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Differential transform method and Adomian decomposition method for free vibration analysis of fluid conveying Timoshenko pipeline

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Abstract. The free vibration analysis of fluid conveying Timoshenko pipeline with different boundary conditions using Differential Transform Method (DTM) and Adomian Decomposition Method (ADM) has not been investigated by any of the studies in open literature so far. Natural frequencies, modes and critical fluid velocity of the pipelines on different supports are analyzed based on Timoshenko model by using DTM and ADM in this study. At first, the governing differential equations of motion of fluid conveying Timoshenko pipeline in free vibration are derived. Parameter for the nondimensionalized multiplication factor for the fluid velocity is incorporated into the equations of motion in order to investigate its effects on the natural frequencies. For solution, the terms are found directly from the analytical solution of the differential equation that describes the deformations of the cross-section according to Timoshenko beam theory. After the analytical solution, the efficient and easy mathematical techniques called DTM and ADM are used to solve the governing differential equations of the motion, respectively. The calculated natural frequencies of fluid conveying Timoshenko pipelines with various combinations of boundary conditions using DTM and ADM are tabulated in several tables and figures and are compared with the results of Analytical Method (ANM) where a very good agreement is observed. Finally, the critical fluid velocities are calculated for different boundary conditions and the first five mode shapes are presented in graphs.

Keywords: adomian decomposition method; critical fluid velocity; differential transform method; fluid conveying pipeline; free vibration; natural frequencies

1. Introduction

The vibration analysis of fluid conveying pipelines play a very important role in the engineering fields, such as, oil transportation, nuclear plant, municipal water supply, aviation, cosmonautics, etc. Therefore, natural frequencies, modes and critical fluid velocity of the pipelines became a hot topic and several studies on vibrations of fluid conveying pipelines have been reported. Chellapilla *et al*. (2007) obtained the critical velocity of a fluid flowing in a pipeline and resting on the Pasternak foundation by using Fourier series and Galerkin methods. Tong *et al*. (2007) investigated the vibration and acoustic radiation characteristics of a submerged structure using direct-boundary element method and finite elements method. In the other study, in-plane vibrations of curved pipes conveying fluid are investigated using the generalized differential quadrature rule proposed by Wang and Qiao (2008). Chang and Lee (2009) studied on free vibration of a single-walled carbon nanotube containing a fluid flow using the Timoshenko beam model. Lee *et al*. (2009) investigated the wave characteristics, divergence stability and dynamics of the oil pipelines conveying internal flow by using the spectral element method. The natural frequency of fluid-structure interaction in pipeline conveying fluid is investigated using eliminated element-Galerkin method, and the natural frequency equations with different boundary conditions are obtained by Yi-min *et al*. (2010). Kubenko *et al*. (2011) analyzed the effect of external static loading (uniform radial pressure and axial compression) on the buckling of cylindrical shells interacting with the fluid flowing inside. Ni *et al*. (2011) analyzed the free vibration problem of Euler-Bernoulli pipes conveying fluid with several typical boundary conditions by using Differential Transform Method. In this study, Euler-Bernoulli beam theory is used, but rotary inertia and transverse shear deformation are not taken into account by the authors. In the other study, the natural frequency equations of fluid-structure interaction in pipeline conveying fluid with both ends supported is investigated using a direct method, and the direct method is derived from Ferrari's method by Yi-min *et al*. (2012). Dai *et al*. (2012) studied on the vibration analysis of three-dimensional pipes conveying fluid with consideration of steady combined force by transfer matrix method. In the other paper, dynamic response of a clamped-clamped pipe conveying fluid is solved using the generalized integral transform technique by Gu *et al*. (2013). Kheiri *et al*. (2014) developed a general linear theoretical model for dynamics of a pipe conveying fluid flexibly restrained at both ends, allowing them to conveniently study the dynamics of pipes with both

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classical and non-classical boundary conditions.

DTM was applied to solve linear and non-linear initial value problems and partial differential equations by many researches. The concept of DTM was first introduced by Zhou (1986) and he used DTM to solve both linear and non-linear initial value problems in electric circuit analysis. Çatal (2006, 2008) suggested DTM for the free vibration analysis of both ends simply supported and one end fixed, the other end simply supported Timoshenko beams resting on elastic soil foundation. Çatal and Çatal (2006) calculated the critical buckling loads of partially embedded Timoshenko pile in elastic soil by DTM. Arikoglu and Ozkol (2010) investigated the vibration analysis of a three layered composite beam with a viscoelastic core by using DTM. The effects of loading on the optimal shape of an Euler-Bernoulli column was investigated by Erdönmez and Özkol (2010) considering four loading conditions which are mainly classified as eccentric compressive and using DTM. In the other studies, Yesilce investigated the free vibration analysis of moving Bernoulli-Euler and Timoshenko beams by using DTM (2010, 2013). Lal and Ahlawat (2015) investigated axisymmetric vibrations and buckling analysis of functionally graded circular plates subjected to uniform in-plane force using DTM. In the other study, nonlocal Euler-Bernoulli beam theory was employed for vibration analysis of functionally graded size-dependent nanobeams using Navier-based analytical method and a semi analytical DTM by Ebrahimi and Salari (2015). Yesilce (2015) described the determination of the natural frequencies and mode shapes of the axial-loaded Timoshenko multiple-step beam carrying a number of intermediate lumped masses and rotary inertias by using Numerical Assembly Technique and DTM. Aydin and Bozdogan (2016) applied DTM to obtain critical buckling load of multistorey structures. Bozyigit and Yesilce (2016) investigated free vibrations of moving Reddy-Bickford beams for different support conditions and velocity values using dynamic stiffness method and differential transformation.

ADM was studied to solve partial differential equations by many researches. The concept of ADM was first introduced by Adomian (1986, 1994). Hsu *et al*. (2008, 2009) investigated free vibration of non-uniform Euler-Bernoulli beams with general elastically end constraints and uniform Timoshenko beams under various supporting boundary conditions using Adomian modified decomposition method. The design of shaped piezoelectric modal sensor for beam with arbitrary boundary conditions using ADM was studied by Mao and Pietrzko (2010*a*). In the other study, Sweilam and Khader (2010) studied on approximate solutions to the nonlinear vibrations of multiwalled carbon nanotubes by using ADM. ADM was employed to investigate the free vibrations of a stepped Euler-Bernoulli beam consisting of two uniform sections by Mao and Pietrzko (2010*b*). In the other study, Mao (2011) investigated the free vibrations of the Euler-Bernoulli beams with multiple cross-section steps by using ADM. In the other study, Mao and Pietrzko (2012) studied on the free vibrations of tapered Euler-Bernoulli beams with a continuously exponential variation of width and a constant thickness along the length under various boundary conditions by using ADM. ADM was employed to investigate the free vibrations of elastically connected parallel Euler-Bernoulli beams by Mao (2012). Tapaswini and Chakraverty (2014) studied on the dynamic response of imprecisely defined beam subject to various loads using ADM. In the other study, Mao (2015) investigated the free vibration of rotating Euler-Bernoulli beams with the thickness and/or width of the cross-section vary linearly along the length by using Adomian modified decomposition method. Jamali *et al.* (2016) used domain decomposition method for buckling analysis of nanocomposite cut out plates.

Since previous studies have shown DTM and ADM to be the efficient tools and they have been applied to solve boundary value problems for many differential equations that are very important in fluid mechanics, viscoelasticity, control theory, acoustics, etc. Besides the variety of the problems to DTM and ADM may be applied, their accuracy and simplicity in calculating the natural frequencies and plotting the mode shapes makes these methods outstanding among many other methods.

The free vibration analysis of simply supported, one end fixed, the other end simply supported and fixed-fixed supported Timoshenko pipeline fluid conveying is performed in this study. At the beginning of the study, the governing equations of motion are obtained. In the next step, the equations of motion, including the parameter for the nondimensionalized multiplication factors for the fluid velocity, are solved using the efficient mathematical techniques, called DTM and ADM. The natural frequencies of Timoshenko pipelines fluid conveying and the critical fluid velocity are calculated, the first five mode shapes are plotted and the effects of the parameter for the nondimensionalized multiplication factor for the fluid velocity and the length of the pipelines are investigated. A suitable example that studies the effects of parameter for the nondimensionalized multiplication factors for the fluid velocity and the length of the pipelines on the free vibration analysis of Timoshenko pipelines fluid conveying using DTM and ADM has not been investigated by any of the studies in open literature so far.

