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Transverse cracking based numerical analysis and its effects on cross-ply laminates strength under thermo-mechanical degradation

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Abstract. Components manufactured from composite materials are frequently subjected to superimposed mechanical and thermal loadings during their operating service. Both types of loadings may cause fracture and failure of composite structures. When composite cross-ply laminates of type $[0_m/90_n]_s$ are subjected to uni-axial tensile loading, different types of damage are set-up and developed such as matrix cracking: transverse and longitudinal cracks, delamination between disoriented layers and broken fibers. The development of these modes of damage can be detrimental for the stiffness of the laminates. From the experimental point of view, transverse cracking is known as the first mode of damage. In this regard, the objective of the present paper is to investigate the effect of transverse cracking in cross-ply laminate under thermo-mechanical degradation. A Finite Element (FE) simulation of damage evolution in composite cross-ply laminates of type $[0_m/90_n]_s$ subjected to uni-axial tensile loading is carried out. The effect of transverse cracking on the cross-ply laminate strength under thermo-mechanical degradation is investigated numerically. The results obtained by prediction of the numerical model developed in this investigation demonstrate the influence of the transverse cracking on the bearing capacity and resistance to damage as well as its effects on the variation of the mechanical properties such as Young's modulus, Poisson's ratio and coefficient of thermal expansion. The results obtained are in good agreement with those predicted by the Shear-lag analytical model as well as with the obtained experimental results available in the literature.

Keywords: cross-ply laminate; thermo-mechanical properties; Shear-lag analysis; transverse cracking; finite element method (FEM)

1. Introduction

The three modes of damage in composite laminates: transverse cracking, longitudinal cracking and delamination had been the subject of thorough theoretical and experimental research. Several theories have studied the initiation and the development of these damage modes and have described their effect on the thermo-mechanical properties degradation of laminates: theory of

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nonlinear elasticity (Gu and Reddy 1992), fracture mechanics approach (Joffe, Krasnikovs *et al*. 2001), (Brighenti, Carpinteri *et al*. 2013), variational approach (Yang *et al.* 2003), (Rebière 2009), (Rebière, Maâtallah *et al*. 2001), (Rebière, Maâtallah *et al*. 2002), (Rebière and Gamby 2004) and (Rebière and Gamby 2008), Shear-lag type analysis (Henaff-Gardin, Lafarie-Frenot *et al*. 1996), (Henaff-Gardin, Lafarie-Frenot *et al*. 1996), (Henaff-Gardin and Lafarie-Frenot 2002), (Brighenti, Carpinteri *et al*. 2016) and finite-element analysis (McCartney, Schoeppner *et al*. 2000). Most of the studies investigate the behavior and the properties of composite laminates with matrix cracking by assuming that the cracks are regularly spaced, thus, the analysis can be focused on a segment containing a crack (elementary cell) (Rebière 2009), (Berthelot, El Mahi *et al*. 1996) and (Leblond, El Mahi *et al*. 1996).

Using an approach based on an Equivalent Constraint Model (ECM), Kashtalyan and Soutis (2000) have analyzed the degradation of the stiffness of cross-ply laminates of type $[0_m/90_n]_s$ due to matrix crack in the two plies at 90° (transverse cracking) and at 0° (longitudinal cracking). They have performed a 2D Shear-lag type analysis to determine the stress field in the damaged cross ply laminates and to describe the degradation of the stiffness. They have found that the stiffness of a damaged laminate depends on the density of cracks of 90° layer and on the density of cracks in the external layers. The effect of matrix cracking on the behavior of a Glass-fiber / Epoxy of type $[0/90]_s$ and of a symmetric unbalanced cross-ply laminate of type $[0/45]_s$ under static loading as well as the theoretical prediction of stiffness reduction due to damage based on ECM which takes account of simultaneous matrix cracking in all laminate layers have been analyzed by Katerlos, Kashtalyan *et al*. (2008).

Henaff-Gardin, Lafarie-Frenot *et al*. Part 1 (1996), Henaff-Gardin, Lafarie-Frenot *et al*. Part 2 (1996) and Henaff-Gardin and Lafarie-Frenot (2002) have used Shear-lag type analysis to study cross-ply laminates damaged by double matrix periodic cracks, i.e., transverse and longitudinal cracking under plane bi-axial and thermal loading. They have assumed that the displacement in the plane in each layer has a parabolic variation through the thickness of the laminate in the normal direction of the crack plane and is constant in the other direction.

Rebière, Maâtallah *et al*. (2001), Rebière, Maâtallah *et al*. (2002), Rebière and Gamby (2004) and Rebière and Gamby (2008) have examined, through a model approach of variation using the complementary minimum energy principle, the effect of transversal and longitudinal cracks on the stress field distribution, the loss of stiffness and the reduction of Poisson's coefficient of cross-ply laminate subjected to tensile and fatigue tests. Their models are intended to predict the evolution of mechanical properties of the cross-ply laminates of type $[0_m/90_n]_s$ with $m+n=8$ for a better understanding of damage evolution. When the transverse damage becomes important, Robiere, Maâtallah *et al.* have observed a reduction in the Poisson's coefficient and, on the other hand, longitudinal damage increases it.

