

## Time-dependent bond transfer length under pure tension in one way slabs

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**Abstract.** In a concrete member under pure tension, the stress in concrete is uniformly distributed over the whole concrete section. It is supposed that a local bond failure occurs at each crack, and there is a relative slip between steel and surrounding concrete. The compatibility of deformation between the concrete and reinforcement is thus not maintained. The bond transfer length is a length of reinforcement adjacent to the crack where the compatibility of strain between the steel and concrete is not maintained because of partially bond breakdown and slip. It is an empirical measure of the bond characteristics of the reinforcement, incorporating bar diameter and surface characteristics such as texture. Based on results from a series of previously conducted long-term tests on eight restrained reinforced concrete slab specimens and material properties including creep and shrinkage of two concrete batches, the ratio of final bond transfer length after all shrinkage cracking, to THE initial bond transfer length is presented.

**Keywords:** bond transfer length; creep; shrinkage; pure tension; one way slab

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### 1. Introduction

Due to the presence of steel in a Reinforced Concrete (RC) member, bar-concrete bond and interaction consideration is inevitable. Concrete-reinforcement bond between one crack and another to carry a certain amount of the tensile force normal to the cracked plane is illustrated by tension stiffening that contributes to the overall stiffness of the member. With a perfect bond, no slip occurs between the concrete and reinforcement, whereas with low bond, relative displacement can occur. Good bond properties increase the stiffening effect (Massicotte *et al.* 1990).

In actual fact, from the tension stiffening concept, concrete does not crack suddenly and completely, but undergoes progressive microcracking (strain softening). Due to the bond between steel and the concrete, the intact concrete between the adjacent primary cracks carries considerable tensile force (Behfarnia 2009). This bond significantly makes the stress-strain characteristics of tensile concrete in cracked reinforced concrete structures different from the stress-strain characteristics of uncracked concrete (Kaklauskas and Ghaboussi 2001). Since the bond stresses arise from the change in the steel force along the length, the effect of bond becomes more pronounced at the end anchorages of reinforcing bars and in the vicinity of cracks (Kwak and

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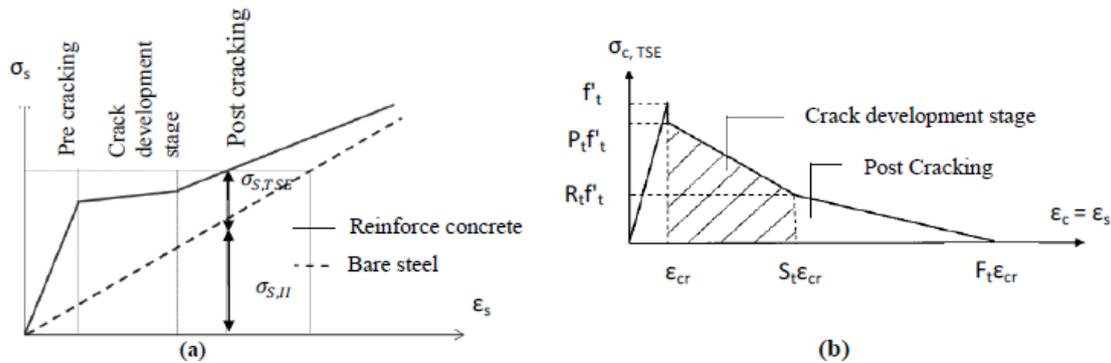


Fig. 1 (a) Tension stiffening model derived from uniaxial tension (b) the equivalent concrete stress-strain relationship