2. The mathematical model and formulation

A uniform Timoshenko pipeline conveying fluid is presented in Fig. 1. The Timoshenko beam theory, which is taken rotary inertia and transverse shear deformation into account, is applied to analyze the flexural vibration of the fluid conveying Timoshenko pipeline. The governing equations of Timoshenko beam are two coupled differential equations expressed in terms of the flexural displacement and the angle of rotation due to bending. The coupled equations for Timoshenko pipeline conveying fluid are (Vedula 1999)

$$\frac{AG}{\bar{k}}\cdot\left(\frac{\partial^2 y(x,t)}{\partial x^2}-\frac{\partial \phi(x,t)}{\partial x}\right)-V^2\cdot m_f\cdot\frac{\partial^2 y(x,t)}{\partial x^2} \\ -m\cdot\frac{\partial^2 y(x,t)}{\partial t^2}-2\cdot V\cdot m_f\cdot\frac{\partial^2 y(x,t)}{\partial x\cdot\partial t}=0 \tag{1}$$

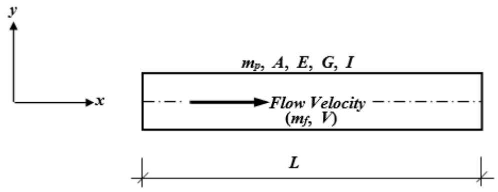


Fig. 1 Timoshenko pipeline conveying fluid

$$EI\cdot\frac{\partial\varphi^2(x,t)}{\partial x^2}-\frac{m_p\cdot I}{A}\cdot\frac{\partial\varphi^2(x,t)}{\partial t^2}+\frac{AG}{\bar{k}}\cdot\left(\frac{\partial y(x,t)}{\partial x}-\varphi(x,t)\right)=0\quad(0\le x\le L)\tag{2}$$

where $y(x,t)$ is the total transverse deflection, $\phi(x,t)$ is the angle of rotation due to bending, m_p is mass per unit length of the pipe, m_f is mass per unit length of the fluid, m is total mass per unit length ($m=m_p+m_f$), L is length of the pipe, A is the cross-section area [$A=\pi\cdot(D^2-d^2)/4$], here, D and d are the outer and inner diameter of pipe, respectively, $\bar{k}$ is the shear coefficient $\left[\bar{k}=2\cdot(1+\mu)/4+3\cdot\mu\right]$,here, μ is the Possion ratio, I is moment of inertia of the pipe [$I=\pi\cdot(D^2-d^2)/64$], V is the fluid velocity, E, G are Young's modulus and shear modulus of the pipe, respectively, x is the pipe position, t is time variable.

The bending moment function $M(x,t)$ and the shear force function $T(x,t)$ of the fluid conveying Timoshenko pipeline are written as

$$M(x,t)=EI\cdot\frac{\partial\varphi(x,t)}{\partial x}\tag{3}$$

$$T(x,t)=\frac{AG}{\bar{k}}\cdot\gamma(x,t)=\frac{AG}{\bar{k}}\cdot\left(\frac{\partial y(x,t)}{\partial x}-\varphi(x,t)\right)\tag{4}$$

where $\gamma(x,t)$ is the associated shearing deformation.

Assuming that the motion is harmonic we substitute for $y(z,t)$ and $\phi(z,t)$ the following

$$y(z,t)=y(z)\cdot e^{i\omega t}\tag{5}$$

$$\varphi(z,t)=\varphi(z)\cdot e^{i\omega t}\quad(0\le z\le 1)\tag{6}$$

where $y(z)$ and $\phi(z)$ are the amplitudes of the total transverse deflection and the angle of rotation due to bending, respectively; ω is the natural circular frequency of the vibrating system and $i=\sqrt{-1}$. Eqs. (1)-(2) can be converted into the ordinary differential equations by using Eqs. (5)-(6) as

$$\left(\frac{AG}{\bar{k}\cdot L^2}-\frac{m_f\cdot V^2}{L^2}\right)\cdot\frac{d^2y(z)}{dz^2}-\left(\frac{2\cdot m_f\cdot i\cdot\omega\cdot V}{L}\right)\cdot\frac{dy(z)}{dz}-\left(\frac{AG}{L\cdot\bar{k}}\right)\cdot\frac{d\varphi(z)}{dz}+\left(m\cdot\omega^2\right)\cdot y(z)=0\tag{7}$$

$$\left(\frac{EI}{L^2}\right)\cdot\frac{d^2\varphi(z)}{dz^2}+\left(\frac{AG}{L\cdot\bar{k}}\right)\cdot\frac{dy(z)}{dz}+\left(\frac{m_p\cdot\omega^2\cdot I}{A}-\frac{AG}{\bar{k}}\right)\cdot\varphi(z)=0\tag{8}$$

where $z=\dfrac{x}{L}$.

It is assumed that the solution is

$$y(z)=C\cdot e^{isz}\tag{9}$$

$$\varphi(z)=P\cdot e^{isz}\tag{10}$$

and substituting Eqs. (9)-(10) into Eqs. (7)-(8) results

$$\left[m\cdot\omega^2-\left(\frac{AG}{\bar{k}\cdot L^2}-\frac{m_f\cdot V^2}{L^2}\right)\cdot s^2+\left(\frac{2\cdot m_f\cdot\omega^2\cdot V}{L}\right)\cdot s\right]\cdot C-\left[\left(\frac{AG}{L\cdot\bar{k}}\right)\cdot i\cdot s\right]\cdot P=0\tag{11}$$

$$\left[\left(\frac{AG}{L\cdot\bar{k}}\right)\cdot i\cdot s\right]\cdot C+\left[\frac{m_p\cdot\omega^2\cdot I}{A}-\frac{AG}{\bar{k}}-\left(\frac{EI}{L^2}\right)\cdot s^2\right]\cdot P=0\tag{12}$$

Eqs. (11)-(12) can be written in matrix form for the two unknowns P and C as

$$\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\cdot\begin{Bmatrix}C\\ P\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix}\tag{13}$$

where

$$A_{11}=m\cdot\omega^2-\left(\frac{AG}{\bar{k}\cdot L^2}-\frac{m_f\cdot V^2}{L^2}\right)\cdot s^2+\left(\frac{2\cdot m_f\cdot\omega^2\cdot V}{L}\right)\cdot s\tag{14}$$

$$A_{21}=-A_{12}=\left(\frac{AG}{L\cdot\bar{k}}\right)\cdot i\cdot s\tag{15}$$

$$A_{22}=\frac{m_p\cdot\omega^2\cdot I}{A}-\frac{AG}{\bar{k}}-\left(\frac{EI}{L^2}\right)\cdot s^2\tag{16}$$

and the non-trivial solution will be when the determinant of the coefficient matrix will be zero. Thus, we have a fourth-order equation with the unknowns, resulting in four values and the general solution functions can be written as:

$$y(z,t)=\left[C_1\cdot e^{is_1z}+C_2\cdot e^{is_2z}+C_3\cdot e^{is_3z}+C_4\cdot e^{is_4z}\right]e^{i\omega t}\tag{17}$$

$$\varphi(z,t)=\left[P_1\cdot e^{is_1z}+P_2\cdot e^{is_2z}+P_3\cdot e^{is_3z}+P_4\cdot e^{is_4z}\right]e^{i\omega t}\tag{18}$$

The eight constants, C_1, ..., C_4 and P_1, ..., P_4 will be found from Eqs. (11), (12) and boundary conditions.

