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Performance evaluation of wavelet and curvelet transforms based-damage detection of defect types in plate structures

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Abstract. This study focuses on the damage detection of defect types in plate structures based on wavelet transform (WT) and curvelet transform (CT). In particular, for damage detection of structures these transforms have been developed since the last few years. In recent years, the CT approach has been also introduced in an attempt to overcome inherent limitations of traditional multi-scale representations such as wavelets. In this study, the performance of CT is compared with WT in order to demonstrate the capability of WT and CT in detection of defect types in plate structures. To achieve this purpose, the damage detection of defect types through defect shape in rectangular plate is investigated. By using the first mode shape of plate structure and the distribution of the coefficients of the transforms, the damage existence, the defect location and the approximate shape of defect are detected. Moreover, the accuracy and performance generality of the transforms are verified through using experimental modal data of a plate.

Keywords: damage detection; defect; rectangular plate; wavelet transform; curvelet transform

1. Introduction

In recent years, the damage identification of structures has received considerable attention in the technical literature and structural applications. Hence, the concept of structural damage detection as a reliable and effective non-destructive damage method has been developed and investigated by many researchers. Furthermore, this concept has eliminated the drawbacks of traditional methods such as destructive and expensive experiments. The techniques of structural damage detection are based on a comparison between the current material-structural state as damaged structure and that of the previous baseline state, which is considered to be the undamaged condition. However, some of the techniques can only detect the structural damage using damaged structure. Most non-destructive damage identification methods consist of two techniques: the local and global damage detection techniques (Doebling, Farrar *et al*. 1996).

Successful application of the wavelet transforms (WTs) in the fields of damage identification and health monitoring has been widely investigated by many researchers (Bombale, Singha *et al*. 2008, Pakrashi and Alan 2009, Gokdag and Kopmaz 2010, Gokdag 2011). Ovanesova and Suaez

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(2004) presented a method based on using WT which is utilized in structures such as beams and plane frames. It was shown that the proposed method can detect the localization of crack by using a response signal from static or dynamic loads. The results of this study demonstrated that by selection of a suitable wavelet, the method is capable to extract damage information from the response signal in a simple, robust and reliable way. In the work of Wang and Deng (1999), a structural damage detection technique based on wavelet analysis of spatially distributed structural response measurements was introduced. In this method, a sudden change in the spatial variation of the transformed response was utilized for the detection of crack in structures. It was shown that this proposed technique required neither any analysis of the complete structure in question, nor any knowledge of the material properties and prior stress states of the structure. Liew and Wang (1998) utilized the wavelet theory for the crack identification in a simply supported beam with a transverse on-edge non-propagating open crack. In this study, a mathematical model of the cracked beam was obtained. Based on the results of this study, crack identification is easily accomplished by using wavelet analysis, while by using the traditional eigenvalue analysis this detection can hardly be done. Douka, Loutridis *et al*. (2002) introduced a method based on wavelet analysis in order to detect the location and depth of crack in bending plates. In this proposed method, the position of the crack is determined by the sudden change in the spatial variation of the transformed displacement response. In order to estimate the depth of the crack an intensity factor is also defined which relates the depth of the crack to the coefficients of WT. In this method, crack depth is accurately predicted by an intensity factor law. Chang and Chen (2004) presented a technique for structure damage detection based on spatial wavelet analysis. In this method, the spatially distributed signals (e.g., the displacements or mode shapes) of the rectangular plate after damage are needed. They showed that distributions of the wavelet coefficients can identify the damage position of rectangular plate by showing a peak at the position of the damage. The results of this study also demonstrated that this method was very sensitive to the damage size. Loutridis, Douka *et al*. (2005) utilized a two-dimensional wavelet transform for identification of cracks in plate structures. In the proposed method, the vibration mode of a cracked plate was transformed to the wavelet domain. The location and extent of the crack were also accurately displayed in the transformed response. Due to the simplicity of its computational implementation and the accuracy of the results, the proposed method was known as attractive method. By using WT, Kim, Kim *et al*. (2006) introduced a vibration-based damage evaluation technique. In the proposed method, only a few of the lower mode shapes were required before and after a small damage event in order to identify the location and size damage of plate-like structures. Fan and Qiao (2009) proposed a two-dimensional (2-D) continuous wavelet transform (CWT)-based damage detection algorithm using Dergauss2d wavelet for plate structures. The proposed algorithm only required the mode shapes of the damaged plates. In this work, the vibration mode shapes of a cantilever plate with different types of damage were presented in order to demonstrate the effectiveness and viability of the proposed method. Yang, Yang *et al*. (2011) utilized the first mode shape of damaged stiffened plates, and detected the damage locations with two-dimensional discrete wavelet analysis. The results of the numerical analysis and experimental investigation revealed that the proposed method was applicable to detect single crack or multi-cracks of a stiffened structure. The experimental results of this study also showed that fewer measurement points were required with the proposed technique in comparison to those presented in the previous studies. Hou, Noori *et al*. (2000) introduced a technique based on structure modeling with several springs and applying harmonic stimulations. The results demonstrated that wavelet was able to identify the sudden damage successfully. Xu, Cao *et al*. (2015) proposed two-dimensional curvature mode shape method based on wavelets and Teager energy for damage detection in plates. The efficiency of the proposed method was demonstrated using finite element simulations and experimentally validated through noncontact measurement by a scanning laser vibrometer. The numerical results showed that its advances of clear mechanism of characterizing damage, robustness against noise, and sensitivity to slight damage were sufficiently corroborated. Recently, He and Zhu (2015) have presented an adaptive-scale damage detection strategy based on a wavelet finite element model for thin plate structures. In this study, equations of motion and corresponding lifting schemes in thin plate structures have derived with the tensor products of cubic Hermite multi-wavelets as the elemental interpolation functions. The numerical results have demonstrated that the proposed method can progressively locate and quantify plate damages.

The curvelet transform (CT) has been introduced as a new multi-scale pyramid representation with many directions and positions at each length scale and needle-shaped elements at fine scale (Candes 1988). CT can overcome inherent limitations of traditional multi-scale representations such as WT. Candes and Donoho (2000) introduced CT which can be used to represent the distribution as a superposition of functions of various lengths. Bagheri, Ghodrati Amiri *et al.* (2009) proposed a new method based on CT to assess the damage location in plate structures. The results of this work demonstrated that CT has good capability in detecting linear damage and its capability of denoising recorded data prior to applying the damage detection method.

The main aim of this study is to evaluate the performance and accuracy of wavelet and curvelet transforms for the damage detection of defect types in plate structures. In order to achieve this purpose, the performance of CT is compared with that of WT by detecting defect types in plate structures. Assuming the horizontal, vertical and oblique linear defect and the curved defect as defect shape, the detection of defect types in rectangular plate is investigated. In the present study, the procedure of damage detection is performed based on using the distribution of coefficients of WT and CT. The results demonstrate that WT and CT accurately identify the damage existence, the failure location and the linear shape of the damage in plate structures. Furthermore, the results of damage detection in plate structure with the curved defect reveal high accuracy and performance generality of WT and CT.

2. Wavelet transforms

Morlet and Grossmann initially proposed wavelet theory, and Meyer developed the mathematical foundations of wavelets. Daubechies (1990), Mallat (1998) modified this by defining the connection between wavelets and digital signal processing. Wavelet transforms (WTs) have replaced the short-time Fourier transform, and have overcome the drawbacks of short-time Fourier transform. WTs have been utilized in a number of areas, including data compression, image processing, and time-frequency spectral estimation.

A mother wavelet $\psi(t)$ is a waveform that is limited duration and an average value of zero, and the wavelet kernel can be expressed as follows

$$\psi_{a,b}(t)=\frac{1}{\sqrt{|b|}}\psi\left(\frac{t-a}{b}\right) \tag{1}$$

where a and b are translation and dilation parameters, respectively. These parameters are real numbers and a must be positive.

2.1 Continuous wavelet transform

A continuous wavelet transform (CWT) has been introduced to divide a continuous-time function into wavelets. Unlike Fourier transform, the main ability of the CWT is to construct a time-frequency representation of a signal that offers very good time and frequency localization. The CWT of a function $f(t)$ at a translational value (a>0) a and scale value b is expressed by the following integral

$$CWT_f^{\psi}(a,b)=\frac{1}{\sqrt{|b|}}\int_{-\infty}^{+\infty} f(t)\psi^*\left(\frac{t-a}{b}\right)dt=\left(f(t),\psi_{a,b}(t)\right) \tag{2}$$

where $\psi_{a,b}(t)$ is a continuous function in both the time domain and the frequency domain called the mother wavelet; and ψ^* represents operation of complex conjugate.

Generally, the concept of the CWT is exactly similar to time-transform concept in short-time Fourier transform which determines the measure of window displacement and clearly possesses the transform time information. Furthermore, in WT the frequency variable is not available directly. In other words, the scale parameter is expressed as the inverse of frequency, i.e., $b=1/f$. According to the presented definition in Eq. (2) as inner product, WT can be expressed as a tool for measuring the similarity between signal and basic function (wavelet). In the CWT concept, a wavelet function is utilized to decompose a time function into time-frequency and in turn to introduce locality into the analysis, which is not the case of Fourier transform. The wavelet function $\psi_{a,b}(t)$, efficiently expresses the fast decay in time domain and the limited bandwidth in frequency.

2.2 Discrete wavelet transform

In the CWT concept, the transform and scale parameters are continuously changed. By discretizing the parameters a and b, a discrete wavelet transform (DWT) is obtained. Hence, the procedure becomes much more efficient if the dyadic values of a and b are expressed as follows

$$a=2^j\ ;\ b=2^j k\ ;\ j,k\in Z \tag{3}$$

where Z is a set of integers.

