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An integrated particle swarm optimizer for optimization of truss structures with discrete variables

Ali Mortazavi[1a], Vedat Toğan[*2] and Ayhan Nuhoğlu[1b]

[1]*Department of Civil Engineering, Ege University, 35100 İzmir, Turkey*
[2]*Department of Civil Engineering, Karadeniz Technical University, 61080 Trabzon, Turkey*

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Abstract. This study presents a particle swarm optimization algorithm integrated with weighted particle concept and improved fly-back technique. The rationale behind this integration is to utilize the affirmative properties of these new terms to improve the search capability of the standard particle swarm optimizer. Improved fly-back technique introduced in this study can be a proper alternative for widely used penalty functions to handle existing constraints. This technique emphasizes the role of the weighted particle on escaping from trapping into local optimum(s) by utilizing a recursive procedure. On the other hand, it guaranties the feasibility of the final solution by rejecting infeasible solutions throughout the optimization process. Additionally, in contrast with penalty method, the improved fly-back technique does not contain any adjustable terms, thus it does not inflict any extra ad hoc parameters to the main optimizer algorithm. The improved fly-back approach, as independent unit, can easily be integrated with other optimizers to handle the constraints. Consequently, to evaluate the performance of the proposed method on solving the truss weight minimization problems with discrete variables, several benchmark examples taken from the technical literature are examined using the presented method. The results obtained are comparatively reported through proper graphs and tables. Based on the results acquired in this study, it can be stated that the proposed method (integrated particle swarm optimizer, iPSO) is competitive with other metaheuristic algorithms in solving this class of truss optimization problems.

Keywords: optimization; truss structures; particle swarm optimization, weighted particle, constraint handling

1. Introduction

During the last five or six decades, the methods which focus directly on the functions domain and numerical evaluation of the functions instead of evaluating their gradients, have been gained considerable interest to researchers. They are commonly categorized as the metaheuristic optimization techniques. These methods provide mathematical simplicity, they are, therefore, proper to apply on the different complex engineering optimization fields (Coello-Coello 2002). For these reasons, many new metaheuristic techniques have been developed and improved by researchers. Among others, the genetic algorithm (GA), which is based on the concept of natural selection, can be recognized as the most widely used optimization method. From the emergence of the GA up to now, it has been employed in the various structural optimization applications (Rajeev and Krishnamoorthy 1992, Hajela and Lee 1995, Erbatur and Hasançebi 2000, Deb and Gulati 2001, Toğan and Daloğlu 2006, He and Hwang 2007, Gholizadeh *et al.* 2008, Rahami *et al.* 2008, Toğan and Daloğlu 2008, Li 2015, Alaimo *et al.* 2016, Dizangian and Ghasemi 2016). So far, many different techniques inspired from the nature have been consecutively proposed and widely applied in distinct optimization problems. Among several techniques presented in this class, one can list water wave optimization (WWO), different variants of particle swarm optimizer (PSO), bat-inspired algorithm (BI), ant colony optimization (ACO), artificial bee colony optimization (ABC), water cycle optimization (WCO) and mine blast algorithm (MBA) (Zheng 2015, Sadollah *et al.* 2015, Fan and Yan 2014, Hasançebi *et al.* 2013, Nickabadi *et al.* 2011, Camp and Bichon 2004, Sönmez 2011). In this regard, the efforts on developing optimization algorithms mimicking the biology, music, physics, social sciences, and natural phenomena are still being continued. The rationale behind this interest is the abilities of these methods on solving complicated optimization problems with different objective functions over cost, time, planning, route finding and etc.

The particle swarm optimization (PSO) is one of the optimization algorithms developed by imitating the social behavior of the flock of birds, bees, and fish. It tries to generate mathematical model by considering these animals' physical movements to avoid predators, and to seek the best food sources (Kennedy and Eberhart 1995). After recognizing affirmative features of the PSO, e.g. faster convergence rate and easier programming, it was employed in various fields. However, it was observed from these applications that the standard PSO has some drawbacks such as staggering of the convergence in later stages of the process. To relieve the shortcomings of the standard PSO, the different forms of this technique were developed to

*Corresponding author, Associate Professor
E-mail: togan@ktu.edu.tr
[a]Ph.D. Student
[b]Assistant Professor

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improve its performances (Fan and Yan 2014, Nickabadi *et al.* 2011, Li *et al.* 2014). He *et al.* (2004) improved the standard PSO algorithm employing the fly-back mechanism to handle the constraints of the mechanical design optimization problems.

Generally, in all metaheuristic methods based on the swarm intelligence, the process of using the information stored in the proper agents (e.g., global best particle) in the colony is very important (Camp and Bichon 2004, Kaveh and Talatahari 2009b, Chen and Zhao 2009, Hasançebi 2008, Perez and Behdinan 2007). However, putting all workload just on some agents means to waste the probable searching capacity of the other agents. So, incorporating of higher number of agents on the navigation of the colony over the search space can play a determinative role on the performance of a metaheuristic method. In this respect, this study presents an integrated particle swarm optimizer (iPSO) with the weighted particle concept and the improved fly-back technique. The iPSO improves the search capability of the standard PSO algorithm by adding these approaches. In this study, the improved fly-back is introduced and used to handle the problem constraints. The improved fly-back approach uses a recursive algorithm on the weighted particle to hold the violated agent(s) into the feasible search space during the optimization process. Thus, it guarantees the feasibility of the final solution. On the other hand, since the weighted particle is the weighted average of all particles, such an approach increases the participation level of all particles in navigating of the swarm. Also, in contrast with standard PSO, which applies just two significant particles (i.e., global best and best prior position of the current particle) to conduct the swarm, iPSO additionally uses the weighted particle to form flying paths for particles. In later sections, the corresponding terms of the proposed method are illustrated in detail. Also, the performance of the proposed iPSO is tested through several trusses weigh minimization problems.

2. Particle swarm optimization

Particle swarm optimization (PSO) inspired form the behavior of animals' colony in the nature like birds and fishes (Kennedy and Eberhart 1995). In this method, a group of particles called swarm are first randomly generated. Each particle is a potential solution to the problem. The swarm iteratively flies over the problem domain for a unit of time. At the end of iteration, every particle finds its own new position. Subsequently, the quality of recent positions of each particle is evaluated via calculating a proper objective function. In the end of iteration, the best particle's location is identified. The position updating process of the *i*th particle in iteration is formulated as below

$$^{t+1}V_i = w\,{}^{t}V_i + c_1 r_1 (x_i^P - {}^{t}x_i) + c_2 r_2 (x^G - {}^{t}x_i) \tag{1}$$

$$^{t+1}x_i = {}^{t}x_i + {}^{t+1}V_i \tag{2}$$

in which, c_1 and c_2 are two positive constants of acceleration, r_1 and r_2 are two uniform random values selected from U (0, l), x_i and V_i represent the position and the velocity of the *i*th particle, respectively. w is the inertia to consider the effect of initial velocity of particle. x_i^P is the previous best position of ith particle so called Pbest and x^G is the global best position among the swarm so called Gbest. Finally, superscripts "t" and "t+1" denote the current step and the next step, respectively.

2.1 Weighted particle

The weighted particle is defined as the weighted average of all particles. Indeed, it is the gravity center of Pbest's swarm, thus, its position is determined as below (Li *et al.* 2014)

$$x^W = INT\left(\sum_{i=1}^{M} \bar{c}_i^W x_i^P\right) \tag{3}$$

$$\bar{c}_i^W = \hat{c}_i^W \Big/ \sum_{i=1}^{M} \hat{c}_i^W \tag{4}$$

$$\hat{c}_i^W = \frac{\max\limits_{1\le k\le M}\left(f(x_k^P)\right) - f(x_i^P) + \varepsilon}{\max\limits_{1\le k\le M}\left(f(x_k^P)\right) - \min\limits_{1\le k\le M}\left(f(x_k^P)\right) + \varepsilon} \tag{5}$$
$$i = 1, 2, ..., M$$

In Eqs. (3)-(5), M is the number of particles, the operator INT(.) returns the integer part of any variable, x^W is the position of the weighted particle, $\hat{c}_i^W$ is the weighted constant of each particle. f(.) indicates the objective function of the problem. $\max\limits_{1\le k\le M}\left(f(x_k^P)\right)$ and $\min\limits_{1\le k\le M}\left(f(x_k^P)\right)$ are the worst and best fitness values in the Pbest, respectively. ε (=0.001) is the small positive number to prevent division by zero condition.