The aim of the present work is the investigation on the stiffness degradation of a cross-ply laminate of type $[0_m/90_n]_s$ due to transverse cracking. A finite element analysis is performed, and the results are compared with experimental and theoretical results under the same conditions for crack crossing the width of an elementary cell characterized by a geometric transverse crack ratio $\bar{a}$ (Eq. (1)).

2. Geometrical model

The plate under study is a cross-ply laminate of type $[0_m/90_n]_s$ composed of two external layers

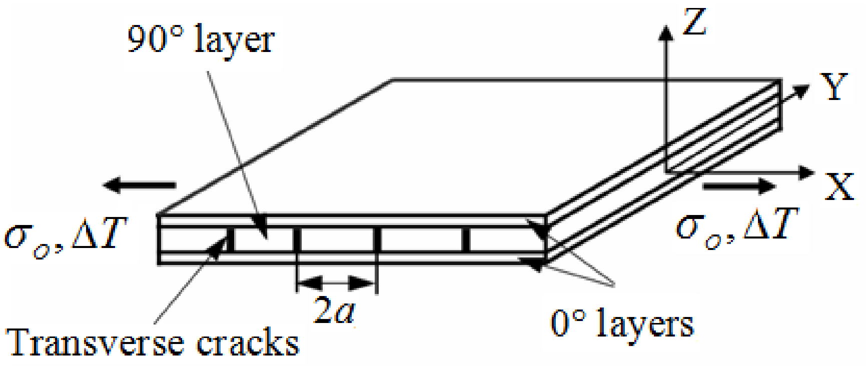

Fig. 1 Plate laminated with transverse cracks in the 90° layer

at 0° and an intermediate layer at 90° as is shown in Fig. 1. The thicknesses of the layers at 0° and 90° are respectively labeled t_0 and $2t_{90}$ with $t_0+t_{90}=h$. The median plan (o, x, y) of the laminate is a symmetry plan.

In Fig. 1, the transverse cracks are assumed to cross all the thickness of the 90° layer. They have a planar rectangular form, normal to the loading axis. Moreover, their distribution is assumed to be in the x direction of the median plan of the laminate. The number of transverse cracks increases with increasing load up to the saturation of the crack density. With the above hypotheses and due to the symmetry, the laminate plate can be treated as an elementary cell as is shown in Fig. 2. Then the analysis of the stiffness degradation is reduced to a 2D approach of the elementary cell. This cell is limited by two consecutive transverse cracks. The origin of the reference system is located at the center of the damaged cell between the two cracks (x direction) and in the median plan of the laminate (z direction). The transverse cracking is characterized by $2a$.

3. Formulation of the problem in the laminates

The applied stress is expressed by: $\sigma_0 = \dfrac{1}{2h} N_x$, where N_x is the applied load by unit of width along the x-axis. To define a model of stress distribution, the spacing between the cracks is assumed equidistant which means that the laminate has a periodic row of cracks in the 90° layer. Indeed, models taking into account non-uniform distribution had been treated via Weibull parameter as in (Sun, Daniel *et al.* 2003), (Li and Wisnom 1997). The uniform spacing considered in this investigation has been applied for symmetry reason and simplicity. Then, the symmetry conditions can be used in a transverse section containing one crack (Fig. 2). The non-dimensional coordinates and the geometrical parameters are introduced by dividing the corresponding quantity by the half of the thickness of the 90° layer

$$\bar{z} = \frac{z}{t_{90}} \;\; ; \;\; \bar{x} = \frac{x}{t_{90}} \;\; ; \;\; \bar{a} = \frac{a}{t_{90}} \;\; ; \;\; \bar{h} = \frac{h}{t_{90}} \qquad (1)$$

$\bar{a}$ is the geometric ratio of the transverse crack. The following analysis is made by assuming a general plane strain condition

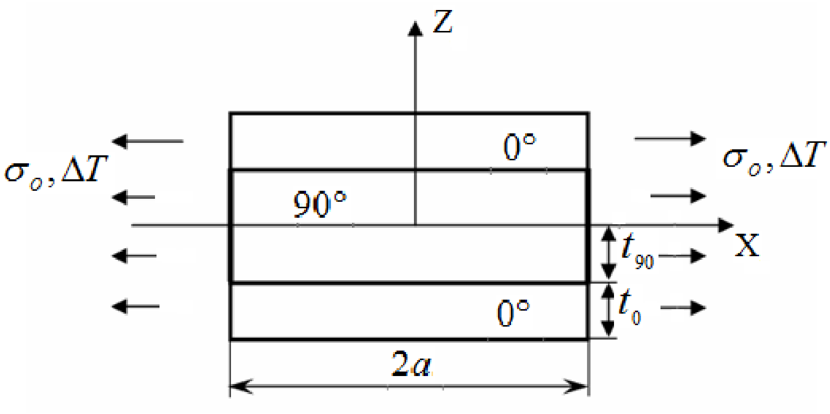

Fig. 2 Elementary cell between two consecutive transverse cracks

$$\varepsilon_y^0 = \varepsilon_y^{90} = \varepsilon_y = const \tag{2}$$

By applying the deformation in the external layers at 0° (which are not damaged and, consequently, the deformations are equal to those of the laminate $\varepsilon_x = \bar{\varepsilon}_x^0$) and by assuming that the thermal residual stresses are null, the longitudinal modulus and the Poisson's coefficient of the damaged laminate can be defined by the following expressions

$$E_x = \frac{\sigma_0}{\bar{\varepsilon}_x^0} \quad ; \quad \nu_{xy} = -\frac{\varepsilon_y}{\bar{\varepsilon}_x^0} \tag{3}$$

