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Series tuned mass dampers in train-induced vibration control of railway bridges

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Abstract. This paper presents the series multiple tuned mass dampers (STMDs) to suppress the resonant vibrations of railway bridges under the passage of high-speed trains (HSTs). A STMD device consisting of two spring-mass-damper units connected each other in series is installed on the bridge. In solution, bridge is modeled as a simply-supported Euler-Bernoulli beam with constant cross-section, and vehicle is simulated as a series of moving forces with constant speed. By the assumed mode method, the governing equations of motion of the beam-TMD device coupled system traversed by a moving train are obtained. The optimum values for the parameters of the STMD device are obtained for the criterion based on the minimization of the maximum dynamic displacement of the beam at its midspan. Single TMD and multiple TMDs in parallel are also considered for demonstration of the STMD device's performance. The results show that STMDs are effective in bridge vibration suppression and robust to parameters' change in the main system and the absorber itself.

Keywords: tuned mass dampers; railway bridges; vibration control; high-speed train; resonant vibrations

1. Introduction

Dynamic response of bridges to the passage of moving vehicles is a subject of great interest to engineers. For railway bridges, repeat actions of train axle loads can cause excessive vibrations on the bridge. These vibrations adversely affect the passenger comfort and traffic safety as well as causing the fatigue phenomenon on the bridge. To control the resonant vibrations of bridges, different devices have been developed such as tuned mass dampers (TMDs). Use of TMDs is a simpler and economical way to suppress the excessive vibrations of bridges.

Most of the previous research on TMDs in reducing bridge vibrations due to moving vehicles is related to the use of single or multiple tuned mass dampers (MTMDs). Kwon *et al*. (1998) used a TMD to suppress the vibration of a three-span bridge induced by high-speed trains (HSTs). They modeled the train as a series of 2-degree-of-freedom (DOF) system, and compared the midpoint deflections and their fast Fourier transforms before and after the TMD installation to show the efficiency of TMD to suppress the bridge vibrations. Wang *et al*. (2003) dealt with the applicability of passive TMDs to control the train-induced vibrations on bridges. They gave a design approach for optimal TMD parameters, and studied the TMD effectiveness. Chen and Huang (2004) proposed a simplified 2-DOF system for the design of TMDs. They emphasized the effectiveness of TMDs for vibration control. Samani *et al*. (2013) investigated the performance of linear and nonlinear dynamic vibration absorbers applied to the specific problem of moving loads or vehicles. Krenk and Høgsberg (2014) developed a simple explicit design procedure for a linear tuned mass absorber with a viscous damper on a flexible structure. Chun *et al*. (2015) focused on the H_∞ optimal design of a dynamic vibration absorber for reduction of vibrations of the damped primary system. They used a damping element which is directly connected to the ground instead of the primary mass unlike traditional dynamic vibration absorber configurations.

Vibration reduction with a single TMD is achieved by tuning the TMD frequency to the vicinity of the fundamental frequency of structure. However, structural frequency estimation and fabrication errors as well as time-variant characteristics of the combined system may lead to the detuning effect. It can be avoided by using MTMD devices because they contain multiple tuning frequencies in a wider frequency bandwidth. Yau and Yang (2004a) proposed a hybrid TMD system in reducing the multiple resonant peaks of the cable-stayed bridges under HSTs. Using the same approach, they also developed a wideband MTMD system for reducing the dynamic response of continuous truss bridges to moving train loads (Yau and Yang 2004b). Lin *et al*. (2005) dealt with the applicability of MTMDs to control train-induced vibrations of bridges. They found that MTMDs are more effective and reliable than a single TMD for suppressing the resonant vibrations of bridges when the train axle arrangement is regular. Li *et al*. (2005) studied vibration control of railway bridges under HSTs using MTMDs. They investigated the performance of MTMDs with considering the parameters of the frequency range and the damping ratio of each TMD, and the total

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number of TMDs in the system. They also proposed the optimal parameters for MTMDs. Luu *et al*. (2012) focused on the optimization of TMD systems to suppress multi-resonant dynamic structural response of high-speed railway bridges. Stăncioiu and Ouyang (2012) proposed an iterative design method based on updating the receptances of a primary structure by controlling the modifications introduced at every step by an individual TMD.

In studies on MTMDs mentioned above, the vibration absorbers are connected to each other in parallel. A new type of TMD system in which multiple absorbers are connected to each other in series was proposed by Zuo (2009). He concluded that the series multiple TMDs (STMDs) are more effective and robust than all the other types of TMDs with the same mass. Li and Zhu (2006) proposed the double TMD device consisting of two spring-mass-damper units connected in series. They found that the double TMDs are highly effective and robust for reduction of undesirable vibrations, especially due to the ground acceleration.

Although the different numbers of TMDs placed in parallel have been extensively studied for suppressing the bridge vibrations under vehicular and seismic loads, researchers have not yet been studied on the application of STMDs to reduce the bridge vibrations induced by moving loads. Therefore, the purpose of this paper is to show the effectiveness and robustness of STMDs to control the resonant vibrations of railway bridges due to train passages.

2. Governing equations

The bridge is modeled as a simply-supported elastic beam with a constant cross-section. It is subject to a series of moving forces with constant speed that represents the train. To reduce excessive vibrations, three different TMD devices (classic TMD, MTMD and STMD) are installed on the beam at $x=x_s$ as shown in Fig. 1. Each TMD device is installed at the midspan of the bridge. Equations of the vertical motion for the bridge can be written as

$$EI\frac{\partial^4 y}{\partial x^4}+c_b\frac{\partial y}{\partial t}+m_b\frac{\partial^2 y}{\partial t^2}=F^{train}+F^{tmd} \tag{1}$$

where $y=y(x,\ t)$, EI, m_b, and c_b denote the deflection, the flexural stiffness, the mass per unit length of the beam, and the damping of the beam, respectively. $F^{train}=F^{train}(x,\ t)$ and $F^{tmd}=F^{tmd}(x_s,t)$ are the external forces acting on the bridge due to the train and the TMD system, respectively. The load F^{train} exerted by the train to the bridge can be defined as

$$\begin{aligned}F^{train}(x,t)=&\sum_{k=1}^{K}P_k\delta[x-(vt-d_k)]\times\\&\left[H\left(t-\frac{d_k}{v}\right)-H\left(t-\frac{d_k+L}{v}\right)\right]\end{aligned} \tag{2}$$

where v is the train speed, d_k is the distance of the kth force from the first one (d_1 = 0), K is the total number of axle forces, P_k is the kth axle force, δ(-) is the Dirac delta function, and H (-) is the Heaviside unit step function.

