Structural Engineering and Mechanics, *Vol. 62, No. 2 (2017) 143-149*
DOI: https://doi.org/10.12989/sem.2017.62.2.143

Wave propagation in functionally graded beams using various higher-order shear deformation beams theories

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(Received September 24, 2016, Revised December 16, 2016, Accepted December 22, 2016)

Abstract. In this work, various higher-order shear deformation beam theories for wave propagation in functionally graded beams are developed. The material properties of FG beam are assumed graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of the constituents, the governing equations of the wave propagation in the FG beam are derived by using the Hamilton's principle. The analytic dispersion relations of the FG beam are obtained by solving an eigenvalue problem. The effects of the volume fraction distributions on wave propagation of functionally graded beam are discussed in detail. The results carried out can be used in the ultrasonic inspection techniques and structural health monitoring.

Keywords: wave propagation; functionally graded beam; transverse shear deformation; higher-order beam theory

1. Introduction

As one of the most important developments in new advanced composite materials, functionally graded materials (FGMs) have attracted immense attention from research and engineering communities since they were first proposed by Japanese scientists (Udupa 2014). FGMs are inhomogeneous composite materials containing two or more constituents whose material composition continuously and smoothly varies along certain direction(s). Compared with conventional homogeneous materials, mechanical properties of FGMs can be appropriately tailored for design purpose (Udupa *et al*. 2014, Rafiee *et al*. 2013).

The FGM is widely used in many structural applications such as aerospace, nuclear, civil, and automotive. When the application of the FGM increases, more accurate beam theories are required to predict the response of functionally graded (FG) beams. Since the shear deformation has significant effects on the responses of functionally graded (FG) beams, shear deformation theories are used to capture such shear deformation effects. The first-order shear deformation theory (FSDBT) accounts for the shear deformation effects by the way of linear variation of in-plane displacements through the thickness. Since the FSDBT violates the conditions of zero transverse shear stresses on the top and bottom surfaces of the beam, a shear correction factor which depends on many parameters is required to compensate for the error due to a constant shear strain assumption through the thickness (Tounsi 2013a, Benzair *et al*. 2008, Heireche *et al*. 2008).

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The higher-order shear deformation theories (HSDTs) account for the shear deformation effects, and satisfy the zero transverse shear stresses on the top and bottom surfaces of the plate, thus, a shear correction factor is not required (Ould Larbi *et al*. 2013, Tounsi *et al*. 2013b, Hadji *et al*. 2016). Generally, HSDTs are proposed assuming a higher-order variations of in-plane displacements or both in-plane and transverse displacements through the thickness (Talha and Singh 2010, Reddy 2011). Bourada *et al*. (2015) studied a new simple shear and normal deformations theory for functionally graded beams. Hebali *et al*. (2015) used a new quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates. Bennoun *et al*. (2016) investigated a novel five variable refined plate theory for vibration analysis of functionally graded sandwich plates. Tounsi *et al*. (2016) studied the buckling and vibration of functionally graded sandwich plate using a new 3-unknowns non-polynomial plate theory. Houari *et al*. (2016) used a new simple three-unknown sinusoidal shear deformation theory for functionally graded plates. Belabed *et al*. (2014) investigated an efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates. Mahi *et al*. (2015) used a new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates. Hamidi *et al*. (2015) proposed a sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates. Ait Amar Meziane *et al*. (2014) used an efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions. Bousahla *et al.* (2014) investigated a novel higher order shear and normal deformation theory based on neutral surface position for

ISSN: 1225-4568 (Print), 1598-6217 (Online)

