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Reliability sensitivities with fuzzy random uncertainties using genetic algorithm

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Abstract. A sensitivity analysis estimates the effect of the change in the uncertain variable parameter on the probability of the structural failure. A novel fuzzy random reliability sensitivity measure of the failure probability is proposed to consider the effect of the epistemic and aleatory uncertainties. The uncertainties of the engineering variables are modeled as fuzzy random variables. Fuzzy quantities are treated using the λ-cut approach. In fact, the fuzzy variables are transformed into the interval variables using the λ-cut approach. Genetic approach considers different possible combinations within the search domain (λ-cut) and calculates the parameter sensitivities for each of the combinations.

Keywords: reliability; fuzzy random variables; sensitivity analysis; λ-cut approach

1. Introduction

It has been well recognized that uncertainty is an integral part of civil engineering problems and it needs to be included in structural reliability to obtain credible estimates of failure probability. In reliability engineering, uncertainty can be viewed as the difference between the present state of knowledge and the complete knowledge (Wang, Lu *et al*. 2013). Although many sources of uncertainty may exist, they may be identified to belong to one of two major categories in reliability engineering, aleatory and epistemic (Kiureghian and Ditlevsen 2009, Jahani, Muhanna *et al*. 2014, Lagaros 2014). Aleatory uncertainty is due to natural variability associated with a structural system, which referred to as irreducible, objective uncertainty and usually is modeled by random variables with Probability Density Function (PDF). The credibility of probabilistic theory relies on the availability of sufficient data to describe accurately the probabilistic distribution of the uncertain variables. Indeed, reliability estimates are very sensitive to small variations of the assumed probabilistic models (Abbasnia, Shayanfar *et al*. 2014, Jiang, Li *et al*. 2014). In contrast to aleatory uncertainty, epistemic uncertainty is knowledge based and is related to our ability to understand, measure, and describe the system under study. There are different approaches to model the epistemic uncertainty, such as interval modeling (Wu, Zhao *et al*. 2005, Thacker and Huyse 2007, Pedroni, Zio *et al*. 2013, Fang, Xiong *et al*. 2014, Debruyne,

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Vandepitte *et al*. 2015), Bayesian modeling (Song, Kang *et al*. 2010, Yin, Lam *et al*. 2010, An, Choi *et al*. 2011), chaos theory (Yang, Li *et al*. 2006, Schoefs, Yáñez-Godoy *et al*. 2011), evidence theory (Liu, Li *et al*. 2009, Jiang, Zhang *et al*. 2014), and fuzzy modeling (Chen, Wei *et al*. 2006, Tutmez, Cengiz *et al*. 2013, Bedirhanoglu 2014). Fuzzy random reliability has been developed in the last two decades (Möller, Graf *et al*. 2000). Möller *et al.* (Möller, Graf *et al*. 2003) introduced a methodology to estimate the membership function of the safety index by considering fuzzy randomness.

The reliability sensitivity analysis is a very useful tool to increase the safety level of a structural system or identify the crucial design or simplify model which decreases computational time and cost. These sensitivities are advantageous because they quantify the importance of distribution parameters such as means, standard deviations and correlations. This quantification helps us assess the validity of the reliability estimates by considering the assumptions made about the choices of the parameter values. Also, it helps us define the roles of the random variables in subsequent analyses (e.g., an unimportant random variable can be treated as a constant). The sensitivities also play fundamental role in problems involving optimization. Most existing reliability sensitivity analysis methods are based on the probability and statistics theories. The methodologies for calculating reliability sensitivity based on probability theory have been well established (Ditlevsen and Madsen 1996, Saltelli, Tarantola *et al*. 2004, Cao, Dai *et al*. 2011). A few investigations (Bae, Grandhi *et al*. 2006, Helton, Johnson *et al*. 2006) have been conducted to explore sensitivity analysis with epistemic uncertainty. Helton *et al*. (Helton, Johnson *et al*. 2006) proposed a three-step sampling-based sensitivity analysis for epistemic uncertainty. Guo and Du (2007) proposed an approach to conduct sensitivity analysis with mixture of aleatory and epistemic uncertainties. In their method, the most important epistemic variables are captured under the framework of the unified uncertainty analysis. A new numerical reliability index has been proposed to cope with the reliability model with mixture of random and fuzzy variables (Cui, Lu *et al*. 2011). It is necessary to explore new reliability sensitivity analysis theories and methods for identifying the most important variables when fuzzy random uncertainties are presented in the model inputs. In this paper, a model with fuzzy random uncertainty is considered and a novel fuzzy random reliability sensitivity measure of the failure probability is proposed.

Here, incomplete knowledge about the distribution parameters is modeled using fuzzy numbers. Therefore, the input basic variables are considered fuzzy random variables and parameter sensitivities are expressed in terms of fuzzy numbers. Fuzzy random reliability sensitivity analysis can be used for inaccuracies and lack of reliability or uncertainty in data, usually faced when the available information is insufficient to provide statistical description of the required data.

Different approaches are available to solve the problem with fuzzy numbers (Kaufmann and Gupta 1988). The λ-cut approach (Möller, Graf *et al*. 2000) is one of the most popular and efficient approaches to propagate fuzziness in systems under consideration. In λ-cut approach, a variety of methods can be used to attain the bounds of the response quantity for each λ-cut. These bounds are usually obtained using search algorithms (Möller and Beer 2008) or Cartesian Product (CP) algorithm (Ferson, Kreinovich *et al*. 2002). In this paper, parameter sensitivities are ranged between lower and upper bounds at each λ-cut.

This paper is organized as follows: Section 2 gives a brief description of the fuzzy sets, and then the reliability in the presence of epistemic and aleatory uncertainties is discussed. The sensitivity analysis is introduced in Section 3, and a novel fuzzy sensitivity measure is illustrated in detail in section 4. Three examples are employed to validate the reasonability of the derivative sensitivity measure and the efficiency of the proposed method in Section 5. Finally, conclusions are drawn in Section 6.

2. Fuzzy random reliability

2.1 Fuzzy sets

Fuzzy set theory is employed to model the epistemic uncertainties. Potentialities of fuzzy set theory in several fields of civil engineering have been underlined in several papers (Ayyub 1991, Valliappan and Pham 1995). Initially, fuzzy sets were considered an effective tool for accounting for subjective information obtained from experienced engineers in structural engineering decision and, if necessary, for merging subjective and objective information (Blockley 1979, Brown and Yao 1983). A fuzzy set $\tilde{A}$ in a universe of discourse X is a set of ordered pairs

$$\tilde{A}=\left\{x,\mu_A(x)\right\}, \qquad x\in X \tag{1}$$

The membership function $\mu_A(x)$ for a fuzzy set $\tilde{A}$ can be defined as

$$\mu_A(x):\ X\rightarrow\left[0,1\right] \tag{2}$$

The membership degree $\mu_A(x)$ quantifies the grade of membership of the element x to the fuzzy set $\tilde{A}$. A key difference between the classical (crisp) and fuzzy sets is that the membership degree of the crisp sets can take only two values (0 or 1), whereas a fuzzy set can have an infinite number of membership degrees between 0 and 1. If the shape of the membership function is triangular, the fuzzy set is called as triangular fuzzy number, and can be expressed in the following form

$$\tilde{A}=[\,a,b,c\,] \tag{3}$$

Where a and c are, respectively, the lower and upper x values of the triangle at μ=0, and b is the x value that corresponds to μ=1. It is possible to evaluate fuzzy sets using λ-cuts or membership levels. The λ-cut of a fuzzy set $\tilde{A}$ can be defined as

$$A_\lambda=\left\{x\in X\,\middle|\,\mu_A(x)\geq\lambda\right\}, \quad \text{for } \lambda\in\left[0,1\right] \tag{4}$$

That the λ-cut of a fuzzy set $\tilde{A}$ is the crisp set that contains all elements of the universal set whose membership grades in $\tilde{A}$ are greater than or equal to the specific value of λ. At each λ-cut level, the variation of the fuzzy variable is defined by a lower bound $X_L(\lambda)$ and upper bound $X_U(\lambda)$.

