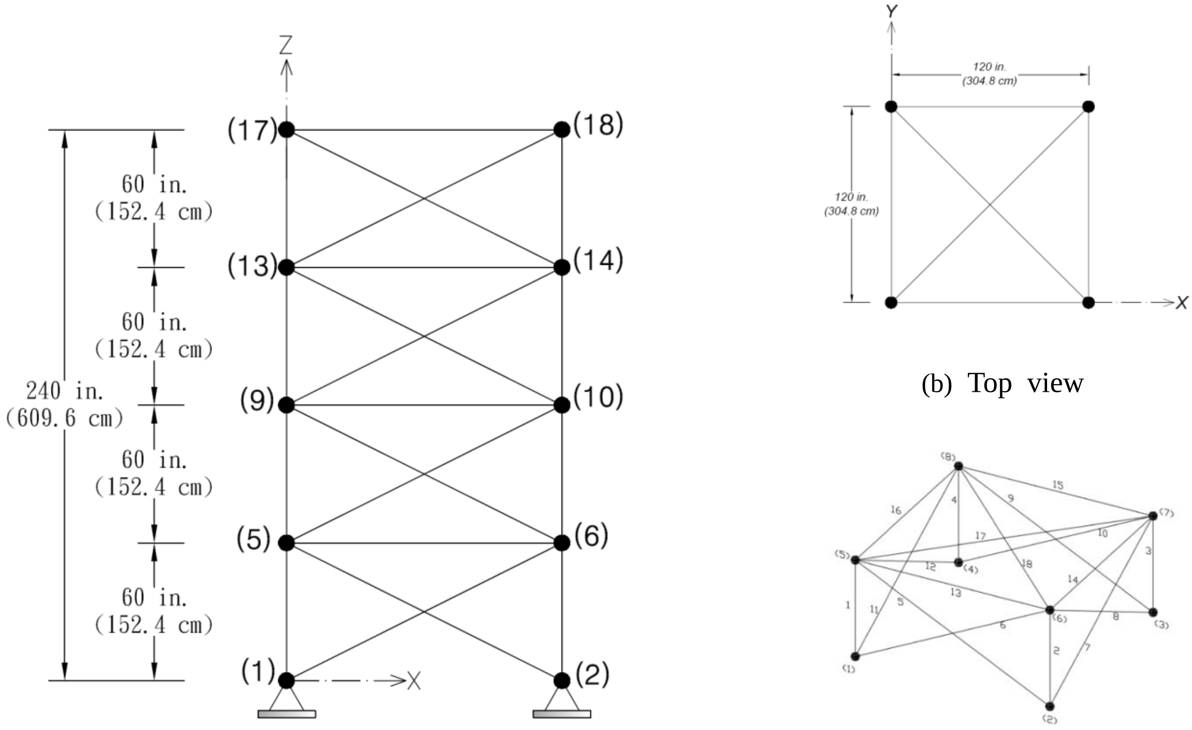


Fig. 10 Convergence trace in the 72-bar truss design

$400.66\ lb$ to $385.54 lb$ while it is obtained $371.95\ lb$ by TLBO and $370.52\ lb$ by OTLBO. Convergence superiority of OTLBO over TLBO is also evident from Fig.10. OTLBO has captured it via 7500 fitness evaluations that is nearly 23% lower than 9700 analyses taken by TLBO up to its last fitness improvement.

Statistical study reveals average results of $371.65\ lb$ vs $378.98\ lb$ and the worst results of $373.91\ lb$ vs $396.24\ lb$ for OTLBO vs TLBO, respectively. It is notable that OTLBO has found these results with a standard deviation of $1.14\ lb$ which is 80% lower than $5.65\ lb$ by TLBO.

Analyzing DI variation in Fig.11, OTLBO shows more stable trend of diversity decrease than TLBO among different optimization runs. Such a dynamic balance between diversification and intensification has resulted in higher quality of final solution of OTLBO, as reported in Table 4.