Equations of the vertical motion for TMD devices installed on the bridge at $x=x_s$ (see Fig. 1), and the load F^{tmd} on the bridge due to TMD can be given as:

(1) For STMD device (see Fig. 1(a))

$$m_2\frac{\partial^2 y_2}{\partial t^2}+c_2\left(\frac{\partial y_2}{\partial t}-\frac{\partial y_1}{\partial t}\right)+k_2(y_2-y_1)=0 \tag{3}$$

$$\begin{aligned}&m_1\frac{\partial^2 y_1}{\partial t^2}+c_1\left(\frac{\partial y_1}{\partial t}-\frac{\partial y}{\partial t}\right)+k_1(y_1-y)\\&-c_2\left(\frac{\partial y_2}{\partial t}-\frac{\partial y_1}{\partial t}\right)-k_2(y_2-y_1)=0\end{aligned} \tag{4}$$

$$F^{tmd}(x_s,t)=\delta(x-x_s)\left[c_1\left(\frac{\partial y_1}{\partial t}-\frac{\partial y}{\partial t}\right)+k_1(y_1-y)\right] \tag{5}$$

(2) For MTMD device (see Fig. 1(b))

$$\begin{aligned}&m_j\frac{\partial^2 y_j}{\partial t^2}+c_j\left(\frac{\partial y_j}{\partial t}-\frac{\partial y}{\partial t}\right)+k_j(y_j-y)=0\\&(j=1,2,...,n)\end{aligned} \tag{6}$$

$$F^{tmd}(x_s,t)=\sum_{j=1}^{n}\delta(x-x_s)\left[c_1(\dot{y}_j-\dot{y})+k_1(y_1-y)\right] \tag{7}$$

(3) For classic TMD device (see Fig. 1(c))

$$m_1\frac{\partial^2 y_1}{\partial t^2}+c_1\left(\frac{\partial y_1}{\partial t}-\frac{\partial y}{\partial t}\right)+k_1(y_1-y)=0 \tag{8}$$

$$F^{tmd}(x_s,t)=\delta(x-x_s)\left[c_1\left(\frac{\partial y_1}{\partial t}-\frac{\partial y}{\partial t}\right)+k_1(y_1-y)\right] \tag{9}$$

In Eqs. (3) to (9), $y_j=y_j(t)$, m_j, c_j and k_j are the vertical displacement, the mass, the damping, and the stiffness of the jth TMD (j=1, 2, …, n), respectively.

3. Method of solution

Governing differential equations of motion given in the previous section for each beam-TMD coupled system can be solved by assuming the deflection of the beam $y(x,\ t)$ as in the following series form

$$y(x,t)=\sum_{i=1}^{M}q_i(t)\varphi_i(x) \tag{10}$$

where M, $q_i(t)$ and $\varphi_i(x)$=sin ($i\pi x/L$) denote the number of modes, the generalized coordinates, and the modal shape function of ith vibration mode for the bridge, respectively. L is the beam length.

Substituting Eq. (10) into Eq. (1), multiplying the resulting expression by $\varphi_m(x)$, carrying out the integration along the beam length, and using the modal orthogonality, the ith modal equation of motion for the beam can be obtained as

$$\ddot{q}_i+2\bar{\xi}_i\bar{\omega}_i\dot{q}_i+\bar{\omega}_i^2q_i=F_i^{train}+F_i^{tmd} \tag{11}$$

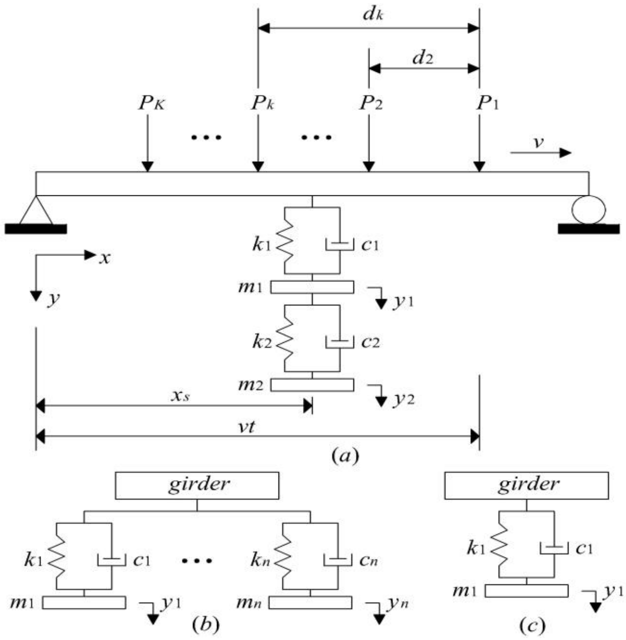

Fig. 1 A simply-supported beam installed with dynamic vibration absorbers subject to moving train loads (a) series multiple TMD (STMD), (b) parallel multiple TMD (MTMD-*n*), and (c) classic TMD (TMD-1)

where $\bar{\xi}_i$ and $\bar{\omega}_i$ are the damping ratio and the natural frequency for the *i*th mode of beam vibrations, respectively. Dot denotes the derivate with respect to time. The load term F_i^{train} due to the train is given by

$$F_i^{train}(x,t)=\sum_{k=1}^{N}\frac{2P_k}{m_b L}\sin\frac{i\pi(vt-d_k)}{L}\times \left[H\left(t-\frac{d_k}{v}\right)-H\left(t-\frac{d_k+L}{v}\right)\right] \tag{12}$$