We note that the initial modulus of the undamaged laminate measured with the same load is E_{x0}=0/σ_{x0} and consequently

$$\frac{E_x}{E_{x0}} = \frac{\varepsilon_{x0}}{\bar{\varepsilon}_x^0} \tag{4}$$

For the calculation of the coefficient of thermal expansion α_x of the laminate, it is assumed that the applied load σ_0 is null, and $\bar{\varepsilon}_x^0$ and α_x are defined by the following expressions

$$\alpha_x = \frac{\bar{\varepsilon}_x^0}{\Delta T} \tag{5}$$

Using Eqs. (A14)-(A15) reported in Joffe *et al*. (2001), the constitutive equations of layers at 90° and 0° are obtained. In these equations we have $\bar{\sigma}_z^{90} = \bar{\sigma}_z^0 = 0$, and the equilibrium of the forces in the z direction is given by

$$\int_{-a}^{+a} \sigma_z^i dx = 0 \qquad i = 90°, 0° \tag{6}$$

The corresponding constitutive equations to the deformation components and of the normal stress in the plan are as follows

$$\begin{Bmatrix} \bar{\varepsilon}_x^0 \\ \varepsilon_y \end{Bmatrix} = \begin{bmatrix} S_{xx}^0 & S_{xy}^0 \\ S_{xy}^0 & S_{yy}^0 \end{bmatrix} \begin{Bmatrix} \bar{\sigma}_x^0 \\ \bar{\sigma}_y^0 \end{Bmatrix} + \begin{Bmatrix} \alpha_x^0 \\ \alpha_y^0 \end{Bmatrix} \Delta T \tag{7}$$

$$\begin{Bmatrix} \bar{\varepsilon}_x^{90} \\ \varepsilon_y \end{Bmatrix} = \begin{bmatrix} S_{22} & S_{12} \\ S_{12} & S_{11} \end{bmatrix} \begin{Bmatrix} \bar{\sigma}_x^{90} \\ \bar{\sigma}_y^{90} \end{Bmatrix} + \begin{Bmatrix} \alpha_2 \\ \alpha_1 \end{Bmatrix} \Delta T \tag{8}$$

The equilibrium equations of a damaged or undamaged laminate are:

a) In the *x*-direction

$$\bar{\sigma}_x^{90} t_{90} + \bar{\sigma}_x^0 t_0 = \sigma_0 \left(t_{90} + t_0 \right) \tag{9}$$

b) In the *y*-direction

$$\bar{\sigma}_y^{90} t_{90} + \bar{\sigma}_y^0 t_0 = 0 \tag{10}$$

Eqs. (7)-(10) contain seven unknowns: four stress components and three strain components. The total number of equations is six. One of these unknowns can be considered dependent on the others. The system is solved with respect to $\bar{\sigma}_x^{90}$

$$\left.\begin{aligned} \varepsilon_y &= g_1 \bar{\sigma}_x^{90} + f_1 \sigma_0 + m_1 \Delta T \\ \bar{\varepsilon}_x^{90} &= g_2 \bar{\sigma}_x^{90} + f_2 \sigma_0 + m_2 \Delta T \\ \bar{\varepsilon}_x^{0} &= g_3 \bar{\sigma}_x^{90} + f_3 \sigma_0 + m_3 \Delta T \end{aligned}\right\} \tag{11}$$

The expressions of g_i, f_i, m_i, i=1, 2, 3 with respect to the laminate geometry and the properties of the constituents are given in Joffe *et al*. (2001).

In order to obtain an expression for the stress $\bar{\sigma}_x^{90}$, we consider the axial disturbance caused by the presence of two cracks. The distribution of the axial stress can be written under the following form Joffe *et al*. (2001), Rebière (2009)

$$\sigma_x^{90} = \sigma_{x0}^{90} - \sigma_{x0}^{90} f_1 \left(\bar{x}, \bar{z} \right) \tag{12}$$

$$\sigma_x^{0} = \sigma_{x0}^{0} + \sigma_{x0}^{90} f_2 \left(\bar{x}, \bar{z} \right) \tag{13}$$