(a) Side view

(b) Top view

(c) 3D view of the first story

Fig. 9 The 72-bar space truss

Table 4 Optimal design comparison for the 72-bar space truss

Element Group	Section Variable	Optimal cross section area in²(cm²) Wu & Chow 1995	Lee & Geem 2004	Li *et al*. 2009	Kaveh & Talatahari 2009	Kaveh & Hosseini 2014	Present work TLBO	Present work OTLBO
1	A1 ~ A4	1.5	1.9	2.1	1.9	2.0	1.8(11.62)	1.9(12.26)
2	A5 ~ A12	0.7	0.5	0.6	0.5	0.5	0.5(3.23)	0.5(3.23)
3	A13 ~ A16	0.1	0.1	0.1	0.1	0.1	0.1(0.65)	0.1(0.65)
4	A17 ~ A18	0.1	0.1	0.1	0.1	0.1	0.1(0.65)	0.1(0.65)
5	A19 ~ A22	1.3	1.4	1.4	1.3	1.3	1.5(9.68)	1.3(8.39)
6	A23 ~ A30	0.5	0.6	0.5	0.5	0.5	0.5(3.23)	0.5(3.23)
7	A31 ~ A34	0.2	0.1	0.1	0.1	0.1	0.1(0.65)	0.1(0.65)
8	A35 ~ A36	0.1	0.1	0.1	0.1	0.1	0.1(0.65)	0.1(0.65)
9	A37 ~ A40	0.5	0.6	0.5	0.6	0.5	0.6(3.87)	0.5(3.23)
10	A41 ~ A48	0.5	0.5	0.5	0.5	0.5	0.5(3.23)	0.5(3.23)
11	A49 ~ A52	0.1	0.1	0.1	0.1	0.1	0.1(0.65)	0.1(0.65)
12	A53 ~ A54	0.2	0.1	0.1	0.1	0.1	0.1(0.65)	0.1(0.65)
13	A55 ~ A58	0.2	0.2	0.2	0.2	0.2	0.1(0.65)	0.1(0.65)
14	A59 ~ A66	0.5	0.5	0.5	0.6	0.6	0.5(3.23)	0.5(3.23)
15	A67 ~ A70	0.5	0.4	0.3	0.4	0.4	0.4(2.58)	0.4(2.58)
16	A71 ~ A72	0.7	0.6	0.7	0.6	0.6	0.5(3.23)	0.6(3.87)
Best (lb)		400.66	387.94	388.94	385.54	385.54	371.95	370.52
Average (lb)		—	—	—	—	—	378.98	371.65
Worst (lb)		—	—	—	—	—	396.24	373.91
Standard deviation (lb)		—	—	—	—	—	5.65	1.14