Using Eq. (10) into the equations of motion for TMD devices considered yields the followings:

(1) For STMD device, from Eqs. (3) to (5)

$$\ddot{y}_1+2\xi_1\omega_1\left[\dot{y}_1-\sum_{m=1}^{M}\dot{q}_m\varphi_m(x_s)\right] +\omega_1^2\left[y_1-\sum_{m=1}^{M}q_m\varphi_m(x_s)\right] -\mu\left[2\xi_2\omega_2(\dot{y}_2-\dot{y}_1)+\omega_2^2(y_2-y_1)\right]=0 \tag{13}$$

$$\ddot{y}_2+2\xi_2\omega_2(\dot{y}_2-\dot{y}_1)+\omega_2^2(y_2-y_1)=0 \tag{14}$$

$$F_i^{tmd}(x_s,t)=\sin\frac{i\pi x_s}{L}2\mu_1\times \begin{bmatrix} 2\xi_1\omega_1\left(\dot{y}_1-\sum_{m=1}^{M}\dot{q}_m\varphi_m(x_s)\right) \\ +\omega_1^2\left(y_1-\sum_{m=1}^{M}q_m\varphi_m(x_s)\right)\end{bmatrix} \tag{15}$$

(2) For MTMD device, from Eqs. (6) and (7)

$$\ddot{y}_j+2\xi_j\omega_j\left[\dot{y}_j-\sum_{m=1}^{M}\dot{q}_m\varphi_m(x_s)\right] +\omega_j^2\left[y_j-\sum_{m=1}^{M}q_m\varphi_m(x_s)\right]=0 \quad (j=1,2,...,n) \tag{16}$$

$$F_i^{tmd}(x_s,t)=\sin\frac{i\pi x_s}{L}\sum_{j=1}^{n}2\mu_j\times \begin{bmatrix} 2\xi_j\omega_j\left(\dot{y}_j-\sum_{m=1}^{M}\dot{q}_m\varphi_m(x_s)\right) \\ +\omega_j^2\left(y_j-\sum_{m=1}^{M}q_m\varphi_m(x_s)\right)\end{bmatrix} \tag{17}$$

(3) For classic TMD device, from Eqs. (8) and (9)

$$\ddot{y}_1+2\xi_1\omega_1\left[\dot{y}_1-\sum_{m=1}^{M}\dot{q}_m\varphi_m(x_s)\right] +\omega_1^2\left[y_1-\sum_{m=1}^{M}q_m\varphi_m(x_s)\right]=0 \tag{18}$$

In above expressions, $\xi_j = c_j/2m_j\omega_j$ and $\omega_j=(k_j/m_j)^{1/2}$ denote the damping ratio and the frequency of the *j*th TMD, respectively. $\mu=m_2/m_1$ is the TMD mass ratio for STMD device, and $\mu_j=m_j/(m_bL)$ is the mass ratio of the *j*th TMD to that of the beam.

Total mass of the beam is defined as m_bL. Where, m_b is mass per unit length of the beam and L is span length of the beam.

$$F_i^{tmd}(x_s,t)=\sin\frac{i\pi x_s}{L}2\mu_1\times \begin{bmatrix} 2\xi_1\omega_1\left(\dot{y}_1-\sum_{m=1}^{M}\dot{q}_m\varphi_m(x_s)\right) \\ +\omega_1^2\left(y_1-\sum_{m=1}^{M}q_m\varphi_m(x_s)\right)\end{bmatrix} \tag{19}$$

The equations of motion for the entire bridge-TMD system under moving train can simply be expressed as in the following matrix form

$$\mathbf{M}\ddot{\mathbf{u}}+\mathbf{C}\dot{\mathbf{u}}+\mathbf{K}\mathbf{u}=\mathbf{F} \tag{20}$$

where the mass (**M**), damping (**C**) and stiffness (**K**) matrices, and the load vector **F** and the unknowns vector **u** are given in Appendix. Eq. (20) can be solved numerically by the Newmark's method to obtain unknowns.

4. Optimization problem

To obtain the optimal TMD parameters, we shall try to minimize the maximum deflections of the bridge by adjusting the mass ratio $\mu=m_2 / m_1$, the damping ratio ξ_j, and the frequency ratio $\beta_j=\omega_j/\bar{\omega}_1$ of STMD. The optimization problem is solved using a MATLAB toolbox so-called *fmincon* which uses the Sequential Quadratic Programming (SQP) method. First, the start points and

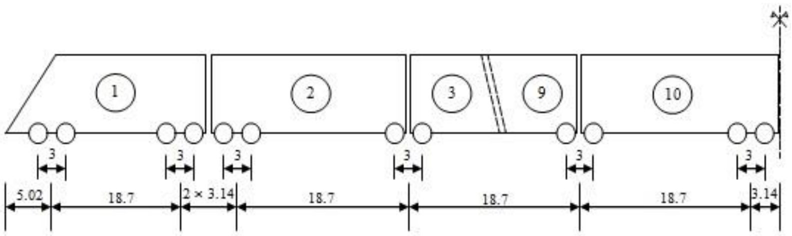

Fig. 2 Configuration of the Eurostar high-speed train (units in m)

Table 1 Optimal parameters for different TMD devices considered in the study (μ_T=2%)