The bending moment and shear force functions of the fluid conveying Timoshenko pipeline can be obtained by using Eqs. (3)-(4) as

$$M(z,t)=\left[\left(\frac{EI}{L}\right)\cdot\frac{d\varphi(z)}{dz}\right]\cdot e^{i\omega t}\tag{19}$$

$$T(z,t)=\left[\left(\frac{AG}{\bar{k}\cdot L}\right)\cdot\frac{dy(z)}{dz}-\left(\frac{AG}{\bar{k}}\right)\cdot\varphi(z)\right]\cdot e^{i\omega t}\tag{20}$$

Table 1 DTM theorems used for equations of motion

Original Function	Transformed Function
$y(z)=u(z)\pm v(z)$	$Y(k)=U(k)\pm V(k)$
$y(z)=a\cdot u(z)$	$Y(k)=a\cdot U(k)$
$y(z)=\dfrac{d^m u(z)}{dz^m}$	$Y(k)=\dfrac{(k+m)!}{k!}\cdot U(k+m)$
$y(z)=u(z)\cdot v(z)$	$Y(k)=\sum_{r=0}^{k}U(r)\cdot V(k-r)$
$y(z)=z^m$	$Y(k)=\delta(k-m)=\begin{cases}0 & \text{if } k\neq m\\ 1 & \text{if } k=m\end{cases}$

Table 2 DTM theorems used for boundary conditions

z=0		*z*=1	
Original Boundary Conditions	Transformed Boundary Conditions	Original Boundary Conditions	Transformed Boundary Conditions
$y(0)=0$	$Y(0)=0$	$y(1)=0$	$\sum_{k=0}^{\infty}Y(k)=0$
$\dfrac{dy}{dz}(0)=0$	$Y(1)=0$	$\dfrac{dy}{dz}(1)=0$	$\sum_{k=0}^{\infty}k\cdot Y(k)=0$
$\dfrac{d^2y}{dz^2}(0)=0$	$Y(2)=0$	$\dfrac{d^2y}{dz^2}(1)=0$	$\sum_{k=0}^{\infty}k\cdot(k-1)\cdot Y(k)=0$
$\dfrac{d^3y}{dz^3}(0)=0$	$Y(3)=0$	$\dfrac{d^3y}{dz^3}(1)=0$	$\sum_{k=0}^{\infty}k\cdot(k-1)\cdot(k-2)\cdot Y(k)=0$

3. Differential Transform Method (DTM)

DTM is a semi-analytic transformation technique based on Taylor series expansion and is a useful tool to obtain analytical solutions of the differential equations. Certain transformation rules are applied and the governing differential equations and the boundary conditions of the system are transformed into a set of algebraic equations in terms of the differential transforms of the original functions in DTM. The solution of these algebraic equations gives the desired solution of the problem. High-order Taylor series method differs from DTM as Taylor series method requires symbolic computation of the necessary derivatives of the data functions and is impractical for large orders. DTM is an iterative procedure to obtain analytic Taylor series solutions of differential equations (Yesilce 2015).

A function $y(z)$, which is analytic in a domain *D*, can be represented by a power series with a center at $z=z_0$, any point in *D*. The differential transform of the function $y(z)$ is given by

$$Y(k)=\frac{1}{k!}\cdot\left(\frac{d^k y(z)}{dz^k}\right)_{z=z_0} \tag{21}$$

where $y(z)$ is the original function and $Y(k)$ is the transformed function. The inverse transformation is defined as

$$y(z)=\sum_{k=0}^{\infty}(z-z_0)^k\cdot Y(k) \tag{22}$$

From Eqs. (21)-(22) we get

$$y(z)=\sum_{k=0}^{\infty}\frac{(z-z_0)^k}{k!}\cdot\left(\frac{d^k y(z)}{dz^k}\right)_{z=z_0} \tag{23}$$

Eq. (23) implies that the concept of the differential transformation is derived from Taylor's series expansion, but the method does not evaluate the derivatives symbolically. However, relative derivative are calculated by iterative procedure that are described by the transformed equations of the original functions. In real applications, the function $y(z)$ in Eq. (22) is expressed by a finite series and can be written as

$$y(z)=\sum_{k=0}^{\bar{N}}(z-z_0)^k\cdot Y(k) \tag{24}$$

Eq. (24) implies that $\sum_{k=\bar{N}+1}^{\infty}(z-z_0)^k Y(k)$ is negligibly small. Where $\bar{N}$ is series size and the value of $\bar{N}$ depends on the convergence of the eigenvalues.

Theorems that are frequently used in differential transformation of the differential equations and the boundary conditions are introduced in Table 1 and Table 2, respectively.

3.1 Using differential transformation to solve motion equations

Eqs. (7)-(8) can be rewritten as follows

$$\begin{aligned}\frac{d^2y(z)}{dz^2}=&\left[\frac{2\cdot i\cdot\omega\cdot\alpha\cdot\bar{k}\cdot L^2\cdot\sqrt{m_f\cdot EI}}{AG\cdot L^2-\bar{k}\cdot\alpha^2\cdot EI}\right]\cdot\frac{dy(z)}{dz}\\&+\left[\frac{AG\cdot L^3}{AG\cdot L^2-\bar{k}\cdot\alpha^2\cdot EI}\right]\cdot\frac{d\varphi(z)}{dz}\\&-\left[\frac{m\cdot\omega^2\cdot\bar{k}\cdot L^4}{AG\cdot L^2-\bar{k}\cdot\alpha^2\cdot EI}\right]\cdot y(z)\end{aligned} \tag{25}$$

$$\begin{aligned}\frac{d^2\varphi(z)}{dz^2}=&-\left[\frac{AG\cdot L}{EI\cdot\bar{k}}\right]\cdot\frac{dy(z)}{dz}\\&+\left[\frac{AG\cdot L^2}{EI\cdot\bar{k}}-\frac{m_p\cdot\omega^2\cdot I\cdot L^2}{EI\cdot A}\right]\cdot\varphi(z)\end{aligned} \tag{26}$$

where

$\alpha=V\cdot L\cdot\sqrt{\dfrac{m_f}{EI}}$;(Nondimensionalized multiplication factor for the fluid velocity) (27)

The differential transform method is applied to Eqs. (25)-(26) by using the theorems introduced in Table 1 and the following expressions are obtained

$$Y(k+2)=\left[\frac{2\cdot i\cdot\omega\cdot\alpha\cdot\bar{k}\cdot L^2\cdot\sqrt{m_f\cdot EI}}{AG\cdot L^2-\bar{k}\cdot\alpha^2\cdot EI}\right]\cdot\frac{Y(k+1)}{(k+2)}$$

$$+\left[\frac{AG\cdot L^3}{AG\cdot L^2-\bar{k}\cdot\alpha^2\cdot EI}\right]\cdot\frac{\Phi(k+1)}{(k+2)}$$
$$-\left[\frac{m\cdot\omega^2\cdot\bar{k}\cdot L^4}{AG\cdot L^2-\bar{k}\cdot\alpha^2\cdot EI}\right]\cdot\frac{Y(k)}{(k+1)\cdot(k+2)} \tag{28}$$

$$\Phi(k+2)=-\left[\frac{AG\cdot L}{EI\cdot\bar{k}}\right]\cdot\frac{Y(k+1)}{(k+2)}$$
$$+\left[\frac{AG\cdot L^2}{EI\cdot\bar{k}}-\frac{m_p\cdot\omega^2\cdot I\cdot L^2}{EI\cdot A}\right]\cdot\frac{\Phi(k)}{(k+1)\cdot(k+2)} \tag{29}$$

where $Y(k)$ is the transformed function of $y(z)$ and $\Phi(k)$ is the transformed function of $\phi(z)$.

The boundary conditions of a simply supported fluid conveying Timoshenko pipeline are given below

$$y(z=0)=0 \tag{30a}$$

$$M(z=0)=0 \tag{30b}$$

$$y(z=1)=0 \tag{30c}$$

$$M(z=1)=0 \tag{30d}$$

Applying the differential transform method to Eqs. (30a)-(30d) and using the theorems introduced in Table 2, the transformed boundary conditions of a simply supported pipeline are obtained as

for $z = 0$;

$$Y(0)=\Phi(1)=0 \tag{31a}$$

for $z = 1$;

$$\sum_{k=0}^{\bar{N}}Y(k)=\sum_{k=0}^{\bar{N}}\bar{M}(k)=0 \tag{31b}$$

where $\bar{M}(k)$ is the transformed function of $M(z)$.