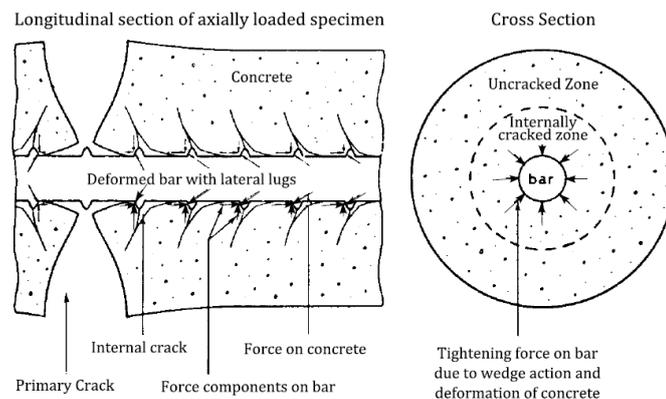


Fig. 2 The schematic deformation of concrete in a tension specimen

Filippou 1990).

Accurate prediction of steel-concrete bond enhances the stress and strain calculations for RC member, however, the problems is that under sustained loading and time-dependent effects of creep and shrinkage, the bond behavior changes. The current study aims to find a relationship between the short-term and long-term bond behavior and the changes along the bond transfer length with time.

Schematic stress-strain diagram for steel and concrete in reinforced concrete under uniaxial tension are divided into three regions of pre-cracking, crack development stage, and post-cracking is shown in Fig. 1. Under sustained load, the concrete stress ( $\sigma_c$ ) gradually reduces due to cracking and bond breakdown caused by drying shrinkage and, to a lesser extent, due to tensile creep. Early shrinkage reduces the cracking load and, under sustained service loads, shrinkage makes additional primary cracks with time and causes a time-dependent decay of the steel-concrete bond (Sokolov *et al.* 2010). This decay is a short-term effect which occurs particularly rapidly in large diameter specimens.

Nilson (1968) developed a finite element model for reinforced concrete considering the influence of reinforcement, progressive cracking, bond stress transfer and non-linear material properties, and provided a significant improvement on the model proposed by Ngo and Scordelis (1967). Goto (1971) applied tension to the reinforcement in a series of uni-axially reinforced

concrete prisms and presented the internal crack mechanisms and the determination of the schematic deformation diagram as shown in Fig. 2.

## 2. Time-dependent effects of creep and shrinkage on bond mechanism

Creep and shrinkage over time affect the behavior of reinforced concrete members, especially in cracked members. Similar results in cracked and uncracked sections are presented in experimental investigation performed by Divakar and Dilger (1988), emphasizing to account for shrinkage and creep effects in the long-term study of RC members.

Tension-stiffening relationships are coupled with shrinkage and accompanying creep effects. In most cases tension stiffening equations are derived from shrunk experimental RC members (Sokolov *et al.* 2010). Concrete shrinkage strain continues to increase with time at a decreasing rate. Other internal events such as bond slip or crack development around the bar reduce the tension stiffening, and there is no reason to suppose that this reduction will occur at the same rate as creep or shrinkage (Vakhshouri and Nejadi 2014). Shrinkage induced stresses significantly drop after cracking and almost disappear with yielding of reinforcement (Kaklauskas and Ghaboussi 2001).

Assuming a constant bond stress and tension stiffening over time, the considerable time-dependent change in the average concrete tensile stress caused by tensile creep and restrained shrinkage, together with the time-dependent breakdown of the bond, which is shrinkage-induced or cover-controlled cracking between the primary cracks, cannot be adequately modeled Gilbert (2008). The time-dependent effects of creep, shrinkage and temperature variation also exceed the nonlinear behavior of RC member due to bond-slip, aggregate interlock at a crack and dowel action of the reinforcing steel crossing a crack (Kwak and Filippou 1990).

## 3. Behavior after cracking and before yielding of steel

Different researchers have investigated the bond behavior and strain changes in steel and concrete, however, the time-dependent changes of bond transfer length are the least understood parameter in this matter. Followings are results of some researchers about the bond changes under tension.