2.2 Improved fly-back technique to handle constraints

In many engineering problems, several boundary conditions should be satisfied to obtain a feasible and meaningful solution. The common way to handle the constraints of the optimization problems is to apply a proper penalty function to penalize the infeasible solutions. Two drawbacks can be considered for such an approach. First, it requires appropriate penalty function which has own tuning parameter(s). Adjusting these parameters is tedious especially when the main metaheuristic algorithm also has its own ad hoc parameters. Second, it does not guarantee the feasibility of the final solution. Some investigations showed that this technique might cause a reduction in the efficiency of the applied optimization algorithm (Coello-Coello 2002). The fly-back mechanism (He *et al.* 2004) is one of the methods proposed to relieve this problem. In this study, the fly-back mechanism is improved with the concept of weighted particle. For implementing this improvement initially, the problem constraints are divided into two distinct categories in this study: characteristic and numeric. The characteristic constraints are those which require a structural analysis to recognize their violations (e.g., constraints on allowable stress or displacement). The numeric constraints are those which do not require any structural analysis to recognize their violations (e.g., constraints on cross sectional areas). By these definitions, improved fly-back can be implemented via the following steps:

Step 1: Initially, the problem constraints are categorized into numeric and characteristic constraints.

Step 2: During the optimization process, if a particle violates the numeric boundary conditions, the corresponding violated components in the particle are then replaced with same components in the weighted particle.

Step 3: The new particle is verified. If it is feasible and gives a better objective value (i.e., has a lower weight), it is replaced with old one. If it is not feasible, the particle is rest to its prior best location saved in the Pbest matrix.

By adopting such an approach, three main purposes are achieved: (i) the survival chance of the violated particles is increased, (ii) the feasibility of the final solution is guaranteed, and (iii) the role of the weighted particle is highlighted. Former two increase the global search ability of the algorithm (i.e., improve exploration), and the last one by emphasizing the role of the weighted particle leads to higher utilization of this specific particle in guidance of the swarm (i.e., improves exploitation).

3. Discrete truss optimization problems

Generally, during a structural optimization, it is attempted to minimize the weight of structure. Hence, in truss optimization problems, the weight of the truss system is taken as the objective function of the problem. However, some constraints should be satisfied to achieve a feasible solution. These constraints can be mathematically expressed with some inequalities. Under these circumstances, a structural optimization problem and its constraints are formulated as below

$$X = \{x_1, x_2, \ldots, x_j, \ldots, x_d\} \quad d = 1, 2, \ldots, D \tag{6}$$

$$g_q(X) \le 0 \quad q = 1, 2, \ldots, Q \tag{7}$$

in which, $x_1, x_2, \ldots, x_j, \ldots, x_d$ are design variables (e.g., components of the particle), X is a vector (e.g., a particle), and D is the problem dimension (i.e. number of design variables). Also, g_q and Q are the constraints of the problem and their number, respectively. In an optimization problem, the main goal is to find the X vector, such that it produces the minimum value of the fitness function, $f(X)$, while satisfies the all $g_q(X)$ limitations. In a discrete optimization problem, the particles can fly continuously over the domain, while they can lie just on the specified integer points. This continues movement is converted into the discrete point by employing an appropriate mapping function. It means that, in a discrete optimization problem, x_j should be an integer number that refers to the sequence number of the specific variable in a predefined set (e.g., S=set of acceptable cross-sectional area numbers). The mapping process is mathematically shown as

$$S = \{s_1, s_2, \cdots, s_i, \cdots, s_q\} \qquad 1 \le i \le q \tag{8}$$

$$h(x_j) = s_i \tag{9}$$

in which, $h(.)$ is the mapping function. It means, x_j is an integer number which $x_j \in \{1,2,3,\ldots,q\}$ and $h(x_j) \in S$.

4. Integrated particle swarm optimization (iPSO)

In the standard PSO algorithm, when a particle stands very close to its own previous best position and/or to the global best particle, the guidance role of one or both particles can be highly reduced or even be vanished. It means if either one or both points are trapped into a local optimum, any particles located much closer to them are also trapped into the local optimum. To prevent such a condition, iPSO uses the weighted particle as a third guidance point. Accordingly, the corresponding formulation for iPSO is formulated for the discrete optimization problems as

$$\begin{aligned} &\text{if } \text{rand}_{0i} \le \alpha \quad \rightarrow \; {}^{t+1}v_i = 0 \\ &\qquad {}^{t+1}X_i = INT\left({}^{t}X_i + \varphi_{4i}\left({}^{t}X^W - {}^{t}X_i\right)\right) \end{aligned} \tag{10}$$

$$\begin{aligned} &\text{if } \text{rand}_{0i} > \alpha \quad \rightarrow \\ &{}^{t+1}v_i = w_i \times {}^{t}v_i + \left(\varphi_{1i} + \varphi_{2i} + \varphi_{3i}\right)\left({}^{t}X_j^P - {}^{t}X_i\right) \\ &\qquad + \varphi_{2i}\left({}^{t}X^G - {}^{t}X_j^P\right) + \varphi_{3i}\left({}^{t}X^W - {}^{t}X_j^P\right) \\ &{}^{t+1}X_i = INT\left({}^{t}X_i + {}^{t+1}v_i\right) \end{aligned} \tag{11}$$

where $\varphi_{1i} = C_1$ x rand_{1i}, $\varphi_{2i} = C_2$ x rand_{2i}, $\varphi_{3i} = C_3$ x rand_{3i}, $\varphi_{4i} = C_4$ x rand_{4i}. The operator $INT(.)$ returns the integer part of any variable. Superscripts "t" and "t+1" denote current and next time steps, respectively. Also, ${}^{t+1}v_i$ is the updated velocity, w_i is the inertia factor of current velocity, and ${}^{t}v_i$ is the current velocity of i^{th} particle. $C_1 = -(\varphi_{2i} + \varphi_{3i})$, C_2=2, C_3=1, and C_4=2 are accelerator factors, rand_{ki} where $k \in \{0,1,2,3,4\}$, is a random number selected from [0, 1] interval, ${}^{t}X_j^P$ is randomly selected particle from the current X^P vector. Also, ${}^{t}X^G$ is the global best particle up to recent step at ${}^{t+1}X_i$ and ${}^{t}X_i$ which are the updated and the current position of the i^{th} particle, respectively. Also, ${}^{t}X^W$ is the weighted particle calculated for the current step. Li *et al.* (2014) tested the performances of this formulation on optimizing the scalar functions with continuous search spaces. Based on those tests, in each iteration, w is randomly selected from [0.5, 0.55] and α=0.4.

The iPSO uses improved fly-back method to handle the constraints of the problem and to keep all solutions in the feasible area during the whole optimization process. The

Table 1 The pseudo-code for iPSO

t=1;
Initialize positions and velocities of all particles randomly;
WHILE (the termination conditions are not occurred)
  Calculate the weighed particle x^w using Eqs. (3)-(5)
    FOR (each particle in the swarm)
      IF ($\mathrm{rand}_{0i} \leq \alpha$)
        Generate velocity vector according Eq. (10)
      ELSEIF ($\mathrm{rand}_{0i} > \alpha$)
        Generate velocity vector according Eq. (11)
      END IF
        Update the position of the current particle (x_i)
    END FOR
  Numeric constraints violation controlling:
    If any component of current particle violates the numeric constraint (e.g., cross sections), replace it by another one, which is selected from corresponding component stored in the weighted particle.
  Characteristic constraints violation controlling:
    If the current particle violates characteristic constraints (e.g., allowable deflections), reset it to its previous best position stored in the Pbest swarm.
  Evaluate objective function for the current particle $f(x_i)$ and also for the weighted particle $f(x^w)$
  IF ($f(x_i)<f(X_i^{Pbest})$) (i.e., X_i^{Pbest} is the previous best position of the current particle)
    Update Pbest (X^P) (i.e., replace X_i^{Pbest} with x_i)
  ELSEIF ($f(x_i)<f(x^G)$)
    Set $x^G = x_i$
  ELSE IF ($f(x^w) < f(x^G)$)
    Set $x^G = x^w$
  END IF
  $t = t + 1$
END WHILE

weighted particle improves the flight path of the particles flying excessively close to either own prior best point stored in Pbest or global best point (Gbest). On the other hand, since the weighted particle is the weighted average of all particles, it makes possible to share the information between all particles in the swarm. The pseudo-code for the proposed iPSO method is shown in Table 1.