Table 5 Optimal design results for the 582-bar tower design

Element Group	Hasançebi *et al*. 2009		Kaveh & Talatahari 2009		Kaveh & Mahdavi 2014		TLBO		OTLBO	
	Section	Area (cm^2)	Section	Area (cm^2)	Section	Area (cm^2)	Section	Area (cm^2)	Section	Area (cm^2)
1	W8X21	39.74	W8X24	45.68	W8X21	39.74	W8X21	39.74	W8X21	39.74
2	W12X79	149.68	W12X72	136.13	W12X79	149.68	W21X83	156.77	W12X72	136.13
3	W8X24	45.68	W8X28	53.16	W8X28	53.23	W8X28	53.23	W8X28	53.23
4	W10X60	113.55	W12X58	109.68	W10X60	90.96	W12X65	123.23	W21X62	118.06
5	W8X24	45.68	W8X24	45.68	W8X24	45.68	W8X24	45.68	W8X24	45.68
6	W8X21	39.74	W8X24	45.68	W8X21	39.74	W8X21	39.74	W8X21	39.74
7	W8X48	90.97	W10X49	92.90	W10X68	128.38	W8X48	90.97	W10X49	92.90
8	W8X24	45.68	W8X24	45.68	W8X24	45.68	W8X24	45.68	W8X24	45.68
9	W8X21	39.74	W8X24	45.68	W8X21	39.74	W8X21	39.74	W8X21	39.74
10	W10X45	85.81	W12X40	75.48	W14X48	90.96	W8X48	90.97	W8X48	90.97
11	W8X24	45.68	W12X30	56.71	W12X26	49.35	W8X21	39.74	W8X21	39.74
12	W10X68	129.03	W12X72	136.13	W21X62	118.06	W14X74	140.64	W12X72	136.13
13	W14X74	140.65	W18X76	143.87	W18X76	143.87	W12X58	109.68	W24X76	144.52
14	W8X48	90.97	W10X49	92.90	W12X53	100.64	W10X49	92.90	W10X49	92.90
15	W18X76	143.87	W14X82	155.48	W14X61	115.48	W14X82	155.48	W12X79	149.68
16	W8X31	55.90	W8X31	58.84	W8X40	75.48	W10X39	74.19	W8X31	58.9
17	W8X21	39.74	W14X61	115.48	W10X54	101.93	W14X74	140.64	W21X62	118.06
18	W16X67	127.10	W8X24	45.68	W12X26	49.35	W8X28	53.23	W8X24	45.68
19	W8X24	45.68	W8X21	39.74	W8X21	39.74	W8X21	39.74	W8X21	39.74
20	W8X21	39.74	W12X40	75.48	W14X43	81.29	W12X50	94.84	W18X46	87.10
21	W8X40	75.48	W8X24	45.68	W8X24	45.68	W10X22	41.87	W8X21	39.74
22	W8X24	45.68	W14X22	41.87	W8X21	39.74	W8X21	39.74	W8X21	39.74
23	W8X21	39.74	W8X31	58.84	W10X22	41.87	W10X30	57.03	W6X25	47.35
24	W10X22	41.87	W8X28	53.23	W8X24	45.68	W8X21	39.74	W8X21	39.74
25	W8X24	45.68	W8X21	39.74	W8X21	39.74	W8X21	39.74	W8X21	39.74
26	W8X21	39.74	W8X21	39.74	W8X21	39.74	W8X21	39.74	W8X21	39.74
27	W8X21	39.74	W8X24	45.68	W8X24	45.68	W8X21	39.74	W8X21	39.74
28	W8X24	45.68	W8X28	53.23	W8X21	39.74	W8X21	39.74	W8X21	39.74
29	W8X21	39.74	W16X36	68.39	W8X21	39.74	W8X21	39.74	W8X21	39.74
30	W8X21	39.74	W8X24	45.68	W6X25	47.35	W8X21	39.74	W8X21	39.74
31	W8X24	45.68	W8X21	39.74	W10X33	62.64	W14X26	49.61	W8X21	39.74
32	W8X24	45.68	W8X24	45.68	W8X28	53.22	W8X21	39.74	W8X21	39.74
Best (m^3)	22.3958		22.0607		21.8376		21.5155		20.9835	
Mean (m^3)	-----		-----		-----		21.9798		21.2646	
Worst (m^3)	-----		-----		-----		22.5555		21.5845	
St. Dev.(m^3)	-----		-----		-----		0.3385		0.2172	

5.4 The 582-bar truss design

As an example of a large-scale problem a 582-bar truss of Fig. 12 (80 m tower) is considered. This optimization problem has already been solved with discrete variables by Hasançebi *et al*. (2009), Kaveh and Talatahari (2009) and Kaveh and Mahdavi (2014).To keep symmetry of the tower around *x*- and *y*-axes its members are considered in 32 independent groups for sizing. A single load case consisting of 5 kN lateral forces in both *x* and *y* directions and vertical forces of 30 kN in the downward z-direction is applied at every node of the tower. The tower is optimized for minimum volume while member cross-sections are to be selected from a list of AISC W-sections based on area and radii of gyration (Hasançebi *et al*. 2009). The corresponding lower and upper bounds of section area are $39.74\ \mathrm{cm}^2$ and