First cracking occurs at the weakest part of the cross-section where the concrete tensile stress reaches the lower characteristic value of the tensile strength, so the stress in the concrete at the crack drops to zero. The bond characteristics of materials determine the position of the subsequent cracks relative to the first crack (Mihai *et al.* 2010). The zero tensile stress of concrete at each crack is rising with distance from the crack due to the steel-concrete bond, to a maximum value ( $\sigma_c$ ) (less than the tensile strength of the concrete) mid-way between adjacent cracks (Bizindavyi and Neale 1999, Wenkenbach 2011). The next crack will then not form within initial slip region ( $s_0$ ) of the first crack as the stresses in the concrete are lower within this limit than outside. Slip at the concrete-steel interface in the region of significant bond stress ( $s_0$  on either side of the crack) causes the crack to open. When the bond from steel to concrete can no longer transfer sufficient tensile force to form an additional crack between two existing cracks, the final established cracking state is reached (Kwak and Filippou 1990).

Scott and Gill (1987) studied seven tensile specimens with gauged to the reinforcing bars and

found a linear variation in steel strain adjacent to a crack location and linear relation of bond stresses with the load. This linear change of the stress implies a constant bond stress, which initially advocates some form of plastic behavior (Piyasena 2002).

Fig. 3 shows the mechanism by which the cracks form in distinct distances ( $d$ ) in a uniaxial reinforced concrete prism under tension. Beeby and Scott (2005) also presented the schematic variation of strain and stress in the region of a crack, as shown in Fig. 4.

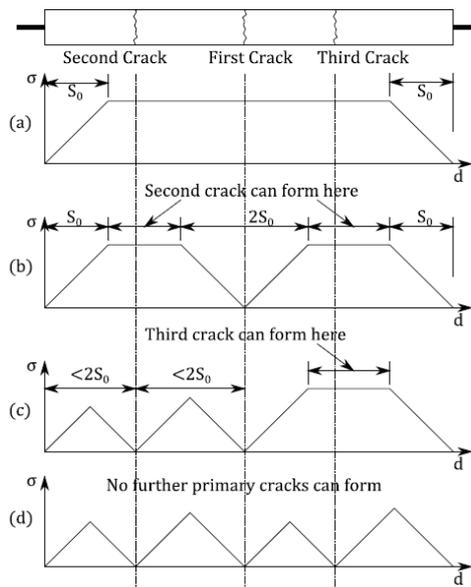


Fig. 3 Cracking mechanism in a uni-axially reinforced concrete prism under tension

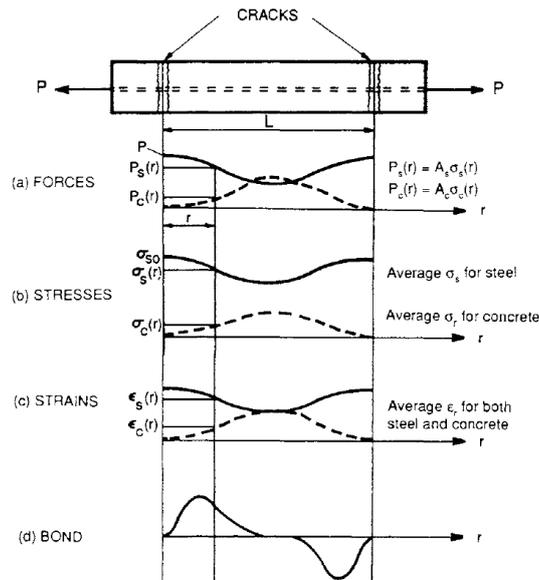


Fig. 4 Schematic distribution of forces, strains, normal stresses, and bond stresses along a racked RC member