bending analysis of advanced composite plates. Bourada *et al*. (2016) studied the buckling of isotropic and orthotropic plates using a novel four variable refined plate theory. Bellifa *et al*. (2016) analyze the bending and free vibration analysis of functionally graded plates using a simple shear deformation theory and the concept the neutral surface position. Bouderba *et al.* (2013) studied the thermomechanical bending response of FGM thick plates resting on Winkler-Pasternak elastic foundations. Bouderba *et al*. (2016) analyze the thermal stability of functionally graded sandwich plates using a simple shear deformation theory. Beldjelili *et al*. (2016) studied the hygro-thermo-mechanical bending of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory. Tounsi *et al*. (2013c) used a refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates. Zidi *et al*. (2014) analyse the bending of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory. Bounouara *et al*. (2016) using a nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation. Tounsi *et al*. (2013d) studied a nonlocal effects on thermal buckling properties of double-walled carbon nanotubes. Besseghier *et al*. (2015) investigated a nonlinear vibration properties of a zigzag single-walled carbon nanotube embedded in a polymer matrix. Benguediab *et al*. (2014) studied the chirality and scale effects on mechanical buckling properties of zigzag double-walled carbon nanotubes. Larbi Chaht *et al*. (2015) analyses the bending and buckling of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect. Ahouel *et al*. (2016) investigated the size-dependent mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept. Zemri *et al*. (2015) studied the mechanical response of functionally graded nanoscale beam: an assessment of a refined nonlocal shear deformation theory beam theory. Al-Basyouni *et al*. (2015) used size dependent bending and vibration analysis of functionally graded micro beams based on modified couple stress theory and neutral surface position. Hadj *et al*. (2015) studied the influence of the porosities on the free vibration of FGM beams.

The study of the wave propagation in the FG structures has received also much attention from various researchers. Sun and Luo (2011a) also studied the wave propagation and dynamic response of rectangular functionally graded material plates with completed clamped supports under impulsive load. Considering the thermal effects and temperature-dependent material properties, Sun and Luo (2011b) investigated the wave propagation of an infinite functionally graded plate using the higher-order shear deformation plate theory. Recently, Ait Yahi *et al*. (2015) studied Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories.

Considering FG structural members, it is evident from the above discussed literature that there is no study on wave propagation in FG beams.

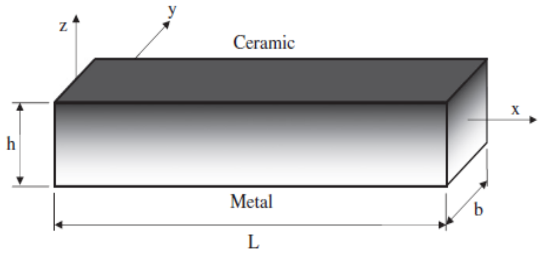

Fig. 1 Geometry and coordinate of a FG beam

Thus, the objective of this work is to investigate the wave propagation of FG beam using various simple higher-order shear deformation theories. The displacement fields of the proposed theories are chosen based on cubic, sinusoidal, hyperbolic, and exponential variation in the displacements through the thickness. By dividing the transverse displacement into the bending and shear parts and making further assumptions, the governing equations of the wave propagation in the FG beam are derived by using the Hamilton's principle. The analytic dispersion relations of the FG beam are obtained by solving an eigenvalue problem. The dispersion, phase velocity and group velocity curves of the wave propagation in FG beams are plotted. The influences of the volume fraction index on the dispersion and phase velocity of the wave propagation in the FG beam are clearly discussed.

2. Functionally graded beams

Consider a functionally graded beam with length L and rectangular cross section $b\times h$, with b being the width and h being the height as shown in Fig. 1. The beam is made of isotropic material with material properties varying smoothly in the thickness direction.

2.1 Material properties

The properties of FGM vary continuously due to the gradually changing volume fraction of the constituent materials (ceramic and metal), usually in the thickness direction only. The power-law function is commonly used to describe these variations of materials properties. The expression given below represents the profile for the volume fraction.

$$V_C = \left(\frac{z}{h} + \frac{1}{2}\right)^p \tag{1a}$$

p is a parameter that dictates material variation profile through the thickness. The value of p equal to zero represents a fully ceramic beam, whereas infinite p indicates a fully metallic beam, and for different values of p one can obtain different volume fractions of metal.

The material properties of FG beams are assumed to vary continuously through the depth of the beam by the rule of mixture (Marur 1999) as

$$P(z) = \left(P_t - P_b\right)\mathrm{V_C} + P_b \tag{1b}$$

where P denotes a generic material property like modulus, P_t and P_b denotes the property of the top and bottom faces of the beam respectively, Here, it is assumed that modules E, G and v vary according to the Eq. (1). However, for simplicity, Poisson's ratio of beam is assumed to be constant in this study for that the effect of Poisson's ratio v on deformation is much less than that of Young's modulus (Delale and Erdogan 1983, Benachour *et al*. 2011).

