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Structural damage identification using gravitational search algorithm

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Abstract. This study aims to present a novel optimization algorithm known as gravitational search algorithm (GSA) for structural damage detection. An objective function for damage detection is established based on structural vibration data in frequency domain, i.e., natural frequencies and mode shapes. The feasibility and efficiency of the GSA are testified on three different structures, i.e., a beam, a truss and a plate. Results show that the proposed strategy is efficient for determining the locations and the extents of structural damages using the first several modal data of the structure. Multiple damages cases in different types of structures are studied and good identification results can be obtained. The effect of measurement noise on the identification results is investigated.

Keywords: damage identification; gravitational search algorithm; vibration data; frequency domain; modal assurance criteria

1. Introduction

In the last few decades, techniques based on vibration response have received extensive attention in the field of structural damage identification and health monitoring. Local damages cause changes in structural physical properties, mainly in stiffness and damping at damaged locations. It will result in changing structural dynamic characteristics, such as natural frequencies and mode shapes, etc. Many researchers made abundant excellent literature in the field of vibration-based damage detection. Doebling, Farrar *et al*. (1998) presented a comprehensive review of the damage detection methods by examining changes in the dynamic responses of a structure. Housner, Bergman *et al*. (1997) provided a summary on the most-advanced techniques in control and health monitoring in civil engineering structures.

The usual model-based damage detection methods concentrate on minimizing an objective function, usually defined as the discrepancies between the vibration data obtained by modal testing and those computed from the analytical data. Traditional optimization methods are gradient-based (Hao and Xia 2002). Swarm intelligence algorithms provide different perspective in optimization problems. Optimization techniques and their improved versions with strong global searching

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ability are needed for more accurate and reliable solution. In these years, genetic algorithm (GA) as a global optimization method has been widely used for damage detection problem (Buezas, Rosales *et al.* 2011, Chou and Ghaboussi 2001, Hao and Xia 2002, He and Hwang 2006, Sahoo and Maity 2007, Vakil-Baghmisheh, Peimani *et al.* 2008). Apart from GA, particle swarm optimization (PSO), as a population-based global optimization, is widely used recently due to its simplicity, wide applicability and outstanding performance (Eberhart and Kennedy 1995, Kennedy and Eberhart 1995, Poli, Kennedy *et al.* 2007). PSO was also introduced in damage detection by many researchers (Begambre and Laier 2009, Kang, Li *et al.* 2012, Mohan, Maiti *et al.* 2013, Vakil Baghmisheh, Peimani *et al.* 2012). More recently, Xu, Ding *et al*. (2015) presented a structural damage detection method based on chaotic artificial bee colony algorithm. Li and Lu (2015) developed a multi-swarm fruit fly optimization algorithm for structural damage identification. It was verified through beam and planar truss structures.

All population-based search algorithms provide satisfactory results for some specific problems but there is no heuristic algorithm that could provide a superior performance than others in solving all optimizing problem. Hence, proposing new high performance heuristic algorithms are welcome (Rashedi, Nezamabadi-pour *et al.* 2009). Rashedi, Nezamabadi-pour *et al.* (2009) proposed a new heuristic search algorithm named Gravitational Search Algorithm (GSA), which is inspired by Newton's law of universal gravitation. The convergence analysis is conducted by Ghorbani and Nezamabadi-pour (2012), then the stability analysis of the algorithm is completed by Farivarand Shoorehdeli (2016). In recently years, the GSA has already been successfully applied to numerous real-world problems in engineering (Bahrololoum, Nezamabadi-Pour *et al.* 2012, Li and Zhou 2011, Rashedi, Nezamabadi-pour *et al.* 2011, Sarafrazi, and Nezamabadi-pour *et al.* 2013, Xu and Zhang 2014, Khatibinia and Naseralavi 2014, Su and Wang 2015, Yuan, Chen *et al.* 2015). But so far, no report has been presented on the application of GSA to structural damage detection. In this study, the GSA is extended to the field of structural damage identification in civil and mechanical engineering. The inverse problem of damage identification is treated as an optimization problem. An objective function for damage detection is established using the natural frequencies and mode shapes of the structure. Three different types of structures are studied to illustrate the correctness and efficiency of the proposed method. Results show that the proposed strategy is efficient on determining the locations and the extents of structural damages. The measurement noise seems to have little effect on the identification results.

2. Theory for structural damage identification

2.1 Parameterization of local damage

The eigenvalue equation for a finite element model of anintact structure can be expressed

$$(\mathbf{K}-\omega_j^2\mathbf{M})\phi_j = 0 \tag{1}$$

where **K** and **M** denote the structural stiffness matrix and mass matrix respectively. ω_j is the jth natural frequency and ϕ_j is the corresponding mode shape.

When a local damage occurs in a structure, it causes the loss in stiffness, while the loss in mass is usually negligible. The damaged stiffness of a structure with *nel* elements can be expressed by a set of damage parameters d_i (i=1,2,...,nel) in the form of the following equation

$$\mathbf{K_d} = \sum_{i=1}^{nel} (1 - d_i) \mathbf{k_i^e} \tag{2}$$

where $\mathbf{K_d}$ is the stiffness matrix of the damaged system, and $\mathbf{k_i^e}$ presents the *i*th elemental stiffness matrix. The positive parameter d_i with value range between 0 and 1 refers to the damage extent of the *i*th element. When d_i=0, it means no damage. While d_i=1, it represents the complete damaged status. Therefore, detection of the local damages in a structure is implemented by quantification of the values of the damage parameter vector $\{d\}$. In this paper, d_i ranges between 0 to 0.99.

2.2 Objective function in frequency domain

It is aforementioned that the changes instructure properties will lead to changes in the dynamic characteristics, such as natural frequencies and mode shapes. In the context of damage detection, the task is to minimize the objective function using the discrepancies between the measured modal data and the calculated ones.

The objective function for damage detection is defined in terms of natural frequencies and modal assurance criteria (MAC)

$$f = \sum_{j=1}^{NF} w_{\omega j} \Delta\omega_j^2 + \sum_{j=1}^{NM} w_{\phi j} (1 - MAC_j) \tag{3}$$

where $w_{\omega j}$ is a weight factor corresponding to *j*th natural frequency, while $w_{\phi j}$ corresponding to *j*th *MAC*. *NF* and *NM* are the numbers of natural frequencies and mode shapes used in calculation, respectively.

The mathematical expression of the differences of natural frequencies in Eq. (3) is expressed as

$$\Delta\omega_j = \left| \frac{\omega_j^C - \omega_j^M}{\omega_j^M} \right| \tag{4}$$

where ω_j^C and ω_j^M are the *j*th calculated and measured natural frequencies, respectively.

The expression of MAC in *j*th modein Eq. (3) is

$$MAC_j = \frac{(\{\phi_j^C\}^T \{\phi_j^M\})^2}{\left\|\{\phi_j^C\}\right\|^2 \left\|\{\phi_j^M\}\right\|^2} \tag{5}$$

where ϕ_j^C and ϕ_j^M are the*j*th calculated and measured mode shapes, respectively. Eq. (5) indicates that value of MAC is 1 when the calculated mode shape equals to the measured one.