5. Numerical examples

Design examples of 10, 25 and 72 and 244 bar truss structures are solved to demonstrate the effectiveness and robustness of the proposed iPSO. The design examples considered were already optimized by researchers using different algorithms, i.e., genetic algorithm (GA), harmony search (HS), particle swarm optimization and heuristic particle swarm optimization (HPSO), mine blast algorithm (MBA) and particle swarm ant colony optimization (DHPSACO). The results obtained using the iPSO and the reported ones are tabulated to make a comparison among them.

In this study, the optimizations process is repeated 20 times by the swarms consisting of 10 and 20 particles for the first three and the last examples, respectively. The best solution is presented as the result of the corresponding

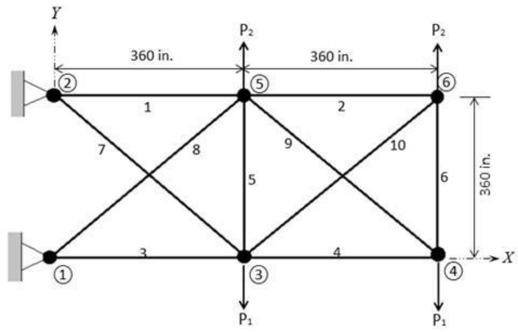


Fig. 1 10-bar truss structure

examples together with the statistical results of 20 independent runs. The algorithm is coded in MATLAB environment, and all computations are performed in a computer with the CORE i7 @ 2.2 GHz and 6.0 MB of RAM. The structural analysis is performed in terms of a program based on the finite element method (FEM).

5.1 10 bar planar truss structure

A 10-bar truss structure shown in Fig. 1 is studied as a first example for comparison purpose. This truss was optimized by Wu and Chow (1995) using GA, by Li et al. (2009) using PSO, PSOPC, and HPSO, and Sadollah et al. (2012) using MBA. Fig. 1 also illustrates the geometry, supporting, and loading conditions for the truss.

The material density is 0.1 lb/in^3, and Young's modulus is 10000 ksi. The stress limitations of the members are considered as ±25 ksi, and the displacement limitations of all nodes in both main directions (x and y) are ±2 in. The loading conditions $P1$=100 and $P2$=0 kips imposed to nodes 3, 4, 5 and 6 in y direction are considered. Two cases of discrete variables are studied for this example. The discrete variables are selected from the set D={1.62, 1.80, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80, 4.97, 5.12, 5.74,7.22, 7.97, 11.50, 13.50, 13.90, 14.20, 15.50, 16.00, 16.90, 18.80, 19.90, 22.00, 22.90, 26.50, 30.00, 33.50} in^2 for Case 1, and from the set D={0.1, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0, 6.5, 7.0, 7.5, 8.0, 8.5, 9.0, 9.5, 10.0, 10.5, 11.0, 11.5, 12.0, 12.5, 13.0, 13.5, 14.0, 14.5, 15.0, 15.5, 16.0, 16.5, 17.0, 17.5, 18.0, 18.5, 19.0, 19.5, 20.0, 20.5, 21.0, 21.5, 22.0, 22.5, 23.0, 23.5, 24.0, 24.5, 25.0, 25.5, 26.0, 26.5, 27.0, 27.5, 28.0, 28.5, 29.0, 29.5, 30.0, 30.5, 31.0, 31.5} in^2 for Case 2.

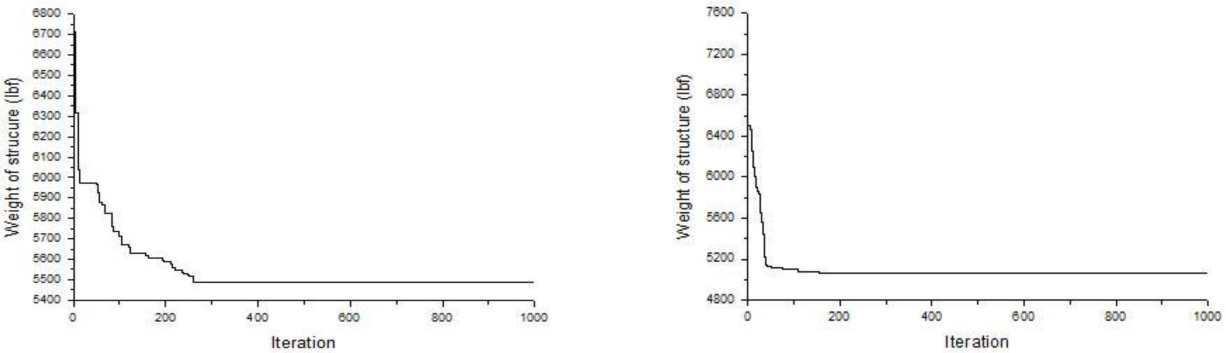
A maximum number of iteration is taken into account as 1000 in the design process of this example. Tables 2-3 show the best designs obtained by using iPSO and the methods employed in the related works for Case 1 and 2, respectively.

The result obtained in this study using the iPSO produces lighter design than other methods for Case 1. For Case 2, the iPSO finds lighter design than PSO, PSOPC, HPSO, slightly difference design than MBA, and heavier than GA.

Typical design histories for the best design of the 10-bar truss are illustrated in Figs. 2(a)-(b) for Case 1 and Case 2, respectively.

Table 2 Optimal design comparison for the 10-bar planar truss structure (Case 1)

Variables (in^2)	Wu and Chow (1995) GA	Li *et al.* (2009) PSO	Li *et al.* (2009) PSOPC	Li *et al.* (2009) HPSO	Sadollah *et al.* (2012) MBA	This study iPSO
A_1	33.50	30.00	30.00	30.00	30.00	33.50
A_2	1.62	1.62	1.80	1.62	1.62	1.62
A_3	22.00	30.00	26.50	22.90	22.90	22.90
A_4	15.50	13.50	15.50	13.50	16.90	15.50
A_5	1.62	1.62	1.62	1.62	1.62	1.62
A_6	1.62	1.80	1.62	1.62	1.62	1.62
A_7	14.20	11.50	11.50	7.97	7.97	7..97
A_8	19.90	18.80	18.80	26.50	22.90	22.00
A_9	19.90	22.00	22.00	22.00	22.90	22.00
A_{10}	2.62	1.80	3.09	1.80	1.62	1.62
Best weight (lb)	5613.84	5581.76	5593.44	5531.98	5507.75	5491.70
Worst weight (lb)	-	-	-	-	5536.965	5517.12
Mean weight (lb)	-	-	-	-	5527.296	5496.33
Standard deviation (lb)	-	64.079	12.842	3.840	11.38	5.75
Objective function evaluations (OFEs)	-	50000	50000	20150	3600	2480



(a) Case 1 (b) Case 2

Fig. 2 Convergence history of the 10-bar truss

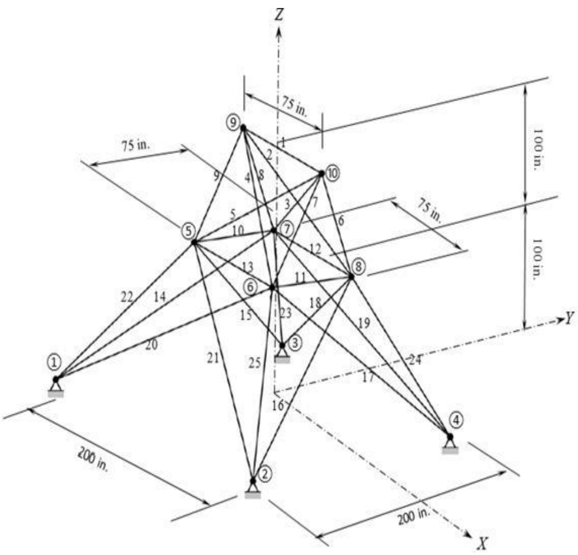


Fig. 3 25-bar truss structure

5.2 25-bar spatial truss structure

Another problem to test the performance of the iPSO is the 25-bar space truss shown in Fig. 3. Members of the truss are divided into 8 groups to reduce the search space. The displacement limitations for all nodes are ±0.35 in in *x*, *y*, and *z* directions. The discrete variables are chosen from the set *D*={0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.6, 2.8, 3.0, 3.2, 3.4} in^2 for Case 1; from the set *D*={0.01, 0.4, 0.8, 1.2, 1.6, 2.0, 2.4, 2.8, 3.2, 3.6, 4.0, 4.4, 4.8, 5.2, 5.6, 6.0} in^2 for Case 2; and from the American Institute of Steel Construction (AISC) Code given in Table 4 for Case 3.