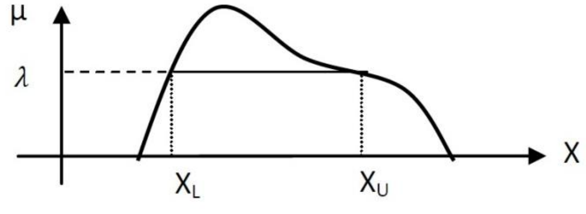
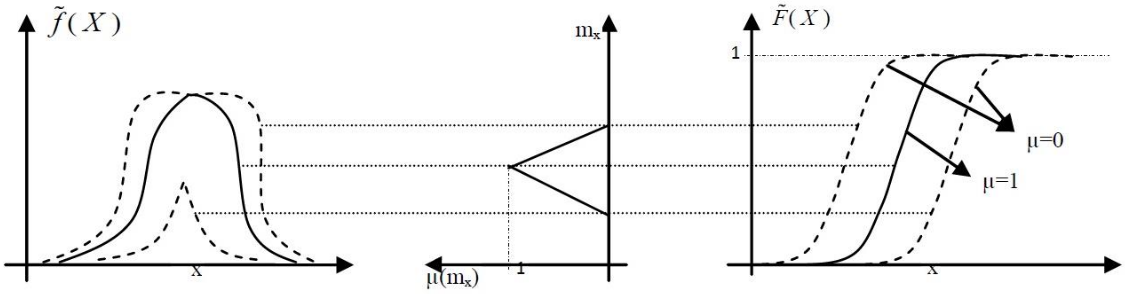


Fig. 1 λ -cut of a fuzzy variable



(a) Fuzzy probability density function (b) Membership degree (c) Fuzzy cumulative distribution function

Fig. 2 The fuzzy distribution functions of the fuzzy random variable

A fuzzy number $\tilde{X}$ and its *λ*-cut of the membership function are shown in Fig. 1.

2.2 Fuzzy random variables

Randomness and fuzziness are usually two alternative representations of uncertainties (Wang, Huang *et al*. 2012). Randomness has been considered due to the probability theory in many studies. However, fuzziness in the randomness exists in most of the engineering problems because of the lack of sufficient data, data with fuzziness, and unknown or non-constant reproduction conditions (Wang, Huang *et al*. 2012). The first ideas and definitions relating to the theory of fuzzy random variables have been discussed in (Kwakernaak 1978, Kwakernaak 1979, Puri and Ralescu 1986). Fuzzy random variable has attracted more attention, for its capacity of uncertainty representation when engineering problems are handled (Möller, Liebscher *et al*. 2008, Debruyne, Vandepitte *et al*. 2015). A fuzzy random variable is a random variable for which the statistical parameters of its distribution (mean and standard deviation) are considered fuzzy numbers. Therefore, there are families of distribution with different membership degrees for each fuzzy random variable. Fig. 2 shows a fuzzy random variable $\tilde{X}$ with fuzzy cumulative distribution function $\tilde{F}(X)$ and the fuzzy probability density function $\tilde{f}(X)$. Based on the use of fuzzy random variables as the basic variables for the reliability problem, the main idea of fuzzy random reliability is appeared.

3. Sensitivity analysis

Sensitivity analysis studies how small variations of parameters around a reference point change the value of the output. Reliability sensitivity is usually measured by the partial derivative of failure probability with respect to the distribution parameter of the basic random variable as it can objectively describe the effect of distribution parameters on the failure probability. Based on the main idea of the parameter sensitivities, we can illustrate a novel fuzzy reliability sensitivity measure of failure probability in the presence of epistemic and aleatory uncertainties.

3.1 Construct the approximate hyperplane g(x)=0

Let $g(x)$ denote the implicit limit state function with n-dimension normal basic random vector X. The approximate linear limit state function $g_L(x)$ is obtained using Taylor's series expansion of linearized safety margin $g(x)$ where

$$g(x) = g(x_1, x_2, \ldots, \mathrm{x}_n) \tag{5}$$

It should be noted that when the basic variables are non-normal, an equivalent normal distribution at design point can be obtained, using the Rosenblatt, Nataf or a similar transformation technique (Melchers 1999).

Using Taylor's series expansion about the design point $X^*=(x_1^*, x_2^*,\ldots, x_n^*)$ results in

$$g(x) = g(\mathrm{x}_1^*, \mathrm{x}_2^*, \ldots, \mathrm{x}_n^*) + \sum_{i=1}^{n} \left(\frac{\partial g}{\partial x_i}\bigg|_{X^*}\right)(\mathrm{x}_i - \mathrm{x}_i^*) + \sum_{i=1}^{n} \left(\frac{\partial^2 g}{\partial {x_i}^2}\bigg|_{X^*}\right)\frac{(\mathrm{x}_i - \mathrm{x}_i^*)^2}{2} + \cdots \tag{6}$$

Recall that $(\frac{\partial g}{\partial x_i})_{X^*}$ means that $\frac{\partial g}{\partial x_i}$ is evaluated at X^*.

Retaining only the linear terms, $g_L(x)$ is defined as follows

$$g_L(x) \approx g(\mathrm{x}_1^*, \mathrm{x}_2^*, \ldots, \mathrm{x}_n^*) + \sum_{i=1}^{n} \left(\frac{\partial g}{\partial x_i}\bigg|_{X^*}\right)(\mathrm{x}_i - \mathrm{x}_i^*) \tag{7}$$

The linear approximation limit state function can be described by $g_L(x)=g(x)=0$ as written below

$$g(x) = a_0 + \sum_{i=1}^{n} a_i x_i = 0 \tag{8}$$

Where (a_0,a_1) are coefficients determined through the Eq. (7). Since it is a linear function, the mean (μ) and standard deviation (σ) of g are readily determined. When x_i is Normal distributed and mutually independent, it is well-known that these are

$$\mu_g = a_0 + \sum_{i=1}^{n} a_i \,.\mu_{X_i} \tag{9}$$

and

$$\sigma_g = [\sum_{i=1}^{n} (\mathrm{a}_i \ \sigma_{x_i})^2]^{1/2} \tag{10}$$

Where the μ_{x_i} are the mean values and the σ_{x_i} are the standard deviations respectively for the random variables x_i. For mutually independent x_i, the relationship of failure probability and reliability index can be denoted as

$$P_f = \phi(-\beta) = \phi(-\mu_g / \sigma_g) \tag{11}$$

where $\phi(\cdot)$ is the cumulative distribution function of the standard normal random variable. With the equivalent hyperplane and the corresponding probability of failure, the sensitivities can be estimated using the FORM approach. As will be seen, this involves estimating the change in failure probability resulting from a change in parameter value, for the given equivalent hyperplane (Melchers and Ahammed 2004).