Where σ_{x0}^{90} and σ_{x0}^{0} are the longitudinal stresses of the undamaged laminate loaded along *x* direction, and are determined by the classical theory of laminates in the 90° and 0° layers respectively; $-\sigma_{x0}^{90} f_1 \left(\bar{x}, \bar{z} \right)$ and $\sigma_{x0}^{90} f_2 \left(\bar{x}, \bar{z} \right)$ are stress perturbations caused by the presence of the cracks. By using the results of the force of equilibrium in the *x* direction (Eq. (9)), we obtain

$$\bar{\sigma}_x^{90} = \sigma_{x0}^{90} - \sigma_{x0}^{90} \frac{1}{2\bar{a}} R\left(\bar{a} \right) \tag{14}$$

$$\bar{\sigma}_x^{0} = \sigma_{x0}^{0} + \sigma_{x0}^{90} \frac{1}{2\bar{a}\lambda} R\left(\bar{a} \right) \tag{15}$$

With

$$R(\bar{a}) = \int_{-\bar{a}}^{+\bar{a}} \int_{0}^{1} f_1(\bar{x}, \bar{z}) d\bar{x} d\bar{z} \tag{16}$$

Eq. (16) is called perturbation function. It is linked to the axial stress perturbation in the 90° layer and is dependent on the crack spacing (crack density). The stress perturbation function is written under the following form (Berthelot and Le Corre 1999), (Amara, Tounsi *et al.* 2006)

$$R(\bar{a}) = \frac{2}{\xi} \tanh(\xi \bar{a}) \tag{17}$$

Where ξ is the shear lag parameter

$$\xi^2 = \bar{G} \frac{t_{90}(t_{90} E^{90} + t_0 E_x^0)}{t_0 E_x^0 E^{90}} \tag{18}$$

The coefficient $\bar{G}$ depends on the hypotheses used for longitudinal displacements and on the shear stress distribution.

- Hypothesis on longitudinal displacements: The variation of the longitudinal displacement is assumed to be parabolic in the thickness of the 90° layer.

$$u_{90}(x, z) = \bar{u}_{90}(x) + \left(z^2 - \frac{t_{90}^2}{3} \right) A_{90}(x) \tag{19}$$

The variation of the longitudinal displacement is determined in the thickness of the 0° layers by

$$u_0(x, z) = \bar{u}_0(x) + f(z) A_0(x) \tag{20}$$

Where $\bar{u}_{90}(x)$ and $\bar{u}_0(x)$ are the average values (estimated through the thickness of the layers) of the longitudinal displacements $u_{90}(x,z)$ and $u_0(x,z)$ in the 90° and 0° layers (Berthelot and Le Corre 2000).

- Hypotheses on shear stresses. It is assumed that the transverse displacement is independent of on the longitudinal coordinate

$$\sigma_{xz}^i = G_{xz}^i \gamma_{xz}^i \quad ; \quad \gamma_{xz}^i = \frac{\partial u_i}{\partial z} + \frac{\partial w_i}{\partial x} \approx \frac{\partial u_i}{\partial z} \quad ; \quad i = 0°, 90° \tag{21}$$

Where G_{xy}^i is the transverse shear modulus in the 0° and 90° layers (Berthelot and Le Corre 2001). Following this assumption, Eq. (21) becomes

$$\sigma_{xz}^i \approx G_{xz}^i \frac{\partial u_i}{\partial z} \quad ; \quad i = 0°, 90° \tag{22}$$

Thus, the shear stresses can be expressed by

$$\sigma_{xz}^0 = G_{xz}^0 f'(z) A_0(x) \tag{23}$$

$$\sigma_{xz}^{90} = 2 G_{xz}^{90} z A_{90}(x) \tag{24}$$

Where $f'(z) = \frac{df}{dz}$

According to these assumptions and their association to the mechanical formulations of continuous medium which govern the elasticity problem in the elementary cell, (Berthelot and Le Corre 1999) have deduced the expressions of the average longitudinal stress (Eqs. (14)-(15)), respectively, in the 90° and 0° layers, as well as the shear stress at the interface between 0° and 90° layers:

$$\overline{\sigma}_x^{90}(x) = \sigma_0 \frac{E^{90}}{E_{x0}} \left(1 - \frac{\cosh \eta \overline{a} \left(\frac{x}{a} \right)}{\cosh \eta \overline{a}} \right) \tag{25}$$

$$\overline{\sigma}_x^{0}(x) = \sigma_0 \frac{E^{0}}{E_{x0}} \left(1 + \frac{t_{90}}{t_0} \frac{E^{90}}{E^{0}} \frac{\cosh \eta \overline{a} \left(\frac{x}{a} \right)}{\cosh \eta \overline{a}} \right) \tag{26}$$

$$\tau(x) = \sigma_0 \frac{E^{90}}{E_{x0}} \eta \frac{\cosh \eta \overline{a} \left(\frac{x}{a} \right)}{\cosh \eta \overline{a}} \tag{27}$$

Where η is the load transfer parameter between two consecutive cracks

$$\eta^2 = 3\left(1 + \frac{t_{90}}{t_0} \right) \frac{E_{x0} G}{E^0 E^{90}} \tag{28}$$

The coefficient $\overline{G}$ in Eq. (18) is determined by

$$\overline{G} = \frac{3G}{t_{90}} \tag{29}$$

The shear modulus of the elementary cell is given by

$$G = \frac{G_{xz}^{90}}{1 - 3 \frac{G_{xz}^{90}}{G_{xz}^{0}} \frac{f(t_{90})}{t_{90} f'(t_{90})}} \tag{30}$$