It can be observed that Eq. (3) is a function with respect to the damage index $\{d\}$. Given a particular damage condition, if one damage index enables the objective function to achieve minimum, theoretically, which is 0, the damage index represents the true damage status.

3. An introduction to gravitational search algorithm (GSA)

In the GSA (Rashedi, Nezamabadi-pour *et al*. 2009), inspired by the lawofuniversal gravitation and Newton's second lawofmotion, particles or agents update their locations in a similar way. Their performances are measured by their masses. Good performances as good solutions mean heavy masses. In other words, the heavier the mass is, the larger force it will apply on others, and the smaller acceleration it will be obtained for motion. In GSA each agent (mass) has four properties: its position, its inertial mass M_i, its active mass M_a and its passive mass M_p. Inertial mass is a measure of resistance ability for an agent to change its state of motion when a force is applied. Active mass is a measure of strength of gravitational field produced by the agent itself. Passive mass means measure of strength of an agent interaction with global gravitational field. And the position of the mass represents a solution of the problem. The gravitational and inertial masses are determined by the fitness function.

For an artificial system with N agents (masses), the position of the ith agent is expressed as

$$X_i = (x_i^1, \ldots, x_i^d, \ldots, x_i^D) \quad for\ i = 1, 2, \ldots. N \tag{6}$$

where x_i^d means the position of the ith agent in the dth dimension, while D is the global dimension of search space. At time t, the force applying on mass i from mass j is written as

$$F_{ij}^d = G(t) \frac{M_{pi}(t) \times M_{ai}(t)}{R_{ij}(t) + \varepsilon} (x_j^d(t) - x_i^d(t)) \tag{7}$$

where M_{aj} is the active gravitational mass related to agent j, M_{pi} is the passive gravitational mass of agent i, $G(t)$ represents gravitational constant at time t, ε is a small constant to guarantee that the denominator is not zero. R_{ij} (t) means Euclidian distance between the agents i and j.

$$R_{ij}(t) = \left\| X_i(t), X_j(t) \right\|_2 \tag{8}$$

The resultant force acting on agent i in dimension d is a randomly weighted sum of dth component of the force from *Kbest* agents

$$F_i^d(t) = \sum\nolimits_{j \in Kbest, j \neq i} rand_j F_{ij}^d(t) \tag{9}$$

where $rand_j$ is a random number between 0 and 1. *Kbest* means the set of first K agents with best fitness value and biggest mass. The concept of *Kbest* set is introduced here to efficiently balance exploitation and exploration. The exploration is the ability to expand search space, while exploitation to find the optimum solution. With the lapse of iteration, exploration must fade out and exploitation must fade in. The *Kbest*, as a function of time, starts with the initial value K_0 and decreases with time, i.e., all agents apply force at the very beginning, then decrease linearly, and in the end, only the biggest mass is retained.

According to the law of motion, the acceleration of the agent i at time t in dimension d is expressed as

$$a_i^d(t) = \frac{F_i^d(t)}{M_{ii}(t)} \tag{10}$$

where M_{ii} is the initial mass of the ith agent. The next velocity and position of the agent are calculated as

$$v_i^d(t+1) = rand_i \times v_i^d(t) + a_i^d(t) \tag{11}$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \tag{12}$$

where $rand_i$ is a random number between 0 and 1.

The gravitational constant G is initialized at the beginning and it will decrease with the lapse of time, with the similar concept of the decreasing weight factor in PSO (Shi and Eberhart 1998). It is expressed as

$$G(t) = G_0 e^{-\alpha \frac{t}{T}} \tag{13}$$

where G_0 is an initial constant and T is the total number of iteration.

The gravitational and inertia masses are simply calculated by the fitness value of each agent. As mentioned above, the heavier mass means better fitness evaluation and slower change on its position. For the simplicity, an assumption is made on the equality of the gravitational and inertia mass. The values of masses are calculated in the following forms

$$M_{ai} = M_{pi} = M_{ii} = M_i, \quad i = 1, 2, \ldots, N \tag{14}$$

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \tag{15}$$

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^{N} m_j(t)} \tag{16}$$

where $fit_i(t)$ indicates the fitness value of the agent$i$ at time t. The $worst(t)$ and $best(t)$ are defined as follows.

For minimization problems:

$$\begin{cases} best(t) = \min\limits_{j \in \{1,\ldots,N\}} fit_j(t) \\ worst(t) = \max\limits_{j \in \{1,\ldots,N\}} fit_j(t) \end{cases} \tag{17}$$

For maximization problems:

$$\begin{cases} best(t) = \max\limits_{j \in \{1,\ldots,N\}} fit_j(t) \\ worst(t) = \min\limits_{j \in \{1,\ldots,N\}} fit_j(t) \end{cases} \tag{18}$$

Damage identification based on the GSA is accomplished in the following steps:
Step 1: Initialize the population (generate a random population of N solutions) in search space. Set boundary values (for damage detection problem, $d_i \in [0, 0.99]$).
Step 2: Evaluate the fitness value in Eq.(3) for each agent using Eqs.(1)~(5).
Step 3: Update $G(t)$, $best(t)$, $worst(t)$, and $M_i(t)$ for i=1,2,...,N.
Step 4: Calculate the total force in different directions for each agent.

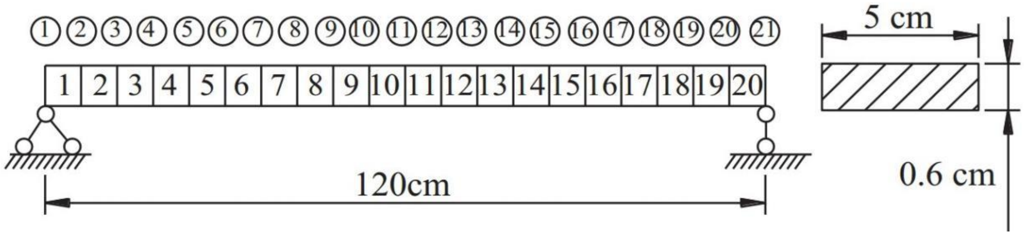

Fig. 1 Sketch of a simply supported beam and cross section (①, ②, …, denote node number of the FEM; 1,2,…,20 denote element number) (Dimensions not scaled)

Step 5: Calculate the acceleration and velocity for each agent.
Step 6: Update the position for each agent.
Step 7: Repeat Steps 2 to 6 until the stop criterion is met.

4. Numerical simulation

4.1 Parameters setting for GSA

The constant G_0 and α in Eq. (13)are taken as 100 and 20, respectively, same as the parameter setting in (Rashedi, Nezamabadi-pour *et al*. 2009). The population N the maximum iteration number $T$are taken 100 and 250, respectively. The dimensional boundary is set to be $d_i \in [0, 0.99]$, i=1,2,...,D. Both $w_{\omega j}$ and $w_{\phi j}$ in Eq. (3) are set to be 1. The final identification results are the average values of 20 independent runs.