Young's modulus is 10000 ksi, the stress limitations of the members are adopted as ±40 ksi, and the material density is 0.1 lb/in^3. Truss structure is subjected to three load cases shown in Table 5.

The best solutions with weights of 484.85 lb, 551.61 lb and 540.03 lb obtained by using iPSO and the other algorithms are presented in Tables 6, 7, and 8, respectively, for Case 1, Case 2, and Case 3. 500 iterations are taken as the maximum iteration number.

For Case 1, the result of iPSO is similar to HS (Lee and Geem 2004), HPSO (Li *et al.* 2009), and MBA (Sadollah *et al.* 2012), but in Cases 2 and 3 the designs obtained by iPSO are better than all results achieved by distinct

Table 3 Optimal design comparison for the 10-bar planar truss structure (Case 2)

Variables (in^2)	Wu and Chow (1995) GA	Li *et al.* (2009) PSO	Li *et al.* (2009) PSOPC	Li *et al.* (2009) HPSO	Sadollah *et al.* (2012) MBA	This study iPSO
A_1	33.50	24.50	25.50	31.50	29.50	29.50
A_2	0.50	0.10	0.10	0.10	0.10	0.10
A_3	16.50	22.50	23.50	24.50	24.00	23.00
A_4	15.00	15.50	18.50	15.50	15.00	16.00
A_5	0.10	0.10	0.10	0.10	0.10	0.10
A_6	0.10	1.50	0.50	0.50	0.50	0.50
A_7	0.50	8.50	7.50	7.50	7.50	7.50
A_8	18.00	21.50	21.50	20.50	21.50	21.00
A_9	19.50	27.50	23.50	20.50	21.50	22.00
A_{10}	0.50	0.10	0.10	0.10	0.10	0.10
Best weight (lb)	4217.30	5243.71	5133.16	5073.51	5067.33	5067.30
Worst weight (lb)	-	-	-	-	-	5072.98
Mean weight (lb)	-	-	-	-	-	5067.11
Standard deviation (lb)	-	-	-	-	0	1.08
Objective function evaluations (OFEs)	-	50000	50000	25000	3000	2050

Table 4 The available cross-section areas from the AISC code

No	in^2	No	in^2	No	in^2	No	in^2
1	0.111	17	1.563	33	3.840	49	11.50
2	0.141	18	1.620	34	3.870	50	13.50
3	0.196	19	1.800	35	3.880	51	13.90
4	0.250	20	1.990	36	4.180	52	14.20
5	0.307	21	2.130	37	4.220	53	15.50
6	0.391	22	2.380	38	4.490	54	16.00
7	0.442	23	2.620	39	4.590	55	16.90
8	0.563	24	2.630	40	4.800	56	18.80
9	0.602	25	2.880	41	4.970	57	19.90
10	0.766	26	2.930	42	5.120	58	22.00
11	0.785	27	3.090	43	5.740	59	22.90
12	0.994	28	1.130	44	7.220	60	24.50
13	1.000	29	3.380	45	7.970	61	26.50
14	1.228	30	3.470	46	8.530	62	28.00
15	1.266	31	3.550	47	9.300	63	30.00
16	1.457	32	3.630	48	10.850	64	33.50

Table 5 The loads Cases 1, 2 and 3 for the 25-bar truss structure

	Load (kips)								
	Case 1			Case 2			Case 3		
Nodes	P_x	P_y	P_z	P_x	P_y	P_z	P_x	P_y	P_z
9	1	-10	-10	0	20	-5	1	10	-5
10	0	-10	-10	0	-20	-5	0	10	-5
7	0.5	0	0				0.5	0	0
5	0.6	0	0				0.5	0	0

optimization methods.

Figs. 4(a)-(c) show the typical weight histories and the convergence rates of the 25-bar truss during the iPSO process.

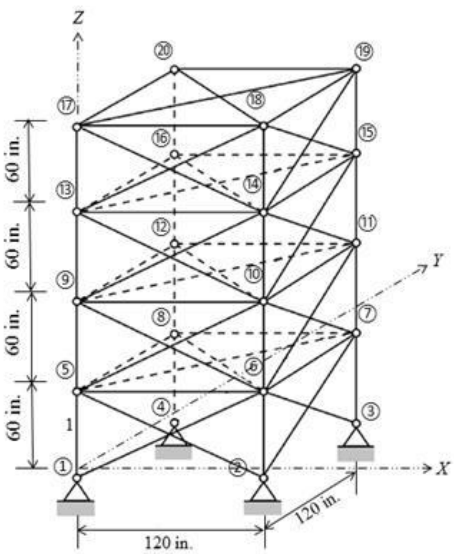


Fig. 4 72-bar truss structure

5.3 72-bar space truss structure

The 72-bar spatial truss structure and its configuration are demonstrated in Fig. 5 with nodes numbering scheme. The material density is 0.1 lb/in^3 and Young's modulus is 10000 ksi for the all members.

The stress limitation is ±25 ksi, and the displacement limitation is ±0.25 for all nodes in three directions. To reduce the size of the search space, the members are divided into 16 groups in symmetrical manner. There are two loading conditions for this example shown in Table 9. For Case 1, the discrete variables are chosen from the set D={0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8,0.9,1.0, 1.1,1.2, 1.3,1.4, 1.5,1.6, 1.7, 1.8, 1.9, 2.0, 2.1,2.2, 2.3,2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 3.0, 3.1, 3.2} in^2. However, the set taken from American Institute of Steel Construction (AISC) Code and shown in Table 4 is considered for Case 2.

Table 6 Optimal design comparison for the 25-bar spatial truss structure (Case 1)

Variables (in^2)	Wu and Chow (1995) GA	Li (2015) GA	Lee and Geem (2004) HS	Li *et al.* (2009) PSO	Li *et al.* (2009) PSOPC	Li *et al.* (2009) HPSO	Sadollah *et al.* (2012) MBA	This study iPSO
A_1	0.10	0.10	0.10	0.40	0.10	0.10	0.10	0.10
A_2 - A_5	0.50	1.80	0.30	0.60	1.10	0.30	0.30	0.30
A_6 - A_9	3.40	2.30	3.40	3.50	3.10	3.40	3.40	3.40
A_{10} - A_{11}	0.10	0.20	0.10	0.10	0.10	0.10	0.10	0.10
A_{12} - A_{13}	1.50	0.10	2.10	1.70	2.10	2.10	2.10	2.10
A_{14} - A_{17}	0.90	0.80	1.00	1.00	1.00	1.00	1.00	1.00
A_{18} - A_{21}	0.60	1.80	0.50	0.30	0.10	0.10	0.50	0.50
A_{22} - A_{25}	3.40	3.00	3.40	3.40	3.50	3.50	3.40	3.40
Best weight (lb)	486.29	546.01	484.85	486.54	490.16	484.85	484.85	484.85
Worst weight (lb)	-	-	-	-	-	-	485.048	484.85
Mean weight (lb)	-	-	-	-	-	-	484.885	484.85
Standard deviation (lb)	-	-	-	256.7491	1.04208	0.02664	7.2E-02	0
OFEs	-	-	-	50000	50000	3750	2150	620

Table 7 Optimal design comparison for the 25-bar spatial truss structure (Case 2)

Variables (in^2)	Wu and Chow (1995) GA	Li (2015) GA	Lee and Geem (2004) HS	Li *et al.* (2009) PSO	Li *et al.* (2009) PSOPC	Li *et al.* (2009) HPSO	Sadollah *et al.* (2012) MBA	Kaveh and Talatahari (2009a) DHPSACO	This study iPSO
A_1	0.40	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
A_2 - A_5	2.00	1.60	2.00	2.00	2.00	2.00	2.00	1.60	1.60
A_6 - A_9	3.60	3.60	3.60	3.60	3.60	3.60	3.60	3.20	3.20
A_{10} - A_{11}	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
A_{12} - A_{13}	0.01	0.01	0.01	0.40	0.01	0.01	0.01	0.01	0.01
A_{14} - A_{17}	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.8	0.8
A_{18} - A_{21}	2.00	2.00	1.60	1.60	1.60	1.60	1.60	2.0	2.0
A_{22} - A_{25}	2.40	2.40	2.40	2.40	2.40	2.40	2.40	2.4	2.4
Best weight (lb)	563.52	568.69	560.59	566.44	560.59	560.59	560.59	551.61	551.61
Worst weight (lb)	-	-	-	-	-	-	560.59	-	551.61
Mean weight (lb)	-	-	-	-	-	-	560.59	-	551.61
Standard deviation (lb)	-	-	-	-	-	-	0	-	0
OFEs	-	-	-	50000	1500	7500	950	5000	820

Table 10 and Fig. 6(a), respectively, show the results and the weight history for 72-bar truss obtained by iPSO for Case 1. In addition, numerical solutions and typical weight history obtained for Case 2 are demonstrated in Table 11 and Fig. 6(b), respectively.