Type	j	μ	ξ_j (%)	$\beta_j = \omega_j / \bar{\omega}_1$
TMD-1	1	-	13.4	0.958
TMD-3	1	-	7.2	0.879
	2			0.98
	3			1.081
TMD-7	1	-	4.1	0.843
	2			0.891
	3			0.939
	4			0.988
	5			1.036
	6			1.085
	7			1.133
STMD	1	0.118	0	1.046
	2		20.5	0.941

bounds are assigned for optimization of design parameters. Then, governing differential equations of the motion for the bridge-train-TMD coupled system are solved numerically using the modal superposition method. The results are used to obtain the objective function. Then, *fmincon* code is applied to update all parameters. The final step is to check whether the objective function is minimum or not. If the objective function is minimized, the optimal parameters are obtained, and the optimization problem has been successfully solved. Within the present concerns, the problem might be basically stated as follows:

$$\min J(p) \quad l_b \le p \le u_b \tag{21}$$

where p=(μ, ξ_j, β_j), $J(p)$, l_b and u_b represent the optimization variables, the objective function, the lower bound and upper bound of the optimization variables, respectively. $J(p)$ is defined as maximum amplitude in frequency response of midspan. Assuming the damping ratio of the beam $\bar{\xi}_i$ and the total TMD mass to the beam mass ratio $\mu_T=\Sigma\mu_j$ are known, we can select the followings to be control parameters such as $0.0<\mu<1.0$, $0.8<\beta_j<1.2$, and $0.0<\xi_j<0.5$. The ranges are determined as the bounds proposed by Yau and Yang (2004), Li and Zhu (2006), Zuo (2009).

For optimization of multiple TMDs in parallel, uniform stiffness and damping are assumed for all TMD units in the system as suggested by Xu and Igusa (1992), since manufacturing of this type of absorbers is much simpler than those with varying stiffness and damping. In optimization of MTMDs, the mass of each TMD units varies as in the work by Bandivadekar and Jangid (2012). In all cases, the mass ratio μ_T is selected to be 2%.

The calculated optimal parameters for TMD devices considered here are given in Table 1.

5. Result and discussion

To show performance of STMDs in suppressing bridge vibrations due to HSTs, some illustrative results are presented and discussed in this section. A MATLAB program has been developed to calculate the deflections at midspan of the bridge with different TMDs under the train passage at resonant speeds. Effectiveness of STMD device compared to other TMD types is investigated as well as its robustness to change in parameters of the bridge and the vibration absorbers, i.e., detuning.

In numerical calculations, the beam properties are assumed as: L=40 m, I=17.9 m^4, E=28.2 GPa, m_b=38,240 kg/m, $\bar{\xi} = 2.5\%$ (Wang *et al*. 2003). The first natural circular frequency of the beam is calculated as $\bar{\omega}_1 = 22.41$rad/sec. The Eurostar train is modeled using the axle arrangements given by Museros and Martinez-Rodrigo (2007). The Eurostar train consists of 48 identical 170kN loads. In the analyses, different types of vibration absorbers such as two TMDs in series (STMD), three TMDs in parallel (TMD-3), seven TMDs in parallel (TMD-7), and single TMD (TMD-1) are considered.

As previously mentioned, the total TMD mass to the bridge mass ratio is assumed to be constant as μ_T=2%. The train moves at a resonant speed over the bridge. As is well known, the resonance due to repetitive nature of train's axle loads strongly depends on the type of train. The rth resonance speed of the ith vibration mode is given by (Fryba 2001)

$$v_{i,r} = \frac{\bar{\omega}_i d}{2\pi r} \quad (i = 1, 2, 3, ..., \; r = 1, 2, 3, ...,) \tag{22}$$

where d is the axle spacing of the train, and r is the resonance number. For the considered train, the first three resonance speeds for the first vibration mode of the bridge can be calculated as 66.70, 33.35, and 22.23 m/sec (or 240.11, 120.05 and 80.04 km/h), respectively.

In Fig. 3, the midpoint deflection time-history of the bridge with and without STMD device for undamped case is given. Analytical results are compared with that of the finite element (FE) solution obtained by SAP2000 software (SAP2000). In FE modelling, the STMD is considered as a

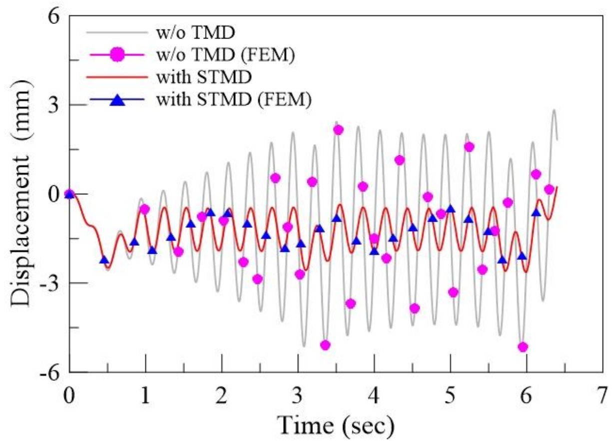

Fig. 3 Comparison of the analytical and numerical dynamic analysis results for midspan deflections of the bridge ($\bar{\xi} = 0.0\%$)

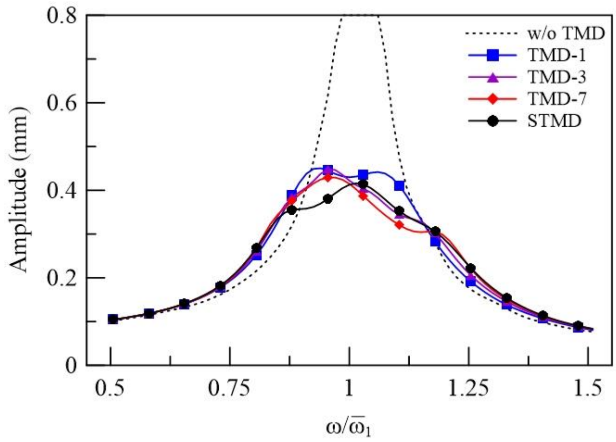

Fig. 4 Variation of the deflection amplitude of the bridge at its midspan with the normalized excitation frequency for different TMD installations