3.2 Sensitivity measure of failure probability

The sensitivity measure $\partial P_f/\partial\theta$ can reflect the effect of the parameter θ (mean, standard deviation…) on the failure probability at the reference point which is generally fixed at the design point u^*. The parameter sensitivity by differentiating Eq. (11) respect to the distribution parameters of the basic random variables and using the chain function derivative rule can be represented as

$$\frac{\partial P_f}{\partial\theta_i}=\frac{\partial P_f}{\partial\beta}\frac{\partial\beta}{\partial\theta_i} \tag{12}$$

Mean and standard deviation of the basic variables are the most interest statistical parameters. The parameter sensitivities of mean and standard deviation are $\partial P_f\ /\partial\mu_{x_i}$ and $\partial P_f\ /\partial\sigma_{x_i}$, respectively. If the random variables are mutually independent, it is easily shown that:

$$\frac{\partial P_f}{\partial\mu_{X_i}}=\frac{\partial\phi(-\beta)}{\partial\mu_{X_i}}=\frac{\partial\phi(-\beta)}{\partial\beta}\frac{\partial\beta}{\partial\mu_{X_i}} \tag{13}$$

And

$$\frac{\partial P_f}{\partial\sigma_{X_i}}=\frac{\partial\phi(-\beta)}{\partial\sigma_{X_i}}=\frac{\partial\phi(-\beta)}{\partial\beta}\frac{\partial\beta}{\partial\sigma_{X_i}} \tag{14}$$

Where

$$\frac{\partial\phi(-\beta)}{\partial\beta}=\frac{\partial\left(\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{-\beta}e^{-\frac{1}{2}x^2}\,\mathrm{dx}\right)}{\partial\beta}=-\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\beta^2} \tag{15}$$

The partial derivatives of $\frac{\partial\beta}{\partial\mu_{X_i}}$ and $\frac{\partial\beta}{\partial\sigma_{X_i}}$ can be calculated as

$$\frac{\partial\beta}{\partial\mu_{X_i}}=\frac{\partial(\frac{\mu_g}{\sigma_g})}{\partial\mu_{X_i}}=\frac{\partial\left(\frac{a_0+\sum_{i=1}^{n}a_i.\mu_{X_i}}{\left[\sum_{i=1}^{n}(\mathrm{a}_i\ \sigma_{X_i})^2\right]^{1/2}}\right)}{\partial\mu_{X_i}}=\frac{a_i}{\sigma_g} \tag{16}$$

And

$$\frac{\partial\beta}{\partial\sigma_{X_i}} = \frac{\partial(\frac{\mu_g}{\sigma_g})}{\partial\sigma_{X_i}} = \frac{\partial\left(\frac{a_0 + \sum_{i=1}^{n} a_i .\mu_{X_i}}{\left[\sum_{i=1}^{n}(\mathrm{a}_i\ \sigma_{X_i})^2\right]^{1/2}}\right)}{\partial\sigma_{X_i}} = -\frac{a_i{}^2\mu_g\sigma_{X_i}}{\sigma_g{}^3} \tag{17}$$

Respectively. Then the reliability sensitivities $\partial P_f\ /\partial\mu_{x_i}$ and $\partial P_f\ /\partial\sigma_{x_i}$, respectively, can be determined by substituting Eqs. (15) and (16) into Eq. (13) and Eqs. (15) and (17) into Eq. (14).

$$\frac{\partial P_f}{\partial\mu_{x_i}} = -\frac{a_i}{\sqrt{2\pi}\sigma_g}\exp[-\frac{1}{2}(\frac{\mu_g}{\sigma_g})^2] \tag{18}$$

$$\frac{\partial P_f}{\partial\sigma_{x_i}} = -\frac{a_i{}^2\sigma_{x_i}\mu_g}{\sqrt{2\pi}\sigma_g{}^3}\exp[-\frac{1}{2}(\frac{\mu_g}{\sigma_g})^2] \tag{179}$$

Eqs. (18) and (19) provide the accurate reliability sensitivities of the linear limit state function with normal basic random variables.

3.3 Gradients

In the standard FORM theory, gradients of P_f (this is, the sensitivity of P_f to the components in X) are an automatic byproduct of the calculation procedure. In the standard normal space u, it follows directly from the FORM theory that the gradients α_i are given by:

$$\frac{\partial\beta}{\partial u_i} = \alpha_i = \frac{a_i\sigma_{x_i}}{\sigma_g} \tag{20}$$

In the FORM theory, the gradients are evaluated at the design point u^*. However, for the special case of normal random variables, a hyperplane in x space transforms to a hyperplane in u space and in this case, α_i is invariant.

4. Fuzzy random sensitivity analysis

Suppose there is a model $g(x)$ where $X=(x_1, x_2,...,x_n)$ (n is the number of input variables) is the set of uncertain input variables. The uncertainties of these input variables are represented by fuzzy probability distributions.

Failure probability P_f is a function of the basic fuzzy-valued distribution parameter $\theta=(\theta_1, \theta_2,..., \theta_n)$ given by $P_f=\psi(\theta_1, \theta_2,..., \theta_n)$.

Based on the main idea of the FORM theory, we can define a novel fuzzy random sensitivity measure of failure probability in the presence of epistemic and aleatory uncertainties. The main idea of fuzzy random reliability sensitivity is based on the use of fuzzy random variables which can reflect aleatory and epistemic uncertainties. When the fuzzy set theory is employed, assigned