By comparing the results reported in Tables 10-11 for both cases, it can be observed that the results obtained with iPSO are similar to MBA for Case 1 and better than the others for Case 2.

5.4 244-bar space truss tower structure

A 244-bar transmission tower displayed in Fig. 7 is studied as the last example to demonstrate the performance of the proposed method. The members of this tower are collected into 26 independent groups. The multiple loading condition and allowable nodal displacement are presented in Table 12. The allowable cross sections are picked from single angle structural profiles given in AISC code, that are tabulated in Table 13.

Young's modulus of the material is taken as 210 kN/mm^2, the allowable tensile stress is considered as 140 N/mm^2 while the compressing stress is limited according to the AISC-ASD89 design code

$$\begin{cases} \sigma_i^+ = 0.6F_y & \sigma_i \geq 0 \\ \sigma_i^- & \sigma_i < 0 \end{cases} \tag{12}$$

where σ_i^+ and σ_i^- are tensile and compressive stresses, respectively. In which σ_i^- varies by the slenderness ratio as follows

$$\sigma_i^- = \left[\left(1 - \frac{\lambda_i^2}{2C_c^2}\right)F_y \Big/ \left(\frac{5}{3} + \frac{3\lambda_i}{8C_c} - \frac{\lambda_i^3}{8C_c^3}\right)\right] \tag{13a}$$

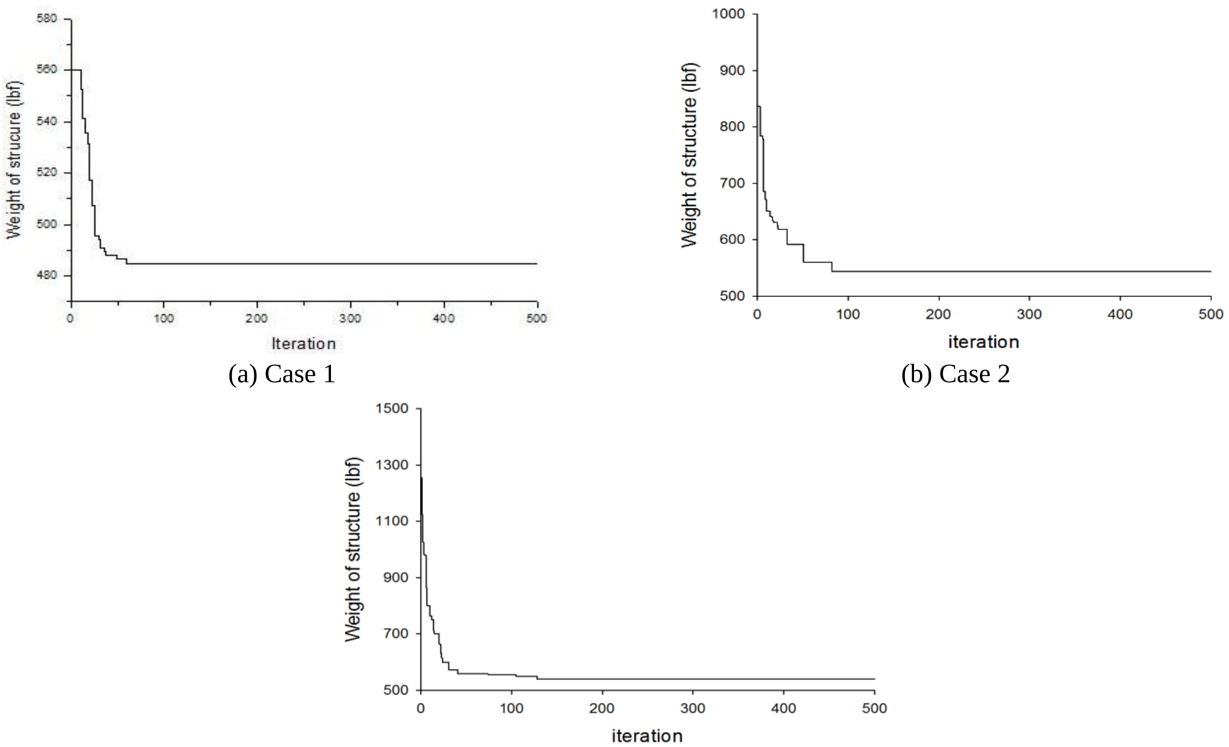
$$for\ \lambda_i < C_c$$

$$\sigma_i^- = \frac{12\pi^2 E}{23\lambda_i^2} \qquad for\ \lambda_i \geq C_c \tag{13b}$$

C_c is the slenderness ratio (k) dividing the elastic and inelastic buckling regions, defined as follows

Table 8 Optimal design comparison for the 25-bar spatial truss structure (Case 3)

Variables (in^2)	Wu and Chow (1995) GA	Li *et al.* (2009) PSO	Li *et al.* (2009) PSOPC	Li *et al.* (2009) HPSO	Sadollah *et al.* (2012) MBA	Kaveh and Talatahari (2009a) DHPSACO	This study iPSO
A_1	0.307	1.0	0.111	0.111	0.111	0.111	0.111
A_2 - A_5	1.990	2.62	1.563	2.130	2.130	2.130	2.38
A_6 - A_9	3.130	2.62	3.380	2.880	2.880	2.880	2.63
A_{10} - A_{11}	0.111	0.25	0.111	0.111	0.111	0.111	0.111
A_{12} - A_{13}	0.141	0.307	0.111	0.111	0.111	0.111	0.111
A_{14} - A_{17}	0.766	0.602	0.766	0.766	0.766	0.766	0.766
A_{18} - A_{21}	1.620	1.457	1.990	1.620	1.620	1.620	1.8
A_{22} - A_{25}	2.620	2.880	2.380	2.620	2.620	2.620	2.63
Best weight (lb)	556.43	567.49	556.90	551.14	551.14	551.14	540.03
Worst weight (lb)	-	-	-	-	-	-	542.21
Mean weight (lb)	-	-	-	-	-	-	540.67
Standard deviation (lb)	-	-	-	-	-	-	0.89
OFEs	-	50000	50000	10000	2400	5000	1340



(a) Case 1

(b) Case 2

(c) Case 3

Fig. 5 Convergence history of the 25-bar truss

Table 9 The loads cases for the 72-bar truss structure

	Load (kips)					
	Case 1			Case 2		
Nodes	P_x	P_y	P_z	P_x	P_y	P_z
17	5.0	5.0	-5.0	0.0	0.0	-5.0
18	0.0	0.0	0.0	0.0	0.0	-5.0
19	0.0	0.0	0.0	0.0	0.0	-5.0
20	0.0	0.0	0.0	0.0	0.0	-5.0

$$C_c = \sqrt{\frac{2\pi^2 E}{F_y}} \tag{14}$$

The maximum allowable slenderness ratio, respectively, is 200 and 300 for compression and tension elements. According to the AISC-ASD code provisions, this constraint can be identified as follows

$$\lambda_i = \frac{k_i l_i}{r_i} \le \begin{cases} 300 & \text{for tension members} \\ 200 & \text{for compression members} \end{cases} \tag{15}$$

Table 10 Optimal design comparison for the 72-bar space truss structure (Case 1)