4.2 A simply supported beam

A simply supported beam studied by Kang, Li *et al.* (2012) is used as a numerical simulation to compare identification results from proposed method with those from IEPSO presented in Kang, Li *et al.* (2012).

The geometry of the beam is shown in Fig. 1. The total number of elements and nodes are 20 and 21, respectively. Young's modulus, E=70 GPa, while density ρ=2.70×10^3 kg/m^3 and Poisson ratio μ=0.33. Two study cases in Kang, Li *et al.* (2012) are re-examined. The parameters used in PSO method are same as the those in reference literature (Kang, Li *et al.* 2012), i.e., p_r=0.05, p_v=0.25, s_p=1.8, b=5. The first 3 natural frequencies and mode shapes are utilized in these two cases same with reference cases. The natural frequencies are contaminated with 1% noise and 10% noise isadded to the modal displacement to simulate the noise contaminated measurements (Kang, Li *et al.* 2012).

4.2.1 Damage case 1

This case presents multiple severe damage cases with the assumption that element 1 and element 9 have 50% and 87.5% reduction in elemental stiffness, respectively.

Both the conditions with and without noise are considered. Fig. 2 shows the identification results for both noise-free and noise-contaminated conditions, using GSA and PSO, respectively. In noise-free condition, the results from GSA and PSO converge to the true values quickly and

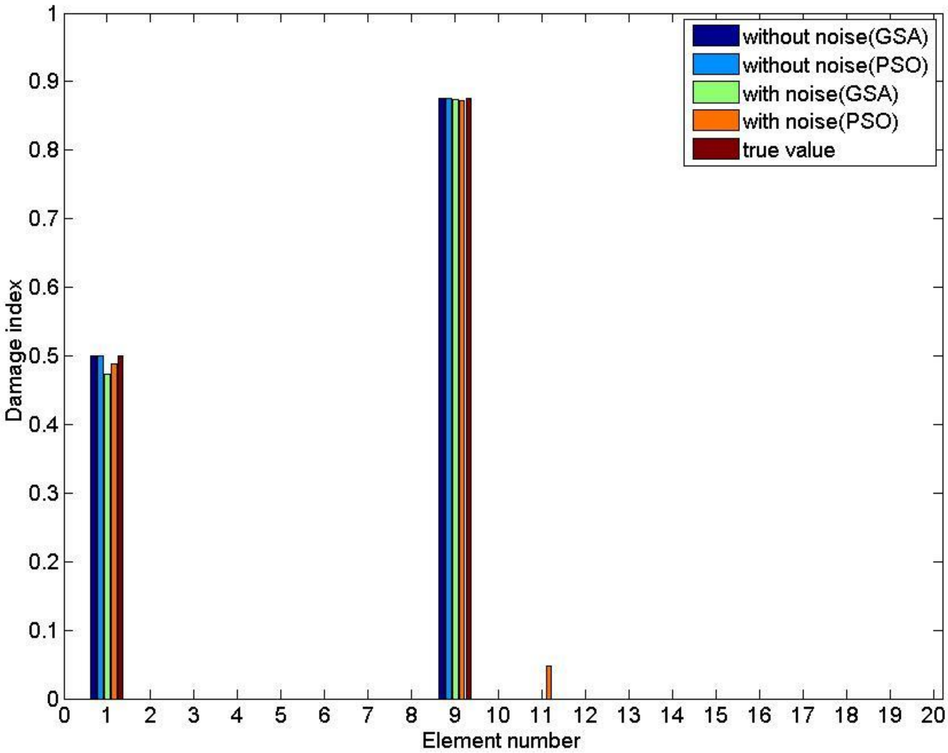

Fig. 2 Comparison on damage detection results for case 1 of the beam

Table 1Damage detection result fora beam structure

Case	Element	Identification result (%)	True value (%)	Error (%)
Case 1	1	GSA 47.36	50	2.64
		PSO 48.85		1.15
	9	GSA 87.37	87.5	1.1
		PSO 87.30		1.6
Case 2	2	8.15	10	1.85
	9	6.21	10	3.79
	16	12.00	15	3.0
	6*	1.12	0	1.12
	8*	2.49	0	2.49

a* denotes false alarm, the same below.

accurately. In noise-contaminated condition, the damage detection results and relative errors are demonstrated in Table 1. The maximum identification error is 2.4% for GSA in element 1. The final converged fitness value is 8.64×10^{-4} for GSA, while 0.0151 for PSO. The evolution processes of damage extent in elements 1 and 9 in noise condition are shown in Fig. 3(a), (b), respectively. It can be observed in both subplots of Fig. 3, the identified results from both PSO and GSA converge to the true damage extents. But the convergence speed of PSO seems be faster than that of GSA in