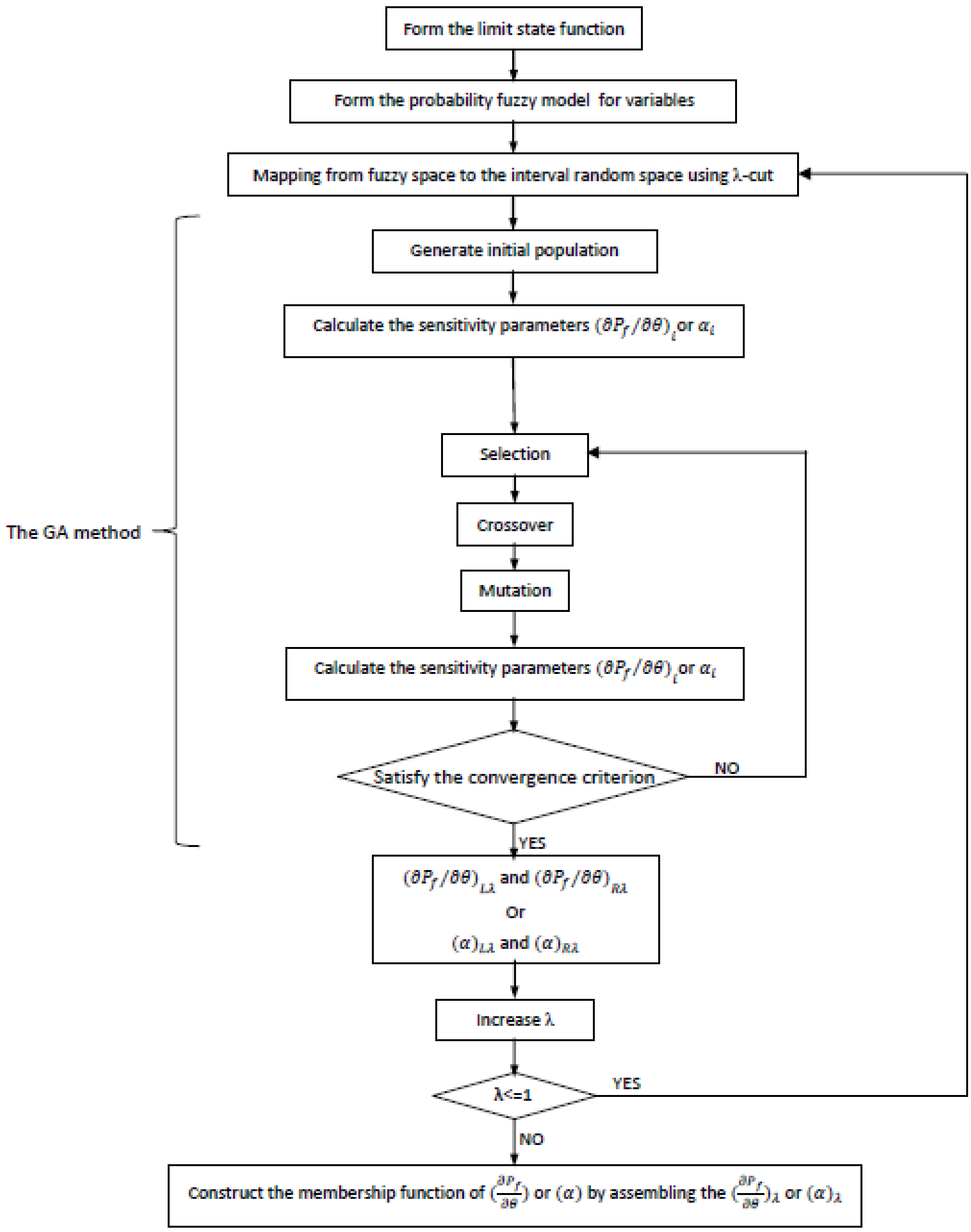


Fig. 3 Flowchart of the proposed algorithm

membership function (MF) is used to model the epistemic uncertainty. Firstly, the bounds of the distribution parameters at each λ-cut are obtained by corresponding membership function. In other words, the fuzzy random space is transformed into an interval random space; hence there are intervals for parameters θ_i^j (jth distribution parameter of ith random variable). Each θ_i^j can reach any value in its interval $\left[\theta^{j}_{iL}(\lambda),\theta^{j}_{iU}(\lambda)\right](i=1,2,...,n;j=1,2)$ then all input variables are sampled randomly from their probability distributions and the model output is calculated for each set of input parameters. In order to find the interval of the model output for the considered λ, the genetic algorithm as one of the well-developed approaches is selected for searching its maximum and minimum. Consequently, the bounds of sensitivity measures $((\frac{\partial P_f}{\partial \theta_i})_L,(\frac{\partial P_f}{\partial \theta_i})_U)$ at various λ-cut can be calculated. Finally, the fuzzy random sensitivity measure is assembled from the resulting intervals at each λ-cut. Details of the proposed method for calculating the fuzzy random sensitivity measures are shown in Fig. 3.

5. Examples

In this section, both numerical and engineering examples are used to demonstrate the proposed algorithm. To calculate the fuzzy random sensitivity as mentioned in section 4, the FORM method is used. All the limit state functions are given in the original x space. λ-cut is used for mapping of the fuzzy space to interval random space. In the examples 1 and 2, λ varies from 0 to 1 by 0.1 steps and in example 3, it varies from 0 to 1 by 0.2 steps. It should be noted that in all examples, the probability of mutation and crossover for the GA are considered 0.1 and 0.9 respectively. In order to validate the reasonability of the fuzzy random sensitivity measure, the CP method is utilized to calculate the exact solution for design point sensitivities in example 1.

5.1 Example 1

Consider a non-linear limit state function with 3 fuzzy random variables that was previously considered by Melchers (Melchers and Ahammed 2004) only for random variables. In the Melchers' method (Melchers and Ahammed 2004), the approximate hyperplane was estimated from 11,751 points falling in the failure domain to give $g_L(x)=-1309.84+47.44x_1+27.19x_2-0.9876x_3$ (Melchers and Ahammed 2004). The mean and standard deviations of the basic variables are considered triangular fuzzy numbers and are presented in Table 1. The λ-cut method is used for mapping of this fuzzy space to the interval random space. Membership degree λ varies from 0 to 1 by step 0.1. At each Membership degree λ, an interval is obtained for each random variable parameter, thus, permissible domain for distribution parameters can be easily obtained. The search goals of the proposed algorithm are the lowest and largest sensitivity measures which are the left $(\frac{\partial P_f}{\partial \theta})_L$ and right $(\frac{\partial P_f}{\partial \theta})_R$ bounds for λ level, respectively. Therefore, $(\frac{\partial P_f}{\partial \theta})_L$ and $(\frac{\partial P_f}{\partial \theta})_R$ can be obtained for all λ levels. Finally, the membership function of the sensitivity parameter is attained by assembling these intervals.

The fuzzy mean value sensitivities $(\frac{\partial P_f}{\partial \mu_{x_1}},\frac{\partial P_f}{\partial \mu_{x_2}},\frac{\partial P_f}{\partial \mu_{x_3}})$, the fuzzy standard deviation sensitivities $(\frac{\partial P_f}{\partial \sigma_{x_1}},\frac{\partial P_f}{\partial \sigma_{x_2}},\frac{\partial P_f}{\partial \sigma_{x_3}})$ and the fuzzy design point sensitivities (α_1, α_2, α_3) are computed by using the

Table 1 Fuzzy random variables and their parameters for the limit state function $g(x)=X_1X_2-X_3=0$

Variables	Mean	Standard deviation	Distribution
X_1	[36.0,40.0,44.0]	[4.5,5.0,5.5]	Normal
X_2	[45.0,50.0,55.0]	[2.0,2.5,3.0]	Normal
X_3	[900.0,1000.0,1100.0]	[180.0,200.0,220.0]	Normal