The boundary conditions of a fixed-fixed fluid conveying Timoshenko pipeline are given below

$$y(z=0)=0 \tag{32a}$$

$$\varphi(z=0)=0 \tag{32b}$$

$$y(z=1)=0 \tag{32c}$$

$$\varphi(z=1)=0 \tag{32d}$$

Applying the differential transform method to Eqs. (32a)-(32d), the transformed boundary conditions of a fixed-fixed pipeline are obtained as

for $z = 0$;

$$Y(0)=\Phi(0)=0 \tag{33a}$$

for $z = 1$;

$$\sum_{k=0}^{\bar{N}}Y(k)=\sum_{k=0}^{\bar{N}}\Phi(k)=0 \tag{33b}$$

The boundary conditions of one end (z=0) fixed and the other end (z=1) simply supported Timoshenko pipeline are given below

$$y(z=0)=0 \tag{34a}$$

$$\varphi(z=0)=0 \tag{34b}$$

$$y(z=1)=0 \tag{34c}$$

$$M(z=1)=0 \tag{34d}$$

Applying the differential transform method to Eqs. (34a)-(34d), the transformed boundary conditions of one end fixed and the other end simply supported pipeline are obtained as

for $z = 0$;

$$Y(0)=\Phi(0)=0 \tag{35a}$$

for $z = 1$;

$$\sum_{k=0}^{\bar{N}}Y(k)=\sum_{k=0}^{\bar{N}}\bar{M}(k)=0 \tag{35b}$$

For simply supported pipeline, substituting the boundary conditions expressed in Eqs. (31a)-(31b) into Eqs. (28)-(29), and taking $Y(1)=c_1$, $\Phi(0)=c_2$; for fixed-fixed supported pipeline, substituting the boundary conditions expressed in Eqs. (33a)-(33b) into Eqs. (28)-(29), and taking $Y(1)=c_1$, $\Phi(1)=c_2$; for one end fixed and the other end simply supported pipeline, substituting the boundary conditions expressed in Eqs. (35a)-(35b) into Eqs. (28)-(29), and taking $Y(1)=c_1$, $\Phi(1)=c_2$; the following matrix expression is obtained

$$\begin{bmatrix}\bar{A}_{11}^{(\bar{N})}(\omega) & \bar{A}_{12}^{(\bar{N})}(\omega)\\ \bar{A}_{21}^{(\bar{N})}(\omega) & \bar{A}_{22}^{(\bar{N})}(\omega)\end{bmatrix}\cdot\begin{Bmatrix}c_1\\ c_2\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix} \tag{36}$$

where c_1 and c_2 are constants and $\bar{A}_{j1}^{(\bar{N})}(\omega)$, $\bar{A}_{j2}^{(\bar{N})}(\omega)$ (j=1, 2) are polynomials of ω corresponding $\bar{N}$.

In the last step, for non-trivial solution, equating the coefficient matrix that is given in Eq. (36) to zero one determines the natural frequencies of the vibrating system as is given in Eq. (37).

$$\begin{vmatrix}\bar{A}_{11}^{(\bar{N})}(\omega) & \bar{A}_{12}^{(\bar{N})}(\omega)\\ \bar{A}_{21}^{(\bar{N})}(\omega) & \bar{A}_{22}^{(\bar{N})}(\omega)\end{vmatrix}=0 \tag{37}$$

The j^{th} estimated eigenvalue, $\omega_j^{(\bar{N})}$ corresponds to $\bar{N}$ and the value of $\bar{N}$ is determined as

$$\left|\omega_j^{(\bar{N})}-\omega_j^{(\bar{N}-1)}\right|\le\varepsilon \tag{38}$$

where $\omega_j^{(\bar{N}-1)}$ is the j^{th} estimated eigenvalue corresponding to ($\bar{N}$ −1) and ε is the small tolerance parameter. If Eq. (38) is satisfied, the j^{th} estimated eigenvalue, $\bar{\omega}_j^{(N)}$ is obtained.

The procedure that is explained below can be used to plot the mode shapes of fluid conveying Timoshenko pipeline. The following equalities can be written by using Eq. (36)

$$\bar{A}_{11}(\omega)\cdot c_1+\bar{A}_{12}(\omega)\cdot c_2=0 \tag{39}$$

Using Eq. (39), the constant c_2 can be obtained in terms of c_1 as follows

$$c_2=-\frac{\bar{A}_{11}(\omega)}{\bar{A}_{12}(\omega)}\cdot c_1 \tag{40}$$

All transformed functions can be expressed in terms of ω, c_1 and c_2. Since c_2 has been written in terms of c_1 above, $Y(k)$, $\Phi(k)$ and $\bar{M}(k)$ can be expressed in terms c_1 and ω as follows

$$Y(k)=Y(\omega,c_1) \tag{41a}$$

$$\Phi(k)=\Phi(\omega,c_1) \tag{41b}$$

$$\bar{M}(k)=\bar{M}(\omega,c_1) \tag{41c}$$

The constants c_1 and c_2 cannot be calculated directly. To plot the mode shapes, the value of one of the constants is assumed as a nonzero value. For instance, c_2 is equated to 1 and the second row and second column of the coefficient matrix are erased. The reduced matrix is multiplied by reduced vector of constants to obtain other constant. After computation of c_1 and knowing that c_2 equals 1, the displacement function is constructed. The mode shapes can be easily plotted by using Eq. (41.a) for different natural frequency values and axial coordinates.

4. Adomian Decomposition Method (ADM)

According to ADM, we consider the differential equation, in the operator form

$$\bar{L}(u)+R(u)+\bar{\bar{N}}(u)=g(z) \tag{42}$$

when $\bar{\bar{N}}$ is a non-linear operator, $\bar{L}$ is the highest-order derivative which is assumed to be invertible [if differential equation describes by M order, where the differential operator $\bar{L}$ is given by $\bar{L}(\cdot)=\frac{d^M(\cdot)}{dx^M}$], R is a linear differential operator of less order than $\bar{L}$ and g is the source term. The method is based on applying the inverse operator $\bar{L}^{-1}$ is therefore considered a M-fold integral operator defined by $\bar{L}^{-1}(*)=\int_0^z\int_0^z\cdots\int_0^z(*)dzdz\cdots dz$ in Eq. (42) to be expressed by

$$\bar{L}^{-1}\bar{L}(u)=\bar{L}^{-1}(g)-\bar{L}^{-1}R(u)-\bar{L}^{-1}\bar{\bar{N}}(u) \tag{43}$$

For the initial value problem, $\bar{L}^{-1}$ can be an integral operator defined from t_0 to t, for the boundary value problems, undefined integration is used and integration constants are encountered. The integration constants are found by applying boundary conditions.

Non-linear term is defined as $\bar{\bar{N}}(u)=\sum_{n=0}^{\infty}A_n$ where $u=\sum_{n=0}^{\infty}u_n$, and the components of A_n are so-called Adomian polynomials, A_i depends on u_0, u_1, …, u_i only ($\forall i$), as follows (Wazwaz 1998, 1999, 2000)

$$\begin{gathered}
A_0=F(u_0)\\
A_1=u_1\cdot F'(u_0)\\
A_2=u_2\cdot F'(u_0)+\frac{(u_1)^2\cdot F''(u_0)}{2}\\
A_3=u_3\cdot F'(u_0)+u_1\cdot u_2\cdot F''(u_0)+\frac{(u_1)^3\cdot F'''(u_0)}{6}\\
\vdots\\
u=A+B\cdot t+\bar{L}^{-1}(g)-\bar{L}^{-1}R(u)-\bar{L}^{-1}\bar{\bar{N}}(u)
\end{gathered} \tag{44}$$

is found.