3. Fundamental equations

3.1 Basic assumptions and constitutive equations

The displacement fields of various shear deformation beam theories are chosen based on following assumptions:

- The axial and transverse displacements are partitioned into bending and shear components;
- The bending component of axial displacement is similar to that given by the classical beam theory (CBT);
- The shear component of axial displacement gives rise to the higher-order variation of shear strain and hence to shear stress through the thickness of the beam in such a way that shear stress vanishes on the top and bottom surfaces.

Based on these assumptions, the displacement fields of various higher-order shear deformation beam theories are given in a general form as

$$u(x,z,t)=u_0(x,t)-z\frac{\partial w_b}{\partial x}-f(z)\frac{\partial w_s}{\partial x} \tag{3a}$$

$$w(x,z,t)=w_b(x,t)+w_s(x,t) \tag{3b}$$

where u, w are displacements in the x, $z$directions, u_0 is the median surface displacements. w_b and w_s are the bending and shear components of transverse displacement, respectively; and $f(z)$ is a shape function determining the distribution of the transverse shear strain and shear stress through the thickness of the beam.

The shape functions $f(z)$ are chosen to satisfy the stress-free boundary conditions on the top and bottom surfaces of the beam, thus a shear correction factor is not required. In this study, these shape functions are chosen based on the third-order shear deformation theory (TSDBT) of Reddy (2000), sinusoidal shear deformation theory (SSDBT) of Touratier (1991), hyperbolic shear deformation theory (HSDBT) of Soldatos (1992), and exponential shear deformation theory (ESDT) of Karama *et al*. (2003), as presented in Table 1.

The strains associated with the displacements in Eq. (3) are

$$\varepsilon_x=\varepsilon_x^0+z\,k_x^b+f(z)\,k_x^s \tag{4a}$$

$$\gamma_{xz}=g(z)\,\gamma_{xz}^s \tag{4b}$$

where

$$\varepsilon_x^0=\frac{\partial u_0}{\partial x},\quad k_x^b=-\frac{\partial^2 w_b}{\partial x^2},\quad k_x^s=-\frac{\partial^2 w_s}{\partial x^2},\ \gamma_{xz}^s=\frac{\partial w_s}{\partial x} \tag{4c}$$

Table 1 Shape functions

model	$f(z)$	$g(z)=1-f'(z)$
Third beam theory (TSDBT)	$\frac{4z^3}{3h^2}$	$1-\frac{4z^2}{h^2}$
Sinusoidal plate theory (SSDBT)	$z-\frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)$	$\cos\left(\frac{\pi z}{h}\right)$
Hyperbolic beam theory (HSDBT)	$z-h\sinh\left(\frac{z}{h}\right)+z\cosh\frac{1}{2}$	$\cosh\left(\frac{z}{h}\right)-\cosh\frac{1}{2}$
Exponential plate theory (ESDBT)	$z-ze^{-2(z/h)^2}$	$\left(1-\frac{4z^2}{h^2}\right)e^{-2(z/h)^2}$

$$g(z)=1-f'(z)\ \text{ and }\ f'(z)=\frac{df(z)}{dz} \tag{4d}$$

The state of stress in the beam is given by the generalized Hooke's law as follows

$$\sigma_x=Q_{11}(z)\,\varepsilon_x\ \text{ and }\ \tau_{xz}=Q_{55}(z)\,\gamma_{xz} \tag{5a}$$

$$Q_{11}(z)=\frac{E(z)}{1-\nu^2}\ \text{ and }\ Q_{55}(z)=\frac{E(z)}{2(1+\nu)} \tag{5b}$$