Table 2 Fuzzy mean value sensitivity

λ	X_1		X_2		X_3	
	$(\frac{\partial P_f}{\partial \mu_{x_1}})_L$	$(\frac{\partial P_f}{\partial \mu_{x_1}})_R$	$(\frac{\partial P_f}{\partial \mu_{x_2}})_L$	$(\frac{\partial P_f}{\partial \mu_{x_2}})_R$	$(\frac{\partial P_f}{\partial \mu_{x_3}})_L$	$(\frac{\partial P_f}{\partial \mu_{x_3}})_R$
0	-1.6433×10^{-2}	-4.5954×10^{-7}	-9.5091×10^{-3}	-2.6115×10^{-7}	9.2663×10^{-9}	3.4363×10^{-4}
0.1	-1.3385×10^{-2}	-1.1504×10^{-6}	-7.6806×10^{-3}	-6.8605×10^{-7}	2.4760×10^{-8}	2.8009×10^{-4}
0.2	-1.0534×10^{-2}	-2.8827×10^{-6}	-6.0324×10^{-3}	-1.6719×10^{-6}	6.0354×10^{-8}	2.1988×10^{-4}
0.3	-8.2107×10^{-3}	-6.8454×10^{-6}	-4.6920×10^{-3}	-3.8159×10^{-6}	1.4009×10^{-7}	1.6726×10^{-4}
0.4	-6.1507×10^{-3}	-1.4707×10^{-5}	-3.5248×10^{-3}	-8.4259×10^{-6}	3.0411×10^{-7}	1.2759×10^{-4}
0.5	-4.5045×10^{-3}	-3.0829×10^{-5}	-2.5756×10^{-3}	-1.7747×10^{-5}	6.5661×10^{-7}	9.3639×10^{-5}
0.6	-3.2217×10^{-3}	-6.0567×10^{-5}	-1.8439×10^{-3}	-3.5526×10^{-5}	1.2648×10^{-6}	6.6107×10^{-5}
0.7	-2.1905×10^{-3}	-1.1695×10^{-4}	-1.2605×10^{-3}	-6.6047×10^{-5}	2.4118×10^{-6}	4.5419×10^{-5}
0.8	-1.4712×10^{-3}	-2.0939×10^{-4}	-8.5005×10^{-3}	-1.1938×10^{-4}	4.3584×10^{-6}	3.0539×10^{-5}
0.9	-9.5095×10^{-4}	-3.6031×10^{-4}	-5.4818×10^{-4}	-2.0678×10^{-4}	7.5431×10^{-6}	1.9899×10^{-5}
1	-5.9739×10^{-4}	-5.9739×10^{-4}	-3.4239×10^{-4}	-3.4239×10^{-4}	1.2436×10^{-5}	1.2436×10^{-5}

Table 3 Fuzzy standard deviation sensitivity

λ	X_1		X_2		X_3	
	$(\frac{\partial P_f}{\partial \sigma_{x_1}})_L$	$(\frac{\partial P_f}{\partial \sigma_{x_1}})_R$	$(\frac{\partial P_f}{\partial \sigma_{x_2}})_L$	$(\frac{\partial P_f}{\partial \sigma_{x_2}})_R$	$(\frac{\partial P_f}{\partial \sigma_{x_3}})_L$	$(\frac{\partial P_f}{\partial \sigma_{x_3}})_R$
0	1.6710×10^{-6}	1.9752×10^{-2}	2.4481×10^{-7}	3.5201×10^{-3}	2.9427×10^{-8}	3.4509×10^{-4}
0.1	4.0725×10^{-6}	1.6891×10^{-2}	6.0673×10^{-7}	3.0143×10^{-3}	7.2026×10^{-8}	2.9302×10^{-4}
0.2	9.7213×10^{-6}	1.4409×10^{-2}	1.4597×10^{-6}	2.5282×10^{-3}	1.7119×10^{-7}	2.5011×10^{-4}
0.3	2.1822×10^{-5}	1.2017×10^{-2}	3.2789×10^{-6}	2.0725×10^{-3}	3.7717×10^{-7}	2.0702×10^{-4}
0.4	4.4891×10^{-5}	9.6872×10^{-3}	6.9724×10^{-6}	1.6810×10^{-3}	7.7986×10^{-7}	1.6833×10^{-4}
0.5	9.1302×10^{-5}	7.6142×10^{-3}	1.4132×10^{-5}	1.2965×10^{-3}	1.5717×10^{-6}	1.3092×10^{-4}
0.6	1.7064×10^{-4}	5.7354×10^{-3}	2.7076×10^{-5}	9.7284×10^{-4}	2.9993×10^{-6}	9.9429×10^{-5}
0.7	3.0655×10^{-4}	4.1749×10^{-3}	4.9619×10^{-5}	7.1209×10^{-4}	5.3098×10^{-6}	7.3415×10^{-5}
0.8	5.2651×10^{-4}	2.9967×10^{-3}	8.6545×10^{-5}	5.0148×10^{-4}	9.2082×10^{-6}	5.1972×10^{-5}
0.9	8.6405×10^{-4}	2.0557×10^{-3}	1.4115×10^{-4}	3.3969×10^{-4}	1.5018×10^{-5}	3.5606×10^{-5}
1	1.3609×10^{-3}	1.3609×10^{-3}	2.2353×10^{-4}	2.2353×10^{-4}	2.3592×10^{-5}	2.3592×10^{-5}

approach outlined in section 4. In the GA approach (Krishnamoorthy 2001), 100 populations and 300 generations are used for each Membership degree λ. The obtained results for $\frac{\partial P_f}{\partial \mu_{x_i}}, \frac{\partial P_f}{\partial \sigma_{x_i}}$ and

Table 4 Fuzzy design point sensitivity

λ	X_1		X_2		X_3	
	$(\alpha_1)_L$	$(\alpha_1)_R$	$(\alpha_2)_L$	$(\alpha_2)_R$	$(\alpha_3)_L$	$(\alpha_3)_R$
0	0.6771	0.8142	0.1582	0.2815	-0.5453	-0.7022
0.1	0.6849	0.8084	0.1635	0.2746	-0.5533	-0.6943
0.2	0.6928	0.8023	0.1688	0.2672	-0.5614	-0.6870
0.3	0.7001	0.7962	0.1744	0.2603	-0.5693	-0.6794
0.4	0.7076	0.7900	0.1799	0.2537	-0.5772	-0.6817
0.5	0.7149	0.7837	0.1856	0.2469	-0.5853	-0.6640
0.6	0.7222	0.7772	0.1912	0.2405	-0.5932	-0.6562
0.7	0.7294	0.7707	0.1970	0.2339	-0.6013	-0.6485
0.8	0.7365	0.7640	0.2030	0.2275	-0.6091	-0.6407
0.9	0.7435	0.7573	0.2090	0.2212	-0.6170	-0.6328
1	0.7505	0.7505	0.2151	0.2151	-0.6249	-0.6249

Table 5 Fuzzy design point sensitivity (comparison with exact solution)

λ	Proposed Method		Exact Value (CP)		Proposed Method		Exact Value (CP)	
	$(\alpha_{1,PM})_L$	$(\alpha_{1,PM})_R$	$(\alpha_{1,CP})_L$	$(\alpha_{1,CP})_R$	$(\alpha_{2,PM})_L$	$(\alpha_{2,PM})_R$	$(\alpha_{2,CP})_L$	$(\alpha_{2,CP})_R$
0	0.6779	0.8141	0.6770	0.8144	0.1584	0.2809	0.1581	0.2817
0.2	0.6926	0.8021	0.6924	0.8025	0.1694	0.2665	0.1688	0.2775
0.4	0.7078	0.7898	0.7074	0.7901	0.1803	0.2532	0.1798	0.2538
0.6	0.7227	0.7771	0.7221	0.7773	0.1918	0.2398	0.1912	0.2405
0.8	0.7367	0.7638	0.7365	0.7641	0.2035	0.2275	0.2030	0.2276
1	0.7505	0.7505	0.7505	0.7505	0.2151	0.2151	0.2151	0.2151

α_i are presented in Tables 2-4, respectively. The results of the proposed method for λ=1 which can be seen in the last row of Tables 2-4 for different parameters are close to those of melchers'method (Melchers and Ahammed 2004) with a very good approximation.