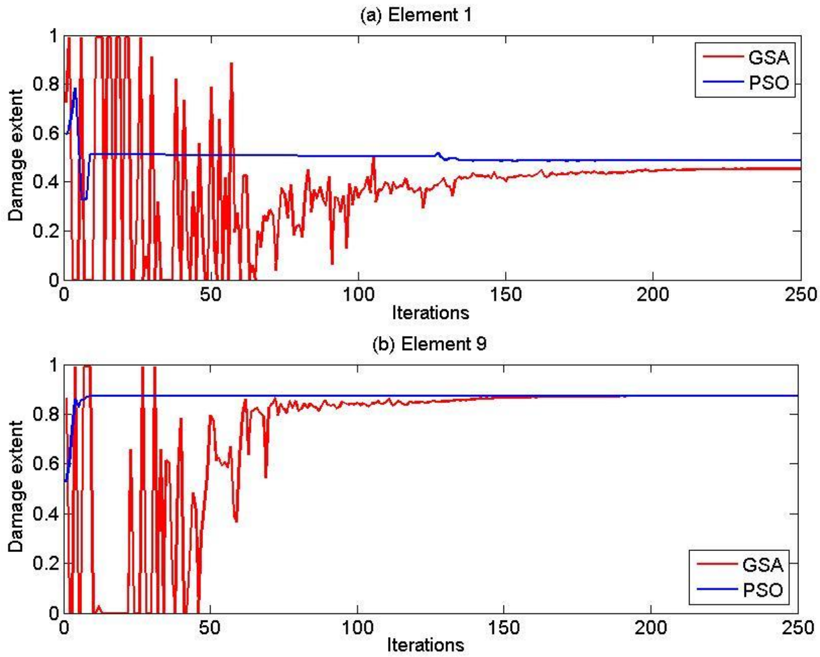

Fig. 3 The evolutionary process of damaged element for case 1 of the beam

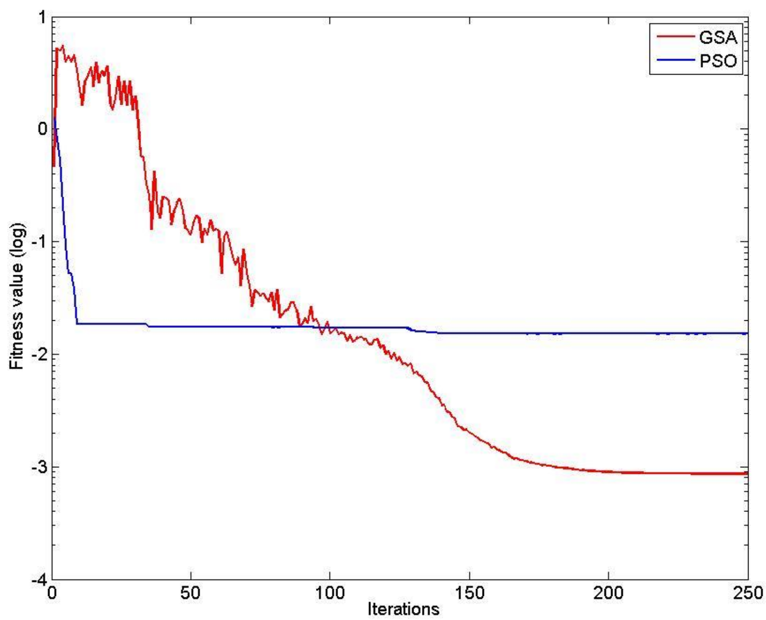

Fig. 4 Evolutionary process of fitness value for case 1 of the beam

this case. In the early stage, the damage indices from GSA fluctuate between boundaries with the decreasing amplitude with the lapse of iterations. The evolutionary process of fitness value is shown in Fig. 4. These results indicate the correctness and effectiveness of proposed approach.

Table 2 Statistical results of the evaluation fitness

Fitness value	GSA	PSO(Kang et al. 2012)
Best	0.000864	0.0151
worst	0.00251	0.0533
Mean	0.00097	0.0324
Mean iterations	200	50

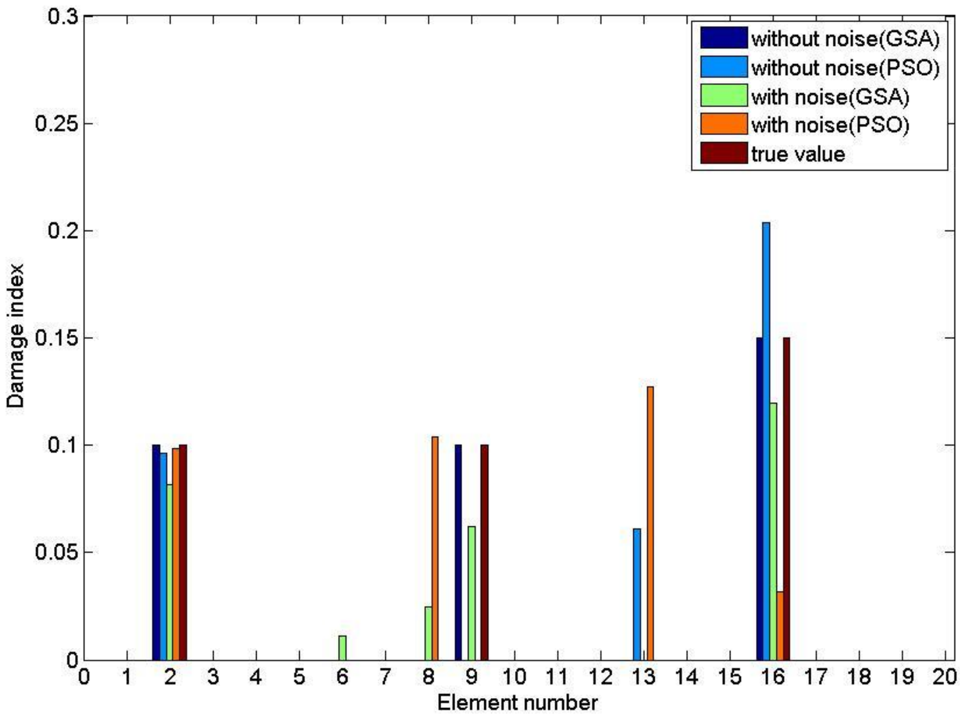

Fig. 5 Comparison on damage detection results from GSA and PSO for case 2 of the beam

Table 2 shows the detailed statistical results of the evaluation fitness for each corresponding algorithm.

4.2.2 Damage case 2

The second case assumes that element 2 and element 9 have both 10% reduction in stiffness meanwhile element 16 has 15%, i.e., d_2=0.1, d_9=0.1 and d_{16}=0.15. Both the conditions with and without noise are considered. Damage detection results are shown in Fig. 5 using both GSA and PSO. In noise-free condition, the identified results from GSA converge to the true value fast and accurately. But due to the premature convergence of PSO, it fails to identify the damaged elements. In the same noise-contaminated condition, GSA identified all three damages successfully with two false alarms. The evolutionary processes in noise condition of damaged elements are illustrated in Fig. 6. Relative errors are listed in Table 1 with maximum identified error 3.79% at element 9. And the evolution process of fitness value in noise condition is shown in Fig. 7. The final converged fitness value in noise condition is 8.13×10^{-4}.