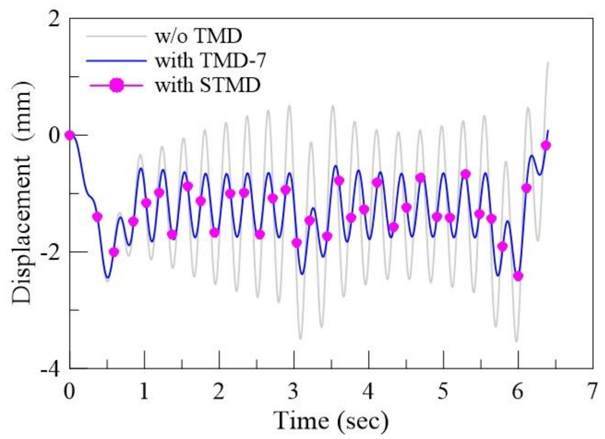

Fig. 5 Comparison of the deflection time-histories at the bridge midspan for TMD-7 and STMD devices

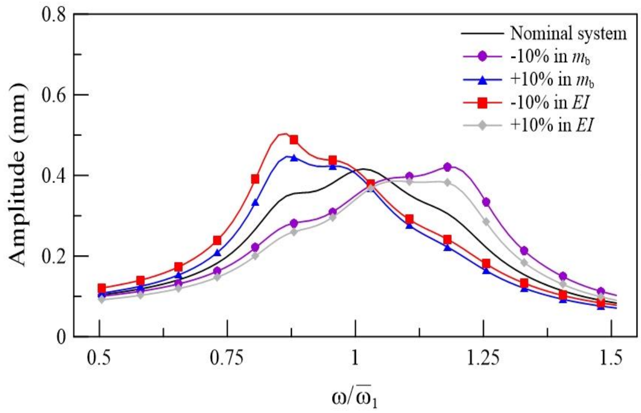

Fig. 6 Robustness of STMD device to the parameter changes in the bridge

two-node link element including spring and dashpot units. In calculations, first five modes are included. As can be seen, the results obtained for both methods agree very well with each other. STMD device shows a good performance to suppress the excessive bridge vibrations.

Fig. 4 gives a comparison of the bridge midspan displacements in frequency domain for different TMD attachments. ω and $\bar{\omega}_1$ represent the excitation frequency and the bridge fundamental frequency, respectively.

It can be seen that all TMD devices are much effective at resonance case, i.e., $\omega / \bar{\omega}_1 = 1.00$, and significantly reduce the structural response. STMD and TMD-7 devices show a better vibration attenuation performance than TMD-1 and TMD-3 devices. For multiple parallel TMDs, the structural response reduces with increasing the number of absorbers, as expected (Li *et al*. 2005, Luu *et al.* 2012).

The time histories of vertical displacement of the bridge with a speed of 66.70 m/s are shown in Fig. 5. The maximum dynamic displacements without and with STMD are 3.53 and 2.45 mm at the speed of 66.70 m/s, respectively. The vibrations are reduced by 30.59%. For TMD-7, the vibrations are reduced by 30.88%. It is noticed that STMD with two absorber units have the approximately same control effectiveness as TMD-7 with seven absorbers, therefore, the use of STMD in bridge vibration suppression may be more economical than that of multiple parallel TMDs.

Since the optimal frequency ratio is the most effective factor on the control performance of TMD devices, any deviation from the optimal or designed frequency of bridge or TMD device may cause significant performance failure. Thus, the parallel multiple TMD devices have been proposed for increasing the robustness of TMD systems.

In the following, robustness analysis has been carried out to understand how the performance of different TMD systems is affected by the changes in the parameters of the main system and the absorber.

We will first consider parameter changes in the bridge. In order to evaluate the robustness of different TMD devices to system parameters' change, let's assume that the stiffness and mass of the bridge deviate ±10% from their design values, which causes approximately ±5% deviation in the bridge fundamental frequency. Fig. 6 shows the robustness of STMD device to deviations in the bridge mass and stiffness. Here, the nominal system represents the bridge with optimally designed STMD. As can be seen from the figure, decrease in the bridge stiffness and increase in