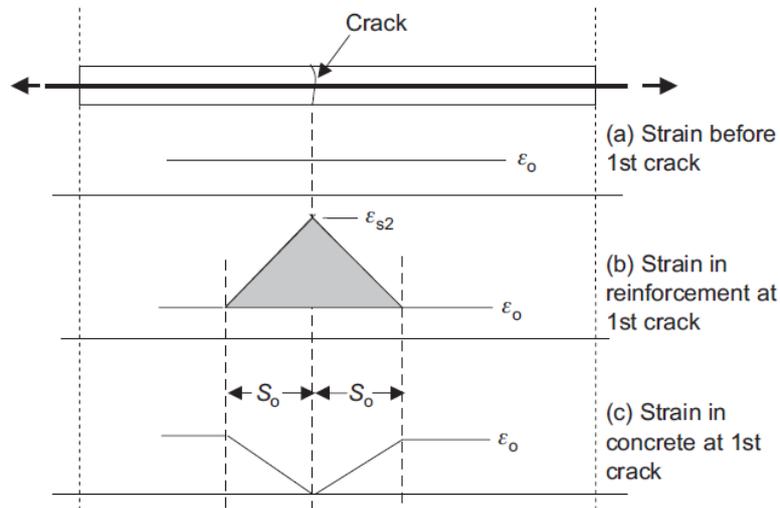


Fig. 4 Continued

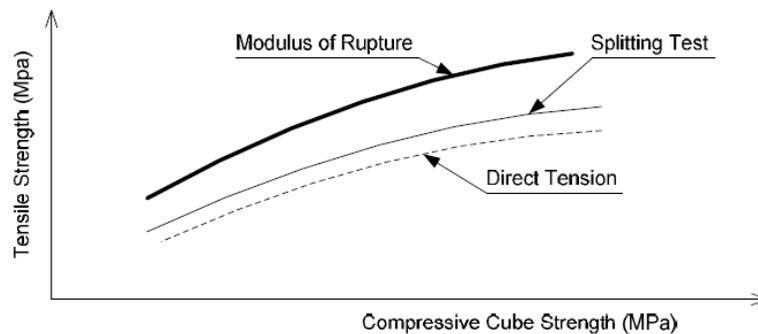


Fig. 5 Relationship between direct and indirect tensile strength measurements and compressive strength

#### 4. Bond transfer length

When the tensile force carried by concrete is transferred to the steel bar, the steel stress at cracked section increase under restriction by the bond forces developed along a certain length of steel bars. This length is called bond transfer length on either side of the crack (Beeby and Scott 2005). There are just limited studies in the short-term definition of bond transfer length while the long-term variation is almost unknown. Service loading gradually moves the transfer region toward the unloaded end, and from the resulting strain curves, more or less bilinear decreasing trend is seen, with a transition point happening at the limit of the initial transfer region (Bizindavyi and Neale 1999).

When investigating the mechanism of primary cracks formation to reach the final pattern, some researches assumed a fixed transfer length ( $s_0$ ) in short-term loading to transfer the bond stress through the concrete to steel (Nie and Cai 2000, Alih and Khelil 2012). Considering the serviceability of structure, time-dependent bond transfer length, significantly affects the crack width, crack distribution and deflection. In case of inadequate development/splice length, bonding may determine the ultimate capacity (Wang 2009).

The measurement of local bond stress and local slip along a stressed steel reinforcement bar in a tension member is difficult and very sensitive to experimental errors (Chan *et al.* 1992). For example, based on different relations, tensile strength will differ because of differences in stress distribution. A comparison is made in Fig. 5 which also shows that the tensile strength increases for an increase in concrete strength, however, not at the same rate (Eigelaar 2010).

Sokolov *et al.* (2010) mentioned that for the beams with average and high reinforcement ratios ( $\rho > 0.5\%$ ), accurate predictions of tension stiffening by the entire methods yield an error (standard deviation) from % 8.8 to %10.3. However, predictions for the lightly reinforced beams ( $\rho < 0.5\%$ ) were far less accurate. These inaccuracies are due to the increased effect of the tensile concrete which is a highly dispersed value.