In order to validate the reasonability of the fuzzy random sensitivity measure, the genetic algorithm with 30 populations and 50 generations is used, and the Cartesian Product (CP) algorithm is then utilized to calculate the exact solution for sensitivity measures at each Membership degree λ. Although CP approach can be used to calculate the sensitivity parameters for different combinations of the basic variables, it obtains the exact solution just for the convex problems and also will be impractical for big problems. Since Eq. (20) is a simple problem, the membership function of the design point sensitivity parameter can be exactly calculated. The results of the CP approach and the GA approach for two design point sensitivities are compared in Table 5. Table 5 shows that the results for two design point sensitivities α_1 and α_2 by the proposed method and the CP method are almost the same, thus, it demonstrates the correctness of the proposed method.

Figs. 4-5 show $\frac{\partial P_f}{\partial \mu_{x_1}}$ and $\frac{\partial P_f}{\partial \sigma_{x_1}}$ respectively, for three different cases of the GA method. It can

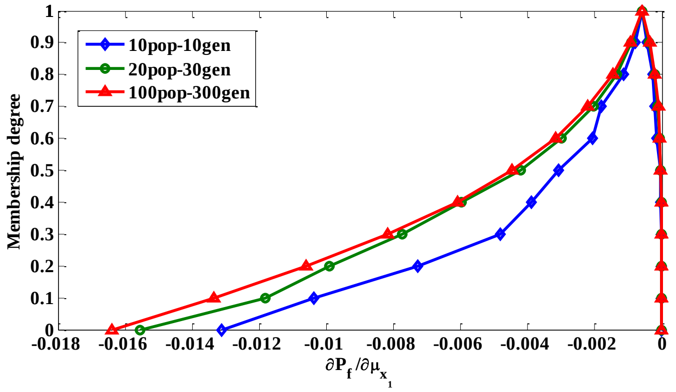


Fig. 4 The fuzzy mean value sensitivity of X_1

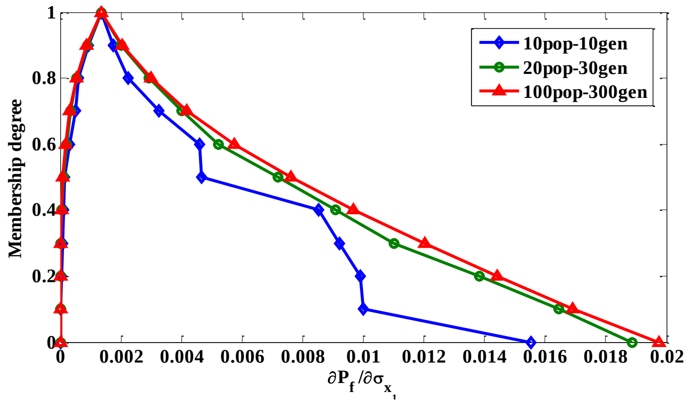


Fig. 5 The fuzzy standard deviation sensitivity of X_1

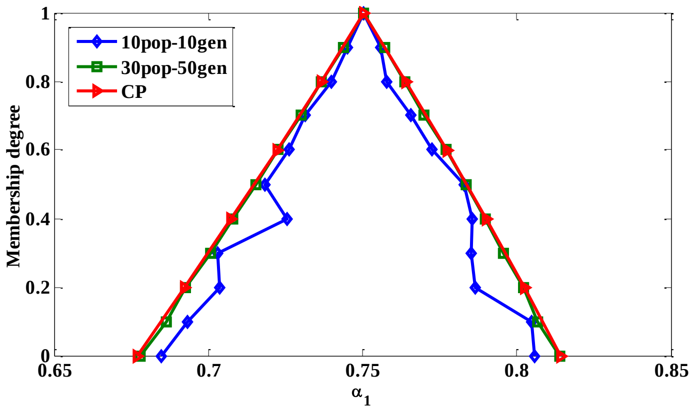


Fig. 6 The fuzzy design point sensitivity of X_1

be seen that the accuracy of the sensitivity parameters are raised by increasing the number of generations of the algorithm.

In Fig. 6, for the GA method, two different cases are considered and compared with the exact solution which shows the accuracy of the design point sensitivity is raised by increasing the

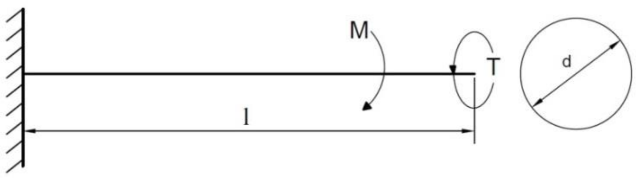


Fig. 7 Circular section cantilever beam

number of generations of the algorithm, such as with 50 generations, the exact solution is achieved in proposed method.

5.2 Example 2

Consider a non-linear limit state function of the model of circular section beam under torsion moment and bending moment (Yanfang, Yanlin *et al*. 2011). The model of beam is shown in Fig. 7. The maximum torsion stress τ and bending stress σ of the beam can be written as follows

$$\tau = \frac{16T}{\pi d^3} \tag{21}$$

$$\sigma = \frac{32M}{\pi d^3} \tag{22}$$

Where *T*, *M* and *d* are the torsion moment, the bending moment and the section diameter of the beam, respectively. Based on Fourth strength theory, complex stress *s* of torsion stress τ and bending stress σ can be written as follow

$$s = \sqrt{\sigma^2 + 3\tau^2} = \frac{16}{\pi d^3}\sqrt{4M^2 + 3T^2} \tag{23}$$

Limit state function of the beam based on stress-strength interface model considering stress failure mode can be defined as

$$g(x) = r - s \tag{24}$$

Where *r* is the material strength of the beam. The approximate hyperplane can be estimated using the Taylor's series expansion around design point.

$$\mathrm{X}^* = (X_r^* = 782.49, X_T^* = 1.43\times 10^4, X_M^* = 1.32\times 10^5, X_d^* = 11.97) \tag{185}$$

Using these values, the linear limit state function can be obtained as follows

$$g_L(x) = -2357.79 + r - 4.80\times 10^{-4} T + 5.91\times 10^{-3} M + 196.91d \tag{196}$$

The mean and standard deviation of the basic variables are considered triangular fuzzy numbers and are presented in Table 6. The λ-cut method is used for mapping of this fuzzy space to the interval random space. Membership degree λ varies from 0 to 1 by step 0.1. At each Membership degree λ, an interval is obtained for each random variable parameter, hence permissible domain for

Table 6 Fuzzy random variables and their parameters for the limit state function $g(x)=r-s=0$

Variables	Mean	Standard deviation	Distribution
r (MPa)	[800,820,840]	[31,32,33]	Normal
T (N.mm)	[14200,14300,14400]	[1200,1300,1400]	Normal
M (N.mm)	[113400,113500,113600]	[9150,9200,9250]	Normal
d (mm)	[11,12,13]	[0.059,0.06,0.061]	Normal