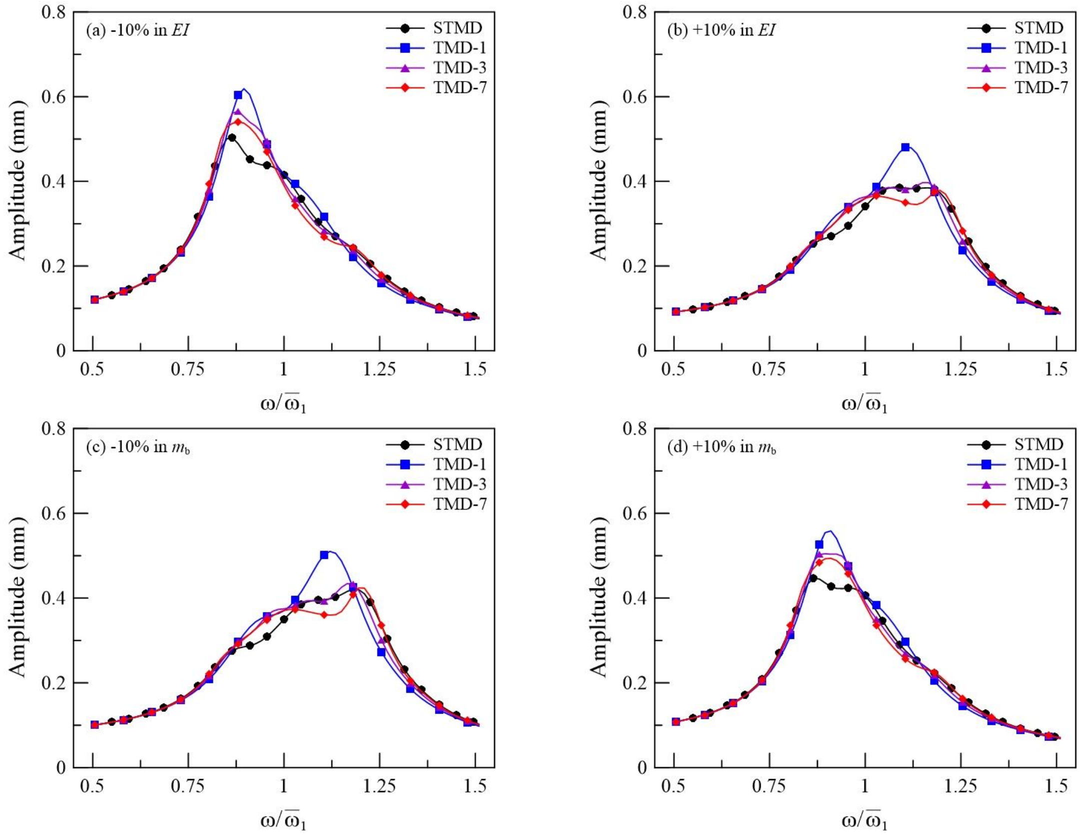

Fig. 7 Comparison of robustness of different TMDs to the parameter changes in the bridge

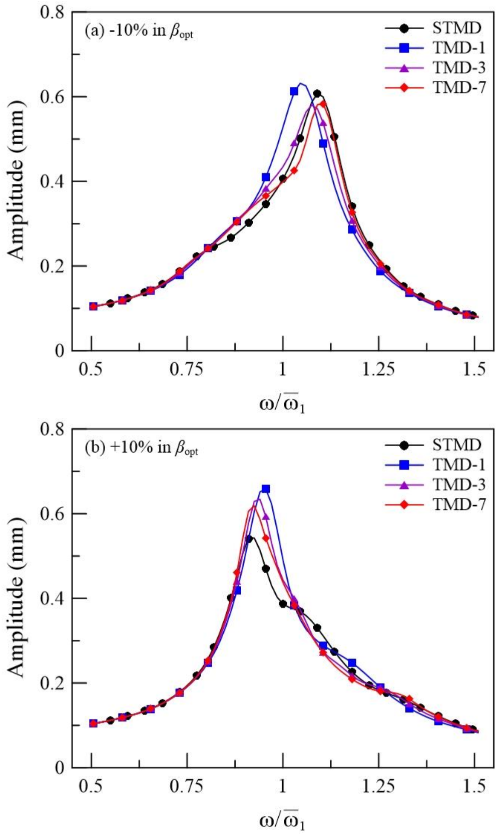

Fig. 8 Comparison of robustness of different TMDs to the parameter changes in the absorber

the bridge mass cause increasing the vibration amplitude of the bridge due to decrease in the natural frequencies. On the other hand, increase in the stiffness and decrease in the mass cause decrease in the vibration amplitude because the natural frequencies increase. Thus, the parameter changes in the bridge significantly affect the STMD's performance, especially for the former case, i.e., increase in m_b, and decrease in *EI*. Comparisons for the robustness performances of different TMDs to the bridge mass and stiffness changes are given in Fig. 7.

Here, all vibration absorbers are considered to be optimally designed. As expected, multiple TMDs (TMD-3, TMD-7 and STMD) have better performance than a single TMD (TMD-1) to detuning due to the main system's parameter changes. We can see that STMD is quite successful as much as multiple TMDs in parallel to control resonant vibrations of the bridge in case of detuning.

Another type of robustness is related with the parameter uncertainties of TMD device. Fig. 8 gives comparisons of displacement amplitudes for different TMDs with $\pm 10\%$ deviations from the optimal tuning frequency. As seen, STMD device is the most robust to detuning due the parameters' uncertainties of TMD devices.

In order for more clear understanding of robustness performances of the considered TMDs, maximum displacement amplitudes of the bridge with different absorber devices for a range of estimation error in the bridge flexural stiffness and the mass are given in Fig. 9. It can be clearly seen that the robustness of TMD-7 and

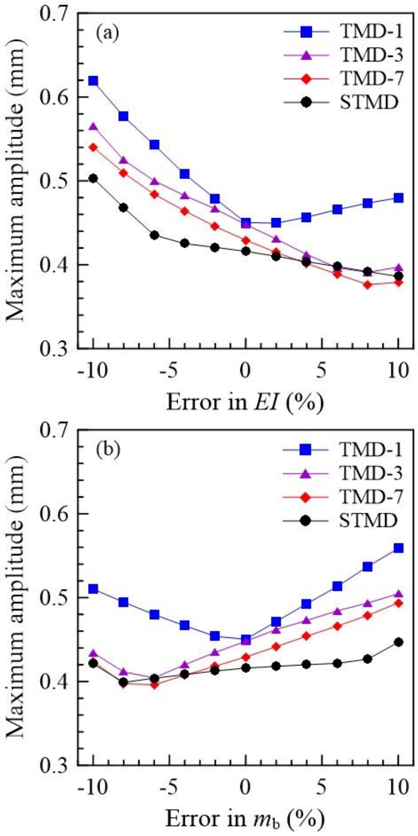

Fig. 9 Effect of deviations in the bridge parameters on the maximum amplitude for different TMDs

Table 2 Comparison of optimal parameters for TMD-1 and STMD devices, and optimal mass distribution in STMD for different μ_T(%)

μ_T(%)	Frequency ratio, β_j			Damping ratio, ξ_j (%)			m_2/m_1
	TMD-1	STMD		TMD-1	STMD		
0.5	0.983	1.006	0.980	6.1	0.0	10.5	0.027
1.0	0.978	1.019	0.963	9.0	0.0	14.6	0.055
1.5	0.967	1.038	0.950	12.6	0.0	20.3	0.097
2.0	0.958	1.046	0.941	13.4	0.0	20.5	0.118
2.5	0.945	1.073	0.925	14.3	0.0	21.0	0.124

STMD are better than that of the other TMD devices considered.

They have almost the same robustness performance. For the detuning due to TMD's parameter change, the robustness of STMD device is better than the others when the error percentage or deviation amount increases as seen in Fig. 10.