This sampling of the coordinates (a,b) is referred to as dyadic sampling because consecutive values of the discrete scales differ by a factor of 2. Using the discrete scales of WT, the DWT can be defined as follows

$$DWT_{f\ (j,k)}^{\psi}=\int_{-\infty}^{\infty} f(t)\psi_{j,k}(t)dt=2^{-j/2}\int_{-\infty}^{\infty} f(t)\psi(2^{-j}t-k)dt \tag{4}$$

where $\psi_{j,k}$ is the wavelet function. The signal resolution is defined as the inverse of the scale $1/a=2^{-j}$, and the integer j is referred to as the level. The signal can be reconstructed from the wavelet coefficients DWT_f^{ψ}; and the reconstruction algorithm is called as the inverse discrete wavelet transform, as follows

$$f(t)=\sum_{j=-\infty}^{+\infty}\sum_{k=-\infty}^{+\infty} DWT_f^{\psi}\, 2^{-j/2}\psi(2^{-j/2}t-k) \tag{5}$$

Therefore, in the discrete wavelet analysis a signal can be represented by its approximations and details.

3. Curvelet transform

In 2000, the curvelet transform (CT) introduced by Candes, Demanet *et al*. (2005) is a multi-scale representation suited for objects which are smooth away from discontinuities across curves. Unlike WT, CT has directional parameters, and the curvelet pyramid contains elements with a very high degree of directional specificity. The main aim of the CT design is to represent edges and other singularities along curves, which is much more efficient than short-time Fourier transform and WT. Furthermore, the advantage of curvelet in comparison with wavelet is as follows (Candes, Demanet *et al*. 2005):

(1) Optimally sparse representation of objects with edges.
(2) Optimal image reconstruction in severely ill-posed problems.
(3) Optimal sparse representation of wave propagators.

3.1 Continues curvelet transform

The second generation of CT as continues curvelet transform has been introduced by Candes, Demanet *et al*. (2005). This transform is implemented in two dimensions, i.e., R^2 with x as spatial variable, ω as frequency domain variable and r and θ as polar coordinates in the frequency domain. A pair of windows $W(r)$ and $V(t)$ are defined as the radial window and angular window, respectively. These are smooth, nonnegative and real-valued, with W taking positive real arguments and is supported on $r\in[1/2,2]$ and V taking real arguments and is supported on $t\in[-1,1]$. These windows are constructed the admissibility conditions (Candes, Demanet *et al*. 2005)

$$\sum_{j=-\infty}^{\infty} W^2(2^j r)=1; \quad r\in(3/4,3/2) \tag{6}$$

$$\sum_{j=-\infty}^{\infty} V^2(\mathrm{t}-l)=1; \quad t\in(-1/2,1/2) \tag{7}$$

for each $j\geq j_0$ (j_0 as initial value), a frequency window U_j is defined in the Fourier domain by

$$U_j(r,\theta)=2^{-(3/4)j}W(2^{-j}r)V(\frac{2^{\lfloor j/2\rfloor}\theta}{2\pi}) \tag{8}$$

where $[j/2]$ is the integer part of $j/2$. Thus, the support of U_j is a polar wedge defined by the support of W and V and is applied with scale dependent window widths in radial and angular directions.

The wave form, $\varphi_j(x)$, is defined by means of its Fourier transform $\varphi_j(x)=U_j(\omega)$. It is also assumed that $U_j(\omega_1, \omega_2)$ is presented as the window defined in the polar coordinate system by Eq. (8). φ_j is defined as the mother curvelet in the sense that all curvelets at scale 2^{-j} are obtained by rotations and translations of φ_j. A sequence of translation parameters $k=(k_1,k_2)\in Z^2$ and rotation angles $\theta_l = 2\pi . 2^{-\lfloor j/2 \rfloor} . l$ are introduced with l=0, 1, 2, … such that $0 \le \theta_l \le 2\pi$ (the spacing between consecutive angles is scale-dependent). The curvelet functions are functions of $x=(x_1,x_2)$ defined at scale 2^{-j}, orientation angle θ_l and position $x_k^{(j,l)} = R_{\theta_l}^{-1}(k_1 2^{-j}, k_2 2^{-j})$ by

$$\varphi_{j,l,k}(x) = \varphi_j \left(R_{\theta_l}(x - x_k^{(j,l)}) \right) \tag{9}$$

where $\varphi_{j,l,k}$ is the curvelet function; R_{θ_l} is the rotational by θ_l radians and $R_{\theta_l}^{-1}$ is its inverse. A curvelet coefficient is defined by the inner product of an element f and a curvelet $\varphi_{j,l,k}$ as follows

$$C(j,l,k) = (f, \varphi_{j,l,k}) = \int_{\boldsymbol{R}^2} f(x).\bar{\varphi}_{j,l,k}(x)\,dx \tag{10}$$

where $\bar{\varphi}_{j,l,k}$ is the conjugate of curvelet $\varphi_{j,l,k}$.

4. Damage detection based on the wavelet and curvelet transforms

This study presents the damage detection of defect types in plate structures based on the wavelet and curvelet transforms. In this section, the application of the 2-D WT and CT in the proposed procedure of the damage detection is expressed.

4.1 The proposed method of the damage detection

In the present study, the procedure of the damage detection in plate structures is performed based on the first mode shape of plate structures and the wavelet and curvelet transforms. For achieving this purpose, the overall framework of the proposed damage detection involves four main steps as follows:

1. The modeling of plate structure with defect: In this study, the ABAQUS software (2011) is utilized for modeling of plate structures with defect. For the simulation of defect, the reduction of material property, cross-sectional area, stiffness, etc has been proposed in literature. In this study, the reduction of Young’s modulus is utilized for the simulation of defect.
2. The structural responses: In the proposed method the structural responses such as mode shapes, displacement, stress, strain, energy, etc are required. In the present study, the first mode shape of plate structure is utilized. It is also noted that the displacement of nodes in the first mode shape is considered as a signal in the procedure of the damage detection.
3. The calculation of transform coefficients: The coefficients of wavelet and curvelet transforms are calculated by using the signal concept which is introduced in the next section.
4. The plot of transform coefficients: In the final step, the damage is detected by the plot of transform coefficients in the wavelet and curvelet transforms.

4.2 Mode shapes of plate

The existence of damage in plate structures can be detected by monitoring and identifying the dynamic responses of the structure such as mode shapes. In order to obtain the mode shapes responses of a plate structure and utilize in the damage detection procedure, the free vibration of a rectangular plate with and without defect are determined using finite element method (FEM).

Based on the Hamilton principle (Hinton 1988), the equations of motion for Mindlin plate are expressed as

$$\boldsymbol{M}\,\ddot{\boldsymbol{u}}(t)+\boldsymbol{K}\,\boldsymbol{u}(t)=\boldsymbol{f}(t) \tag{11}$$

where $\boldsymbol{M}$, $\boldsymbol{K}$ and $\boldsymbol{f}$ are the system mass and stiffness matrices, and the force vector, respectively, and $\ddot{\boldsymbol{u}}$ and $\boldsymbol{u}$ are the accelerations and displacements. It is noted that in Eq. (11) the effect of damping is ignored.

Assuming a harmonic motion, the natural frequencies and the modes of vibration are obtained by solving the generalized eigen problem (Hinton 1988):

$$(\boldsymbol{K}-\omega_i^2\boldsymbol{M})\boldsymbol{\varphi}_i=\boldsymbol{0}\,,\quad i=1,2,\ldots,n_m \tag{12}$$

where ω_i is the natural frequency; and $\boldsymbol{\varphi}_i$ is the ith mode shape vector of vibration. n_m is the number of structural modes. It is noted that in this study the first mode shape $\boldsymbol{\varphi}_1$ is utilized in the damage detection process.

4.3 Application of 2-D wavelet transform

In the present study, the 2-D WTs are utilized for the damage detection of plate structures. For achieving this purpose, the application of the 2-D signal concept based on WTs is required, which in image processing the concept is expressed as an image. Based on the mathematical theory of the image processing, an image is described by a matrix. In fact, an image is simulated by the number of rows and columns of matrix. Hence, each row or column of matrix is considered as a one-dimensional signal and the amplitude of signal shows amount of brightness of available points in the specific row and column.

According to the idea, WT is applied on a row or column of the matrix separately. Hence, for applying the 2-D WT on image, the 1-D wavelet transform is utilized twice. For this purpose, first, the 1-D wavelet transform is applied on rows, and then applied on columns. Thus, four sub-bands are obtained as the coefficients of WT. As shown in Fig. 1, the first sub-band of the coefficients of WT is related to the approximation coefficients. In addition the approximation sub-band, three detail sub-bands are related to horizontal, vertical and diagonal detail in image, respectively. The dimension of each matrix is the half of the initial matrix.

The decompoition of the first level of the 2-D DWT is shown in Fig. 1. For the decompotion of the secon level, each of sub-bands of the first level is considered as the main signal, and WT should be applied on each of them. Also, each signal is decomposited in four sub-bands. Therefore, in the second level of decomposition there are sixteen sub-bands and this procedure can be repeated for the higher level.

Accorrding to the decompostion procedure expressed in above, the dagmage detection

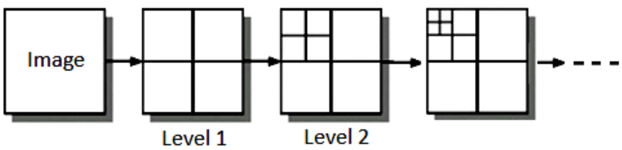


Fig. 1 The decomposed of the levels of DWT

procedure of plate structures are performed as follows: First, plate structure with defect is analyzed and its first mode shape as the structural responces are obtained. Then, based on the mesh of plate structure, the structural responces are put in a matrix. It is noted that the displacement of each node of plate in the first mode shape is considered as an element of matrix. After creating the matrix, the coefficients of WT are obtained based on the concept described in Section 2. In this work, the coffecients of the first level of WT are utlilized in the damage dectection procedure. Finally, the location of damage is identified by the coefficients of WT. In order to investigate the effect of wavelet type in the damage detection procedure, the Haar, Daubechies and Symlet wavelets are chosen and used in this study.

Haar wavelet: The Haar wavelet is a sequence of rescaled “square-shaped” functions which together form a wavelet family or basis. Its mother wavelet and its scaling function are shown in Fig. 2 (Aboufadel and Schlicker 2009).