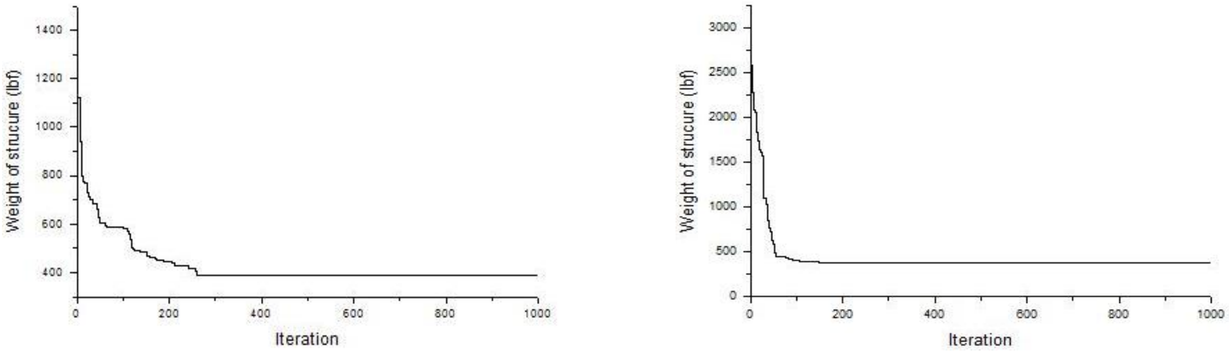
Variables (in^2)	Wu and Chow (1995) GA	Li *et al.* (2005) HS	Li *et al.* (2009) PSO	Li *et al.* (2009) PSOPC	Li *et al.* (2009) HPSO	Sadollah *et al.* (2012) MBA	This study iPSO
A_1- A_4	1.5	1.9	2.6	3.0	2.1	2.0	2.0
A_5 - A_{12}	0.7	0.5	1.5	1.4	0.6	0.6	0.5
A_{13} - A_{16}	0.1	0.1	0.3	0.2	0.1	0.4	0.1
A_{17} - A_{18}	0.1	0.1	0.1	0.1	0.1	0.6	0.1
A_{19} - A_{22}	1.3	1.4	2.1	2.7	1.4	0.5	1.3
A_{23} - A_{30}	0.5	0.6	1.5	1.9	0.5	0.5	0.5
A_{31} - A_{34}	0.2	0.1	0.6	0.7	0.1	0.1	0.1
A_{35} - A_{36}	0.1	0.1	0.3	0.8	0.1	0.1	0.1
A_{37} - A_{40}	0.5	0.6	2.2	1.4	0.5	1.4	0.5
A_{41} - A_{48}	0.5	0.5	1.9	1.2	0.5	0.5	0.5
A_{49} - A_{52}	0.1	0.1	0.2	0.8	0.1	0.1	0.1
A_{53} - A_{54}	0.2	0.1	0.9	0.1	0.1	0.1	0.1
A_{55} - A_{58}	0.2	0.2	0.4	0.4	0.2	1.9	0.2
A_{59} - A_{66}	0.5	0.5	1.9	1.9	0.5	0.5	0.6
A_{67} - A_{70}	0.5	0.4	0.7	0.9	0.3	0.1	0.4
A_{71} - A_{72}	0.7	0.6	1.6	1.3	0.7	0.1	0.6
Best weight (lb)	400.66	387.94	1089.88	1069.8	388.94	385.542	385.543
Worst weight (lb)	-	-	-	-	-	390.615	389.09
Mean weight (lb)	-	-	-	-	-	387.665	387.11
Standard deviation (lb)	-	-	-	-	-	1.62	0.62
OFEs	-	-	50000	50000	12500	9450	2450

Table 11 Optimal design comparison for the 72-bar space truss structure (Case 2)

Variables (in^2)	Wu and Chow (1995) GA	Li *et al.* (2009) PSO	Li *et al.* (2009) PSOPC	Li *et al.* (2009) HPSO	Sadollah *et al.* (2012) MBA	Kaveh and Talatahari (2009a) DHPSACO	This study iPSO
A_1- A_4	0.196	7.220	4.490	4.970	0.196	1.800	1.800
A_5 - A_{12}	0.602	1.800	1.457	1.228	0.563	0.442	0.442
A_{13} - A_{16}	0.307	1.130	0.111	0.111	0.442	0.141	0.111
A_{17} - A_{18}	0.766	0.196	0.111	0.111	0.602	0.111	0.111
A_{19} - A_{22}	0.391	3.090	2.620	2.880	0.442	1.228	1.266
A_{23} - A_{30}	0.391	0.785	1.130	1.457	0.442	0.563	0.602
A_{31} - A_{34}	0.141	0.563	0.196	0.141	0.111	0.111	0.111
A_{35} - A_{36}	0.111	0.785	0.111	0.111	0.111	0.111	0.111
A_{37} - A_{40}	1.800	3.090	1.266	1.563	1.266	0.563	0.563
A_{41} - A_{48}	0.602	1.228	1.457	1.228	0.563	0.563	0.563
A_{49} - A_{52}	0.141	0.111	0.111	0.111	0.111	0.111	0.111
A_{53} - A_{54}	0.307	0.563	0.111	0.196	0.111	0.250	0.111
A_{55} - A_{58}	1.563	1.990	0.442	0.391	1.800	0.196	0.196
A_{59} - A_{66}	0.766	1.620	1.457	1.457	0.602	0.563	0.563
A_{67} - A_{70}	0.141	1.563	1.228	0.766	0.111	0.442	0.391
A_{71} - A_{72}	0.111	1.266	1.457	1.563	0.111	0.563	0.563
Best weight (lb)	427.203	1209.48	941.8	933.09	390.73	393.80	389.87
Worst weight (lb)	-	-	-	-	399.49	-	397.77
Mean weight (lb)	-	-	-	-	395.432	-	394.01
Standard deviation (lb)	-	-	-	-	3.04	-	2.80
OFEs	-	50000	50000	12500	9450	10650	1980

where λ_i is slenderness ration of the ith member; l_i and r_i are the element length and the radius of gyration, respectively. If the constraint on slenderness ratio of compression elements is not satisfied, the allowable stress must not exceed the value obtained by $\left(\frac{12\pi^2 E}{23\lambda_i^2}\right)$ (AISC-ASD, 1989).

Table 14 shows the statistical data of solution process and optimum cross-sectional areas obtained in this study. In



(a) Case 1 (b) Case 2

Fig. 6 Convergence history of the 72-bar truss

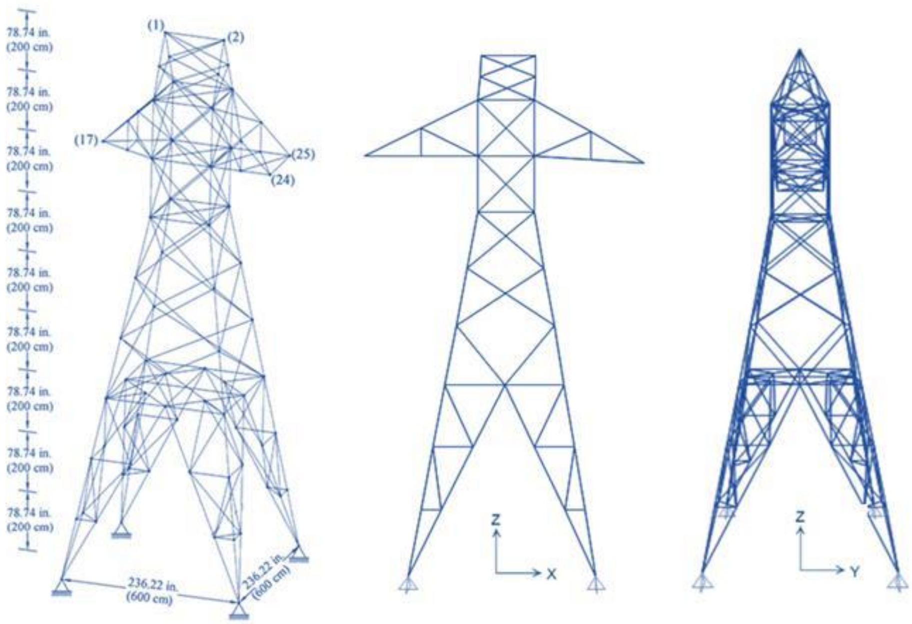


Fig. 7 244-bar truss tower structure

Table 12 Load cases and allowable nodal displacements for 244-bar transmission tower

	Case 1				Case 2			
	Loading (kN)		Displacement limitation (mm)		Loading (kN)		Displacement limitation (mm)	
Joint number	*x*	*z*	*x*	*y*	*x*	*z*	*x*	*y*
1	10	−30	45	15	0	−360	45	15
2	10	−30	45	15	0	−360	45	15
17	35	−90	30	15	0	−180	30	15
24	175	−45	30	15	0	−90	30	15
25	175	−45	30	15	0	−90	30	15

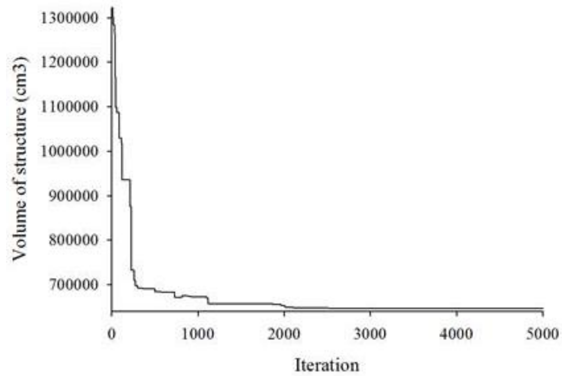


Fig. 8 Volume history of the 244-bar truss tower

addition, Table 14 also compares the results achieved with those available in the literature.