Two analytical functions have been considered and deduced basing on the results of Berthelot and Le Corre (1999) for $f(z)$:

• A complete parabolic model

$$f(z) = z^2 - 2(t_0 + t_{90})z + \frac{2}{3} t_0^2 + 2 t_0 t_{90} + t_{90}^2 \tag{31}$$

• A progressive shear model

$$f(z) = \frac{\sin \frac{t_0}{t_{90}} \eta_t}{\frac{t_0}{t_{90}} \eta_t} - \cosh \eta_t \left(1 + \frac{t_0}{t_{90}} - \frac{z}{t_{90}} \right) \tag{32}$$

Where η_t is the transverse shear parameter through the thickness of 0° layers expressed by

$$\eta_t = \frac{E_x^0}{G_{xz}^0} \frac{1}{\bar{a}} \tag{33}$$

The function $f(z)$ is similar to that used in shear deformation beam/plate theory and has been used in many works (Farahani and Barati 2015), (Bourada, Kaci *et al*. 2015), (Atteshamuddin *et al*. 2015), (Nguyen, Thai *et al*. 2015) and (Ait Yahia, Ait Atmane *et al*. 2015) to name a few. By substituting Eq. (14) in Eq. (11), the result obtained contains two terms. The first term is equal to the deformation of the classical theory of laminate, the second is a new term related to the shear perturbation function $R(\bar{a})$

$$\varepsilon_y = \varepsilon_{y0} - \sigma_{x0}^{90} g_1 \frac{1}{2\bar{a}} R(\bar{a}) \tag{34}$$

$$\bar{\varepsilon}_x^{90} = \varepsilon_{x0} - \sigma_{x0}^{90} g_2 \frac{1}{2\bar{a}} R(\bar{a}) \tag{35}$$

$$\bar{\varepsilon}_x^{0} = \varepsilon_{x0} - \sigma_{x0}^{90} g_3 \frac{1}{2\bar{a}} R(\bar{a}) \tag{36}$$

The stress σ_{x0}^{90} in the 90° layer of an undamaged laminate under thermo-mechanical loading can be calculated using the classical theory of laminates

$$\sigma_{x0}^{90} = Q_{22}\left(\varepsilon_{x0} - \alpha_2 \Delta T\right) + Q_{12}\left(\varepsilon_{y0} - \alpha_1 \Delta T\right) \tag{37}$$

In Eq. (37), ε_{x0} and ε_{y0} are the deformations provoked by the combined thermal and mechanical loads. This equation can be rewritten for the case of a pure mechanical load or a pure thermal load as follows:

• For mechanical load (ΔT=0)

$$\sigma_{x0}^{90} = Q_{22}\varepsilon_{x0}\left(1 - \nu_{12}\nu_{xy0}\right) \tag{38}$$

• For thermal load (σ_0=0)

$$\varepsilon_{x0} = \alpha_{x0}\Delta T \quad \text{and} \quad \varepsilon_{x0} = \alpha_{y0}\Delta T \tag{39}$$

Eq. (37) can be expressed as follows

$$\sigma_{x0}^{90} = Q_{22}\left[\alpha_{x0} - \alpha_2 + \nu_{12}\left(\alpha_{y0} - \alpha_1\right)\right]\Delta T \tag{40}$$

ν_{xy0} is the Poisson's coefficient of the undamaged laminate.

By substituting Eqs. (34)-(36) in Eq. (3) and considering Eq. (38), we obtain the following fundamental expressions:

$$\frac{E_x}{E_{x0}} = \frac{1}{1 + a^*\bar{\rho}R(\bar{a})} \quad \text{and} \quad \frac{\nu_{xy}}{\nu_{xy0}} = \frac{1 - c\bar{\rho}R(\bar{a})}{1 + a^*\bar{\rho}R(\bar{a})} \tag{41}$$

Where $\bar{\rho} = \frac{1}{2\bar{a}}$ is the normalized crack density and a^*, c are known functions which depend on the geometry and elastic properties of layers at 90° and 0°:

$$a^* = \frac{E_2 t_{90}}{E_x^0 t_0}\left(\frac{1-\nu_{12}\nu_{xy}^0}{1-\nu_{12}\nu_{21}}\right)\left(1+\nu_{xy}^0\frac{S_{xy}^0 t_{90}+S_{12}t_0}{S_{yy}^0 t_{90}+S_{11}t_0}\right) \quad ; \quad c = \frac{E_2 t_{90}}{\nu_{xy}^0}\left(\frac{1-\nu_{12}\nu_{xy}^0}{1-\nu_{12}\nu_{21}}\right)\left(\frac{S_{xy}^0 S_{11}-S_{12}S_{yy}^0}{S_{yy}^0 t_{90}+S_{11}t_0}\right)$$