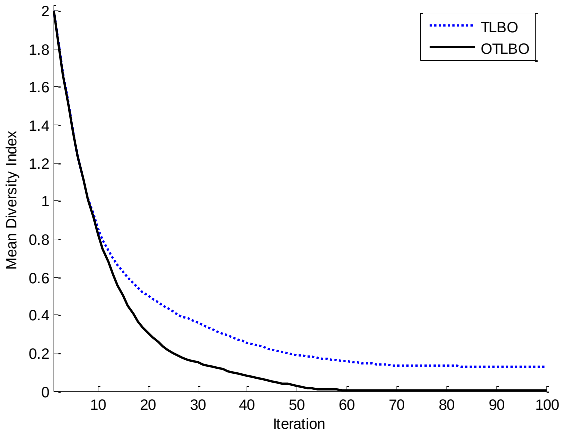


Fig. 11 Mean DI for the 72-bar truss design

1387.09 cm^2, respectively. Nodal displacements are limited to 8.0 cm (3.15 in.) in each direction.

The allowable tensile and compressive stresses are calculated due to the ASD_AISC (1989) provisions as

$$\begin{array}{lll} \sigma_i^+ = 0.6\,Fy & for & \sigma_i \geq 0 \\ \sigma_i^- & for & \sigma_i < 0 \end{array} \tag{15}$$

and σ_i^- is calculated according to the slenderness ratio

$$\begin{cases} \left[\left(1-\dfrac{\lambda_i^2}{2C_c^2}\right) Fy\right] \Big/ \left(\dfrac{5}{3}+\dfrac{3\,\lambda_i}{8\,C_c}-\dfrac{\lambda_i^3}{8\,C_c^3}\right) & for\ \lambda_i < C_c \\ \dfrac{12\,\pi^2 E}{23\,\lambda_i^2} & for \quad \lambda_i \geq C_c \end{cases} \tag{16}$$

whereas E (the modulus of elasticity) is 203893.6 MPa and Fy (the yield stress of steel) is taken 253.1 MPa. λ_i is the slenderness ratio $\left(\lambda_i = \frac{k\,L_i}{r_i}\right)$ where L_i stands for the length of members and r_i is the corresponding minimal radius of gyration. $C_c = \sqrt{2\pi^2 E/Fy}$ denotes the slenderness measure by which the elastic and inelastic buckling regions are distinguished from each other. Furthermore, the maximum slenderness ratio λ_m for tension and compression members is limited to 300 and 200, respectively.

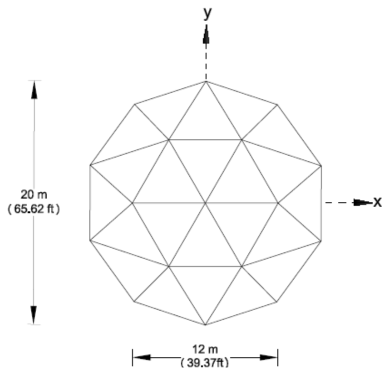
$$\begin{cases} \lambda_m \leq \ 300 \quad for\ tension\ members \\ \lambda_m \leq \ 200 \quad for\ compression\ members \end{cases} \tag{17}$$

The tower is optimized to obtain its minimum volume by a population of 120 classmates. Fig. 13 shows superior convergence history of OTLBO with respect to TLBO in this example. OTLBO has captured the global optimum by 80400 fitness evaluations which is 36% lower than 125520 by TLBO.

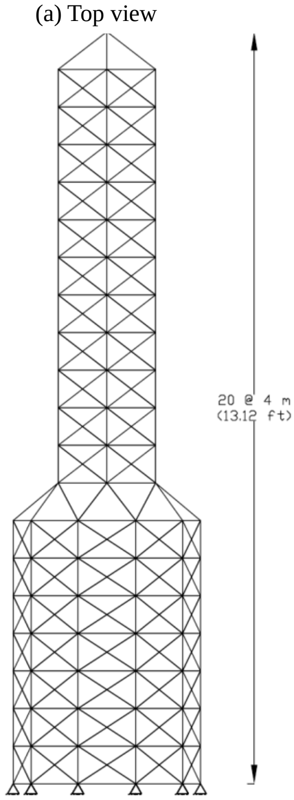
Table 5 provides the results for comparison of OTLBO with TLBO and the other literature works which have treated this problem. According to the reported results, OTLBO has revealed the best quality of final solution among the others. It has outperforms TLBO not only in the best but also in average and worst results. Besides, OTLBO has obtained its solutions with less standard deviation. It confirms more stable trend of DI variation for OTLBO with respect to TLBO, as shown in Fig. 14. The mean DI is similar for both the algorithms at early iterations but takes lower values for OTLBO as its search progresses toward the final higher quality solution.