Weight history for this example is given in Fig. 8. Based on the provided data, the iPSO finds optimum volume as 649,109.41 cm^3 which is smaller than other cited results used different algorithms. Standard deviation of the entire process is 9.31 cm^3 over total number of runs, which indicates that deviation of the solutions around the mean value is in acceptable range for such a truss sizing optimization problem. Also, it is important to point out that since the discrete set of available sizing variables is employed in this example, the obtained result has more practical validity.

Table 13 Available cross-sections for 244-bar truss tower

No	Section	*A in*^{2} (*mm*^{2})	*r in* (*mm*)	No	Section	*A in*^{2} (*mm*^{2})	*r in* (*mm*)
1	L 6×6×1	11.0 (7096.76)	1.17 (29.72)	24	L $3^{1/2}$×$3^{1/2}$×1/2	3.25 (2096.77)	0.683 (17.35)
2	L 6×6×7/8	9.73 (6277.41)	1.17 (29.72)	25	L $3^{1/2}$×$3^{1/2}$×7/16	2.87 (1851.61)	0.684 (17.37)
3	L 6×6×3/4	8.44 (5445.15)	1.17 (29.72)	26	L $3^{1/2}$×$3^{1/2}$×3/8	2.48 (1600.00)	0.687 (17.45)
4	L 6×6×5/8	7.11 (4587.09)	1.18 (29.97)	27	L $3^{1/2}$×$3^{1/2}$×5/16	2.09 (1348.38)	0.690 (17.53)
5	L 6×6×9/16	6.43 (4148.38)	1.18 (29.97)	28	L $3^{1/2}$×$3^{1/2}$×1/4	1.69 (1090.32)	0.694 (17.63)
6	L 6×6×1/2	5.75 (3709.67)	1.18 (29.97)	29	L 3×3×1/2	2.75 (1774.19)	0.584 (14.83)
7	L 6×6×7/16	5.06 (3264.51)	1.19 (30.23)	30	L 3×3×7/16	2.43 (1567.74)	0.585 (14.86)
8	L 6×6×3/8	4.36 (2812.90)	1.19 (30.23)	31	L 3×3×3/8	2.11 (1361.29)	0.587 (14.91)
9	L 6×6×5/16	3.65 (2354.83)	1.20 (30.48)	32	L 3×3×5/16	1.78 (1148.38)	0.589 (14.96)
10	L 5×5×7/8	7.98 (5148.38)	0.973 (24.71)	33	L 3×3×1/4	1.44 (929.03)	0.592 (15.04)
11	L 5×5×3/4	6.94 (4477.41)	0.975 (24.77)	34	L 3×3×3/16	1.09 (703.22)	0.596 (15.14)
12	L 5×5×5/8	5.86 (3780.64)	0.978 (24.84)	35	L $2^{1/2}$×$2^{1/2}$×1/2	2.25 (1451.61)	0.487 (12.37)
13	L 5×5×1/2	4.75 (3064.51)	0.983 (24.97)	36	L $2^{1/2}$×$2^{1/2}$×3/8	1.73 (1116.13)	0.487 (12.37)
14	L 5×5×7/16	4.18 (2696.77)	0.986 (25.04)	37	L $2^{1/2}$×$2^{1/2}$×5/16	1.46 (941.93)	0.489 (12.42)
15	L 5×5×3/8	3.61 (2329.03)	0.990 (25.15)	38	L $2^{1/2}$×$2^{1/2}$×1/4	1.19 (767.74)	0.491 (12.47)
16	L 5×5×5/16	3.03 (1954.83)	0.944 (25.25)	39	L $2^{1/2}$×$2^{1/2}$×3/16	0.902 (581.93)	0.495 (12.57)
17	L 4×4×3/4	5.44 (3509.67)	0.778 (19.76)	40	L 2×2×3/8	1.36 (877.42)	0.389 (9.88)
18	L 4×4×5/8	4.61 (2974.19)	0.779 (19.79)	41	L 2×2×5/16	1.15 (741.93)	0.390 (9.91)
19	L 4×4×1/2	3.75 (2419.35)	0.782 (19.86)	42	L 2×2×1/4	0.938 (605.16)	0.391 (9.93)
20	L 4×4×7/16	3.31 (2135.48)	0.785 (19.94)	43	L 2×2×3/16	0.715 (461.29)	0.394 (10.00)
21	L 4×4×3/8	2.86 (1845.16)	0.788 (20.02)	44	L 2×2×1/8	0.484 (312.26)	0.398 (10.11)
22	L 4×4×5/16	2.40 (1548.38)	0.791 (20.09)	45	L $1^{1/4}$×$1^{1/4}$×3/16	0.434 (280.00)	0.244 (6.198)
23	L 4×4×1/4	1.94 (1251.61)	0.795 (20.19)				

Table 14 Optimal design comparison for the 244-bar space truss tower

Element group	Togan and Daloglu (2008) AGA	Kaveh *et al.* (2016) MMSM	This study iPSO
1	-	L $1^{1/4}$×$1^{1/4}$×3/16	L $1^{1/4}$×$1^{1/4}$×3/16
2	-	L 4×4×3/8	L 4×4×3/8
3	-	L $2^{1/2}$×$2^{1/2}$×3/16	L 2×2×3/16
4	-	L 4×4×5/16	L 4×4×1/4
5	-	L 3×3×3/16	L 3×3×3/16
6	-	L 5×5×7/16	L 4×4×5/16
7	-	L $1^{1/4}$×$1^{1/4}$×3/16	L $1^{1/4}$×$1^{1/4}$×3/16
8	-	L 6×6×3/8	L 5×5×7/16
9	-	L $2^{1/2}$×$2^{1/2}$×3/16	L 2×2×1/8
10	-	L 3×3×3/16	L 2×2×1/8
11	-	L 4×4×7/16	L 4×4×7/16
12	-	L 5×5×3/8	L 5×5×3/8
13	-	L $2^{1/2}$×$2^{1/2}$×3/16	L 3×3×1/4
14	-	L 2×2×1/8	L 2×2×1/8
15	-	L 6×6×3/4	L 6×6×9/16
16	-	L 4×4×5/16	L 4×4×5/16
17	-	L 2×2×1/8	L $1^{1/4}$×$1^{1/4}$×3/16
18	-	L 2×2×1/8	L $1^{1/4}$×$1^{1/4}$×3/16
19	-	L $2^{1/2}$×$2^{1/2}$×3/16	L $1^{1/4}$×$1^{1/4}$×3/16
20	-	L 5×5×7/8	L 6×6×9/16
21	-	L $3^{1/2}$×$3^{1/2}$×1/4	L $3^{1/2}$×$3^{1/2}$×1/4
22	-	L $2^{1/2}$×$2^{1/2}$×3/16	L $1^{1/4}$×$1^{1/4}$×3/16
23	-	L $2^{1/2}$×$2^{1/2}$×3/16	L $1^{1/4}$×$1^{1/4}$×3/16
24	-	L 2×2×1/8	L $1^{1/4}$×$1^{1/4}$×3/16