The Haar wavelet's mother wavelet function, $\psi(t)$, is described as follows

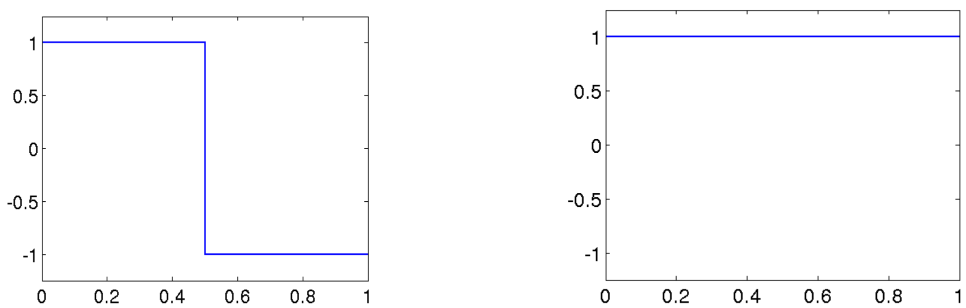
$$\psi(x) = \begin{cases} 1 & 0 \le x < 0.5 \\ -1 & 0.5 \le x < 1 \\ 0 & otherwise \end{cases} \tag{13}$$

and its scaling function is expressed as

$$\varphi(x) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & otherwise \end{cases} \tag{14}$$

Daubechies wavelet: The Daubechies wavelets, based on the work of Daubechies, are a family of orthogonal wavelets defining a discrete wavelet transform and characterized by a maximal number of vanishing moments for some given support (Daubechies 1992). The names of the Daubechies family wavelets are written dbN, where N is the order, and db the “surname” of the wavelet. The db1 wavelet is the same as Haar wavelet. The mother wavelet and scaling function are shown in Fig. 3.

Symlet wavelet: The scaling/Wavelet functions of Daubechies wavelets are far from symmetry because Daubechies wavelets select the minimum phase square root such that the energy concentrates near the starting point of their support, while Symlets select each other set of roots to have closer symmetry with linear complex phase. Symlets are nearly symmetrical wavelets

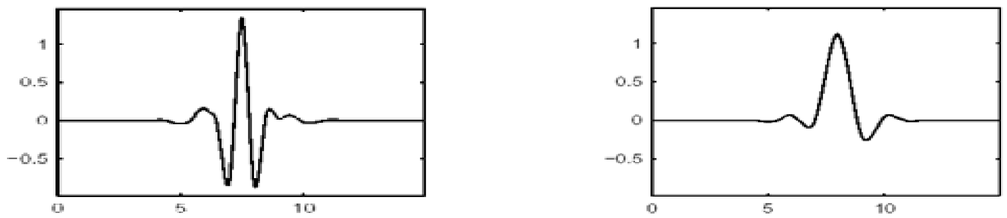


(a) Mother function (b) Scale function

Fig. 2 Haar wavelet function

(a) Mother function (b) Scale function

Fig. 3 db8 wavelet function



(a) Mother function (b) Scale function

Fig. 4 sym8 wavelet function

proposed by Daubechies as modifications to the db family, apart from the symmetry, remaining other properties of Daubechies and Symlet families are similar (Daubechies 1992).The mother wavelet and scaling function are shown in Fig. 4.

To reduce calculation, the DWT with $a=2^j$ and $b=k2^j$ is also utilized. Furthermore, signals are expressed in time domain. In the procedure of the damage detection the time parameter, t, is replaced by the space parameter, x. Therefore, the mother wavelet and the wavelet coefficients in the space frequency transform are expressed as follows

$$\psi_{a,b}(x)=\frac{1}{\sqrt{b}}\psi(\frac{x-a}{b}) \tag{15}$$

$$DWT_{f\ (j,k)}^{\psi}=\int_{-\infty}^{\infty} f\ (x)\psi_{j,k}\ (x)dx\ =2^{-j/2}\int_{-\infty}^{\infty} f\ (x)\psi\left(2^{-j}x-k\right)dx \tag{16}$$

The local analysis of a signal is known as the main advantage of WT. In other words, WT can focus on a time or location interval.

4.4 Application of discrete curvelet transform

The damage detection procedure based on the CT concept is similar to the WT concept. For the damage detection of plate structures by using CT, the plate structure is considered as an image, and the elements of matrix consist on the displacements of structure in the first shape mode (as structural responses). In the damage detection procedure by using CT, the initial matrix decomposes in to a number of sub-matrixes. It is noted that the implementation of CT decomposition in comparing to WT is different. In this study, CT based on the wrapping method

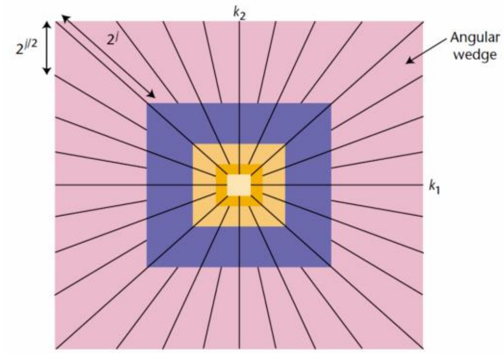


Fig. 5 Curvelet basic digital tiling

(Candes, Demanet *et al*. 2005) is utilized for the damage detection of plate structures. In fact, this method can efficiently overcome the drawbacks of WTs such as the missing directional selectivity of conventional DWT. The CT concept can also allow an almost optimal sparse representation of objects with singularities along smooth curves in the 2-D case. Unlike the isotropic elements of wavelets, the needle-shaped elements of CT possess very high directional sensitivity and anisotropy. The elements are very efficient in representing line-like edges. The wrapping method introduced as the discrete CT is based on the idea of representing a curve as superposition of functions of various length and width obeying the curvelet scaling law. Fig. 5 presents the discrete curvelet analysis method.

As shown in Fig. 5, the discrete curvelet partitioning of the 2-D Fourier converts second dyadic coronae (boxes) and sub-partitioning of the coronae into angular wedges (radiating lines). For example, the tiling corresponds to 5 scales, 16 angles at the second coarsest level and 64 angles at the finest (fifth) scale. The angles double every other scale. In the upper left numbers indicate that if the length of an angular wedge segment is proportional to 2^j (j is an integer), then the end wedge dimension is proportional to $2^{j/2}$.

Based on formula of Plancherel's theorem (Candes, Demanet *et al*. 2005), in the damage detection procedure curvelet coefficients are exactly the inner product of the image matrix, $f(x)$, with mother curvelet at different scales, angles, and positions

$$C(j,l,k) = (f, \varphi_{j,k,l}) = \int_{R^2} f(x).\overline{\varphi}_{j,k,l}(x)dx \qquad (17)$$

Since digital CTs operate in the frequency domain and the rectangular grid is used in the damage detection procedure, curvelet coefficients are expressed by the inner product as the integral over the frequency plane

$$C(j,l,k) = \int \hat{f}(\omega)\tilde{U}_j(S_{\theta l}^{-1}\omega)e^{i\langle b,\omega\rangle}d\omega \qquad (18)$$

where $\hat{f}(\omega)$ is the 2D FFT of the object f; $\tilde{U}_j(\cdot)$ is the window function; $S_{\theta l}$ is the shear matrix and $b=(k_1 2^{-j}, k_2 2^{-j})$; Also, $\langle\ \rangle$ expresses this inner product.

The more detail of the wrapping method is described in the work of Candes *et al*. 2005. Thus, the general procedure of the discrete CT is implemented as follows:

1. Take the 2D FFT of the signal f and obtain $\hat{f}$.

2. Multiply $\hat{f}$ with the window $\tilde{U}_j(\cdot)$.
3. Take the inverse Fourier transform on the appropriate Cartesian grid $b = (k_1 2^{-j}, k_2 2^{-j})$.

5. Numerical examples

In this section, the applicability and efficiency of the damage detection based on WT and CT are demonstrated through five numerical examples. In the examples, a fixed support rectangular plate is considered with 400 cm width, 400 cm height, and the thickness 5 cm. The material assumed to be steel with Young's modulus E=21×10^6 kg/cm^2, mass density 0.00785 kg/cm^3, and Poisson's ratio v=0.3. Young's modulus of defect is assumed to be 1.8×10^6 kg/cm^2. Finally, the performance of the damage identification method based on these transforms is validated using the experimental data of a steel plate measured by Rucka and Wilde (2006).

5.1 The rectangular plate with horizontal linear defect

The fixed support rectangular plate with horizontal linear defect shown in Fig. 6 is presented as the first example. The defect with 40 cm length and 5 cm width is considered in the location shown in Fig. 6. The main aim of this example is that the applicability and efficiency of the wavelet and curvelet transforms are investigated in the identification of linear defect.

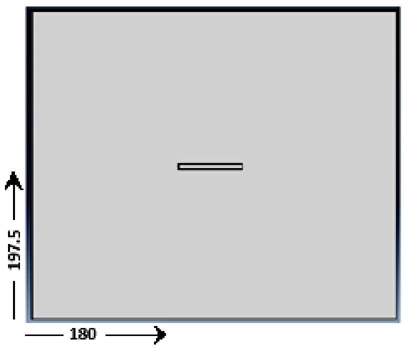


Fig. 6 Geometry of the plate structure with the horizontal linear damage

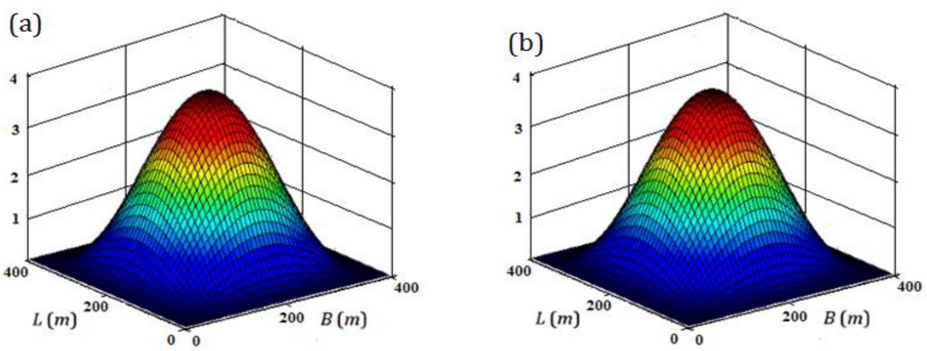
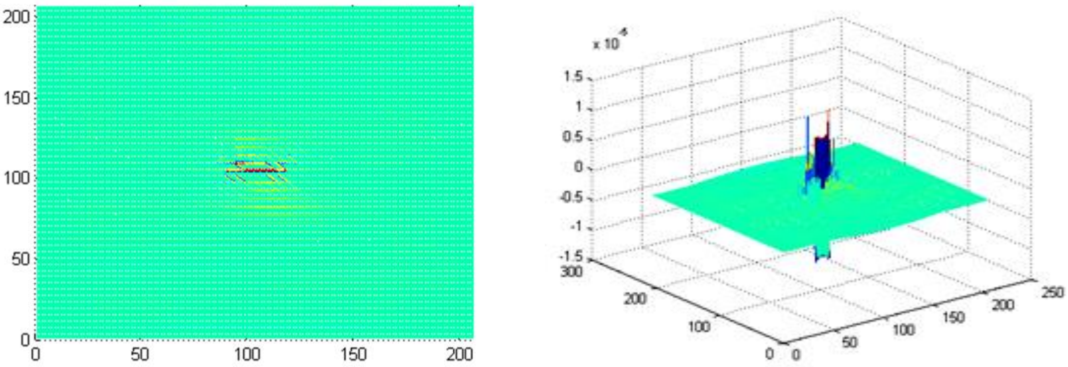