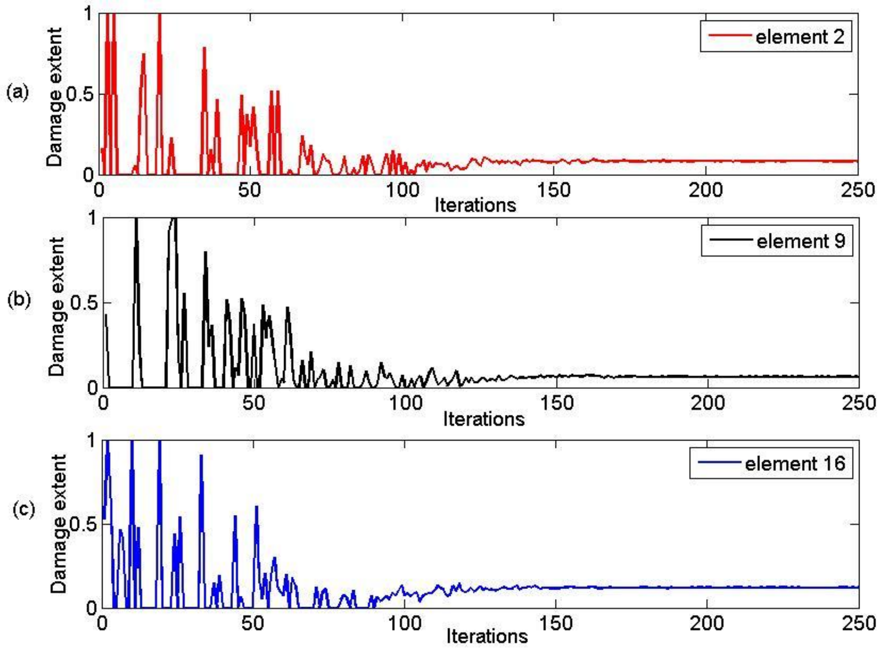

Fig. 6 Evolutionary processes of damaged elements by GSA for case 2 of the beam

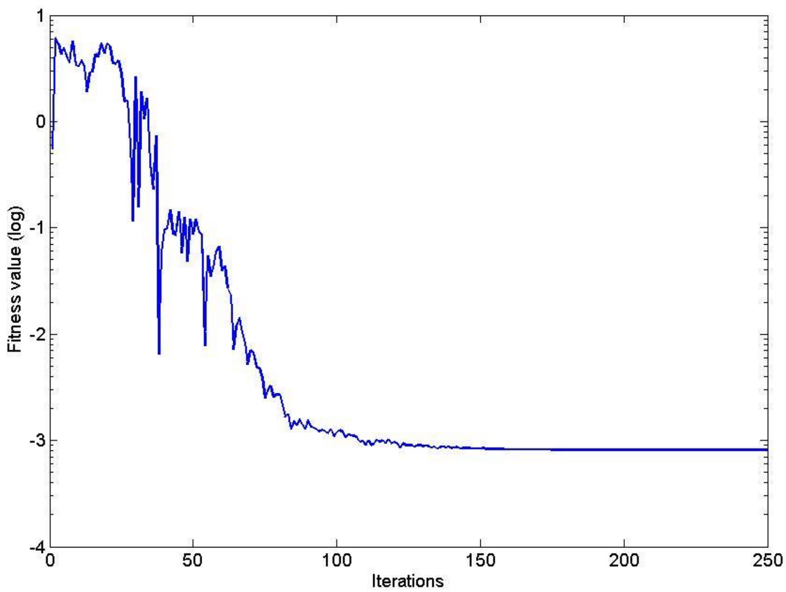

Fig. 7 Process of evolution of fitness value for case 2 of the beam

4.3 A truss structure

A 31-bar truss structure shown in Fig. 8 is studied as an example. The length of each exteriorbar is l=1 m, and length of each interior bar is 1.41 m. The section area of each bar is

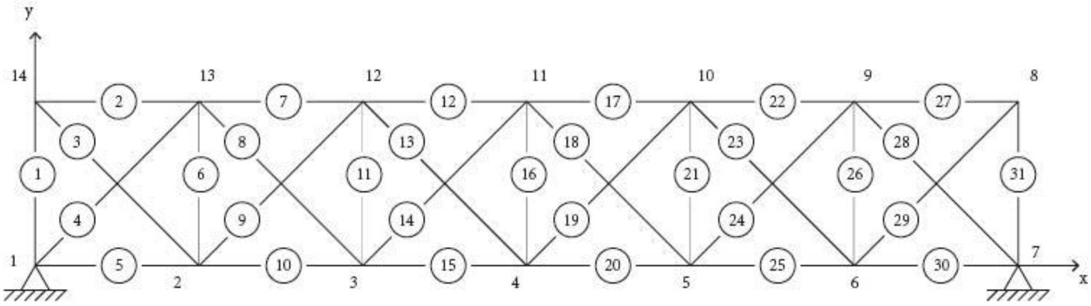

Fig. 8A 31-bar truss structure

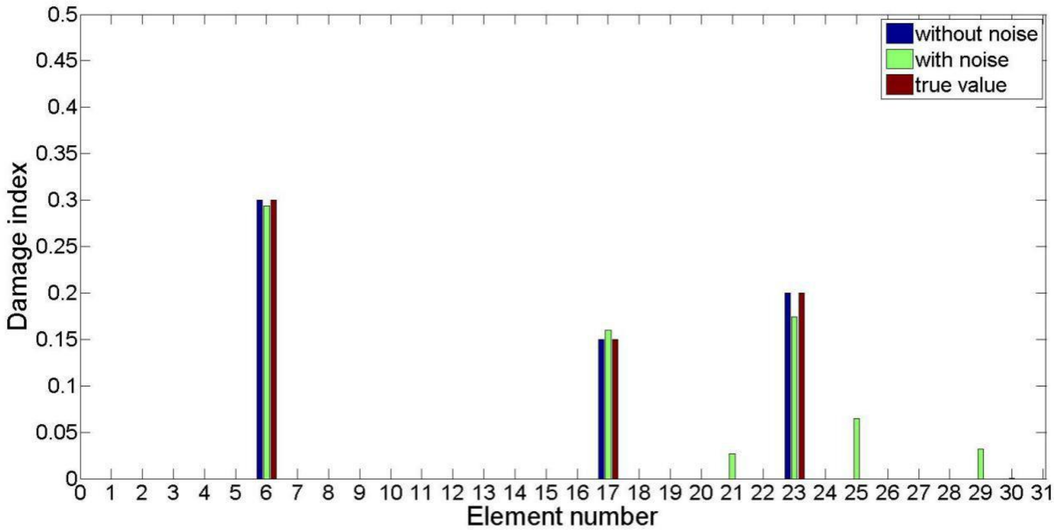

Fig. 9 Damage detection results for case 1 of the truss structure

A=0.004 m^2. Young's modulus E=200 GPa, and density ρ=7800 kg/m^3.Total number of elements and nodes are 31 and 14, respectively. The first 5 natural frequencies and modal shapes are adopted in the damage identification. The natural frequencies are contaminated with 1% uniform noise and 10% uniform noise is added to the modal displacement to simulate the noise contaminated measurement.

4.3.1 Damage case 1

Damages are assumed to locate in element 6, 17 and 23, with a reduction of 30%, 15% and 20% in each stiffness parameter, respectively. Both the conditions with and without noise are considered. The identified results are shown in Fig. 9. When the measurement noise is free, all damages have been identified successfully with no identified error and no false alarms. In noise-contaminated condition, the three damaged elements have been located successfully with a maximum identified error of 2.6% at element 23. And there are three false alarms at elements 21, 25 and 29. The relative errors for the identification are shown in Table 3. Meanwhile, the evolution processes of element damage indices in noise condition are illustrated in Fig. 10. In the noise-polluted environment, final converged fitness value is 0.0015.

Table 3 Damage detection results for a truss structure

Case	Element	Identification results (%)	True value (%)	Error (%)
Case 1	6	29.37	30	0.9
	17	15.98	15	1.2
	23	17.39	20	2.6
	21*	2.72	0	2.7
	25*	6.46	0	6.46
	29*	3.21	0	3.21
Case 2	2	30.56	30	0.56
	10	42.29	40	2.29
	17	21.16	20	0.84
	18	20.65	25	4.35
	23	34.09	35	0.91
	28	20.41	23	2.59
	12*	3.25	0	3.25
	29*	4.42	0	4.42