Another method is presented to calculate the average strains in a tension member, based on bond transfer length and crack spacing for short-term loading as shown in Eq. (1) (Beeby and Scott 2005)

$$S_c = s_0 \varepsilon_{s2} \quad (1)$$

Where;  $S_c$  is the crack spacing,  $\varepsilon_{s2}$  is the strain in the reinforcement at a crack (concrete carries no tension), and  $s_0$  is bond transfer length.

According to investigation by (Beeby and Scott 2005), the stiffness of an axially reinforced tension member is directly related to the number of cracks. They also found that bond transfer length is shown to be proportional to the cover thickness, and the variation in reinforcement strain is linear in the region where it is affected by a crack. These results are in agreement with their previous studies. The number and the extent of cracks are controlled by the size and placement of the reinforcing steel (Kwak and Filippou 1990).

## 5. Experimental program

As part of a long-term cracking study at the University of New South Wales (UNSW), Nejadi and Gilbert (2004) monitored a total eight of fully restrained slab specimens with four different reinforcement layouts for 150 days. They measured the effect of shrinkage on the development of time-dependent direct tension cracking due to restrained deformation. Details of bar arrangement and concrete cover for the specimens are shown in Table 1. Fig. 6 displays the plan and elevation of the test specimens under pure tension. According to Fig. 6 the slab specimen are effectively

Table 1 Details of test specimens (reinforcement and dimensions)

Slab	No. of bars	Bar dia.(mm)	$A_{st}$ (mm <sup>2</sup> )	$C_s$ (mm)	$C_b$ (mm)	$S$ (mm)
RS1-a	3	12	339	109	44	185
RS1-b	3	12	339	109	44	185
RS2-a	3	10	236	110	46	185
RS2-b	3	10	236	110	46	185
RS3-a	2	10	157	145	46	300
RS3-b	2	10	157	145	46	300
RS4-a	4	10	314	115	46	120
RS4-a	4	10	314	115	46	120

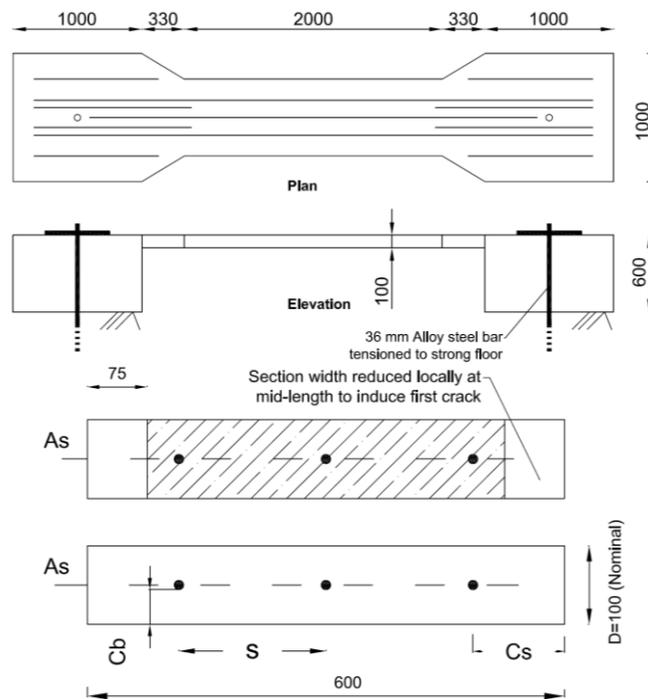


Fig. 6 Section details and plan and elevation of the test specimens

Table 2 Material properties for concrete batch 1 (B.1) and batch 2 (B.2)

	Age (days)									
	3		7		14		21		28	
Concrete batch	B.1	B.2	B.1	B.2	B.1	B.2	B.1	B.2	B.1	B.2
Compressive strength (MPa)	8.17	10.7	13.7	17.4	20.7	25	22.9	27.5	24.3	28.4
Flexural tensile strength (MPa)	1.91	2.47	3.15	3.1	3.43	3.77	3.77	3.97	3.98	4.04
Indirect tensile strength (MPa)			1.55	1.6					1.97	2.1
Modulus of elasticity (MPa)	13240	16130	17130	18940	21080	21750	22150	22840	22810	23210

anchored at both ends to ensure the complete restraining of specimens. At the mid-span of each specimen, the section was locally reduced to make sure that first cracking always occurred at this location.