The maximum peak amplitudes or the minimized objective functions for optimal STMD versus the mass ratio is plotted and compared with optimal TMD-1 and TMD-7 devices in Fig. 11. As can be seen, when mass ratio increases, the maximum amplitude decreases, thus, the control effectiveness increases for all absorbers. STMD and TMD-7 have the same effectiveness to reduce the bridge vibrations under train loads. As also seen in the figure, the value of minimized objective function does not change significantly, thus, the total mass ratio within 2.0-2.5% range is more appropriate for obtaining good vibration reduction performance of TMD.

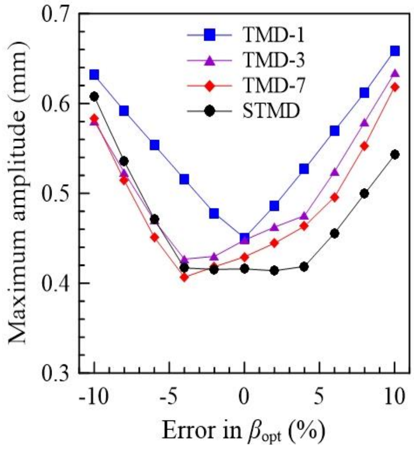

Fig. 10 Effect of deviation in the tuning frequency on the maximum deflection amplitude for different TMDs

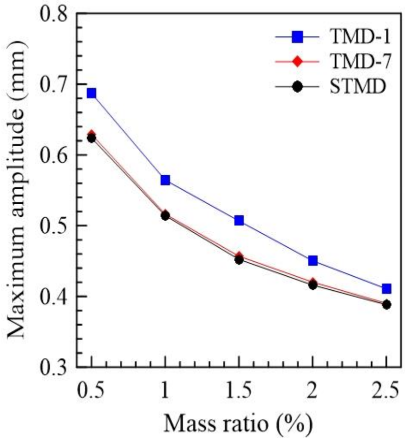

Fig. 11 Variation of the minimized objective function with the mass ratio for different TMDs

A comparison of the optimal frequency ratio β_j, and the damping ratio ξ_j (j =1, 2, …, n) for TMD-1 and STMD devices are given in Table 2 by depending on the total mass ratio μ_T (%).

When the total mass of TMD device increases, the optimal frequency ratio of TMD-1 decreases whereas its optimal damping ratio increases. For STMD device, the optimal damping of the upper absorber has been obtained to be zero. For the second or lower absorber, the optimal damping ratio increases with increasing the total TMD mass similar to the case of TMD-1.

The optimal frequency of the upper absorber increases with increasing the total TMD mass while the lower one decreases. The optimal frequency of TMD-1 always stays within two optimal frequencies of STMD device. For STMD, the damping ratio of the lower absorber is always greater than that of TMD-1. Table 2 also shows the optimal mass ratio μ=m_2/m_1 for STMD device depending on μ_T. As seen, the optimal mass distribution in STMD increases when the total mass of TMD increases. Here, it is assumed the upper TMD has always larger mass than that of the lower one.

6. Conclusion

In this study, the efficiency of STMDs on the other types of TMD devices (single TMD, and parallel multiple TMDs) in suppressing the resonant vibrations of railway bridges under HSTs is studied. According to the results obtained from the research, the following conclusions can be drawn:

- STMD with two absorber units have the similar control effectiveness as TMD-7 with seven absorbers. Therefore, the use of STMD in bridge vibration suppression may be more economical than that of multiple parallel TMDs.
- STMD device is robust to the main system parameters' change as much as MTMD devices.
- STMD device is more robust to the absorber's parameter changes than MTMD devices.
- The optimum damping ratio ξ_1 for the upper TMD unit, i.e., the larger one, in STMD is obtained to be zero. Thus, there is no need to attach a damper to the larger mass m_1.
- In design of a TMD device, the total mass ratio should be selected within 2.0-2.5% range for the best vibration control performance of TMDs.

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Appendix

In the following, **M**, **C** and **K** matrices, and **F** and **u** vectors for the different TMDs installed on the bridge are given.

(1) For STMD device consisting of two TMDs in series

$$\mathbf{M}=\begin{bmatrix}\mathbf{m}_b & \mathbf{0}\\ \mathbf{0} & \mathbf{m}_t\end{bmatrix},\quad \mathbf{C}=\begin{bmatrix}\mathbf{c}_b & \mathbf{0}\\ \mathbf{0} & \mathbf{c}_t\end{bmatrix}+(4\xi_1\omega_1\mu_1)\mathbf{\Phi},\quad \mathbf{K}=\begin{bmatrix}\mathbf{k_b} & \mathbf{0}\\ \mathbf{0} & \mathbf{k_t}\end{bmatrix}+(2\mu_1\omega_1^2)\mathbf{\Phi} \tag{A.1}$$

$$\mathbf{u}=\{q_1\quad q_2\quad \cdots\quad q_M\quad y_1\quad y_2\}^T,\quad \mathbf{F}=\{F_1\quad F_2\quad \cdots\quad F_M\quad 0\quad 0\}^T \tag{A.2}$$

where

$$\mathbf{m}_b=diag(1\quad 1\quad \cdots\quad 1),\quad \mathbf{m}_t=2\begin{bmatrix}\mu_1 & 0\\ 0 & \mu_2\end{bmatrix} \tag{A.3}$$

$$\mathbf{c}_b=diag(2\bar{\xi}_1\bar{\omega}_1\quad 2\bar{\xi}_2\bar{\omega}_2\quad \cdots\quad 2\bar{\xi}_M\bar{\omega}_M),\quad \mathbf{c}_t=4\begin{bmatrix}\xi_1\omega_1\mu_1+\xi_2\omega_2\mu_2 & -\xi_2\omega_2\mu_2\\ -\xi_2\omega_2\mu_2 & \xi_2\omega_2\mu_2\end{bmatrix} \tag{A.4}$$