4.1 Using ADM to solve motion equations

By using Eqs. (25)-(26), the equation of motion can be rewritten in terms of $y(z)$ as

$$\begin{aligned}
&(\bar{A}\cdot\bar{F})\cdot\frac{d^4y(z)}{dz^4}+(\bar{B}\cdot\bar{F})\cdot\frac{d^3y(z)}{dz^3}\\
&+(\bar{A}\cdot\bar{H}+\bar{F}\cdot\bar{C}-\bar{E}\cdot\bar{D})\cdot\frac{d^2y(z)}{dz^2}\\
&+(\bar{B}\cdot\bar{H})\cdot\frac{dy(z)}{dz}+(\bar{C}\cdot\bar{H})\cdot y(z)=0
\end{aligned} \tag{45}$$

where

$$\bar{A}=\frac{AG}{\bar{k}\cdot L^2}-\frac{m_f\cdot V^2}{L^2} \tag{46a}$$

$$\bar{B}=-\frac{2\cdot m_f\cdot i\cdot\omega\cdot V}{L} \tag{46b}$$

$$\bar{C}=m\cdot\omega^2 \tag{46c}$$

$$\bar{D}=-\bar{E}=-\frac{AG}{\bar{k}\cdot L} \tag{46d}$$

$$\bar{F}=\frac{EI}{L^2} \tag{46e}$$

$$\bar{H}=\frac{m_p\cdot\omega^2\cdot I}{A}-\frac{AG}{\bar{k}} \tag{46f}$$

Similarly, by using Eqs. (25)-(26), $\phi(z)$ can be written in terms of $y(z)$ as

$$\varphi(z)=\left(\frac{\bar{A}\cdot\bar{F}}{\bar{D}\cdot\bar{H}}\right)\cdot\frac{d^3y(z)}{dz^3}+\left(\frac{\bar{B}\cdot\bar{F}}{\bar{D}\cdot\bar{H}}\right)\cdot\frac{d^2y(z)}{dz^2}+\left(\frac{\bar{C}\cdot\bar{F}}{\bar{D}\cdot\bar{H}}-\frac{\bar{E}}{\bar{H}}\right)\cdot\frac{dy(z)}{dz} \tag{47}$$

Eq. (45) can be rewritten as

$$\frac{d^4y(z)}{dz^4}=-\left(\frac{\bar{B}}{\bar{A}}\right)\cdot\frac{d^3y(z)}{dz^3}-\frac{\left(\bar{A}\cdot\bar{H}+\bar{F}\cdot\bar{C}-\bar{E}\cdot\bar{D}\right)}{\left(\bar{A}\cdot\bar{F}\right)}\cdot\frac{d^2y(z)}{dz^2}-\frac{\left(\bar{B}\cdot\bar{H}\right)}{\left(\bar{A}\cdot\bar{F}\right)}\cdot\frac{dy(z)}{dz}-\frac{\left(\bar{C}\cdot\bar{H}\right)}{\left(\bar{A}\cdot\bar{F}\right)}\cdot y(z)=0 \tag{48}$$

The inverse operator $\bar{L}^{-1}$, where the inverse operator is $\int_0^z\int_0^z\int_0^z\int_0^z(*)dzdzdzdz$, is applied to both sides of Eq. (48) following expression is obtained

$$y(z)=y(0)+z\cdot y'(0)+\frac{z^2}{2!}\cdot y''(0)+\frac{z^3}{3!}\cdot y'''(0)+\bar{L}^{-1}\left\{\begin{array}{l}-\left(\frac{\bar{B}}{\bar{A}}\right)\cdot\frac{d^3y(z)}{dz^3}\\ -\frac{\left(\bar{A}\cdot\bar{H}+\bar{F}\cdot\bar{C}-\bar{E}\cdot\bar{D}\right)}{\left(\bar{A}\cdot\bar{F}\right)}\cdot\frac{d^2y(z)}{dz^2}\\ -\frac{\left(\bar{B}\cdot\bar{H}\right)}{\left(\bar{A}\cdot\bar{F}\right)}\cdot\frac{dy(z)}{dz}-\frac{\left(\bar{C}\cdot\bar{H}\right)}{\left(\bar{A}\cdot\bar{F}\right)}\cdot y(z)\end{array}\right\} \tag{49}$$

If desired solution function instead of the series expansions of the solution function is obtained in writing the following equation

$$y(z)=\sum_{n=0}^{\infty}y_n(z) \tag{50}$$

Eq. (50) is substituted in Eq. (49), it is obtained

$$\begin{aligned}\sum_{n=0}^{\infty}y_n(z)=&\left[\begin{array}{l}1+\left(\frac{\bar{B}}{\bar{A}}\right)\cdot z+\frac{\left(\bar{A}\cdot\bar{H}+\bar{F}\cdot\bar{C}-\bar{E}\cdot\bar{D}\right)}{\left(\bar{A}\cdot\bar{F}\right)}\cdot\frac{z^2}{2!}\\ +\frac{\left(\bar{B}\cdot\bar{H}\right)}{\left(\bar{A}\cdot\bar{F}\right)}\cdot\frac{z^3}{3!}\end{array}\right]\cdot y(0)\\ &+\left[z+\left(\frac{\bar{B}}{\bar{A}}\right)\cdot\frac{z^2}{2!}+\frac{\left(\bar{A}\cdot\bar{H}+\bar{F}\cdot\bar{C}-\bar{E}\cdot\bar{D}\right)}{\left(\bar{A}\cdot\bar{F}\right)}\cdot\frac{z^3}{3!}\right]\cdot y'(0)\\ &+\left[\frac{z^2}{2!}+\left(\frac{\bar{B}}{\bar{A}}\right)\cdot\frac{z^3}{3!}\right]\cdot y''(0)+\frac{z^3}{3!}\cdot y'''(0)\\ &\left\{\begin{array}{l}-\left(\frac{\bar{B}}{\bar{A}}\right)\cdot\int_0^z\sum_{n=0}^{\infty}y_n(z)\cdot dz-\\ \frac{\left(\bar{A}\cdot\bar{H}+\bar{F}\cdot\bar{C}-\bar{E}\cdot\bar{D}\right)}{\left(\bar{A}\cdot\bar{F}\right)}\cdot\int_0^z\int_0^z\left[\sum_{n=0}^{\infty}y_n(z)\right]\cdot(dz)^2\\ -\frac{\left(\bar{B}\cdot\bar{H}\right)}{\left(\bar{A}\cdot\bar{F}\right)}\cdot\int_0^z\int_0^z\int_0^z\left[\sum_{n=0}^{\infty}y_n(z)\right]\cdot(dz)^3\\ -\frac{\left(\bar{C}\cdot\bar{H}\right)}{\left(\bar{A}\cdot\bar{F}\right)}\cdot\int_0^z\int_0^z\int_0^z\int_0^z\left[\sum_{n=0}^{\infty}y_n(z)\right]\cdot(dz)^4\end{array}\right\}\end{aligned} \tag{51}$$

ADM is applied to Eq. (51) by using the inverse operator, following steps are obtained

for n=0;

$$\begin{aligned}y_0(z)=&\left[\begin{array}{l}1+\left(\frac{\bar{B}}{\bar{A}}\right)\cdot z+\frac{\left(\bar{A}\cdot\bar{H}+\bar{F}\cdot\bar{C}-\bar{E}\cdot\bar{D}\right)}{\left(\bar{A}\cdot\bar{F}\right)}\cdot\frac{z^2}{2!}\\ +\frac{\left(\bar{B}\cdot\bar{H}\right)}{\left(\bar{A}\cdot\bar{F}\right)}\cdot\frac{z^3}{3!}\end{array}\right]\cdot y(0)\\ &+\left[z+\left(\frac{\bar{B}}{\bar{A}}\right)\cdot\frac{z^2}{2!}+\frac{\left(\bar{A}\cdot\bar{H}+\bar{F}\cdot\bar{C}-\bar{E}\cdot\bar{D}\right)}{\left(\bar{A}\cdot\bar{F}\right)}\cdot\frac{z^3}{3!}\right]\cdot y'(0)\\ &+\left[\frac{z^2}{2!}+\left(\frac{\bar{B}}{\bar{A}}\right)\cdot\frac{z^3}{3!}\right]\cdot y''(0)+\frac{z^3}{3!}\cdot y'''(0)\end{aligned} \tag{52}$$

for $n\geq 0$;

$$\begin{aligned}y_{n+1}(z)=&-\left(\frac{\bar{B}}{\bar{A}}\right)\cdot\int_0^z\sum_{n=0}^{\infty}y_n(z)\cdot dz\\ &-\frac{\left(\bar{A}\cdot\bar{H}+\bar{F}\cdot\bar{C}-\bar{E}\cdot\bar{D}\right)}{\left(\bar{A}\cdot\bar{F}\right)}\cdot\int_0^z\int_0^z\left[\sum_{n=0}^{\infty}y_n(z)\right]\cdot(dz)^2\\ &-\frac{\left(\bar{B}\cdot\bar{H}\right)}{\left(\bar{A}\cdot\bar{F}\right)}\cdot\int_0^z\int_0^z\int_0^z\left[\sum_{n=0}^{\infty}y_n(z)\right]\cdot(dz)^3\\ &-\frac{\left(\bar{C}\cdot\bar{H}\right)}{\left(\bar{A}\cdot\bar{F}\right)}\cdot\int_0^z\int_0^z\int_0^z\int_0^z\left[\sum_{n=0}^{\infty}y_n(z)\right]\cdot(dz)^4\end{aligned} \tag{53}$$

Terms derived from Eqs. (52)-(53), the desired function is obtained when the solution is placed in the Eq. (50).