The coefficient of thermal expansion α_x of the damaged laminate can be obtained by substituting Eq. (36) in Eq. (5) and assuming that the applied mechanical stress is null (σ_0=0)

$$\frac{\alpha_x}{\alpha_{x0}} = 1 + e\bar{\rho}R(\bar{a}) \tag{42}$$

Where e is a function of the elastic constants and geometry:

$$e = \frac{E_2 t_{90}}{E_x^0 t_0}\left(\frac{1-\dfrac{\alpha_T}{\alpha_{x0}}+\nu_{12}\dfrac{(\alpha_{y0}-\alpha_L)}{\alpha_{x0}}}{1-\nu_{12}\nu_{21}}\right)\left(1+\nu_{xy}^0\frac{S_{xy}^0 t_{90}+S_{12}t_0}{S_{yy}^0 t_{90}+S_{11}t_0}\right)$$

4. Finite element model analysis

Evaluations of the stiffness degradation have been investigated by a finite element analysis for the case of regularly spaced cracks characterized by the elementary cell as is shown in Fig. 2. The finite element model calculations are made in plane stress condition. The 2D simulation of the stress state is carried out using ANSYS code.

For symmetry reason, only one fourth of the elementary cell is modeled (Fig. 3(b)). The conditions of symmetry on the sides x=-a are check, $z\in[t_{90}, h]$ and $x\in[-a, 0]$, z=0. Traction free conditions are on z=h and on the crack surface x=-a, $z\in[0, t_{90}]$. A constant displacement is applied along the x direction at x=0 (Fig. 3(b)). Linear and quadratic 2D planar rectangular finite elements

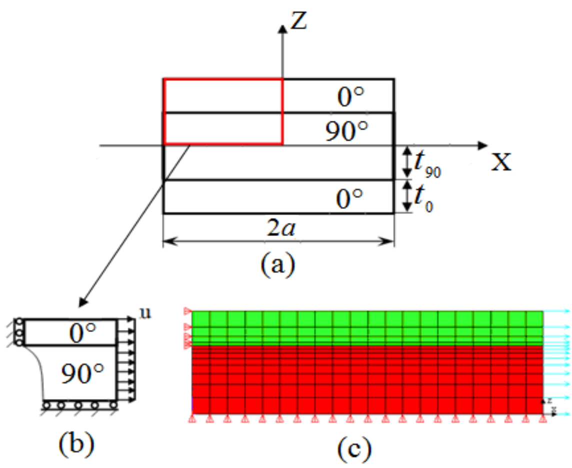

Fig. 3 Elementary cell model: (a) repeated transverse cracks in the 90° layer; (b) boundary conditions of modeled region and (c) finite element modeled region

Table 1 Material properties of cross-ply $[0_m/90_n]_s$ laminates

Properties	Glass-Fibre / Epoxy (Joffe, Krasnikovs *et al.* 2001), (Joffe and Varna 1999)	Graphite-Fibre / Epoxy (AS4-3502) (Groves, Haris *et al.* 1987)
E_1, GPa	44.73	144.78
E_2, GPa	12.76	9.58
ν_{12}	0.297	0.31
ν_{23}	0.420	0.55
G_{12}, GPa	5.80	4.785
G_{23}, GPa	4.49	3.09
α_1, 1/°C	8.6×10^{-6}	-0.72×10^{-6}
α_2, 1/°C	22.1×10^{-6}	27×10^{-6}
ΔT, °C	-105	-147

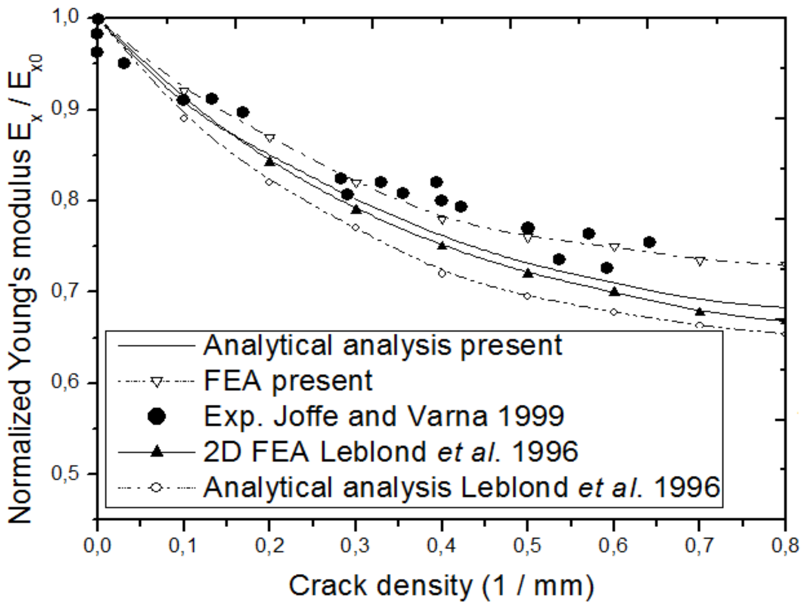

Fig. 4 Variation of the longitudinal Young's modulus as a function of the transverse cracking density for Glass-fibre / Epoxy laminate $[0_2/90_4]_s$ (a=5)

have been used for meshing (Fig. 3(c)).