$$\mathbf{k}_b=diag(\bar{\omega}_1^2\quad \bar{\omega}_2^2\quad \cdots\quad \bar{\omega}_M^2),\quad \mathbf{k}_t=2\begin{bmatrix}\mu_1\omega_1^2+\mu_2\omega_2^2 & -\mu_2\omega_2^2\\ -\mu_2\omega_2^2 & \mu_2\omega_2^2\end{bmatrix} \tag{A.5}$$

$$\mathbf{\Phi}=\begin{bmatrix}\phi_1\phi_1 & \phi_1\phi_2 & \cdots & \phi_1\phi_M & -\phi_1 & 0\\ \phi_2\phi_1 & \phi_2\phi_2 & \cdots & \phi_2\phi_M & -\phi_2 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots\\ \phi_M\phi_1 & \phi_M\phi_2 & \cdots & \phi_M\phi_M & -\phi_M & 0\\ -\phi_1 & -\phi_2 & \cdots & -\phi_M & 0 & 0\\ 0 & 0 & \cdots & 0 & 0 & 0\end{bmatrix} \tag{A.6}$$

In above expressions, F_i values can be calculated by Eq. (12). **0** denotes a matrix all elements of which are zero, and *diag*(−) is a diagonal matrix.

(2) For TMD device with n=1, 2, … parallel TMD units

$$\mathbf{M}=\begin{bmatrix}\mathbf{m}_b & \mathbf{0}\\ \mathbf{0} & \mathbf{m}_t\end{bmatrix},\quad \mathbf{C}=\begin{bmatrix}\mathbf{c}_b & \mathbf{0}\\ \mathbf{0} & \mathbf{c}_t\end{bmatrix}+\mathbf{\Phi}_1,\quad \mathbf{K}=\begin{bmatrix}\mathbf{k_b} & \mathbf{0}\\ \mathbf{0} & \mathbf{k_t}\end{bmatrix}+\mathbf{\Phi}_2 \tag{A.7}$$

$$\mathbf{u}=\{q_1\quad q_2\quad \cdots\quad q_M\quad y_1\quad \cdots\quad y_n\}^T,\quad \mathbf{F}=\{F_1\quad F_2\quad \cdots\quad F_M\quad 0\quad \cdots\quad 0\}^T \tag{A.8}$$

where

$$\mathbf{m}_b=diag(1\quad 1\quad \cdots\quad 1),\quad \mathbf{m}_t=diag(2\mu_1\quad 2\mu_2\quad \cdots\quad 2\mu_n) \tag{A.9}$$

$$\mathbf{c}_b=diag(2\bar{\xi}_1\bar{\omega}_1\quad 2\bar{\xi}_2\bar{\omega}_2\quad \cdots\quad 2\bar{\xi}_M\bar{\omega}_M),\quad \mathbf{c}_t=diag(4\xi_1\omega_1\mu_1\quad 4\xi_2\omega_2\mu_2\quad \cdots\quad 4\xi_n\omega_n\mu_n) \tag{A.10}$$

$$\mathbf{k}_b=diag(\bar{\omega}_1^2\quad \bar{\omega}_2^2\quad \cdots\quad \bar{\omega}_M^2),\quad \mathbf{k}_t=diag(2\mu_1\omega_1^2\quad 2\mu_2\omega_2^2\quad \cdots\quad 2\mu_n\omega_n^2) \tag{A.11}$$

$$\mathbf{\Phi}_1=\left[\begin{array}{c:c} \mathrm{A}\begin{bmatrix}\phi_1\phi_1 & \cdots & \phi_1\phi_M\\ \vdots & \ddots & \vdots\\ \phi_M\phi_1 & \cdots & \phi_M\phi_M\end{bmatrix} & A_1\\ \hdashline A_2 & \begin{bmatrix}0 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & 0\end{bmatrix}\end{array}\right] \tag{A.12}$$

$$A_1=\begin{bmatrix}-4\xi_1\omega_1\mu_1\phi_1 & \cdots & -4\xi_n\omega_n\mu_n\phi_1\\ \vdots & \ddots & \vdots\\ -4\xi_1\omega_1\mu_1\phi_M & \cdots & -4\xi_n\omega_n\mu_n\phi_M\end{bmatrix},\quad A_2=\begin{bmatrix}-4\xi_1\omega_1\mu_1\phi_1 & \cdots & -4\xi_1\omega_1\mu_1\phi_M\\ \vdots & \ddots & \vdots\\ -4\xi_n\omega_n\mu_n\phi_1 & \cdots & -4\xi_n\omega_n\mu_n\phi_M\end{bmatrix} \tag{A.13}$$

$$\mathbf{\Phi}_2=\left[\begin{array}{c:c} \mathrm{B}\begin{bmatrix}\phi_1\phi_1 & \cdots & \phi_1\phi_M\\ \vdots & \ddots & \vdots\\ \phi_M\phi_1 & \cdots & \phi_M\phi_M\end{bmatrix} & B_1\\ \hdashline B_2 & \begin{bmatrix}0 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & 0\end{bmatrix}\end{array}\right] \tag{A.14}$$

$$B_1=\begin{bmatrix}-2\mu_1\omega_1^2\phi_1 & \cdots & -2\mu_n\omega_n^2\phi_1\\ \vdots & \ddots & \vdots\\ -2\mu_1\omega_1^2\phi_M & \cdots & -2\mu_n\omega_n^2\phi_M\end{bmatrix},\quad B_2=\begin{bmatrix}-2\mu_1\omega_1^2\phi_1 & \cdots & -2\mu_1\omega_1^2\phi_M\\ \vdots & \ddots & \vdots\\ -2\mu_n\omega_n^2\phi_1 & \cdots & -2\mu_n\omega_n^2\phi_M\end{bmatrix} \tag{A.15}$$

where $\mathrm{A}=\sum_{j=1}^{n}4\xi_j\omega_j\mu_j$ and $\mathrm{B}=\sum_{j=1}^{n}2\mu_j\omega_j^2$.