Table 6 reports *TR* for the treated examples in two cases: first, it is calculated for the same total N_p iterations but in the second case the elapsed time of each algorithm up to its optimal-design capture, is employed. Note that after such a capture of final optimum, no fitness improvement occurs for the corresponding algorithm.

It can be realized that variation of the total elapsed time within N_p iterations, is negligible between OTLBO and TLBO. However, according to the second row of Table 6 the time required for the capture of final optimum has been considerably lower for the proposed OTLBO specially for larger-scale problems.



(a) Top view



(b) Side View

Fig. 12 The 582-bar tower truss

Table 6 CPU time comparison between OTLBO and TLBO

TR	10-bar Truss	52-bar Truss	72-bar Truss	582-bar Truss
Up to the last iteration	0.970	1.005	0.991	1.013
Up to the first capture	0.826	0.828	0.766	0.649

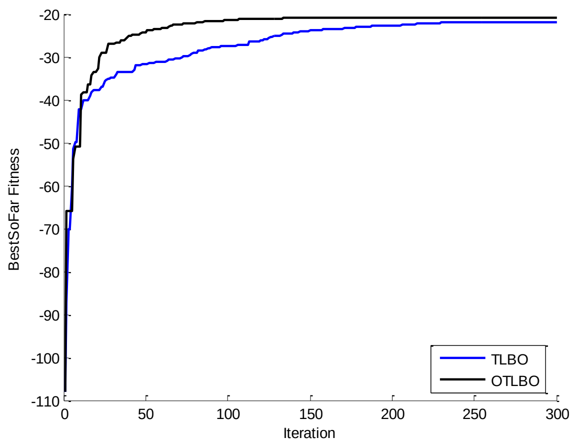


Fig. 13 Convergence trace in the 582-bar truss design

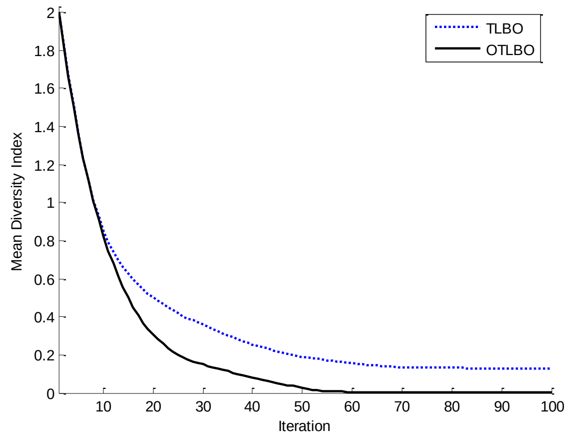


Fig. 14 Mean DI for the 582-bar truss design

6. Conclusions

In the present work, an improved variant of TLBO is developed by introducing it a new artificial agent called the observer. It is constructed by memory exploitation over the current population of classmates. In this regard, the teacher phase is split into the teacher and the observer phases without adding any extra parameters. It is declared that the standard TLBO takes benefit of vector-sum movements and does not include the proposed method of information exchanges via the observer phase. The matter provides theoretical support for expecting performance improvement of the method especially for the discrete problems when such memory exploitations are effective. It was further confirmed by numerical simulation in the present work. An elitist strategy was also employed in both TLBO and OTLBO in order not to loose the already-found elite solution during next iterations.

As a rewarding structural application, discrete sizing problem was formulated and solved by the proposed method. An integer coding is thus suited introducing a proper design vector. Performance of the utilized TLBO and OTLBO was then evaluated via a number of literature benchmarks. Consequently, OTLBO exhibited superior capability in capturing high quality solutions with respect to the other reported results in the treated examples. Besides, the proposed OTLBO revealed rapid convergence toward the optimum in less number of iterations and also less fitness evaluations than TLBO. The higher the number of structural members, the more difference in the required function calls is generally observed.