Fig. 7 The first mode shape for the plate: (a) without damage (b) with damage

In the finite element process, the geometry of plate is divided into a square mesh of 400×400 elements. After the analysis of the undamaged and damaged plate, the first mode shape of damaged and intact plate is depicted in Fig. 7. As observed from Figs. 7(a) and 7(b), the first mode shape of damaged and intact plate are completely the same and it is quite difficult to reveal the difference.

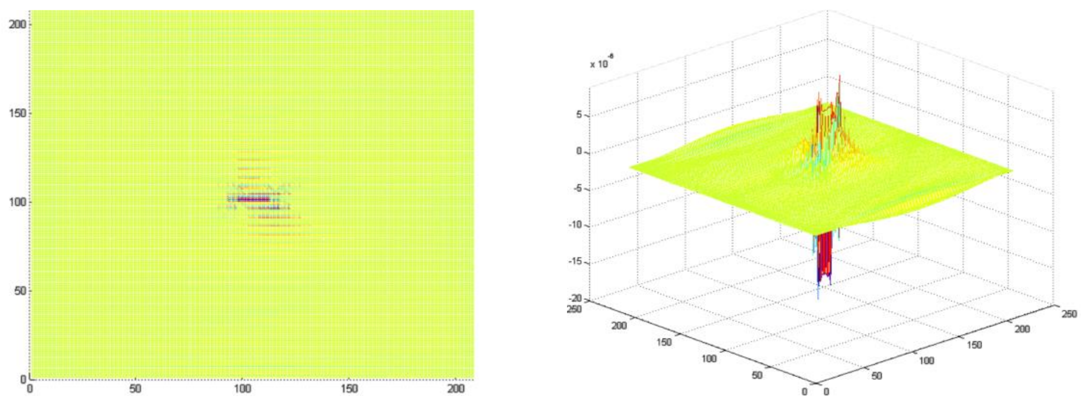
Hence, the methodology based on the wavelet and curvelet transforms can reveal the difference between the undamaged and damaged plate structure. In the first stage, the damage detection process is implemented based on the Haar, Daubechies and Symlet wavelets. In Figs. 8 to 10, the coefficients of the wavelets obtained by the second 2-D and 3-D sub-band of these wavelets are depicted.

As reveal from Figs. 8(b), 9(b) and 10(b), the peak values of the coefficients are observed at the exact damage location created. Thus, these wavelets can efficiently detect the location of the damage in the plate. Furthermore, the results show that in this study the damage detection process is not sensitive to the chosen of the wavelet type. Hence, the damage detection in the next examples are investigated by the Haar wavelet. It is noted that the plot of horizontal wavelet coefficients is required because the linear defect is horizontal.



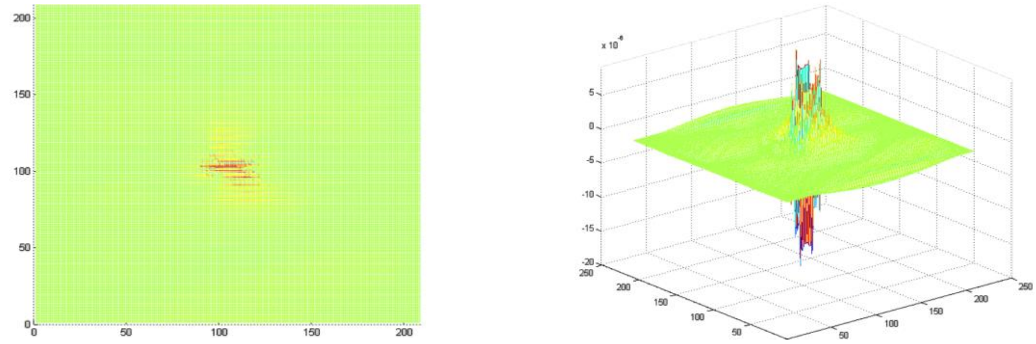
(a) First 2-D sub-band (b) First 3-D sub-band

Fig. 8 The cofficients of the Haar wavelet



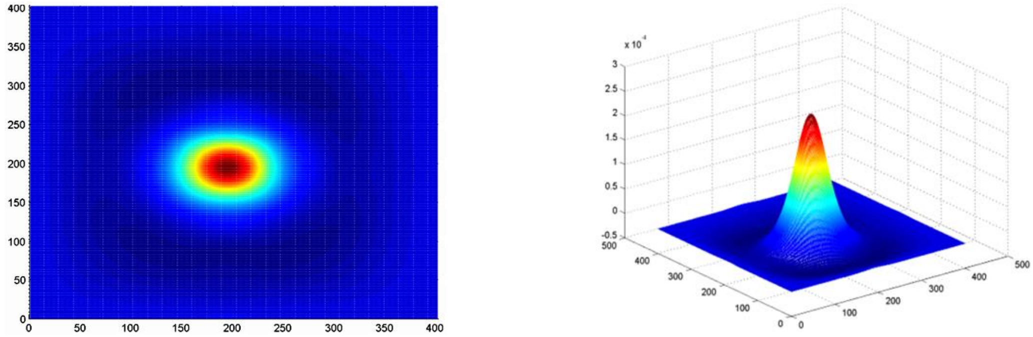
(a) Second 2-D sub-band (b) Second 3-D sub-band

Fig. 9 The cofficients of the Daubechies wavelet



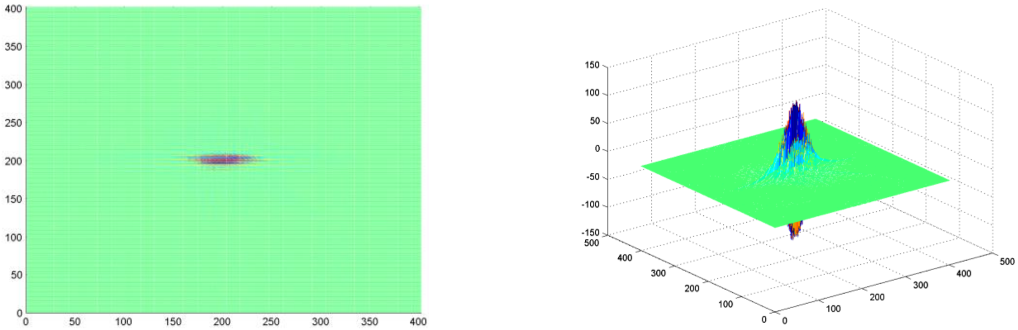
(a) Second 2-D sub-band (b) Second 3-D sub-band

Fig. 10 The cofficients of the Symlet wavelet



(a) First 2-D sub-band (b) First 3-D sub-band

Fig. 11 The cofficients of the first sub-band of CT



(a) Second 2-D sub-band (b) Second 3-D sub-band

Fig. 12 The cofficients of the second sub-band of CT

In second stage, the cofficients for two sub-bands of curvelet coefficients are also depicted in Figs. 11 and 12.It can be seen form Figs. 11(b) and 12(b) that the peak values of the cofficients are created at the exact damage location. Thus, the damage location of the plate is obviously detected using CT. Finally, it is concluded that based on the proposed damage detection procedure the wavelet and curvelet transforms can identify the location of damage. Hence, the transforms can be utilized as an efficient tool for the detection of the horizontal linear damage in plate structures.

5.2 The rectangular plate with vertical linear defect

This example deals with a fixed support rectangular plate with vertical linear defect shown in Fig. 13. The defect with 40*cm* length and 5*cm* width is considered in the 125*cm* distance of edge.

The results of the damage detection procedure based on the Haar wavelet and curvelet transforms are obtained and depicted in Figs. 14 to 17. According to the WT procedure, the sub-bands of the wavelet coefficients are calculated and are shown in Figs. 14 and 15. As reveal from Figs. 14(b) and 15(b), the peak values at the location of damage are obviously ocurred. Hence, the results of the WT concept demonstrat the capability of WT for the identification of damage in the plate structure. In this example, the plot of the vertical wavelet coefficients is required because the linear defect is vertical.

The sub-bands of curvelet coefficients are also depicted in Figs. 16 and 17. For the comparsion of capability of the wavelet and curvelet transforms, the two sub-bands of CT are shown in Figs. 16 and 17. It can be seen from Figs. 16(b) and 17(b) that the peak values are occurred in the location of damage. Thus, the CT concept has the capability of the damage detection in plate structures. Finally, the results of WT and CT show that the transforms can identify the shape and size of damage.