3.2 Governing equations

Hamilton's principle is used herein to derive the equations of motion. The principle can be stated in analytical form as Reddy (2002)

$$\delta\int_{t_1}^{t_2}(U-T)dt=0 \tag{6}$$

where t is the time; t_1 and t_2 are the initial and end time, respectively; δU is the virtual variation of the strain energy; and δT is the virtual variation of the kinetic energy. The variation of the strain energy of the beam can be stated as

$$\begin{aligned}\delta U&=\int_0^L\int_{-\frac{h}{2}}^{\frac{h}{2}}(\sigma_x\delta\,\varepsilon_x+\tau_{xz}\delta\,\gamma_{xz})dzdx\\&=\int_0^L\left(N\frac{d\delta\,u_0}{dx}-M_b\frac{d^2\delta\,w_b}{dx^2}-M_s\frac{d^2\delta\,w_s}{dx^2}+Q\frac{d\delta\,w_s}{dx}\right)dx\end{aligned} \tag{7}$$

where N, M_b, M_s and Q are the stress resultants defined as

$$(N,M_b,M_s)=\int_{-\frac{h}{2}}^{\frac{h}{2}}(1,z,f)\,\sigma_x dz_{ns}\ \text{ and }\ Q=\int_{-\frac{h}{2}}^{\frac{h}{2}}g\,\tau_{xz}dz \tag{8}$$

The variation of the kinetic energy can be expressed as

$$\begin{aligned}\delta T&=\int_0^L\int_{-\frac{h}{2}}^{\frac{h}{2}}\rho(z)[\dot{u}\delta\,\dot{u}+\dot{w}\delta\,\dot{w}]dzdx\\&=\int_0^L\left\{I_0[\dot{u}_0\delta\dot{u}_0+(\dot{w}_b+\dot{w}_s)(\delta\dot{w}_b+\delta\dot{w}_s)]-I_1\left(\dot{u}_0\frac{d\delta\dot{w}_b}{dx}+\frac{d\dot{w}_b}{dx}\delta\,\dot{u}_0\right)\right.\end{aligned}$$

$$+I_2\left(\frac{d\dot{w}_b}{dx}\frac{d\delta\,\dot{w}_b}{dx}\right)-J_1\left(\dot{u}_0\frac{d\delta\dot{w}_s}{dx}+\frac{d\,\dot{w}_s}{dx}\delta\,\dot{u}_0\right)+K_2\left(\frac{d\dot{w}_s}{dx}\frac{d\delta\,\dot{w}_s}{dx}\right)$$
$$+J_2\left(\frac{d\dot{w}_b}{dx}\frac{d\delta\,\dot{w}_s}{dx}+\frac{d\dot{w}_s}{dx}\frac{d\delta\,\dot{w}_b}{dx}\right)\Bigg\}dx \tag{9}$$

where dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z)$ is the mass density; and (I_0, I_1, J_1, I_2, J_2, K_2) are the mass inertias defined as

$$\left(I_0,I_1,J_1,I_2,J_2,K_2\right)=\int_{-\frac{h}{2}}^{\frac{h}{2}}\left(1,z,f,z^2,zf,f^2\right)\rho(z)dz \tag{10}$$

Substituting the expressions for δU and δT from Eqs. (7) and (9) into Eq. (6) and integrating by parts versus both space and time variables, and collecting the coefficients of δu_0, δw_b, and δw_s the following equations of motion of the functionally graded beam are obtained

$$\delta\,u_0:\ \frac{dN}{dx}=I_0\ddot{u}_0-I_1\frac{d\ddot{w}_b}{dx}-J_1\frac{d\ddot{w}_s}{dx} \tag{11a}$$

$$\delta\,w_b:\frac{d^2M_b}{dx^2}=I_0(\ddot{w}_b+\ddot{w}_s)+I_1\frac{d\ddot{u}_0}{dx}-I_2\frac{d^2\ddot{w}_b}{dx^2}-J_2\frac{d^2\ddot{w}_s}{dx^2} \tag{11b}$$

$$\delta\,w_s:\ \frac{d^2M_s}{dx^2}+\frac{dQ}{dx}=I_0(\ddot{w}_b+\ddot{w}_s)+J_1\frac{d\ddot{u}_0}{dx}-J_2\frac{d^2\ddot{w}_b}{dx^2}-K_2\frac{d^2\ddot{w}_s}{dx^2} \tag{11c}$$