Table 7 Fuzzy mean value sensitivity

λ	r		T		M		d	
	$(\frac{\partial P_f}{\partial \mu_r})_L$	$(\frac{\partial P_f}{\partial \mu_r})_R$	$(\frac{\partial P_f}{\partial \mu_T})_L$	$(\frac{\partial P_f}{\partial \mu_T})_R$	$(\frac{\partial P_f}{\partial \mu_M})_L$	$(\frac{\partial P_f}{\partial \mu_M})_R$	$(\frac{\partial P_f}{\partial \mu_d})_L$	$(\frac{\partial P_f}{\partial \mu_d})_R$
0	-2.4453×10^{-3}	-9.2822×10^{-12}	4.3953×10^{-15}	1.1789×10^{-6}	5.4043×10^{-14}	1.4440×10^{-5}	-4.7938×10^{-1}	-1.7904×10^{-9}
0.1	-2.1471×10^{-3}	-1.1974×10^{-10}	5.7595×10^{-14}	1.0341×10^{-6}	6.7872×10^{-13}	1.2718×10^{-5}	-4.2172×10^{-1}	-2.3076×10^{-8}
0.2	-1.8658×10^{-3}	-1.2570×10^{-9}	5.9630×10^{-13}	8.9074×10^{-7}	7.5082×10^{-12}	1.1027×10^{-5}	-3.6599×10^{-1}	-2.4859×10^{-7}
0.3	-1.6118×10^{-3}	-1.1470×10^{-8}	5.3891×10^{-12}	7.7388×10^{-7}	6.6538×10^{-11}	9.4718×10^{-6}	-3.1669×10^{-1}	-2.1894×10^{-6}
0.4	-1.3657×10^{-3}	-8.6079×10^{-8}	4.1274×10^{-11}	6.6114×10^{-7}	5.0092×10^{-10}	8.1092×10^{-6}	-2.6955×10^{-1}	-1.6973×10^{-5}
0.5	-1.1584×10^{-3}	-5.4301×10^{-7}	2.6417×10^{-10}	5.5748×10^{-7}	3.1898×10^{-9}	6.8652×10^{-6}	-2.2861×10^{-1}	-1.0638×10^{-4}
0.6	-9.7414×10^{-4}	-2.8560×10^{-6}	1.3842×10^{-9}	4.6492×10^{-7}	1.6978×10^{-8}	5.7629×10^{-6}	-1.9211×10^{-1}	-5.6757×10^{-4}
0.7	-8.1075×10^{-4}	-1.2960×10^{-5}	6.2239×10^{-9}	3.8725×10^{-7}	7.6606×10^{-8}	4.7555×10^{-6}	-1.5928×10^{-1}	-2.5498×10^{-3}
0.8	-6.6486×10^{-4}	-4.9749×10^{-5}	2.3777×10^{-8}	3.1942×10^{-7}	2.9464×10^{-7}	3.9267×10^{-6}	-1.3046×10^{-1}	-9.7390×10^{-3}
0.9	-5.4193×10^{-4}	-1.6032×10^{-4}	7.6771×10^{-8}	2.6093×10^{-7}	9.4566×10^{-7}	3.2027×10^{-6}	-1.0680×10^{-1}	-3.1604×10^{-2}
1	-4.3820×10^{-4}	-4.3820×10^{-4}	2.1048×10^{-7}	2.1048×10^{-7}	2.5884×10^{-6}	2.5884×10^{-6}	-8.6284×10^{-2}	-8.6284×10^{-2}

distribution parameters can be easily obtained.

The fuzzy mean value sensitivities $(\frac{\partial P_f}{\partial \mu_r}, \frac{\partial P_f}{\partial \mu_T}, \frac{\partial P_f}{\partial \mu_M}, \frac{\partial P_f}{\partial \mu_d})$, the fuzzy standard deviation sensitivities $(\frac{\partial P_f}{\partial \sigma_r}, \frac{\partial P_f}{\partial \sigma_T}, \frac{\partial P_f}{\partial \sigma_M}, \frac{\partial P_f}{\partial \sigma_d})$ and the fuzzy design point sensitivities (α_r, α_T, α_M, α_d) are computed by using the approach outlined in section 4. For the GA method, 100 populations and 100 generations are used for each Membership degree λ. The obtained results are presented in Tables 7-9.

5.3 Example 3

A roof truss is shown in Fig. 8; the top boom and the compression bars are reinforced by concrete, and the bottom boom and the tension bars are steel. Assume that the uniformly distributed load q is applied on the roof truss and the uniformly distributed load can be transformed into the nodal load $p=qL/4$. Taking the safety and applicability into account, the perpendicular deflection Δ_C of the peak of structure node C not exceeding 2.8 cm is taken as the constraint condition and the performance response function can be constructed by $g(x)=0.028-\Delta_C$, where Δ_C is the function of the basic random variables, and $\Delta_C = \frac{ql^2}{2}(\frac{3.81}{A_C E_C} + \frac{1.13}{A_S E_S})$, A_C, A_S, E_C,

Table 8 Fuzzy standard deviation sensitivity

λ	r		T		M		d	
	$(\frac{\partial P_f}{\partial \sigma_r})_L$	$(\frac{\partial P_f}{\partial \sigma_r})_R$	$(\frac{\partial P_f}{\partial \sigma_T})_L$	$(\frac{\partial P_f}{\partial \sigma_T})_R$	$(\frac{\partial P_f}{\partial \sigma_M})_L$	$(\frac{\partial P_f}{\partial \sigma_M})_R$	$(\frac{\partial P_f}{\partial \sigma_d})_L$	$(\frac{\partial P_f}{\partial \sigma_d})_R$
0	2.9018×10^{-11}	1.6974×10^{-3}	2.5569×10^{-16}	1.6798×10^{-8}	3.0247×10^{-13}	1.6766×10^{-5}	2.1262×10^{-9}	1.2273×10^{-1}
0.1	3.3852×10^{-10}	1.5932×10^{-3}	3.2058×10^{-15}	1.5549×10^{-8}	3.5485×10^{-12}	1.5705×10^{-5}	2.4895×10^{-8}	1.1492×10^{-1}
0.2	3.3503×10^{-9}	1.4642×10^{-3}	3.1905×10^{-14}	1.4288×10^{-8}	3.5021×10^{-11}	1.4506×10^{-5}	2.6203×10^{-7}	1.0596×10^{-1}
0.3	2.8392×10^{-8}	1.3297×10^{-3}	2.6575×10^{-13}	1.2843×10^{-8}	2.9311×10^{-10}	1.3191×10^{-5}	2.0962×10^{-6}	9.6439×10^{-2}
0.4	2.0056×10^{-7}	1.2016×10^{-3}	1.8168×10^{-12}	1.1559×10^{-8}	2.0337×10^{-9}	1.1889×10^{-5}	1.4607×10^{-5}	8.6376×10^{-2}
0.5	1.1547×10^{-6}	1.0719×10^{-3}	1.0802×10^{-11}	1.0192×10^{-8}	1.1843×10^{-8}	1.0575×10^{-5}	8.4179×10^{-5}	7.7206×10^{-2}
0.6	5.6110×10^{-6}	9.4058×10^{-4}	5.2110×10^{-11}	8.9430×10^{-9}	5.7014×10^{-8}	9.3498×10^{-6}	4.0948×10^{-4}	6.7949×10^{-2}
0.7	2.2745×10^{-5}	8.1842×10^{-4}	2.1125×10^{-10}	7.7432×10^{-9}	2.2798×10^{-7}	8.1490×10^{-6}	1.6523×10^{-3}	5.9068×10^{-2}
0.8	7.6581×10^{-5}	7.0474×10^{-4}	7.1830×10^{-10}	6.6325×10^{-9}	7.7688×10^{-7}	7.0333×10^{-6}	5.6269×10^{-3}	5.0907×10^{-2}
0.9	2.1652×10^{-4}	5.9831×10^{-4}	2.0202×10^{-9}	5.6241×10^{-9}	2.1693×10^{-6}	5.9849×10^{-6}	1.5758×10^{-2}	4.3402×10^{-2}
1	5.0333×10^{-4}	5.0333×10^{-4}	4.7178×10^{-9}	4.7178×10^{-9}	5.0491×10^{-6}	5.0491×10^{-6}	3.6591×10^{-2}	3.6591×10^{-2}