We may approximate the above solution by the K-term truncated series, Eq. (50) can be rewritten as

$$y(z)=\sum_{n=0}^{K-1}y_n(z) \tag{54}$$

Eq. (54) implies that $\sum_{n=K}^{\infty}y_n(z)$ is negligibly small. The number of the series summation limit K is determined by convergence requirement in practice.

From the boundary conditions of a simply supported Timoshenko pipeline, we get the conditions below

$$y(z=0)=0\rightarrow y(0)=0 \tag{55}$$

From the conditions $M(z=0)=0$, $y(z=1)=0$ and $M(z=1)=0$, we get the three equations which are depend on $y'(0)$, $y''(0)$ and $y'''(0)$ terms. Solving three equations are found $y'(0)$, $y''(0)$ and $y'''(0)$ terms. Finally, putting these terms in Eq. (54), desired solution is found. Consequently, the boundary value problem solution is obtained by converting the initial value problem.

Table 3 The first five natural frequencies of the simply supported fluid conveying Timoshenko pipeline for different values of L and α

L (m)	ω_α (rad/sec)	METHOD	α			
			0.00	0.25	0.50	0.75
3.0	ω_1	DTM, ADM	552.2780	549.7708	542.2060	529.4496
		ANM	**552.2780**	**549.7708**	**542.2060**	**529.4496**
	ω_2	DTM, ADM	1743.2269	1740.0957	1730.6831	1714.9324
		ANM	**1743.2269**	**1740.0957**	**1730.6831**	**1714.9324**
	ω_3	DTM, ADM	3091.3681	3086.9946	3073.8554	3051.8954
		ANM	**3091.3681**	**3086.9946**	**3073.8554**	**3051.8954**
	ω_4	DTM, ADM	4452.6499	4446.8753	4429.5286	4400.5407
		ANM	**4452.6499**	**4446.8753**	**4429.5286**	**4400.5407**
	ω_5	DTM, ADM	5798.9734	5791.7543	5770.0697	5733.8364
		ANM	**5798.9734**	**5791.7543**	**5770.0697**	**5733.8364**
5.0	ω_1	DTM, ADM	213.6885	212.8658	210.3849	206.2053
		ANM	**213.6885**	**212.8658**	**210.3849**	**206.2053**
	ω_2	DTM, ADM	761.4476	760.6098	758.0929	753.8875
		ANM	**761.4476**	**760.6098**	**758.0929**	**753.8875**
	ω_3	DTM, ADM	1483.5996	1482.5748	1479.4981	1474.3633
		ANM	**1483.5996**	**1482.5748**	**1479.4981**	**1474.3633**
	ω_4	DTM, ADM	2276.2921	2275.0144	2271.1791	2264.7798
		ANM	**2276.2921**	**2275.0144**	**2271.1791**	**2264.7798**
	ω_5	DTM, ADM	3091.3681	3089.8029	3085.1049	3077.2671
		ANM	**3091.3681**	**3089.8029**	**3085.1049**	**3077.2671**
7.0	ω_1	DTM, ADM	111.4726	111.0646	109.8342	107.7618
		ANM	**111.4726**	**111.0646**	**109.8342**	**107.7618**
	ω_2	DTM, ADM	417.4662	417.0848	415.9390	414.0252
		ANM	**417.4662**	**417.0848**	**415.9390**	**414.0252**
	ω_3	DTM, ADM	857.1313	856.7046	855.4239	853.2872
		ANM	**857.1313**	**856.7046**	**855.4239**	**853.2872**
	ω_4	DTM, ADM	1374.4353	1373.9360	1372.4377	1369.9388
		ANM	**1374.4353**	**1373.9360**	**1372.4377**	**1369.9388**
	ω_5	DTM, ADM	1931.9382	1931.3492	1929.5818	1926.6344
		ANM	**1931.9382**	**1931.3492**	**1929.5818**	**1926.6344**

From the boundary conditions of one end (z=0) fixed and the other end (z=1) simply supported Timoshenko pipeline

$$y(z=0)=0 \rightarrow y(0)=0 \tag{56}$$

is obtained.

From the conditions $\phi(z=0)=0$, $y(z=1)=0$ and $M(z=1)=0$, we get the three equations which are depend on $y'(0)$, $y''(0)$ and $y'''(0)$ terms. Solving three equations are found $y'(0)$, $y''(0)$ and $y'''(0)$ terms. Thus, putting these terms in Eq. (54), solution is found for the boundary conditions of one end (z=0) fixed and the other end (z=1) simply supported Timoshenko pipeline.

From the boundary conditions of a fixed-fixed supported Timoshenko pipeline, we get the condition below

$$y(z=0)=0 \rightarrow y(0)=0 \tag{57}$$

From the conditions $\phi(z=0)=0$, $y(z=1)=0$ and $\phi(z=1)=0$, we get the three equations which are depend on $y'(0)$, $y''(0)$ and $y'''(0)$ terms. Solving three equations are found $y'(0)$, $y''(0)$ and $y'''(0)$ terms. Thus, putting these terms in Eq. (54), solution is found for the boundary conditions of a fixed-fixed supported Timoshenko pipeline.

5. Numerical analysis and discussions

In this paper; the simply supported, the fixed-fixed supported and one end fixed, the other end simply supported fluid conveying Timoshenko pipelines are considered for numerical analysis. The first five natural frequencies, ω_i (i = 1,…,5) of Timoshenko pipelines are calculated by using computer programs prepared in Matlab by the authors. Natural frequencies are found by determining values for which the determinant of the coefficient matrix is equal to zero.

The numerical results of this paper are obtained based on uniform Timoshenko pipelines with the following data as:

m_f = 0.277 kN.sec^2/m; m_p = 0.311 kN.sec^2/m; D=720 mm;

Table 4 The first five natural frequencies of one end fixed, the other end simply supported fluid conveying Timoshenko pipeline for different values of L and α

L (m)	ω_α (rad/sec)	METHOD	α			
			0.00	0.25	0.50	0.75
3.0	ω_1	DTM, ADM	745.0523	742.8703	736.3021	725.2804
		ANM	**745.0523**	**742.8703**	**736.3021**	**725.2804**
	ω_2	DTM, ADM	1884.8044	1881.8485	1872.9653	1858.1084
		ANM	**1884.8044**	**1881.8485**	**1872.9653**	**1858.1084**
	ω_3	DTM, ADM	3166.4095	3162.1124	3149.2050	3127.6387
		ANM	**3166.4095**	**3162.1124**	**3149.2050**	**3127.6387**
	ω_4	DTM, ADM	4489.0713	4483.3710	4466.2464	4437.6262
		ANM	**4489.0713**	**4483.3710**	**4466.2464**	**4437.6262**
	ω_5	DTM, ADM	5814.8697	5807.7005	5786.1679	5750.1962
		ANM	**5814.8697**	**5807.7005**	**5786.1679**	**5750.1962**
5.0	ω_1	DTM, ADM	312.4215	311.7428	309.7013	306.2803
		ANM	**312.4215**	**311.7428**	**309.7013**	**306.2803**
	ω_2	DTM, ADM	878.9420	878.1767	875.8785	872.0415
		ANM	**878.9420**	**878.1767**	**875.8785**	**872.0415**
	ω_3	DTM, ADM	1580.3295	1579.3412	1576.3745	1571.4243
		ANM	**1580.3295**	**1579.3412**	**1576.3745**	**1571.4243**
	ω_4	DTM, ADM	2344.8902	2343.6378	2339.8786	2333.6067
		ANM	**2344.8902**	**2343.6378**	**2339.8786**	**2333.6067**
	ω_5	DTM, ADM	3137.1042	3135.5580	3130.9171	3123.1750
		ANM	**3137.1042**	**3135.5580**	**3130.9171**	**3123.1750**
7.0	ω_1	DTM, ADM	167.9129	167.5827	166.5896	164.9263
		ANM	**167.9129**	**167.5827**	**166.5896**	**164.9263**
	ω_2	DTM, ADM	499.2323	498.8913	497.8673	496.1584
		ANM	**499.2323**	**498.8913**	**497.8673**	**496.1584**
	ω_3	DTM, ADM	940.8361	940.4316	939.2177	937.1929
		ANM	**940.8361**	**940.4316**	**939.2177**	**937.1929**
	ω_4	DTM, ADM	1447.2125	1446.7278	1445.2735	1442.8481
		ANM	**1447.2125**	**1446.7278**	**1445.2735**	**1442.8481**
	ω_5	DTM, ADM	1989.9701	1989.3918	1987.6561	1984.7618
		ANM	**1989.9701**	**1989.3918**	**1987.6561**	**1984.7618**

d=600 mm; μ = 0.30; EI = 190955.1031 kN.m^2;