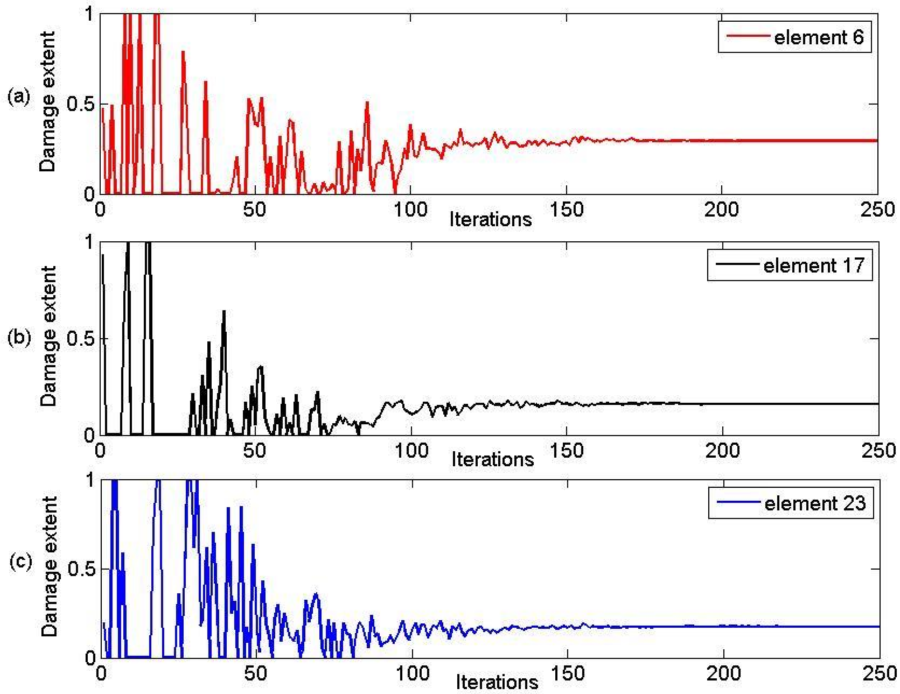

Fig. 10 Evolutionary processes of damaged elements for case 1 of the truss structure

4.3.2 Damage case 2

In this case, 6 damages are assumed to locate at elements 2, 10, 17, 18, 23 and 28, with reduction in stiffness by 30%, 40%, 20%, 25% 35% and 23%, respectively. Element 17 and 18 are

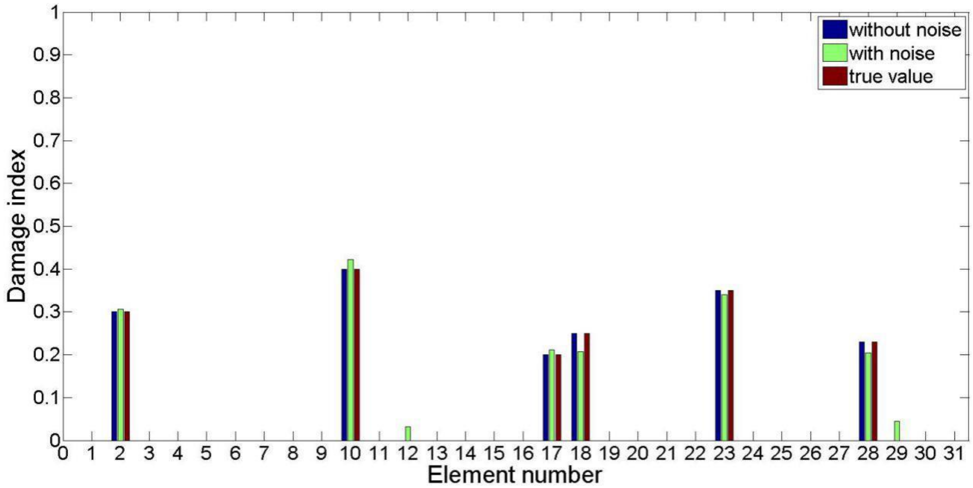

Fig. 11 Identified results for case 2 of the truss structure

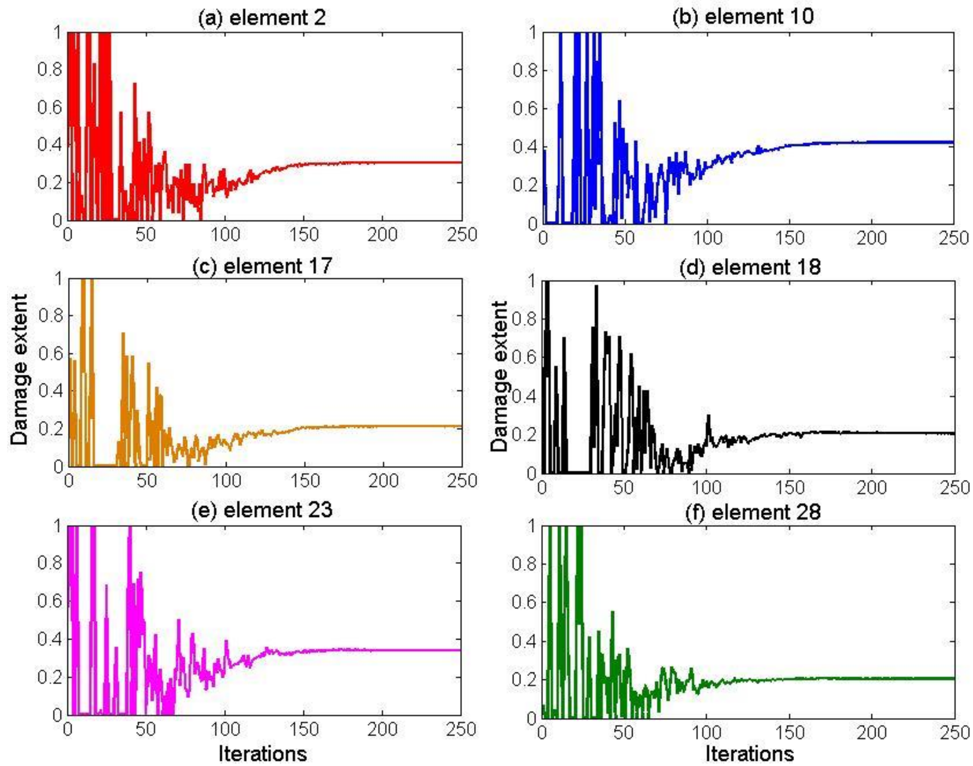

Fig. 12 Evolutionary processes of damaged elements for case 2 of the truss structure

two adjacent elements. The detection results are shown in Fig. 11 for both noise-free and noise-contaminated conditions. When the measurement noise is free, the identified results converge to the true values without false alarms. In noise condition, the six damages have been identified successfully with maximum identified error of 4.35% at element 18. The relative identified errors