Further statistical study confirmed superior effectiveness of OTLBO over TLBO not only in the best but also in the worst and mean results. In addition, lower standard deviation of final fitness has been achieved by OTLBO that means higher succeed rate among various runs of the algorithm.

Tracing the trend of the diversity index, OTLBO is found capable of providing considerable diversification in early iterations and great intensification in the final iterations. The DI trace and the resulted standard deviation, exhibits more stable convergence by OTLBO with respect to TLBO among independent runs. The matter has been confirmed treating several truss sizing examples.

It is worth mentioning that the proposed hybridization preserves simplicity and parameter-less structure of TLBO so that the total run time remains constant in the same number of iterations. However, OTLBO requires less computational effort and function calls than TLBO to capture the same quality design. It is important for sizing problems when structural analyses constitute major part of time consumption during optimization. Besides, the computational time ratio declared superiority of the proposed method over TLBO for larger-scale structures.

Comparison of final results by the present work with those already reported in literature confirmed capability of OTLBO in capturing high quality global optima in discrete problems. Proper hybridization of evolutionary and vector-sum search operators has empowered the proposed method to overcome search refinement as a general challenge of parameter-less methods. In the light of the revealed theoretical points and numerical tests, the proposed OTLBO can be offered as a powerful optimization algorithm for structural design that combines quality of solution with computational efficiency.

References

AISC-ASD (1989), Manual of steel construction-allowable stress design, 9th ed., Chicago, IL, USA.

Arora, J.S. (2004), *Introduction to optimum design*, Elsevier, US.

Baghlani, A. and Makiabad, M. (2013), “Teaching-learning-based optimization algorithm for shape and size optimization of truss structures with dynamic frequency constraints”, *IJST Trans. Civil Eng*., **37**(C), 409-421.