AG = 1337782.705 kN; $\bar{k} = 1.55$; L = 3.0 m,

5.0 m and 7.0 m; α = 0.00, 0.25, 0.50 and 0.75

Using DTM and ADM, the frequency values of the simply supported Timoshenko pipeline for the first five modes are presented in Table 3, the first five frequency values of one end fixed, the other end simply supported Timoshenko pipeline are presented in Table 4 and the fixed-fixed Timoshenko pipeline's the first five frequency values are presented in Table 5 being compared with the frequency values obtained by using analytical method for the different values of the nondimensionalized multiplication factor for the fluid velocity and the length of the pipeline.

For L=7.0 m and α=0.75; Fig. 2 shows the first five mode shapes of Timoshenko pipeline with simply supported boundary condition, Fig. 3 shows the first five mode shapes of Timoshenko pipeline with one end fixed, the other end simply supported boundary condition and Fig. 4 shows the first five mode shapes of Timoshenko pipeline with fixed-fixed boundary condition.

For all boundary conditions, as the nondimensionalized multiplication factor for the fluid velocity is increased for the other variable (L) is being constant, the natural frequency values of all fluid conveying Timoshenko pipelines decreased. This result indicates that, the increasing for the fluid velocity leads to reduction in natural frequency values for all boundary conditions. This result is very important for the effect of the fluid velocity.

A decrease is observed in natural frequency values of all fluid conveying Timoshenko pipelines for the condition of α being constant and the values of the length of the pipeline are increased. This result indicates that, the increasing for the length of the pipeline leads to reduction in natural frequency values for all boundary conditions.

In application of DTM, the natural frequency values of the fluid conveying Timoshenko pipelines are calculated by increasing series size $\bar{N}$. For simply supported and fixed-fixed supported Timoshenko pipelines, when the series size is taken 60; for one end fixed, the other end simply

Table 5 The first five natural frequencies of the fixed-fixed fluid conveying Timoshenko pipeline for different values of L and α

L (m)	ω_α (rad/sec)	METHOD	α 0.00	0.25	0.50	0.75
3.0	ω_1	DTM, ADM	939.8725	937.8720	931.8561	921.7809
		ANM	**939.8725**	**937.8720**	**931.8561**	**921.7809**
	ω_2	DTM, ADM	2004.6776	2001.8982	1993.5468	1979.5836
		ANM	**2004.6776**	**2001.8982**	**1993.5468**	**1979.5836**
	ω_3	DTM, ADM	3238.4886	3234.1997	3221.3203	3199.8119
		ANM	**3238.4886**	**3234.1997**	**3221.3203**	**3199.8119**
	ω_4	DTM, ADM	4522.3483	4516.7935	4500.0980	4472.1684
		ANM	**4522.3483**	**4516.7935**	**4500.0980**	**4472.1684**
	ω_5	DTM, ADM	5832.7115	5825.4276	5803.5826	5767.1884
		ANM	**5832.7115**	**5825.4276**	**5803.5826**	**5767.1884**
5.0	ω_1	DTM, ADM	421.3659	420.7832	419.0323	416.1051
		ANM	**421.3659**	**420.7832**	**419.0323**	**416.1051**
	ω_2	DTM, ADM	987.8029	987.0949	984.9683	981.4197
		ANM	**987.8029**	**987.0949**	**984.9683**	**981.4197**
	ω_3	DTM, ADM	1669.7206	1668.7569	1665.8644	1661.0389
		ANM	**1669.7206**	**1668.7569**	**1665.8644**	**1661.0389**
	ω_4	DTM, ADM	2408.9821	2407.7529	2404.0634	2397.9079
		ANM	**2408.9821**	**2407.7529**	**2404.0634**	**2397.9079**
	ω_5	DTM, ADM	3180.8191	3179.2881	3174.6932	3167.0282
		ANM	**3180.8191**	**3179.2881**	**3174.6932**	**3167.0282**
7.0	ω_1	DTM, ADM	233.4623	233.1869	232.3595	230.9769
		ANM	**233.4623**	**233.1869**	**232.3595**	**230.9769**
	ω_2	DTM, ADM	579.8212	579.5115	578.5821	577.0316
		ANM	**579.8212**	**579.5115**	**578.5821**	**577.0316**
	ω_3	DTM, ADM	1020.3629	1019.9751	1018.8114	1016.8706
		ANM	**1020.3629**	**1019.9751**	**1018.8114**	**1016.8706**
	ω_4	DTM, ADM	1515.7182	1515.2457	1513.8278	1511.4632
		ANM	**1515.7182**	**1515.2457**	**1513.8278**	**1511.4632**
	ω_5	DTM, ADM	2044.9414	2044.3720	2042.6631	2039.8136
		ANM	**2044.9414**	**2044.3720**	**2042.6631**	**2039.8136**

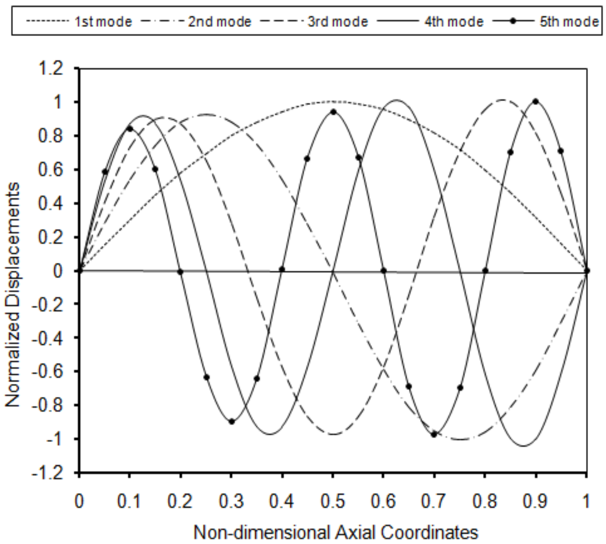


Fig. 2 The first five mode shapes of the simply supported fluid conveying Timoshenko pipeline, L=7.00 m and α=0.75

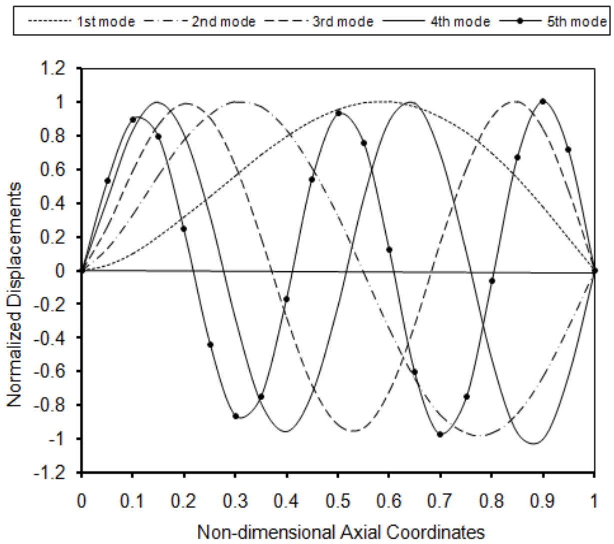


Fig. 3 The first five mode shapes of one end fixed, the other end simply supported fluid conveying Timoshenko pipeline, L=7.00 m and α =0.75