Eq. (11) can be expressed in terms of displacements (u_0,w_b,w_s) by using Eqs. (3), (4), (5) and (8) as follows

$$A_{11}\frac{\partial^2u_0}{\partial x^2}-B_{11}\frac{\partial^3w_b}{\partial x^3}-B_{11}^s\frac{\partial^3w_s}{\partial x^3}=I_0\ddot{u}_0-I_1\frac{d\ddot{w}_b}{dx}-J_1\frac{d\ddot{w}_s}{dx} \tag{12a}$$

$$B_{11}\frac{\partial^3u_0}{\partial x^3}-D_{11}\frac{\partial^4w_b}{\partial x^4}-D_{11}^s\frac{\partial^4w_s}{\partial x^4}=I_0(\ddot{w}_b+\ddot{w}_s)+I_1\frac{d\ddot{u}_0}{dx}$$
$$-I_2\frac{d^2\ddot{w}_b}{dx^2}-J_2\frac{d^2\ddot{w}_s}{dx^2} \tag{12b}$$

$$B_{11}^s\frac{\partial^3u_0}{\partial x^3}-D_{11}^s\frac{\partial^4w_b}{\partial x^4}-H_{11}^s\frac{\partial^4w_s}{\partial x^4}+A_{55}^s\frac{\partial^2w_s}{\partial x^2}=I_0(\ddot{w}_b+\ddot{w}_s)$$
$$+J_1\frac{d\ddot{u}_0}{dx}-J_2\frac{d^2\ddot{w}_b}{dx^2}-K_2\frac{d^2\ddot{w}_s}{dx^2} \tag{12c}$$

where A_{11}, B_{11}, D_{11}, etc., are the beam stiffness, defined by

$$\left(A_{11},B_{11},D_{11},B_{11}^s,D_{11}^s,H_{11}^s\right)=\int_{-\frac{h}{2}}^{\frac{h}{2}}Q_{11}\left(1,z,z^2,f(z),z\,f(z),f^2(z)\right)dz \tag{13a}$$

and

$$A_{55}^s=\int_{-\frac{h}{2}}^{\frac{h}{2}}Q_{55}[g(z)]^2\,dz \tag{13b}$$

4. Dispersion relations

We assume solutions for u_0, w_b and w_s representing propagating waves in the x direction with the form

$$\begin{Bmatrix}u_0(x,t)\\w_b(x,t)\\w_s(x,t)\end{Bmatrix}=\begin{Bmatrix}U\exp[i(k_1x-\omega t)]\\W_b\exp([i(k_1x-\omega t)])\\W_s\exp([i(k_1x-\omega t)])\end{Bmatrix} \tag{14}$$

where U, W_b and W_s are the coefficients of the wave amplitude, k is the wave number of wave propagation along x-axis direction, ω is the frequency.

Substituting Eq. (14) into Eq. (12), we obtain

$$\left([C]-\omega^2[M]\right)\{\Delta\}=0 \tag{15}$$

where $\{\Delta\}=\{U_m, W_b, W_m\}^T$, and $[C]$ and $[M]$ are the symmetric matrixes given by

$$[C]=\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{bmatrix},\ [M]=\begin{bmatrix}m_{11}&m_{12}&m_{13}\\m_{21}&m_{22}&m_{23}\\m_{31}&m_{32}&m_{33}\end{bmatrix} \tag{16a}$$

in which

$$\begin{aligned}a_{11}&=-A_{11}k_1^2\\a_{12}&=ik_1^3B_{11}\\a_{13}&=ik_1^3B_{11}^s\\a_{21}&=-ik_1^3B_{11}\\a_{22}&=-D_{11}k_{11}^4\\a_{23}&=-D_{11}^Sk_1^4\\a_{31}&=-ik_1^3B_{11}^s\\a_{32}&=-D_{11}^sk_1^4\\a_{33}&=-H_{11}^sk_1^4-A_{55}^sk_1^2\\m_{11}&=-I_0\\m_{12}&=iI_1k_1\\m_{13}&=iJ_1k_1\\m_{21}&=-iI_1k_1\\m_{22}&=-\left(I_0+I_2k_1^2\right)\\m_{23}&=-\left(I_0+J_2k_1^2\right)\\m_{31}&=-iJ_1k_1\\m_{32}&=m_{23}\\m_{33}&=-\left(I_0+K_2k_1^2\right)\end{aligned}, \tag{16b}$$

The dispersion relations of wave propagation in the functionally graded plate are given by

$$\left|[C]-\omega^2[M]\right|=0 \tag{17}$$

Assuming k_1=k, the roots of Eq. (17) can be expressed as

$$\omega_1=W_1(k),\quad \omega_2=W_2(k)\text{and}\quad \omega_3=W_3(k) \tag{18}$$

They correspond with the wave modes M_0, M_1 and M_2 respectively. The wave modes M_0 and M_2 correspond to the flexural wave, the wave mode M_1 correspond to the extensional wave.