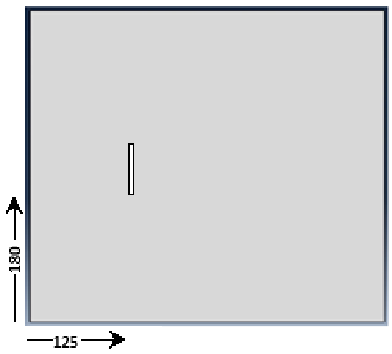
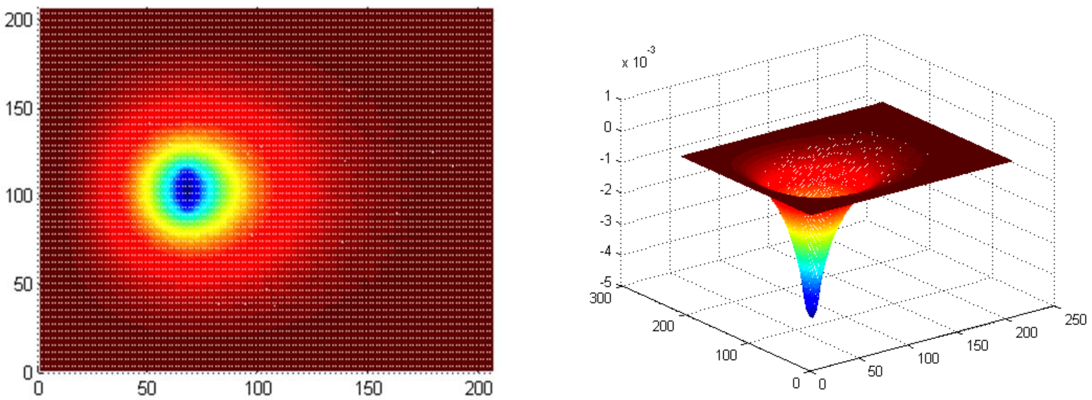
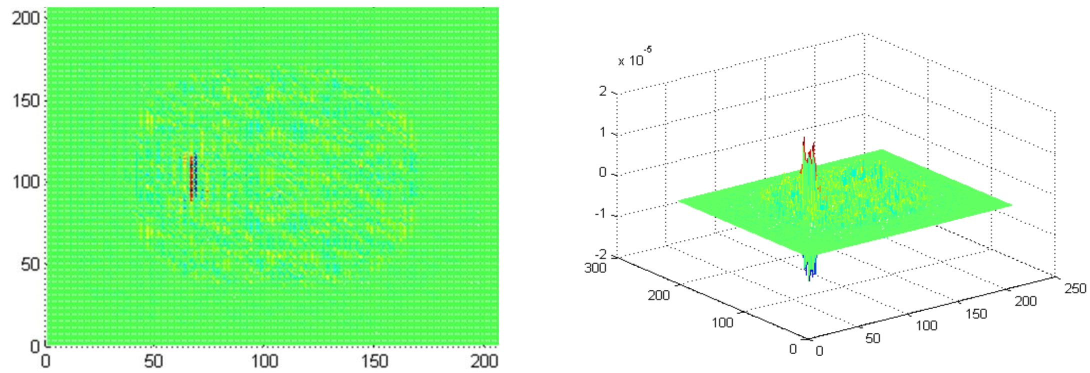


Fig. 13 Geometry of the plate structure with the vertical linear damage



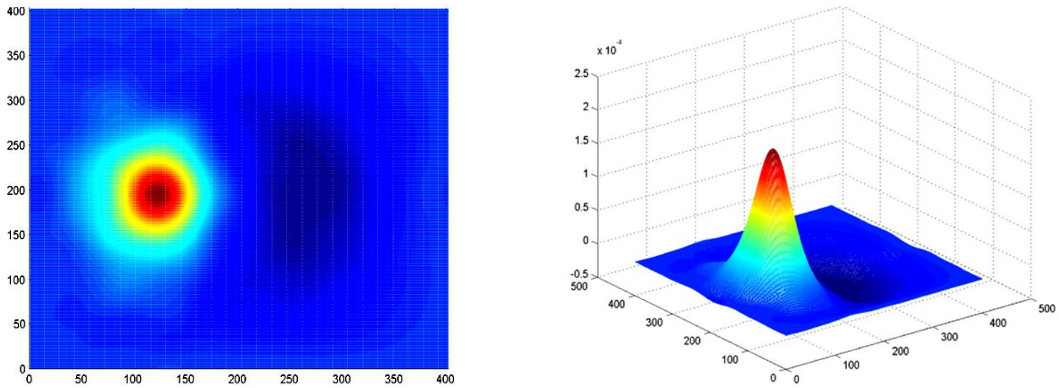
(a) First 2-D sub-band (b) First 3-D sub-band

Fig. 14 The cofficients of the first sub-band of WT



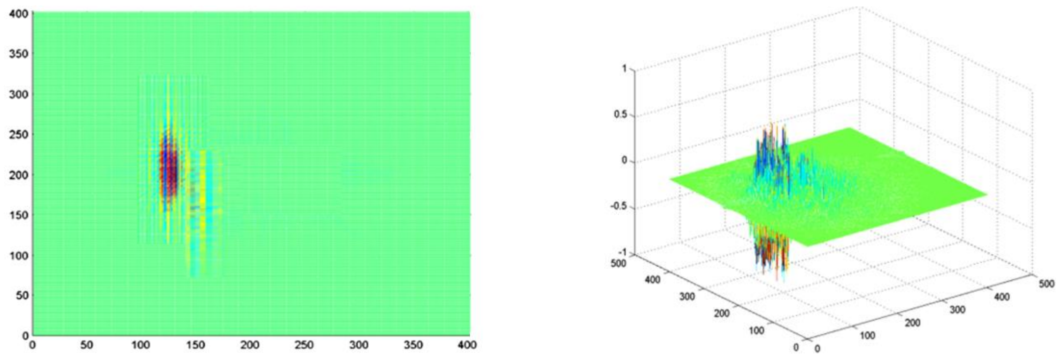
(a) Second 2-D sub-band (b) Second 3-D sub-band

Fig. 15 The cofficients of the second sub-band of WT



(a) First 2-D sub-band (b) First 3-D sub-band

Fig. 16 The cofficients of the first sub-band of CT



(a) Second 2-D sub-band (b) Second 3-D sub-band

Fig. 17 The cofficients of the second sub-band of CT

5.3 The rectangular plate with the oblique linear defect

A fixed support rectangular plate with oblique linear defect shown in Fig. 18 is considered. The

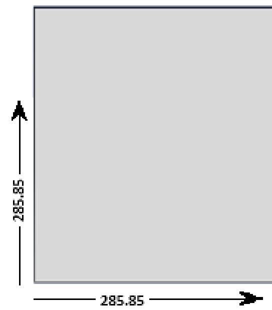
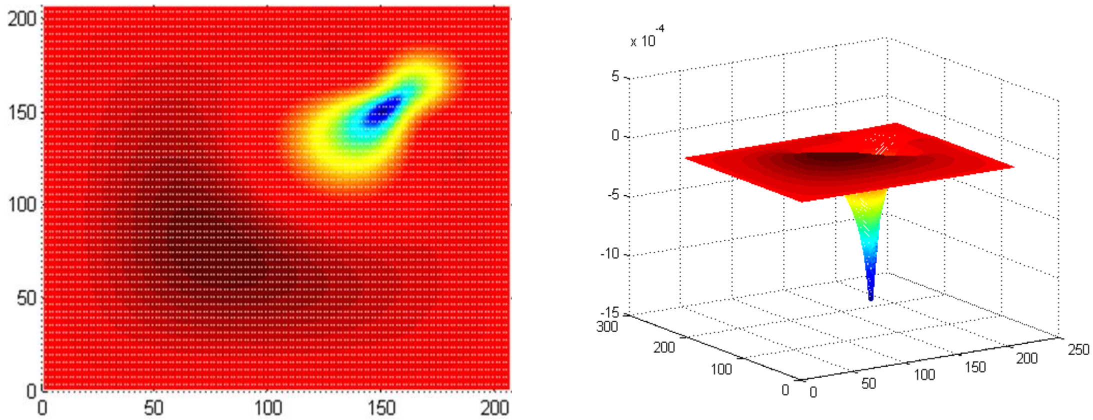
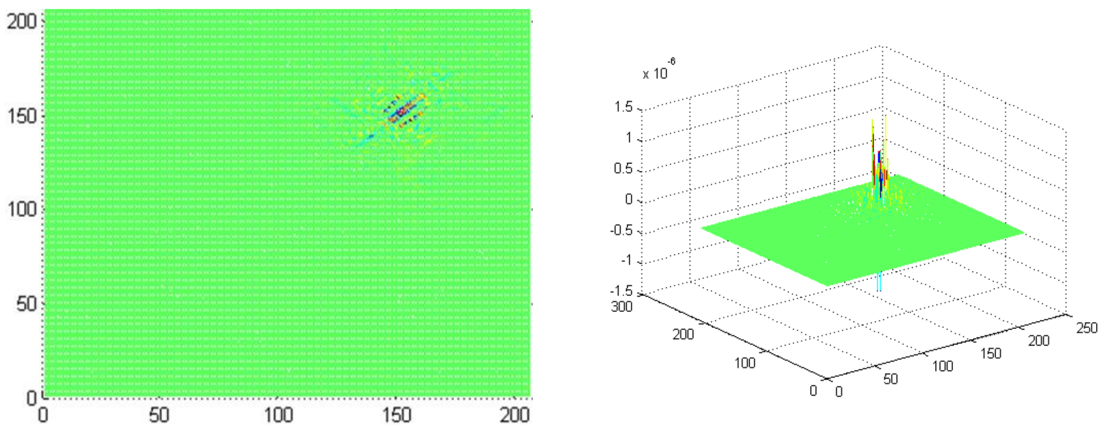