Camp, C.V. and Bichon, B.J. (2004), “Design of space trusses

using ant colony optimization", *J. Struct. Eng.*, **130**(5), 741-751.
Crepinsek, M., Liu, S.H. and Mernik, M. (2012), "Note on teaching-learning-based optimization algorithm", *Info. Sci.*, **212**, 79-93.
Crepinsek, M., Liu, S.H. and Mernik, M. (2013), "Exploration and exploitation in evolutionary algorithms: A survey", *ACM Comput. Surv.*, **45**(3), Article 35.
Deb, K. (2000), "An efficient constraint handling method for genetic algorithms", *Comput. Meth. Appl. Mech. Eng.*, **186**(2), 311-338.
Degertekin, S.O. and Hayalioglu, M.S. (2013), "Sizing truss structures using teaching-learning-based optimization", *Comput. Struct.*, **19**, 177-188.
Fister, I., Yang, X.S., Fister, I., Brest, J. and Fister, D. (2013), "A brief review of nature-inspired algorithms for optimization", *Elektroteh. Vest.*, **80**(3), 1-7.
Geem, Z.W. (2009), *Music-Inspired Harmony Search Algorithm*, Springer.
Hasançebi, O., Çarbas, S., Dogan, E., Erdal, F. and Saka, M.P. (2009), "Performance evaluation of meta-heuristic search techniques in the optimum design of real size pin jointed structures", *Comput. Struct*., **87**(5-6), 284-302.
Kaveh, A. (2014), *Advances in Meta-heuristic Algorithms for Optimal Design of Structures*, Springer, Switzerland.
Kaveh, A. and Hosseini, P. (2014), "A simplified dolphin echolocation optimization method for optimum design of trusses", *Int. J. Opt. Civ. Eng.*, **4**, 381-397.
Kaveh, A. and Mahdavi, V.R. (2014), "Colliding bodies optimization: A novel meta-heuristic method", *Comput. Struct.*, **139**, 18-27.
Kaveh, A. and Shahrouzi, M. (2005), "Direct index coding for discrete sizing optimization of structures by genetic algorithms", *Int. J. Civ. Eng.*, **3-4**(3), 166-181.
Kaveh, A. and Talatahari, S. (2009), "A particle swarm ant colony optimization for truss structures with discrete variables", *J. Const. Steel Res.*, **65**(8), 1558-1568.
Kaveh, A. and Zolghadr, A. (2014), "Comparison of nine meta-heuristic algorithms for optimal design of truss structures with frequency constraints", *Adv. Eng. Soft.*, **76**, 9-30.
Kennedy, J. and Eberhart, R. (2001), *Swarm intelligence*, Academic Press, London, UK.
Kripka, M. (2004), "Discrete optimization of trusses by simulated annealing", *J. Braz. Soc. Mech. Sci. Eng.*, **26**(2), 170-173.
Lee, K.S. and Geem, Z.W. (2004), "A new structural optimization method based on the harmony search algorithm", *Comput. Struct*., **82**(9-10), 781-798.
Li, L.J., Huang, Z.B. and Liu, F. (2009), "A heuristic particle swarm optimization method for truss structures with discrete variables", *Comput. Struct*., **87**(7-8), 435-443.
Makiabad, M., Baghlani, A., Rahnema, H. and Hadianfard, M. (2013), "Optimal design of truss bridges using teaching-learning-based optimization algorithm", *Int. J. Opt. Civ. Eng.*, **3**(3), 499-510.
Pholdee, N. and Bureerat, S. (2014), "Comparative performance of metaheuristic algorithms for mass minimisation of trusses with dynamic constraints", *Adv. Eng. Soft.*, **75**, 1-13.
Shahrouzi, M. (2011), "A new hybrid genetic and swarm optimization for earthquake accelerogram scaling", *Int. J. Opt. Civ. Eng*., **1**(1), 127-140.
Shahrouzi, M. and Pashaei, M. (2013), "Pseudo-random directional search: a new heuristic for optimization", *Sci. Iranica*, **20**(4), 1124-1132.
Rao, R.V. (2016), *Teaching Learning Based Optimization algorithm and its engineering applications*, Springer, Switzerland.
Rao, R.V. and Patel, V. (2012), "An elitist teaching-learning-based optimization algorithm for solving complex constrained optimization problems", *Int. J. Indust. Eng. Comput.*, **3**(4), 535-560.
Rao, R.V. and Patel, V. (2013), "An improved teaching-learning-based optimization algorithm for solving unconstrained optimization problems", *Sci. Iranica*, **20**(3), 710-720.
Rao, R.V., Savsani, V.J. and Vakharia, D.P. (2011), "Teaching-learning-based optimization: A novel method for constrained mechanical design optimization problems", *Comput.-Aid. Des.*, **43**(3), 303-315.
Rao, R.V., Savsani, V.J. and Vakharia, D.P. (2012), "Teaching-learning-based optimization: an optimization method for continuous non-linear large scale problems", *Info. Sci.*, **183**(1), 1-15.
Rajasekhar, A., Rani, R., Ramya, K. and Abraham, A. (2012), "Elitist Teaching Learning Opposition based algorithm for global optimization", *SMC IEEE*, 1124-1129.
Sonmez, M. (2011), "Discrete optimum design of truss structures using artificial bee colony algorithm", *Struct. Multi Disc. Opt.*, **43**(1), 85-97.
Togan, V. (2012), "Design of planar steel frames using teaching-learning Based Optimization", *Eng. Struct.* **34**, 225-232.
Turkkan, N. (2003), "Discrete optimization of structures using a floating point genetic algorithm", *Ann. Conf. Can. Soc. Civ. Eng.*, Monckton, Canada.
Vecek, N., Crepinsek, M. and Mernik, M. (2014), "A chess rating system for evolutionary algorithms: A new method for the comparison and ranking of evolutionary algorithms", *Info. Sci.*, **277**, 656-679.
Wu, S.J. and Chow, P.T. (1995), "Steady-state genetic algorithms for discrete optimization of trusses", *Comput. Struct*., **56**(6), 979-991.
Yang, X.S. (2010), *Engineering optimization, an introduction with meta-heuristic application,* Cambridge University Press.

PL