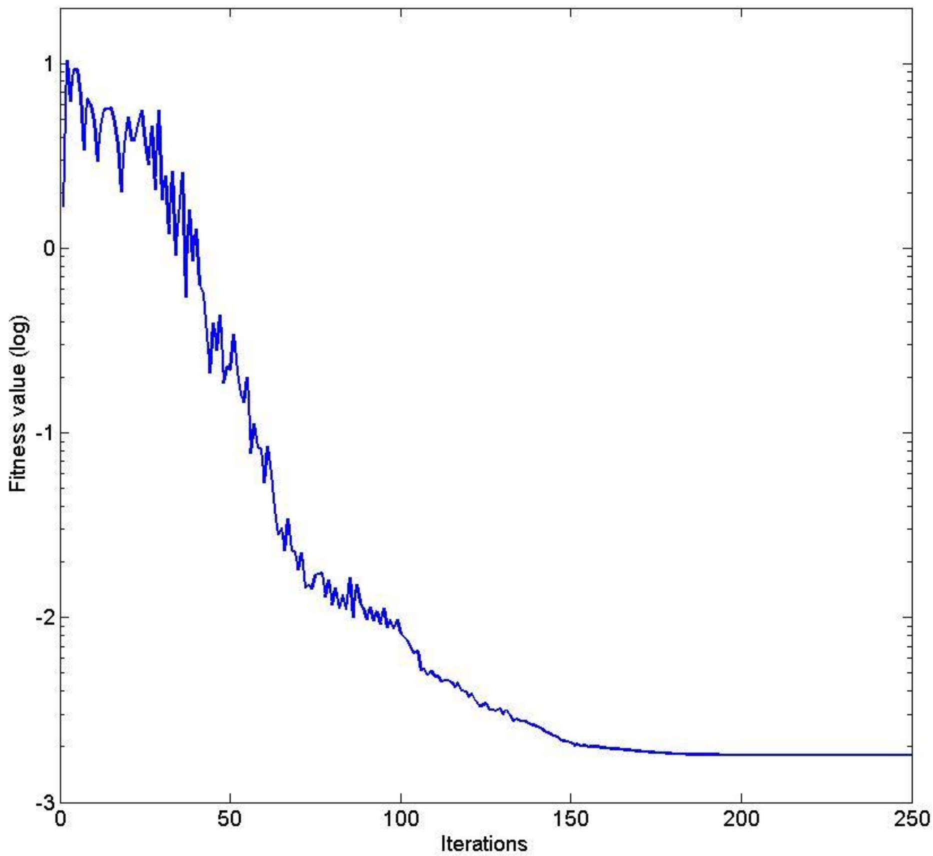

Fig. 13 Process of evolution of fitness value in case 2 of the truss structure

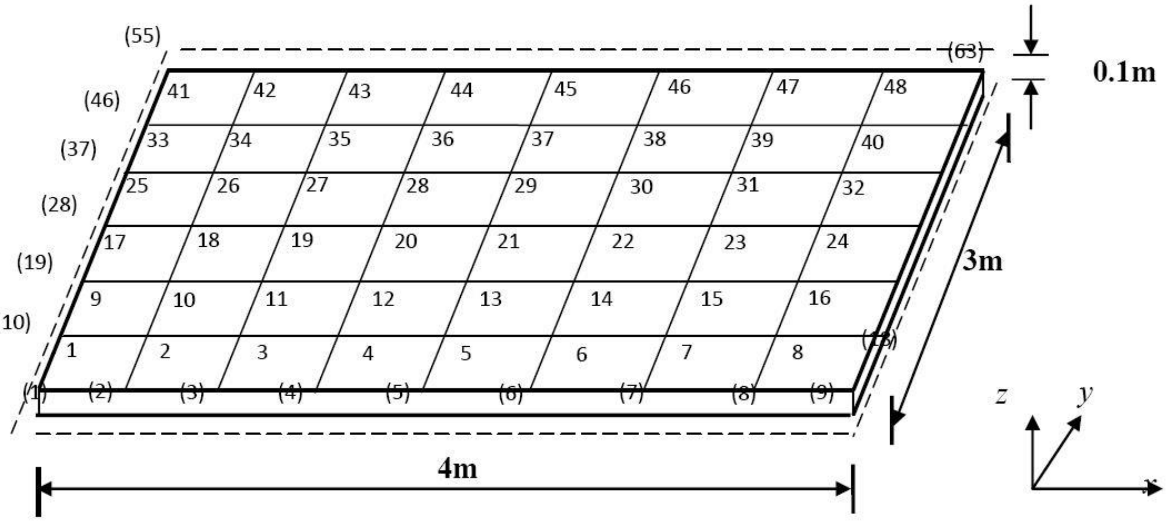

Fig. 14 Sketch of a four-edge simply supported plate ((1), (2), …, (63) denote node number of the FEM; 1,2,…,48 denote element number) (Dimensions not scaled)

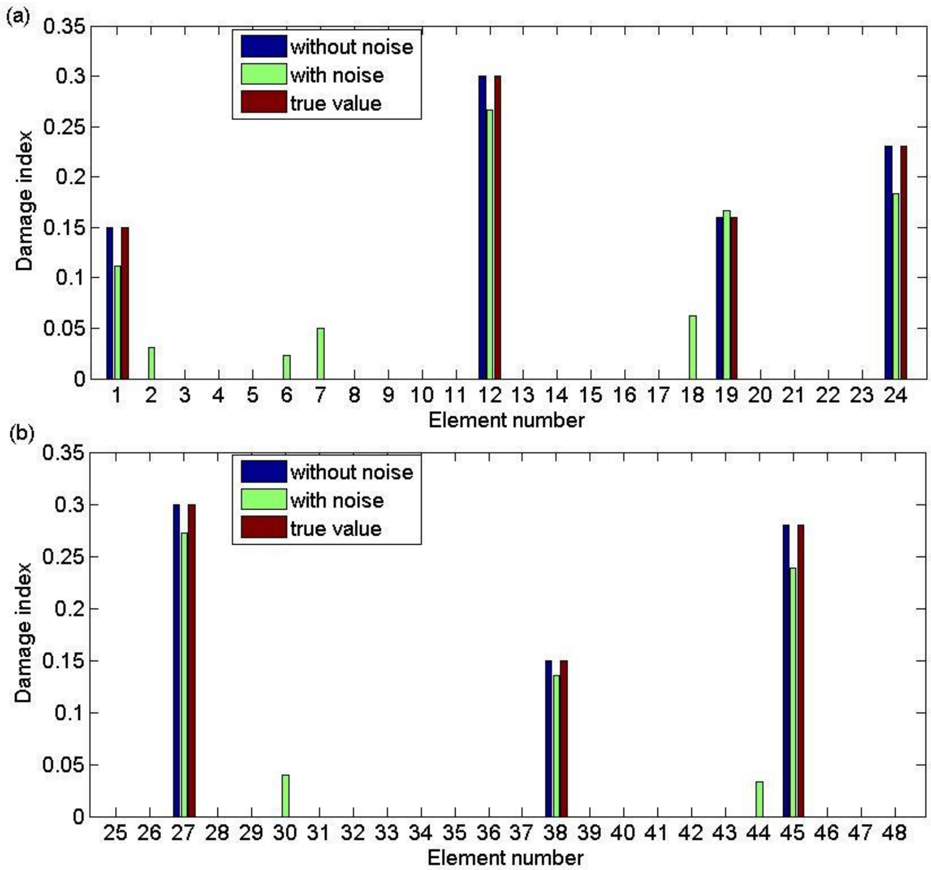

Fig. 15 The damage detection results in the plate

are listed in Table 3. Fig. 12 shows the evolution processes of all damaged elements in noise condition. In the same noise-polluted environment, evolution process of fitness value is demonstrated in Fig. 13 with final converged fitness value 0.0018.