Table 9 Fuzzy standard deviation sensitivity

λ	r		T		M		d	
	$(\alpha_r)_L$	$(\alpha_r)_R$	$(\alpha_T)_L$	$(\alpha_T)_R$	$(\alpha_M)_L$	$(\alpha_M)_R$	$(\alpha_d)_L$	$(\alpha_d)_R$
0	0.4847	0.5125	-0.0088	-0.0106	-0.8385	-0.8552	0.1791	0.1893
0.1	0.4861	0.5111	-0.0090	-0.0105	-0.8395	-0.8544	0.1796	0.1887
0.2	0.4875	0.5098	-0.0090	-0.0104	-0.8402	-0.8535	0.1801	0.1882
0.3	0.4890	0.5083	-0.0091	-0.0103	-0.8411	-0.8527	0.1806	0.1877
0.4	0.4903	0.5070	-0.0092	-0.0102	-0.8419	-0.8519	0.1811	0.1872
0.5	0.4917	0.5056	-0.0093	-0.0102	-0.8428	-0.8510	0.1816	0.1867
0.6	0.4932	0.5042	-0.0094	-0.0101	-0.8436	-0.8502	0.1821	0.1862
0.7	0.4946	0.5029	-0.0095	-0.0100	-0.8444	-0.8494	0.1826	0.1857
0.8	0.4959	0.5015	-0.0096	-0.0099	-0.8453	-0.8486	0.1831	0.1851
0.9	0.4973	0.5001	-0.0096	-0.0098	-0.8461	-0.8478	0.1836	0.1846
1	0.4987	0.4987	-0.0097	-0.0097	-0.8469	-0.9469	0.1841	0.1841

E_S, l respectively are sectional area, elastic modulus, length of the concrete and steel bars. The approximate hyperplane can be estimated using the Taylor's series expansion around design point

$$\mathrm{X}^* = (X_q^* = 2.16\times10^4, X_L^* = 12.04, X_{A_s}^* = 9.37\times10^{-4}, X_{A_c}^* = 3.68\times10^{-2}, X_{E_s}^* = 9.54\times10^{10}, X_{E_c}^* = 1.96\times10^{10}) \tag{207}$$

Using these values the following function can be obtained:

$$g_L(x) = 2.8\times10^{-2} - 1.29\times10^{-6} q - 4.65\times10^{-3} L + 21.09 A_S + 2.24\times10^{-1} A_C + 2.07\times10^{-13} E_S + 4.19\times10^{-13} E_C \tag{218}$$

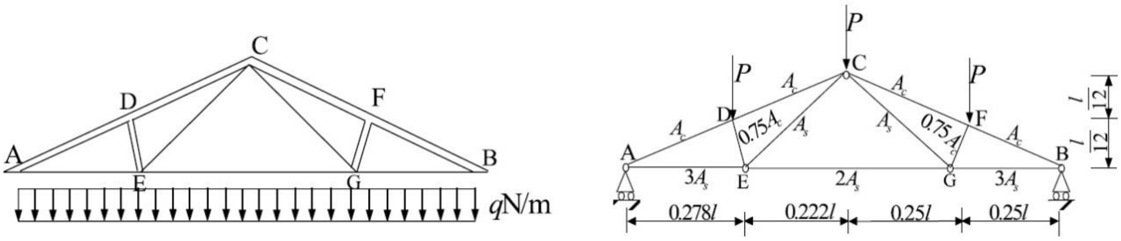


Fig. 8 The schematic diagram of a roof truss

Table 10 Fuzzy random variables and their parameters for the limit state function $g(x)$=0.028−Δ_C=0

Variables	Mean	Standard deviation	Distribution
q (N/m)	[18000,20000,22000]	[1260,1400,1540]	Normal
L (m)	[10.8,12,13.2]	[0.108,0.12,0.132]	Normal
A_s (m^2)	[8.84×10^{-4},9.82×10^{-4},1.1×10^{-3}]	[5.30×10^{-5},5.892×10^{-5},6.48×10^{-5}]	Normal
A_c (m^2)	[0.036,0.04,0.044]	[0.0043,0.0048,0.0053]	Normal
E_s (N/m^2)	[9×10^{10},1×10^{11},1.1×10^{11}]	[5.4×10^{9}, 6×10^{9},6.6×10^{9}]	Normal
E_c (N/m^2)	[1.8×10^{10},2×10^{10},2.2×10^{10}]	[1.08×10^{9},1.2×10^{9},1.32×10^{9}]	Normal

Table 11 Fuzzy design point sensitivity

λ	q		A_S		E_S	
	$(\alpha_q)_L$	$(\alpha_q)_R$	$(\alpha_{A_S})_L$	$(\alpha_{A_S})_R$	$(\alpha_{E_S})_L$	$(\alpha_{E_S})_R$
0	-0.6457	-0.6310	0.4295	0.4431	0.4295	0.4432
0.2	-0.6440	-0.6326	0.4308	0.4418	0.4309	0.4418
0.4	-0.6427	-0.6339	0.4322	0.4405	0.4322	0.4404
0.6	-0.6412	-0.6353	0.4336	0.4391	0.4336	0.4390
0.8	-0.6397	-0.6368	0.4350	0.4377	0.4350	0.4377
1	-0.6383	-0.6383	0.4363	0.4363	0.4363	0.4363
λ	L		A_C		E_E	
	$(\alpha_L)_L$	$(\alpha_L)_R$	$(\alpha_{A_C})_L$	$(\alpha_{A_C})_R$	$(\alpha_{E_C})_L$	$(\alpha_{E_C})_R$
0	-0.1996	-0.1924	0.3709	0.3833	0.1736	0.1802
0.2	-0.1988	-0.1930	0.3720	0.3821	0.1743	0.1795
0.4	-0.1982	-0.1938	0.3733	0.3808	0.1749	0.1789
0.6	-0.1975	-0.1945	0.3746	0.3795	0.1756	0.1782
0.8	-0.1967	-0.1952	0.3758	0.3783	0.1762	0.1775
1	-0.1960	-0.1960	0.3771	0.3771	0.1769	0.1769

The mean and standard deviation of independent normal basic variables are considered triangular fuzzy numbers and are presented in Table 10. Sensitivity parameters are obtained using the approach outlined in section 4 with 100 populations and 100 generations in the GA approach for each membership degree λ. Membership degree λ varies from 0 to 1 by step 0.2. The results of design point sensitivities are presented in Table 11.

6. Conclusions

In this article, the aleatory and epistemic uncertainties of the distribution parameters are considered. The effect of the fuzzy random parameters on the structural failure probability is investigated. In the presence of aleatory and epistemic uncertainties, the sensitivity parameters are expressed in fuzzy form. A new approach is proposed to calculate the fuzzy random sensitivity measures. In the proposed approach, the fuzzy space is transformed into an interval random space using λ-cuts and then the bounds of the fuzzy random sensitivity measures are obtained using the search algorithm (the GA method). Finally, the fuzzy sensitivity parameters are formed by assembling these bounds.

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