Fig. 18 Geometry of the plate structure with the oblique linear damage



(a) First 2-D sub-band (b) First 3-D sub-band

Fig. 19 The cofficients of the first sub-band of WT

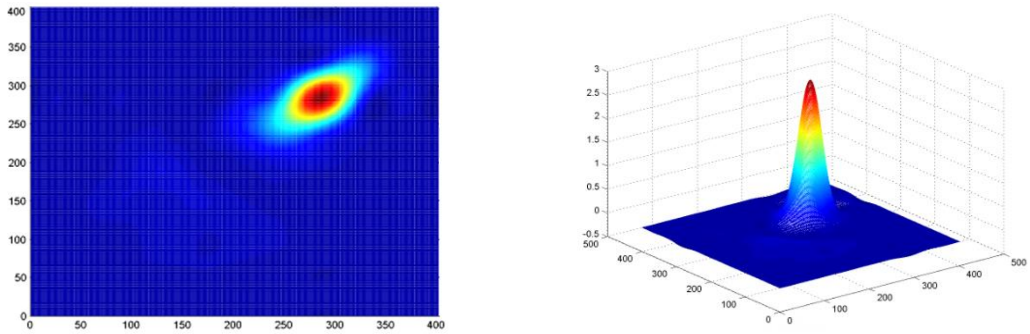


(a) Second 2-D sub-band (b) Second 3-D sub-band

Fig. 20 The cofficients of the second sub-band of WT

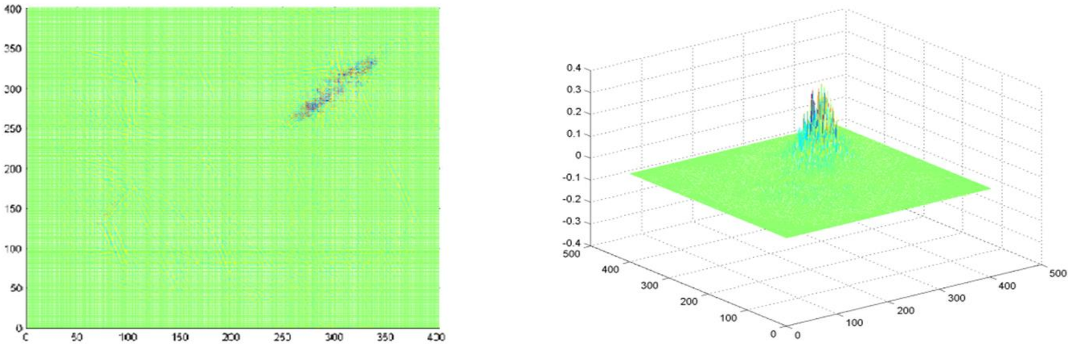
defect with 30 cm length, 3 cm width and 45° is located in 285.85 cm distance of edge of the plate.

Based on the Haar WT procedure, the sub-bands of the wavelet coefficients are calculated and are shown in Figs. 19 and 20. As reveal from Figs. 19(b) and 20(b), the peak values at the location



(a) First 2-D sub-band (b) First 3-D sub-band

Fig. 21 The cofficients of the first sub-band of CT



(a) Second 2-D sub-band (b) Second 3-D sub-band

Fig. 22 The cofficients of the second sub-band of CT

of damage are obviously ocurred. Hence, the results of the WT concept demonstrat the capability of WT for the identification of damage in the plate structure.

It is noted that the plot of diagonal wavelet coefficients is required because the linear defect is oblique. Furthermore, the sub-bands of curvelet coefficients is obtained based on the CT concept and depicted in Figs. 21 and 22. It can be seen from Figs. 21(b) and 22(b) that the peak values are occurred in the location of damage. Thus, the CT concept can be utilized for the detection of oblique linear damage. Finally, the results of Figs. 19 to 22 demonstrate that the wavelet and curvelet transforms can identify the shape and size of damage.

5.4 The rectangular plate with the curved linear defect

A fixed support rectangular plate with curved defect shown in Fig. 23 is chosen. The curved defect with 73 cm length and 3 cm width is assumed to be a part of a circle, and the started location coordinates of the damage is x=275 cm and y=200 cm

Using the damage detection process based on the wavelet and curvelet transforms, the results of the damage detection are shown in Figs. 24 and 25.

For comparison of the wavelet and curvelet transforms, a sub-band of the transforms with the best resolution is selected and shown in Fig. 24 and 25. As seen in Fig. 24(b), a peak as node is created in the location of damage. Since the peak do not cover the total domain of defect, the image of defect shape is not seen. The 2-D and 3-D figuers of a sub-band of CT are depicted in Figs. 25(a) and 25(b). It is clearly seen from Fig. 25(b) that the CT can identify the location and shape of defect. Therefore, CT in comparison with WT can be reliably utilized in the identification

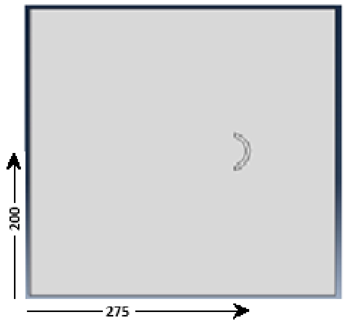
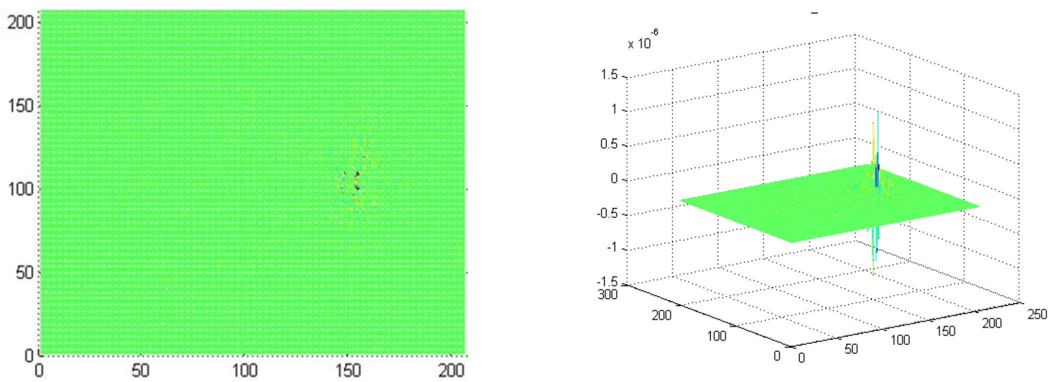
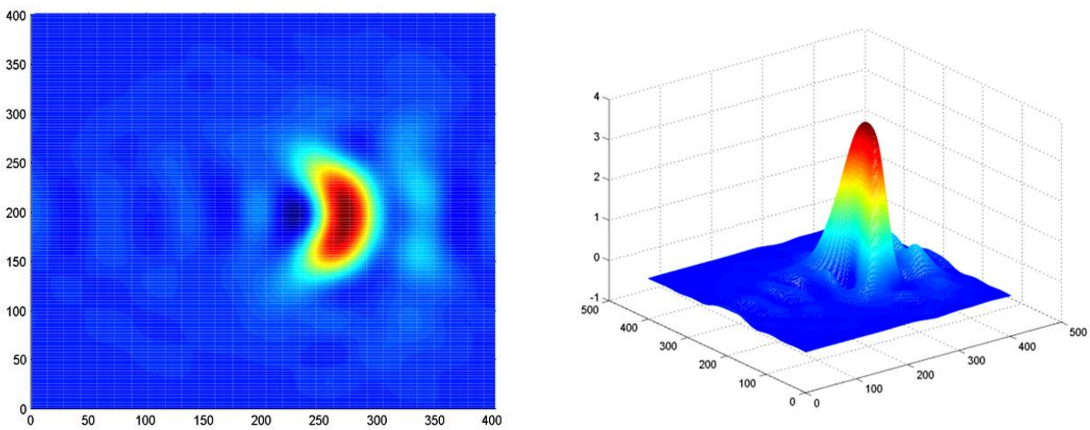


Fig. 23 Geometry of the plate structure with the curved damage



(a) Second 2-D sub-band (b) Second 3-D sub-band

Fig. 24 The cofficients of the Second sub-band of WT



(a) First 2-D sub-band (b) First 3-D sub-band

Fig. 25 The cofficients of the first sub-band of CT of defect with arbitrary shape.

5.5 The rectangular plate with multi-linear defect

A fixed support rectangular plate with multi-linear defect shown in Fig. 26 is considered as the last example in order to investigate the capability of WT and CT in the damage detection. For this purpose, in three regions of plate linear defects with 20 cm length and 2 cm width consist on horizontal, vertical and oblique shape. The location of defects is shown in Fig. 26. The percent damage is also set equal to 5%.

The results of the damage detection procedure based on WT and CT are shown in Figs. 27 to 32. The approximate coefficients of WT are depicted in Fig. 27. As obvious from Fig. 27, the location of damages is clearly detected using the WT concept. Hence, the WT concept has the capability of the multi-damage detection.

Furthermore, the three sub-bands of the wavelet coefficients of the details are indicated in Figs. 28 to 30. As seen from Figs. 28 to 30, the locations of multi-damage are clearly identified. However, the following results are concluded from these figures: 1) As observed from the 2-D

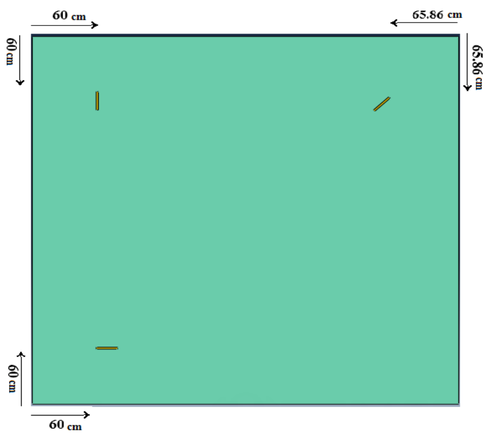
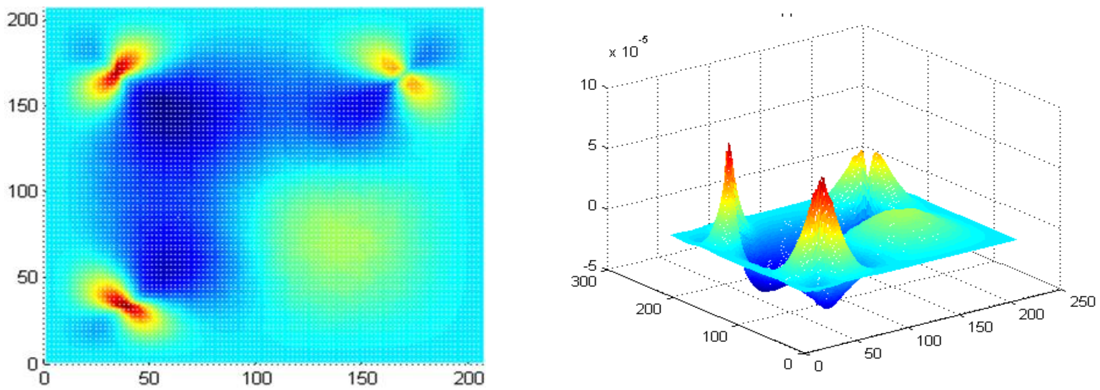
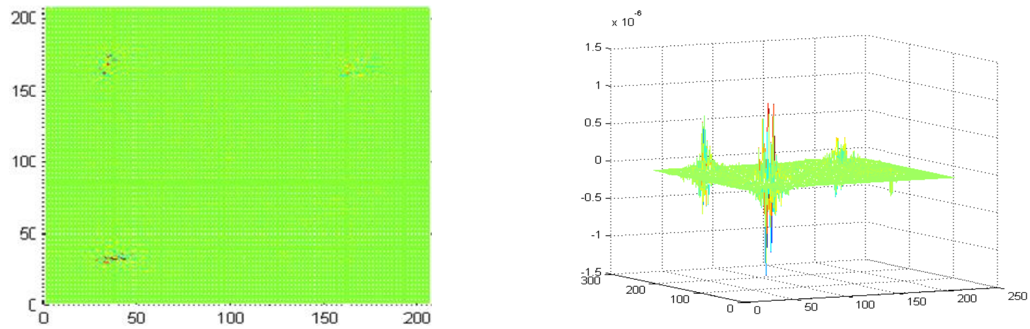