4.4 A four-edge simply supported plate

Fig. 14 shows a thin plate with four edges simply supported and with dimensions of 4 m×3 m×0.1 m. The physical material properties of the plate are: Young's modulus E=210 GPa, mass density ρ=7850 kg/m^3 and Poisson's ratio υ=0.3. In the finite element model, the plate was discretized into 48four-node Kirchhoff plate elements.

Seven damages are assumed to locate at elements 1, 12, 19, 24, 27, 38 and 45 with reduction in stiffness by 15%, 30%, 16%, 23%, 30%, 15% and 18%, respectively. The first 10 natural frequencies and mode shapes are used in the damage identification. They are also polluted by 1% and 10% uniform noise, respectively. The identified results are shown in Fig. 15(a), (b) for both noise-free and noise-contaminated conditions. Again, when the measurement noise is free, the identified results converge to the true values and without false alarm. In noise condition, these seven damages have been identified successfully with maximum identified error of 4.66% at

Table 4 Damage detection result for a plate structure

Element	Identification results (%)	True value (%)	Error (%)
1	11.16	15	3.84
12	26.64	30	3.36
19	16.61	16	0.61
24	18.34	23	4.66
27	27.27	30	2.73
38	13.53	15	1.47
45	23.82	28	4.28
2*	3.11	0	3.1
6*	2.33	0	2.3
7*	4.98	0	5.0
18*	6.26	0	6.3
30*	3.98	0	4.0
44*	3.35	0	3.4

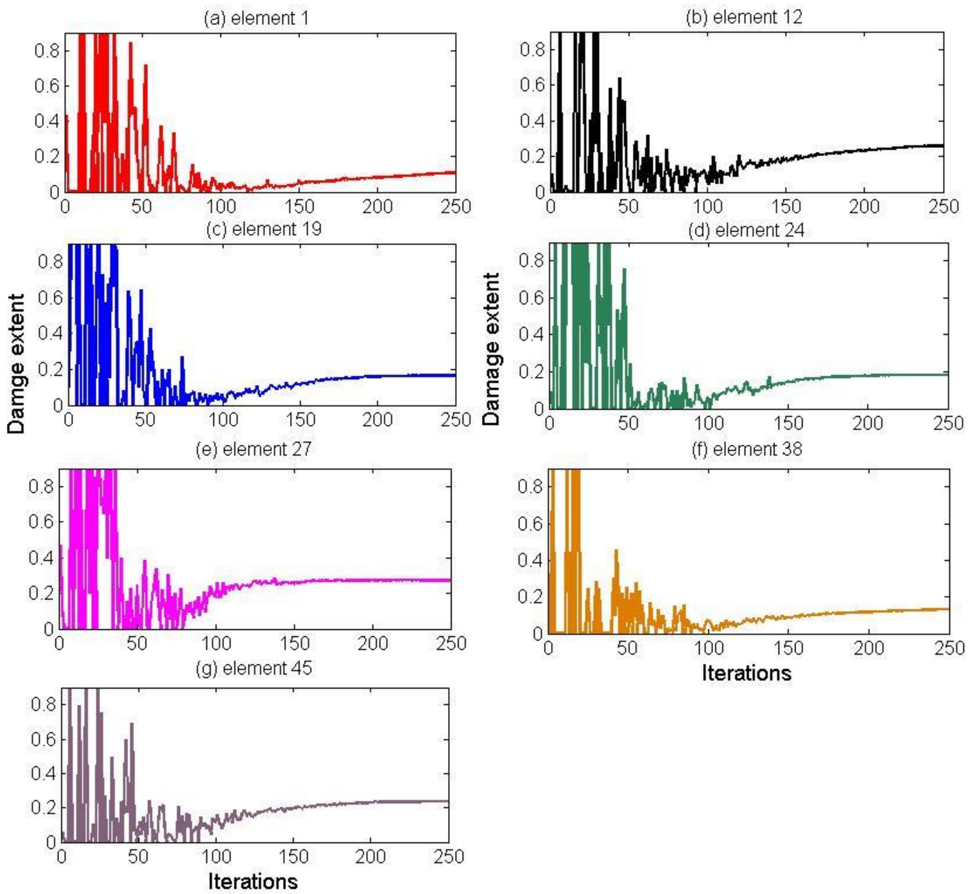

Fig. 16 Evolutionary processes of damaged elements in the plate

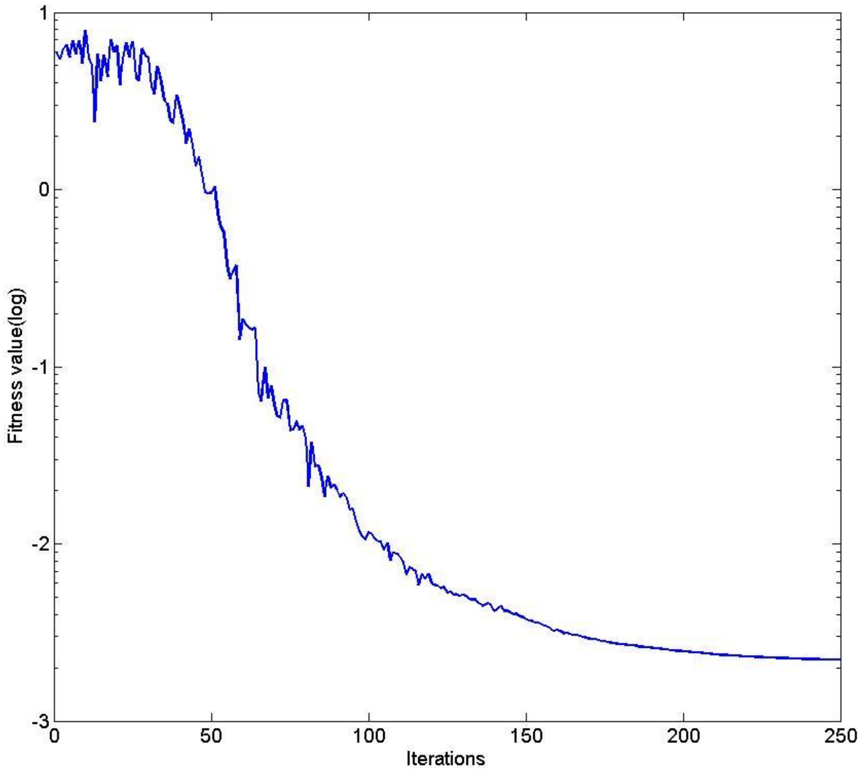

Fig. 17 Evolutionary process of fitness value for the plate

element 24. Relative errors of elements are listed in Table 4. Fig. 16 illustrates the processes of evolution of damaged elements in noise condition. The process of evolution of fitness value is shown in Fig. 17. This example further illustrates the correctness and robustness of the proposed method.

5. Conclusions

A damage identification method based on vibration data using gravitational search algorithm is proposed in this study. An objective function based on natural frequencies and mode shapes is established for structural damage identification. Three different structures are studied to illustrate correctness and efficiency of the proposed methods. Good identification results can be obtained even with noise-contaminated measurement data. Study shows that the proposed method is not sensitive to measurement noise. The advantage of the proposed method is that it is easy to implement as only the first few modal data are needed in the identification.

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