In the finite element computation, an average stress of 45.35 MPa is applied to the elementary cell along the x direction (Figs. 3(b)-(c)). It is assumed that the interface between the layers is a perfect bond. The calculations have been carried out for two types of cross ply laminates: Graphite-fiber/Epoxy laminates AS4-3502 (Groves, Haris *et al.* 1987) and Glass-fiber/Epoxy laminate GF/GP (Joffe, Krasnikovs *et al.* 2001), (Joffe and Varna 1999). The results are compared to those of the analytical model and experimental tests results. The material properties used in this analysis at ambient temperature are shown in Table 1. The dimensions are: t_{90}=1 mm and t_0=0.5 mm for these cross ply laminates.

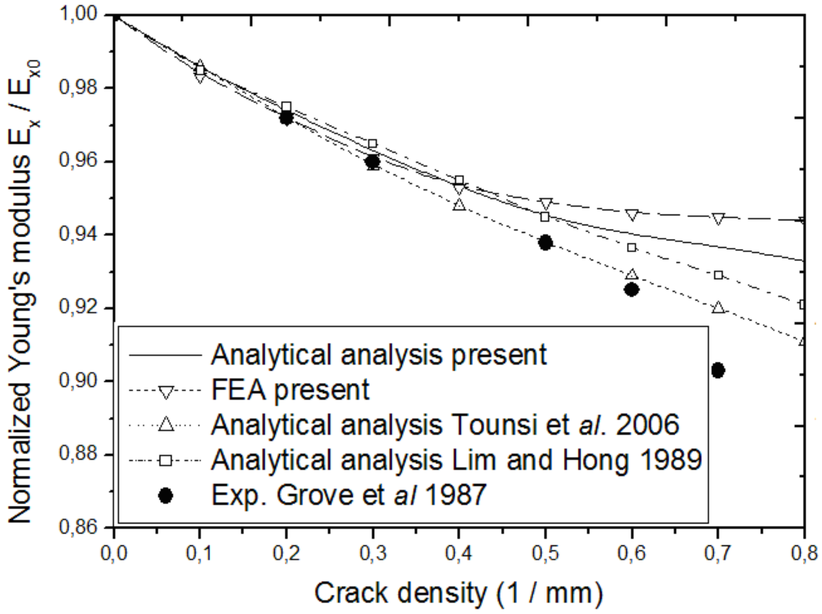

Fig. 5 Variation of the longitudinal Young's modulus as a function of the transverse cracking density for Graphite-fiber / Epoxy $[0/90_3]_s$ laminate (a=5)

5. Results and discussion

The reduction of the longitudinal modulus as a function of transverse crack density has been widely studied. The obtained results are represented in Figs. 4-5 and are compared with the experimental results for the two different types of laminates (Glass-fiber / Epoxy $[0_2/90_4]_s$ (Fig. 4) and Graphite-fiber / Epoxy $[0/90_3]_s$ (Fig. 5)).

Fig. 4 shows the variation of the normalized longitudinal modulus (E_x/E_{x0}) as a function of the crack density. The results deduced from the analysis of the Shear lag analytical model and the 2D FEM (Leblond *et al*. 1996) are also reported. A reduction of 25% is obtained for a crack density of 0.7 to 0.8 for FEA model which agrees with experimental results (Joffe and Varna 1999), while the reduction is 30% for the others models. This reduction depends on the respective stiffnesses of 0° and 90°.

In Fig. 5, the variation of the normalized longitudinal modulus (E_x/E_{x0}) as a function of crack density for the Graphite-fiber / Epoxy $[0/90_3]_s$ laminate is presented. The finite element results are compared with experimental data by Groves, Haris *et al*. (1987) and the results deduced from the analytical models: parabolic Shear-lag analysis of present study, progressive (Tounsi, Amara *et al*. 2006) and modified (Lim and Hong 1989) Shear-lag analysis. The variations of the modulus appear a slight degradation as the transverse matrix crack density increases compared with Glass-fiber/Epoxy laminate (Fig. 4). Indeed, for crack density of 0.8 crack/mm, the longitudinal Young's modulus of Glass-fiber / Epoxy $[0_2/90_4]_s$ laminate is reduced by about 25%, whereas reduction in Graphite-fiber/Epoxy $[0/90_3]_s$ ones is almost 10%. The agreement between the models and experimental data (Groves, Haris *et al*. 1987) is reasonable. However, for a density greater than 0.3, the analytical curves of present study and numerical curves diverge from the experimental results. This gap is due to the initiation and development of the transverse crack progression