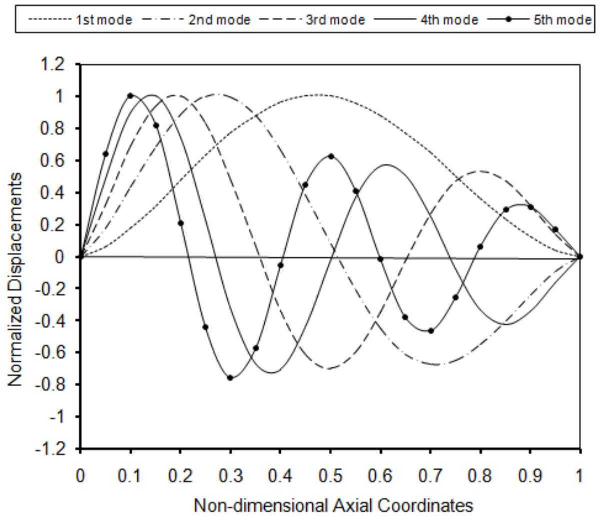


Fig. 4 The first five mode shapes of the fixed-fixed fluid conveying Timoshenko pipeline, L=7.00 m and α=0.75

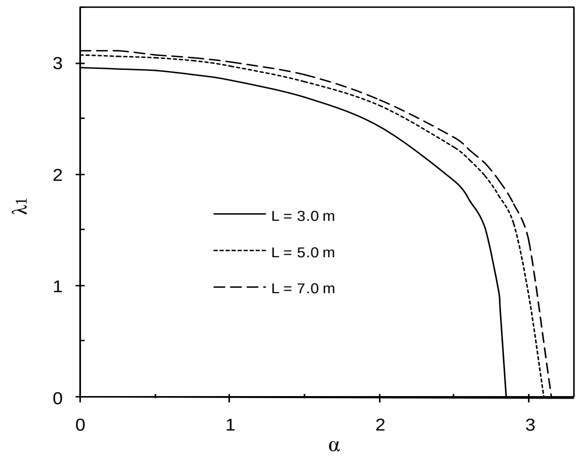
supported Timoshenko pipeline, when the series size is taken 62, the natural frequency values of the first five modes can be appeared in application of DTM. Additionally, here it is seen that higher modes appear when more terms are taken into account in DTM applications. Thus, depending on the order of the required mode, one must try a few values for the term number at the beginning of the calculations in order to find the adequate number of terms.

Similarly, the natural frequency values of the fluid conveying Timoshenko pipelines are calculated by increasing series size K, in application of ADM. For all boundary conditions, when the series size is taken as 26, the natural frequency values of the first five modes of the fluid conveying Timoshenko pipelines can be appeared in application of ADM. When more terms are taken into account in ADM applications, the natural frequencies of the following modes can be appeared.

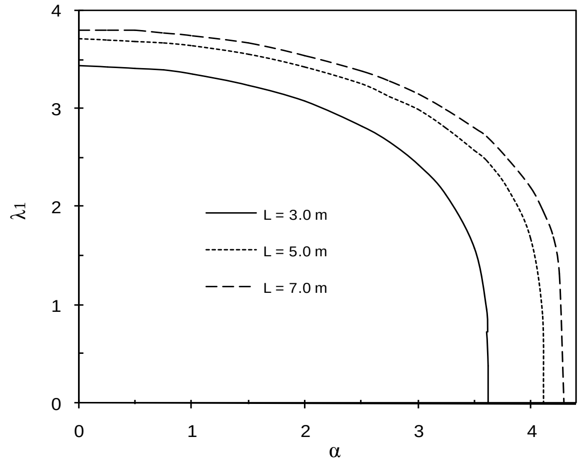
For the different values of α and L, the variations of the first frequency factors $\left[\lambda_1 = \sqrt[4]{\dfrac{m\cdot\omega_1^2\cdot L^4}{EI}}\right]$ for the simply supported, one end fixed, the other end simply supported and fixed-fixed boundary conditions are presented in Fig. 5(a), Fig. 5(b) and Fig. 5(c), respectively.

For all boundary conditions, Figs. 5(a)-(c) show the first natural frequency as function of the fluid velocity, for the different values of the length of the pipelines. It must be stressed that the divergence instability that corresponds to the critical fluid velocity at which the natural frequency tends to zero is based on Timoshenko beam theory whose validity in the vicinity of the instability may become questionable. Nevertheless, the extrapolated value of the critical fluid velocity computed from the linear theory provides useful information as an indicative towards capturing the instability.

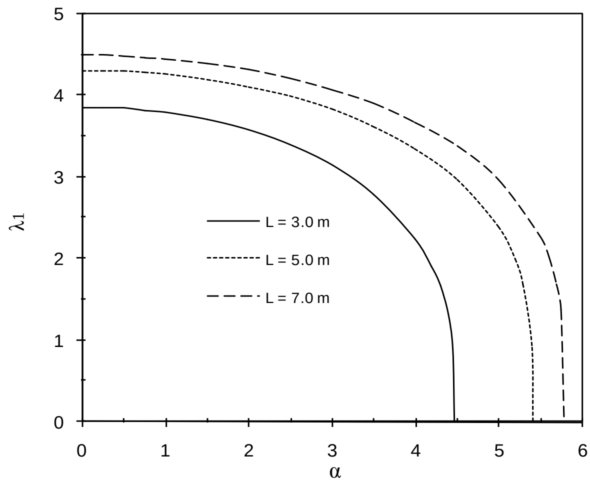
In this numerical analysis, for the different values of the length of the pipelines and boundary conditions; the values of the critical fluid velocity (V_{cr}) are calculated and



(a) Simply supported fluid conveying pipeline



(b) One end fixed, the other end simply supported fluid conveying pipeline



(c) Fixed-fixed fluid conveying pipeline

Fig. 5 Effect of flow velocity on the first frequency factor for the fluid conveying pipeline

presented in Table 6.

As expected, the length of the pipeline increases the fixed-fixed boundary condition`ns have resulted in higher critical fluid velocities than the simply supported and one end fixed, the other end simply supported conditions.

Table 6 The values of the critical fluid velocity (V_{cr}) for the different values of the length of the pipelines and boundary conditions

L (m)	Boundary Condition	V_{cr} (m/sec)
3.0	Simply supported	780.742
	One end fixed, the other end simply supported	990.584
	Fixed-fixed supported	1227.272
5.0	Simply supported	509.537
	One end fixed, the other end simply supported	675.548
	Fixed-fixed supported	889.224
7.0	Simply supported	369.825
	One end fixed, the other end simply supported	503.667
	Fixed-fixed supported	679.774

Similarly, the critical fluid velocity of the one end fixed, and the other simply supported pipeline is higher than the simply supported pipeline's critical fluid velocity.

6. Conclusions

In this study, starting from the governing differential equations of motion in free vibration of the fluid conveying pipelines, DTM algorithms are developed by using Timoshenko beam theory and the iterative-based computer programs are developed for the solution of linear-homogeneous frequency equation set relating to free vibration of the fluid conveying Timoshenko pipelines with simply supported, fixed-fixed and one end fixed, the other end simply supported boundary conditions. Variation in free vibration natural frequencies for the first five modes of Timoshenko pipelines is investigated for the different values of the nondimensionalized multiplication factor for the fluid velocity and the length of the pipeline. The calculated natural frequencies of Timoshenko pipelines by using DTM and ADM are compared with the results of the analytical solution, the critical fluid velocities are calculated and the first five mode shapes are plotted for different boundary conditions.

All the steps of the DTM and ADM are very straightforward and the application of the DTM and ADM to both the equations of motion and the boundary conditions seem to be very involved computationally. However, all the algebraic calculations are finished quickly using symbolic computational software. Besides all these, the analysis of the convergence of the results show that DTM and ADM solutions converge fast. When the results of the DTM and ADM are compared with the results of ANM, very good agreement is observed. The advantages of DTM and ADM are their fast convergence of the solution and their high degree of accuracy.

Thus, the present work has demonstrated that DTM and ADM have high precision and computational efficiency in vibration analysis of fluid conveying Timoshenko pipelines. These methods may be further extended to the analysis for nonlinearly static and dynamic responses of fluid conveying Timoshenko pipelines. Moreover, the fully coupled fluid-structure interaction of fluid conveying pipelines can be investigated using DTM and ADM.

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