Fig. 26 Geometry of the plate structure with the multi-damage



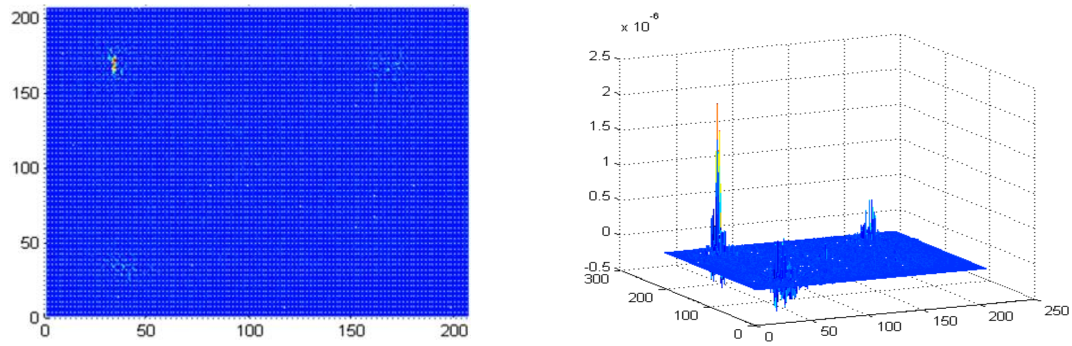
(a) Approximation 2-D sub-band (b) Approximation 3-D sub-band

Fig. 27 The approximation sub-band of WT



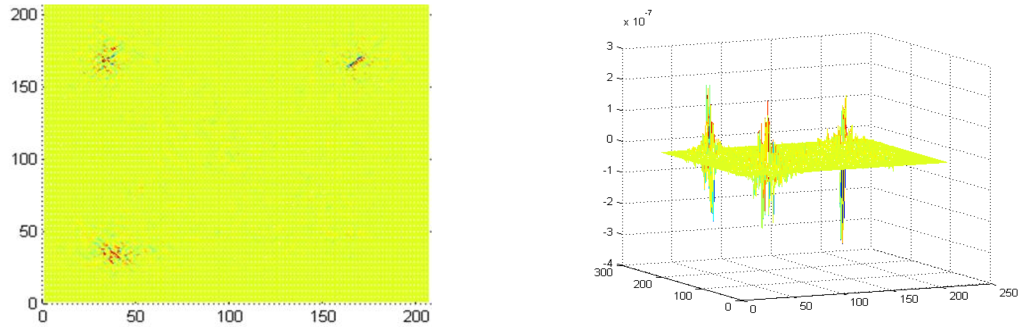
(a) Horizontal 2-D sub-band (b) Horizontal 3-D sub-band

Fig. 28 The cofficients of the horizontal sub-band of WT



(a) Vertical 2-D sub-band (b) Vertical 3-D sub-band

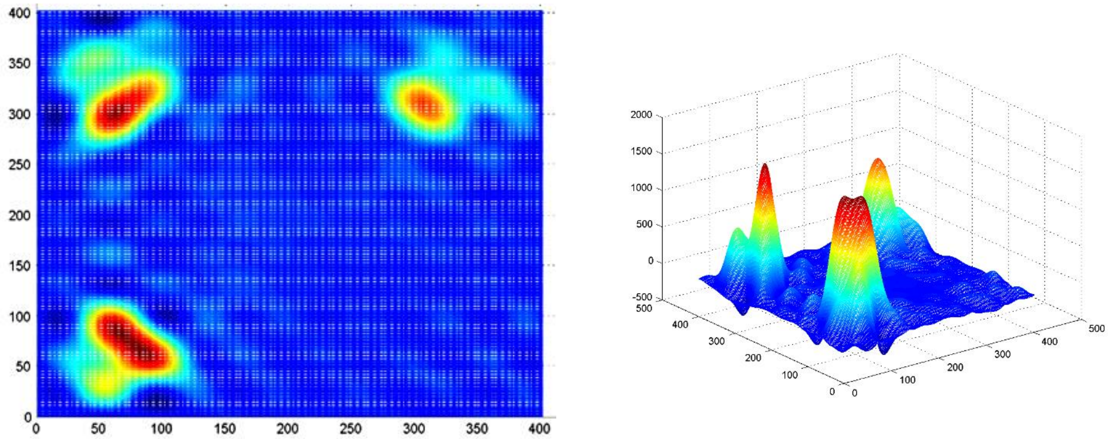
Fig. 29 The cofficients of the vertical sub-band of WT



(a) Diagonal 2-D sub-band (b) Diagonal 3-D sub-band

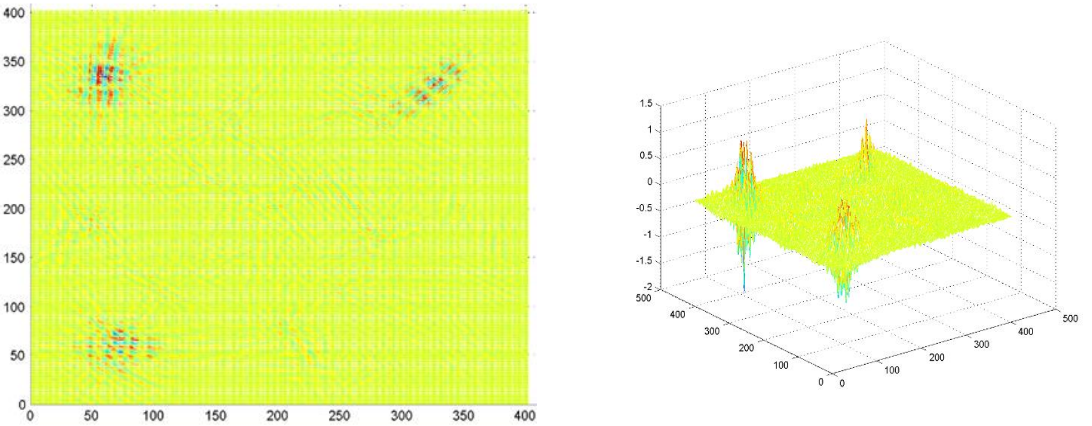
Fig. 30 The cofficients of the diagonal sub-band of WT

horizontal detail of WT shown in Fig. 28(a), the direction of horizontal defect is accurately detected, but for the vertical and oblique defects the direction of defect is not recognizable. 2) By considering Fig. 28(b) related to the 3-D details of WT, in the horizontal defect the height of peak in the location of damage has greater than that of the vertical and oblique defects. Furthermore, the abovementioned descriptions can be applied for Figs. 29 and 30 related to vertical and diagonal coefficients, respectively.



(a) First 2-D sub-band (b) First 3-D sub-band

Fig. 31 The cofficients of the first sub-band of CT



(a) Second 2-D sub-band (b) Second 3-D sub-band

Fig. 32 The cofficients of the second sub-band of CT

For the comparsion of the CT performance with that of WT, two sub-bands are shown in Figs. 31 and 32. As seen from Figs. 31 and 32, that the peak values are created in the location of defects.In other words, in the case of multi-damage the CT concept efficiently detects the location of defects.

5.6 Experimental validation study

In this section, the performance and capability of WT and CT in the damage detection are demonstrated through an experimental study implemented on a realistically damaged plate. To achieve this purpose, the vibration response data measured from a steel plate tested by Rucka and Wilde (2006) is considered and applied in the damage detection procedure. The steel plate of length 56 cm, width 48 cm and height 0.2 cm is shown in Fig. 33. The material properties of the plate include Young's modulus of E=192 GPa, mass density 7430 kg/m^3 and Poisson's ratio of