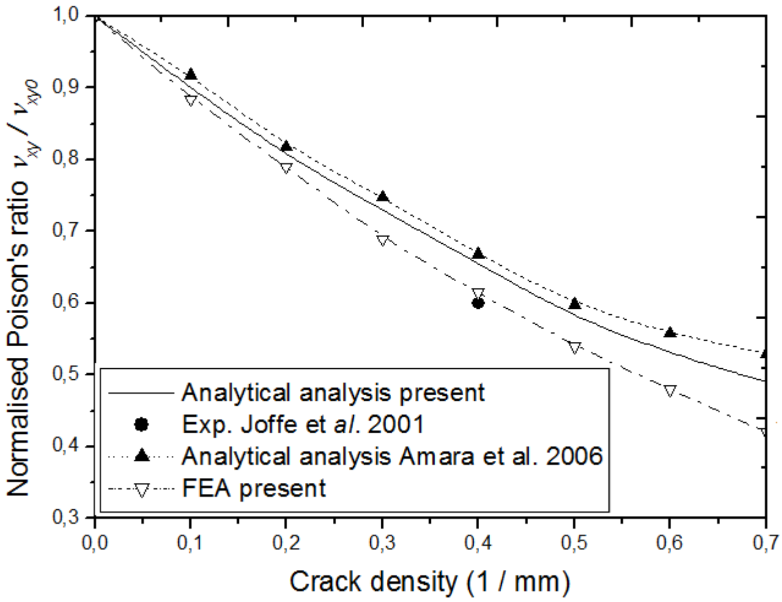

Fig. 6 Poisson's ratio variation due to transverse cracks density in a $[0_2 / 90_4]_s$ Glass-fibre / Epoxy laminate

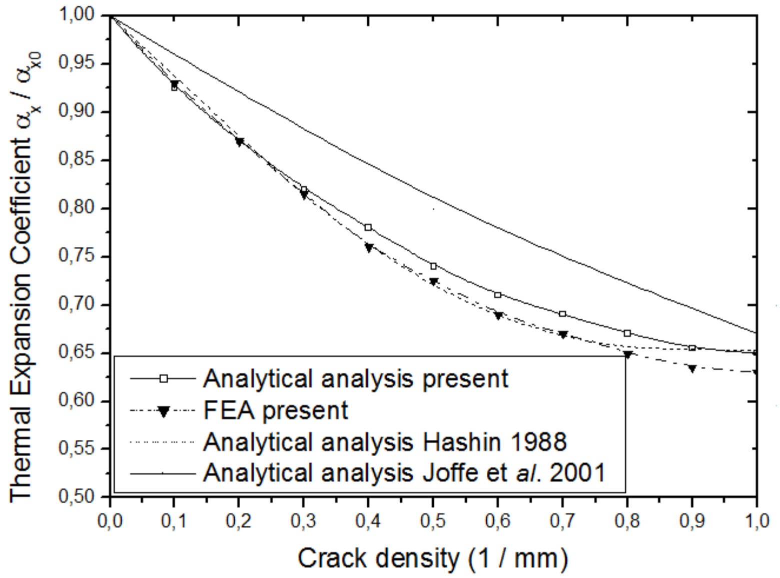

Fig. 7 Change of longitudinal thermal expansion coefficient as a function of crack density for a Glass-fibre / Epoxy $[0 / 90_4]_s$ laminate

simultaneously with a new damage process (longitudinal cracking and interface delamination) which are not taken into consideration in this analysis.

Fig. 6 shows the degradation of the normalized Poisson's ratio as a function of transverse crack density. The figure illustrates comparison with the predictions provided by progressive shear approach (Amara, Tounsi *et al*. 2006) and experimental data (Joffe, Krasnikovs *et al*. 2001). It is seen that the agreement between the experimental results and those predicted using the analytical models is fairly satisfactory, while numerical results reveal a good agreement with test results (Joffe, Krasnikovs *et al*. 2001).

Predicted changes of the thermal expansion coefficient (TEC) with respect to the crack density are shown in Fig. 7. It is seen that the cracks have a fairly significant effect on the TEC of the laminate. Indeed, we estimate the reduction between (32÷37)%. The plot presents the same form with a slight difference for higher densities, whereas, the variation for Joffe, Krasnikovs *et al*. (2001) takes a linear form. FEM analysis agrees well with Hashin (1988), till 0,75 mm^{-1}, after that these curves moving away gradually from each other, but they show an overestimated changes of the TEC. However the shear lag model of present study underestimats the TEC.

6. Conclusions

A simple Finite element analysis method has been applied to the investigation of the stiffness degradation in cross-ply $[0_m/90_n]_s$ laminates due to transverse cracking in the 90° layer. The results are compared with existing experimental data and analytical results using a Shear-lag analysis. On conclusions the basis of the present results, the following conclusion can be drawn:

- The stiffness reduction of Graphite-fiber/Epoxy $[0/90_3]_s$ is lower than that of Glass-fiber / Epoxy $[0_2/90_4]_s$.
- A new damage process (longitudinal cracking and/or interface delamination) initiated and developed simultaneously with transverse crack progression has been observed for a density greater than 0.3 in case of Graphite-fiber/Epoxy $[0/90_3]_s$ laminates. This result suggests that it is necessary and possible to take into consideration the other modes of damage for better prevision of the stiffness reduction.
- Generally FEA slightly underestimates stiffness reduction.
- The intra-laminar cracks significantly reduce the Poisson's ratio and TEC of the laminate.
- The present FEA method is simple, yet its results show reasonable agreement with analytical results and experimental data.

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