The phase velocity of wave propagation in the functionally graded beam can be expressed as

$$C_i=\frac{W_i(k)}{k},\quad (i=1,2,3) \tag{19}$$

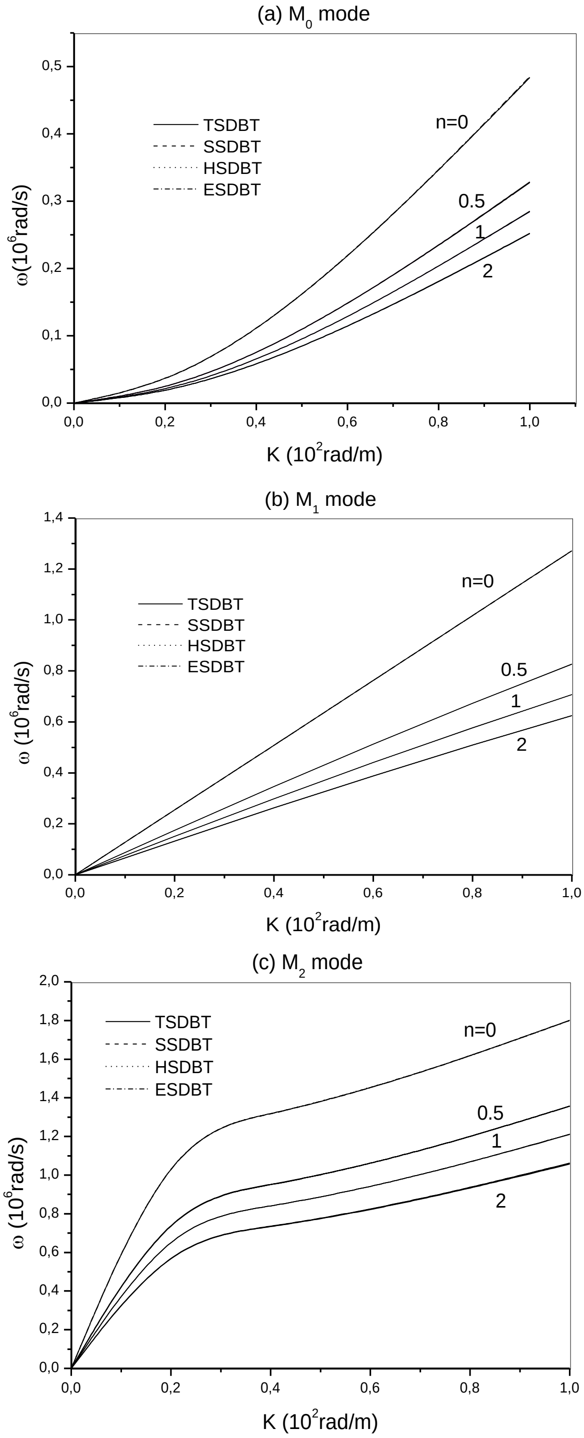

Fig. 2 The dispersion curves of the different functionally graded beams

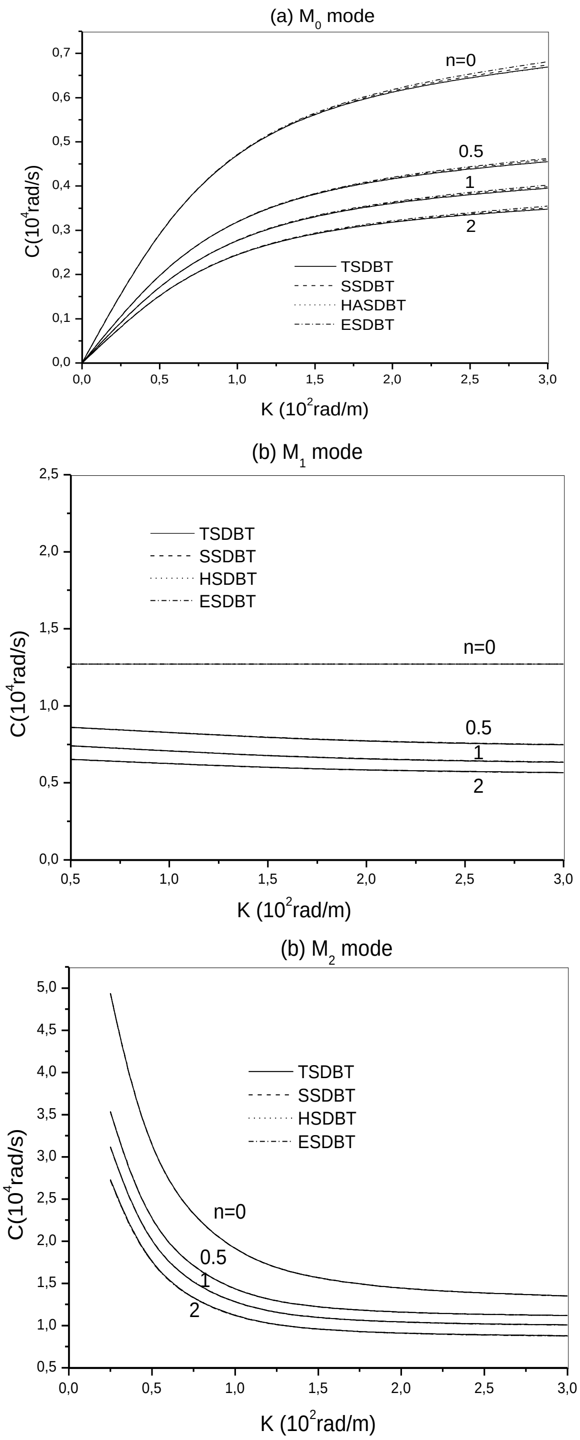

Fig. 3 The phase velocity curves of the different functionally graded beams

5. Numerical results and discussion

In this section, a FG plate made from Si3N4/SUS304; whose material properties are: E=380.43 GPa, ρ=2370 kg/m^3, ν=0.3 for Si_3N_4 and E=201.04 GPa, ρ=8166 kg/m^3, ν=0.3 for SUS_30_4; are chosen for this work. The thickness of the FG plate is 0.02 m. The analysis based on the present TSDBT, SSDBT, HSDBT, and ESDBT are carried out using MAPLE.

Fig. 2 plots the dispersion curves of the different FG beams using various shear deformation plate theories. It can be seen that the dispersion curves predicted by all proposed beam theories are almost identical to each other and this regardless the power law index n and wave modes (M_0, M_1 and M_2).

For the same k, the frequency of the wave propagation in the FG beam decreases with the increase of the power law index n whatever the wave modes. Also the frequency of the wave propagation becomes maximum in the homogeneous beam (n=0).

Fig. 3 shows the phase velocity curves of the different FG beams predicted using various shear deformation beam theories. It can be seen that the phase velocity of the wave propagation in the FG beam decreases as the power law index n increases for the same wave number k.

The phase velocity for the extensional wave mode M_1 of the beam is a constant (n=0), but it is not a constant for the beam ($\neq$0), In the case of the homogeneous beam (n=0), the phase velocity takes the maximum among those of all FG beams. Also, it can be seen that the phase velocity curves predicted by all proposed plate theories are almost identical to each other.

6. Conclusions

The wave propagation of functionally graded beam is analyzed using various higher-order shear deformation beam theories. The main advantage of the proposed theories over the existing higher-order shear deformation theories is that the present ones account for higher-order variation of transverse shear strain through the depth of the beam and satisfies the zero traction boundary conditions on the surfaces of the beam without using shear correction factors. The governing equations of the wave propagation in the FG beams are established. The analytic dispersion relation of the functionally graded beam is obtained by solving an eigenvalue problem. From the present work, it can be concluded that the influence of the volume fraction distributions on wave propagation in the FG beam is significant. An improvement of present formulation will be considered in the future work to account for the thickness stretching effect by employing quasi-3D shear deformation models (Hebali *et al*. 2014, Houari *et al*. 2014, Larbi Chaht *et al*. 2014, Swaminathan and Naveenkumar 2014, Sayyad and Ghugal 2014).

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