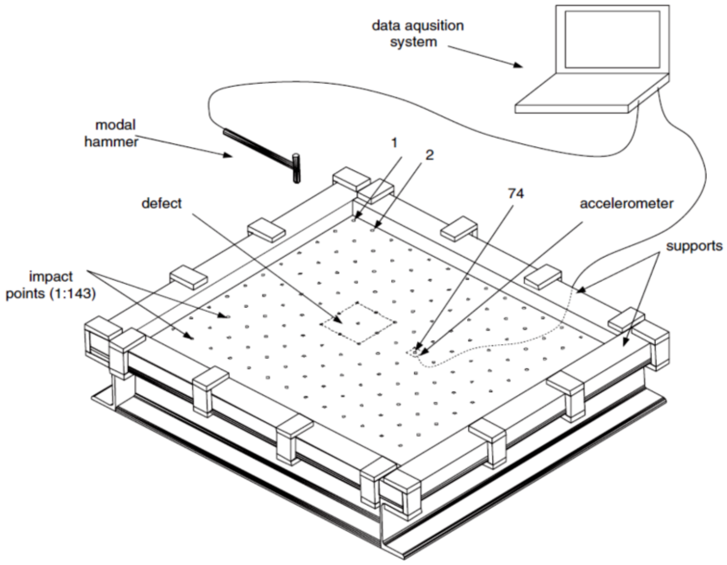
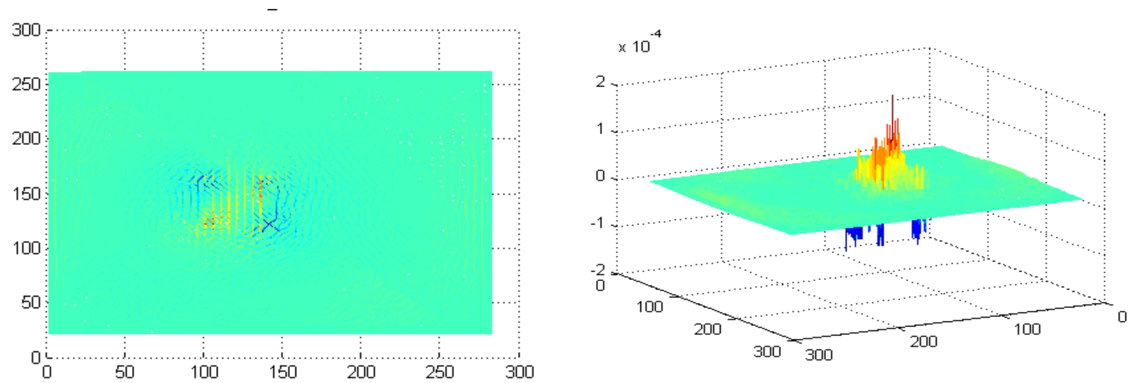


Fig. 33 Experimental set-up (Rucka and Wilde 2006)



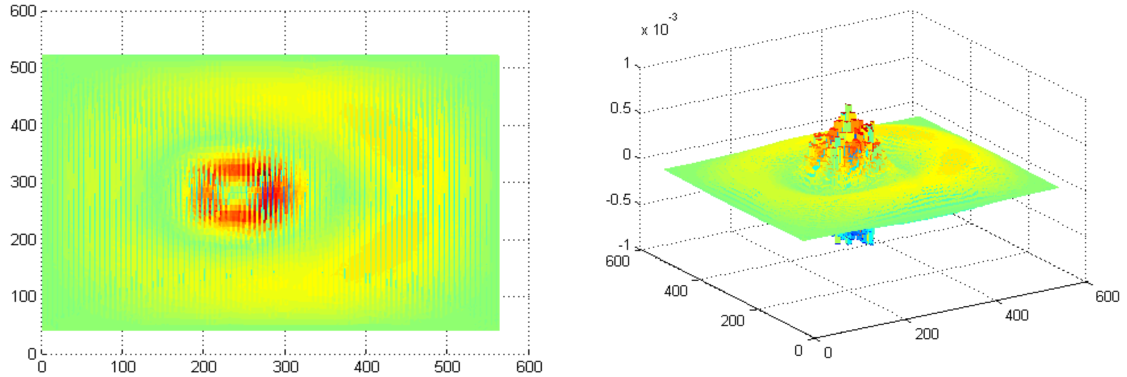
(a) Second 2-D sub-band (b) Second 3-D sub-band

Fig. 34 The cofficients for the Second sub-band of WT

v=0.25 The plate contains a rectangular defect of length 8 cm, width 8 m and height 0.05 cm. The started location coordinates of the damage is *x*=20 cm and *y*=20 cm

Since the mode shape of the plate was measured with sampling distance of 4*cm*, the mode shape of the plate is interpolated to decrease sampling distance from 4 to 1*cm*, in the first stage. In the second stage, the damage detection method based on WT and CT is applied to detect the damage in the plate. Finally, the results of the damage detection procedure are shown in Figs. 34 and 35.

The obtained results indicated that the damage detection based on the wavelet and curvelet transforms can be adopted as the efficient and applicable methods for damage detection of actual structures.



(a) First 2-D sub-band (b) First 3-D sub-band

Fig. 35 The cofficients for the First sub-band of CT

6. Conclusions

This study deals with the comparison of performance of wavelet and curvelet transforms for the detection of defect types in plate structures. Hence, the detection of defect types through the defect shape in rectangular plate is considered. By using the first mode shape of plate structure and the distribution of coefficients of the transforms, the damage existence, the failure location and the approximate shape of the damage are investigated.

By the comparison of the results obtained using the wavelet and curvelet transforms, the advantages of these transforms can be expressed as follows:

• In structures, the damage existence causes the small discontinuity of the structural responses in the domain of damage. The discontinuity cannot be revealed by the observation of the structural responses, but using the wavelet and curvelet transforms can clearly reveal the discontinuity. In fact, in the wavelet and curvelet transforms the distribution of the transform coefficients can identify the discontinuity and creates peaks in the location of damage.

• In the total cases of linear defect, these transforms can efficiently reveal the structural damage. Furthermore, in the damage detection procedure of plate with the linear defect the performance of WT and CT is same.

• The CT concept in comparison with the WT concept can efficiently reveal the damage with the curved shape. In other words, by using of WT entity of peak is not created in the total domain of damage. Three sub-bands of details of WT as horizontal, vertical and diagonal can also identify the structural damages in three directions. However, because of the polar structure CT can detect the types of the structural damages with arbitrary shape.

• In the case of the multi-damages, WT and CT can detect the defects with trivial percent damage and small area. Furthermore, these transforms can simultaneously detect a number of damage in plate structures.

References

ABAQUS (2011), "ABAQUS user's manual V 6.11", University of Cambridge.

Aboufadel, E. and Schlicker, S. (2009), *Discovering wavelets,* Wiley, New York.

Bagheri, A., Ghodrati Amiri, G. and Seyed Razzaghi, S.A. (2009), "Vibration-based damage identification of plate structures via curvelettransform", *J. Sound. Vib*., **327**, 593-603.

Bombale, B.S., Singha, M.K. and Kapuria, S. (2008), "Detection of delamination damage in composite beams and plates using wavelet analysis", *Struct. Eng. Mech.,* **30**(6), 699-712.

Candes, E. (1988), "Ridgelet: theory and application", Ph.D. Dissertation*,* Department of Statistics, Stanford University.

Candes, E. and Donoho, D. (2000), "Curvelets a surprisingly effective non adaptive representation for objects with edges", Technical report, Department of Statistics, Stanford University.

Candes, E., Demanet, L., Donoho, D. and Ying, L., (2005), *Fast discrete curvelet transforms*, Caltech, Pasadena, CA.

Chang, Ch. and Chen, L.W. (2004), "Damage detection of a rectangular plate by spatial wavelet based approach", *J. Appl. Acoust.*, **65**, 819-832.

Daubechies, I. (1990), "Wavelet transform, time-frequency localization and signal analysis", *IEEE. T. Inform. Theory*, **36**(5), 961-1005.

Daubechies, I. (1992), *Ten Lectures on Wavelets*, CBMS-NSF Conference Series in Applied Mathematics, New York.

Doebling, S.W., Farrar, C.R., Prime, M.B. and Shevitz, D.W. (1996), "Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: A Literature Review", Los Alamos National Laboratory RTeport.

Douka, E., Loutridis, S. and Trochidis, A. (2002), "Crack identification in plates using wavelet analysis", *J. Sound. Vib*., **270**, 279-295.

Fan, W. and Qiao, P. (2009), "A 2-dcontinuous wavelet transform of mode shape data for damage detection of plate structures", *Int. J. Solid. Struct.*, **46**, 4379-4395.

Gokdag, H. (2011), "Wavelet-based damage detection method for a beam-type structure carrying moving mass", *Struct. Eng. Mech.*, **38**(1), 81-97.

Gokdag, H. and Kopmaz O. (2010), "A new structural damage detection index based on analyzing vibration modes by the wavelet transform", *Struct. Eng. Mech*., **35**(2), 257-260.

He, W.Y. and Zhu, S. (2015), "Adaptive-scale damage detection strategy for plate structures based on wavelet finite element model", *Struct. Eng. Mech*., **45**(2), 239-256.

Hinton, E. (1988), *Numerical methods and software for dynamic analysis of plates and shells*, Pineridge Press, Swansea.

Hou, Z., Noori, M. and Amand, St. (2000), "Wavelet-based approach for structural damage detection", *J. Eng. Mech.*, ASCE, **124**(10), 1059-1066.

Kim, B.H., Kim, H. and Park, T. (2006), "Nondestructive damage evaluation of plates using the multi-resolution analysis of two-dimensional haar wavelet", *J. Sound. Vib*., **292**, 82-104.

Liew, K.M. and Wang, Q. (1998), "Application of wavelet theory for crack identification in structures", *J. Eng. Mech.*, ASCE, **124**(2), 152-157.

Loutridis, S., Douka, E., Hadjileontiadis, L.J. and Trochidis, A. (2005), "A two-dimensional wavelet transform for detection of cracks in plates", *J. Eng. Struct.*, ASCE, **27**(9), 1327-1338.

Mallat, S. (1998), *A wavelet tour of signal processing*, Academic Press, Stéphane Mallat, USA.

Ovanesova, A. and Suarez, L.E. (2004), "Application of wavelet transforms to damage detection in frame structure", *J. Eng. Struct.*, ASCE, **26**(1), 39-49.

Pakrashi, V. and Alan, O. (2009), "A comparative analysis of structural damage detection techniques by wavelet, kurtosis and pseudo fractal methods", *Struct. Eng. Mech*, **32**(4), 489-500.

Rucka, M. and Wilde, K. (2006), "Application of continuous wavelet transform in vibration based damage detection method for beams and plates", *J. Sound. Vib.,* **297**, 536-550.

Wang, Q. and Deng, X. (1999), "Damage detection with spatial wavelets", *Int. J. Solid. Struct.*, **36**(23), 3443-3468.

Xu, W., Cao, M., Ostachowicz, W., Radzieński, M. and Xia, N. (2015), "Two-dimensional curvature mode shape method basedon wavelets and Teager energy for damage detection in plates", *J. Sound. Vib.,* **7**, 266-

278.
Yang, J.M., Yang, Z.W. and Tseng, C.M. (2011), "Damage detection in stiffened plates by wavelet transform", *Int. J. Nav. Arch. Ocean.*, **3**, 126-135.

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