To delay the onset of drying shrinkage, the specimen was kept in their moulds for three days and moist-cured continually in lab conditions. All slabs were anchored to the support at the age of 3 days and drying also commenced after the same period. The time-dependent development of drying shrinkage strain is also measured on unrestrained specimens of similar dimension and thickness to the slab specimens.

All tests of companion specimens were carried out following the relevant parts of the

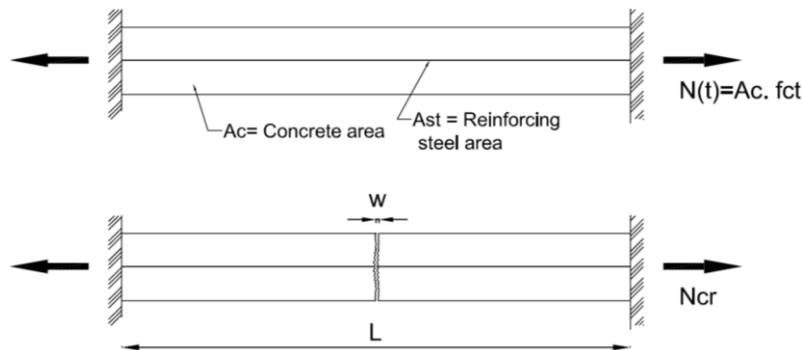


Fig. 7 First cracking in a restrained direct tension member (Gilbert 1992)

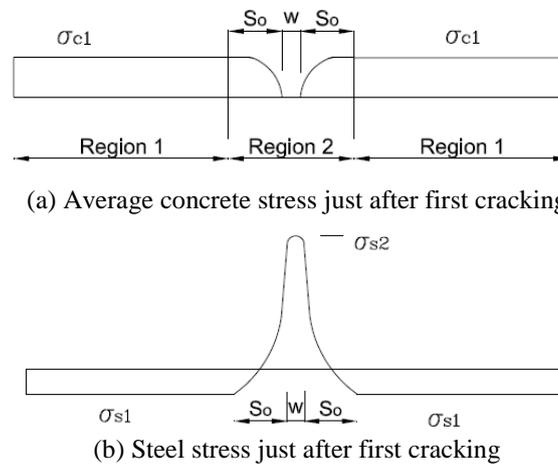


Fig. 8 Variation of average stress in steel and concrete

Australian Standard AS-1012. Two different ready mix batches of concrete were used for slabs through the test according to Table 2.

## 6. Analytical model

In a fully restrained RC member (Fig. 7), as concrete shrinks, the restraining force  $N(t)$  gradually increases until the first crack appears. The resulting force reduces to  $N_{cr}$ , immediately after the first crack appears and the stress away from the crack is less than the tensile strength of concrete  $f_{ct}(t)$ . On either side of the crack, the concrete shortens elastically, allowing the crack to open to a width  $w$ .

The slight increase in crack width with distance from the steel bar can be obtained by integrating the reduction in tensile elastic strain in the concrete (from that at the level of the bar) over the half crack spacing on either side of the crack.