Table 14 Continued

Element group	Togan and Daloglu (2008) AGA	Kaveh *et al.* (2016) MMSM	This study iPSO
26	-	L $1^{1/4}$×$1^{1/4}$×3/16	L $1^{1/4}$×$1^{1/4}$×3/16
Best volume (cm^3)	920050	757637.35	649109.41
Worst volume (cm^3)	-	-	649137.81
Mean volume (cm^3)	-	-	649157.22
Stand. eviation (cm^3)			9.31
OFEs			3580

6. Conclusions

In this study, an integrated particle swarm optimization (iPSO) algorithm is presented. In comparison with standard PSO, the iPSO uses a new concept on velocity updating process as well as a new approach to handle the problem constraints. They are, respectively, the weighted particle concept and improved fly-back approach. Use of the weighted particle as a supplementary particle provides more efficient flight path for the particles especially for those which lie so close to the global best (X^G) and/or its prior best (${}^tX_i^P$) points. Also, since the weighted particle is the weighted average of all agents of the colony, all particles based on their objective values play the role on guidance of the other particles. On the other hand, iPSO uses improved fly-back technique to handle the problem constraints. This technique gives an extra chance to the particle(s) which violate the existing numeric constraints via changing their inappropriate components (which cause the violation) with those available in the weighted particle. In contrast with conventional penalty approach, improved fly-back does not contain any adjustable ad hoc parameters, thus, it does not inflict any extra adjustable parameters to the main optimizer engine. Additionally, improved fly-back presented as a distinct module can be integrated with any other optimization method to handle the constraints.

It is notable that since improved fly-back keeps all particle inside the feasible region, by applying this approach obtained the final solution also is feasible. To evaluate the performance of proposed iPSO algorithm a series of test problems over the truss structures with discrete design variables are solved using the present method. Achieved results show that the new algorithm is competitive with other metaheuristic algorithms on this class of problems.

References

Alaimo, A., Milazzo, A. and Orlando, C. (2016), "Nonlinear model based particle swarm optimization of PID shimmy damping control", *Adv. Aircr. Spacecrt. Sci.*, **3**(2), 211-214.

Camp, C.V. and Bichon, B.J. (2004), "Design of space trusses using ant colony optimization", *J. Struct. Eng.*, **130**, 741-751.

Chen, D. and Zhao, C. (2009), "Particle swarm optimization with adaptive population size and its application", *App. Soft Comput.*, **9**, 39-48.

Coello Coello, C.A. (2002), "Theoretical and numerical constraint-handling techniques used with evolutionary algorithms: a survey of the state of the art", *Comput. Meth. Appl. Mech. Eng.*, **191**, 1245-1287.

Deb, K. and Gulati, S. (2001), "Design of truss-structures for minimum weight using genetic algorithms", *Finite Elem. Anal. Des.*, **37**, 447-465.

Dizangian, B. and Ghasemi, M.R. (2016), "An efficient method for reliable optimum design of trusses", *Steel Compos. Struct.*, **21**(5), 1069-1084.

Erbatur, F., Hasançebi, O., Tütüncü, İ. and Kılıç, H. (2000), "Optimal design of planar and space structures with genetic algorithms", *Comput. Struct.*, **75**, 209-224.

Fan, Q. and Yan, X. (2014), "Self-adaptive particle swarm optimization with multiple velocity strategies and its application for p-Xylene oxidation reaction process optimization", *Chemom. Intell. Lab. Syst.*, **139**, 15-25.

Gholizadeh, S., Salajegheh, E. and Torkzadeh, P. (2008), "Structural optimization with frequency constraints by genetic algorithm using wavelet radial basis function neural network", *J. Sound Vib.*, **312**, 316-331.

Hajela, P. and Lee, E. (1995), "Genetic algorithms in truss topological optimization", *Int. J. Solid. Struct.*, **32**, 3341-3357.

Hasançebi, O. (2008), "Adaptive evolution strategies in structural optimization: Enhancing their computational performance with applications to large-scale structures", *Comput. Struct.*, **86**, 119-132.

Hasançebi, O. and Erbatur, F. (2002), "On efficient use of simulated annealing in complex structural optimization problems", *Acta Mech.*, **157**, 27-50.

Hasançebi, O., Teke, T. and Pekcan, O. (2013), "A bat-inspired algorithm for structural optimization", *Comput. Struct.*, **128**, 77-90.

He, R.S. and Hwang, S.F. (2007), "Damage detection by a hybrid real-parameter genetic algorithm under the assistance of grey relation analysis", *Eng. Appl. Artif. Intell.*, **20**, 980-992.

He, S., Prempain, E. and Wu, Q.H. (2004), "An improved particle swarm optimizer for mechanical design optimization problems", *Eng. Optim.*, **36**, 585-605.

Kaveh, A. and Talatahari, S. (2009a), "A particle swarm ant colony optimization for truss structures with discrete variables", *J. Constr. Steel Res.*, **65**, 1558-1568.

Kaveh, A. and Talatahari, S. (2009b), "Size optimization of space trusses using Big Bang-Big crunch algorithm", *Comput. Struct.*, **87**, 1129-1140.

Kaveh, A., Kalatjari, V. and Talebpour, M. (2016), "Optimal design of steel towers using a multi-metaheuristic based search method", *Period. Polytech. Civil Eng.*, doi:10.3311/PPci.8222.

Kennedy, J. and Eberhart, R. (1995), "Particle swarm optimization", *Proceedings of IEEE International Conference on Neural Networks*, Perth, WA.

Lee, K.S. and Geem, Z.W. (2004), "A new structural optimization method based on the harmony search algorithm", *Comput. Struct.*, **82**, 781-798.

Lee, K.S., Geem, Z.W., Lee, S.H. and Bae, K.W. (2005), "The harmony search heuristic algorithm for discrete structural optimization", *Eng. Optim.*, **37**, 663-684.

Li, J.P. (2015), "Truss topology optimization using an improved species-conserving genetic algorithm", *Eng. Optim.*, **47**, 107-128.

Li, L.J., Huang, Z.B. and Liu, F. (2009), "A heuristic particle swarm optimization method for truss structures with discrete variables", *Comput. Struct.*, **87**, 435-443.

Li, N.J., Wang, W.J., James Hsu, C.C., Chang, W., Chou, H.G. and Chang, J.W. (2014), "Enhanced particle swarm optimizer incorporating a weighted particle", *Neurocomput.*, **124**, 218-227.

Nickabadi, A., Ebadzadeh, M.M. and Safabakhsh, R. (2011), "A novel particle swarm optimization algorithm with adaptive inertia weight", *Appl. Soft Comput.*, **11**, 3658-3670.

Perez, R.E. and Behdinan, K. (2007), "Particle swarm approach for structural design optimization", *Comput. Struct.*, **85**, 1579-1588.

Rahami, H., Kaveh, A. and Gholipour, Y. (2008), "Sizing, geometry and topology optimization of trusses via force method and genetic algorithm", *Eng. Struct.*, **30**, 2360-2369.

Rajeev, S. and Krishnamoorthy, C. (1992), "Discrete optimization of structures using genetic algorithms", *J. Struct. Eng.*, **118**, 1233-1250.

Sadollah, A., Bahreininejad, A., Eskandar, H. and Hamdi, M. (2012), "Mine blast algorithm for optimization of truss structures with discrete variables", *Comput. Struct.*, **102-103**, 49-63.

Sadollah, A., Eskandar, H., Bahreininejad, A. and Kim, J.H. (2015), "Water cycle, mine blast and improved mine blast algorithms for discrete sizing optimization of truss structures", *Comput. Struct.*, **149**, 1-16.

Sonmez, M. (2011), "Artificial bee colony algorithm for optimization of truss structures", *Appl. Soft Comput.*, **11**, 2406-2418.

Toğan, V. and Daloğlu, A.T. (2006), "Optimization of 3d trusses with adaptive approach in genetic algorithms", *Eng. Struct.*, **28**, 1019-1027.

Toğan, V. and Daloğlu, A.T. (2008), "An improved genetic algorithm with initial population strategy and self-adaptive member grouping", *Comput. Struct.*, **86**, 1204-1218.

Wu, S.J. and Chow, P.T. (1995), "Steady-state genetic algorithms for discrete optimization of trusses", *Comput. Struct.*, **56**, 979-991.

Zheng, Y.J. (2015), "Water wave optimization: a new nature-inspired metaheuristic", *Comput. Oper. Res.*, **55**, 1-11.

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