According to Fig. 8, Gilbert (1992) derived the following expressions in Eqs. (2) to (9) for the restraining force  $N_{cr}$ , the concrete and steel stresses away from the crack  $\sigma_{c1}$  and  $\sigma_{s1}$  respectively, and the steel stress at the crack  $\sigma_{s2}$ .

$$N_{cr} = \frac{n\rho f_{ct} A_{ct}}{C_1 + n\rho(1+C_1)} \tag{2}$$

$$\sigma_{c1} = \frac{N_{cr} - \sigma_{s1} A_{st}}{A_c} = \frac{N_{cr}(1+C_1)}{A_c} \tag{3}$$

$$\sigma_{s1} = -\frac{2s_0}{3L-2s_0} \sigma_{s2} = -C_1 \sigma_{s2} \tag{4}$$

$$\sigma_{s2} = \frac{N_{cr}}{A_{st}} \tag{5}$$

$$C_1 = \frac{2s_0}{3L-2s_0} \tag{6}$$

$$\frac{\sigma_{s1}^*}{E_s} L + m \frac{\sigma_{s2}^* - \sigma_{s1}^*}{E_s} \left( \frac{2}{3} s_0 + w \right) = \Delta u \tag{7}$$

$$\sigma_{s1}^* = \frac{-2s_0 m}{3L-2s_0 m} \sigma_{s2}^* + \frac{3\Delta u E_s}{3L-2s_0 m} \tag{8}$$

$$\sigma_{s2}^* = \frac{N(\infty)}{A_{st}} \tag{9}$$

The bond transfer length ( $s_0$ ), as illustrated in Eq. (10), is related to bar diameter and ratio of reinforcement in this study. This expression was proposed earlier by Faver *et al.* (1983) for a member that contained deformed bars or welded wire mesh. Base and Murray (1982) used a similar expression.

$$s_0 = \frac{d_b}{10\rho} \tag{10}$$

Where;  $d$  is bar diameter and  $\rho$  is the tensile reinforcement ratio in the section.

The code alterations regarding the tension stiffening proposed by Beeby *et al.* (2005) are supporting the effect of bar diameter and reinforcement ratio on bond transfer length. A Good bond between steel and concrete increase the stiffening effect and is more significant for low reinforcement ratios than for higher ones, reported by Massicotte *et al.* (1990) in Fig. 9.

Fig. 10 (a), (b) and (c) respectively shows the portion of a fully restrained member under direct tension when all the shrinkage has occurred, and the final pattern of cracks has been established; it illustrates the average stresses in the concrete and steel caused by shrinkage.

Gilbert (1992) derived expressions for the final average spacing  $s_r$  and width in a fully restrained member, the final restraining force in a member, and the final concrete and steel stresses

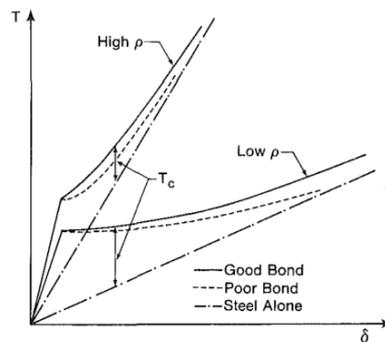


Fig. 9 Tension stiffening effect in strain-stress diagram respect to bond quality and steel ratio

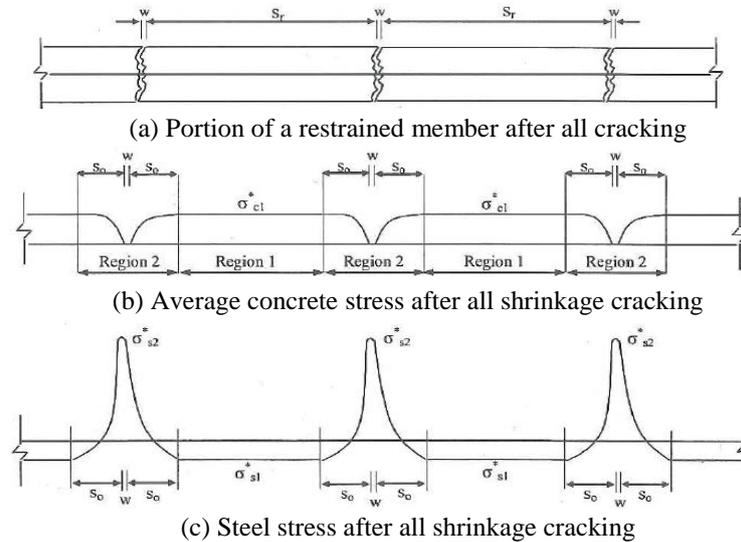


Fig. 10 Final concrete and steel stresses after direct tension cracking (Gilbert 1992)

Table 3 Measured no-bond length after first and all shrinkage cracking

Slab specimen	Measured bond transfer length (mm)							
	RS1- a	RS1- b	RS2- a	RS2- b	RS3- a	RS3- b	RS4- a	RS4- b
After first cracking ( $s_0$ )	261	215	228	201	246	342	256	259
After 150 days shrinkage ( $s_0^*$ )	366	290	290	292	323	454	332	319
$s_0^*/s_0$	1.4	1.35	1.27	1.45	1.31	1.33	1.29	1.23

(as shown in Figs. 10 (b), (c)) by enforcing the requirements for compatibility and equilibrium. With these derivations,  $s_0$  was assumed to remain constant over time, and the supports of the member were presumed to be immovable. However, these assumptions may bring significant error into the calculations. Because the results from this experimental program show deterioration of the bond at the concrete-steel interface and a gradual increase in  $s_0$  with time due to shrinkage effects.

In the present study, the measured steel strains in region 1 as defined in Fig. 8 (measured by a demec gauge) and obtaining the average value for these regions along the steel bar, the steel stress remote from the cracks immediately after the first crack  $\sigma_{s1}$ , and, after all, shrinkage cracking  $\sigma_{s1}^*$ , was calculated. Similarly, from the measured steel strains in region 2 (measured by electric strain gauges) and obtaining the average value for these strain gauges between the reinforcement bars, the steel stress in the vicinity of the first crack immediately after the first cracking of  $\sigma_{s2}$ , and, after all the shrinkage cracking  $\sigma_{s2}^*$  was also calculated.

Using Eqs. (2) to (9), the bond transfer length immediately after the first cracks  $s_0$ , and, after all the shrinkage cracking,  $s_0^*$ , was obtained. The results and ratio of  $s_0^*/s_0$  are presented in Table 3. Batches B1 and B2 are related to indexing  $a$  and  $b$  respectively.

From Table 3, the average ratio of the final bond transfer length  $s_0^*$  to the initial value after first cracking  $s_0$  is 1.33, so the final bond transfer length for extended time calculations may be expressed as Eq. (11)

$$s_0^* = 1.33 s_0 \quad (11)$$

From the test results, the extent to which shrinkage cracking can be controlled depends on limiting the desired crack width and the amount and distribution of bonded reinforcement across the crack.

The final crack width, crack spacing and steel stress at the crack, are dependent on the steel area (or more precisely, the reinforcement ratio). An increase in the steel area reduces the final crack width and with more cracks developing, reduces the crack spacing.

With an increase in the steel area, the loss of stiffness at first cracking reduces and, therefore, the restraining force after cracking is greater, but stress in the steel decreases at each crack. With a larger restraining force, the stress in the concrete away from a crack tends to be higher and consequently further cracking is more likely

## 7. Conclusions

The following conclusions can be made from the study:

- Despite worldwide research on cracking behavior of RC members, there are limited publications in bond transfer length study, especially under sustained loading and long-term monitoring of tension stiffening under pure tension.
- Experimental study on fully restrained slabs presented in this study illustrates that due to the random nature of cracking, high accuracy in calculating the crack width and crack spacing is not achievable.
- Shrinkage causes deterioration in bond at the concrete-steel interface.
- There is a gradual increase in the bond transfer length ( $s_0$ ) with time.
- The final value of the bond transfer length, after all the shrinkage has occurred is 1.33 times the initial value after first cracking.

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