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Deformation of a rectangular plate with an arbitrarily located circular hole under in-plane pure shear loading

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Abstract. Exact solutions for stresses, strains, displacements, and the stress concentration factors of a rectangular plate perforated by an arbitrarily located circular hole subjected to in-plane pure shear loading are investigated by two-dimensional theory of elasticity using the Airy stress function. The hoop stresses, strains, and displacements occurring at the edge of the circular hole are computed and plotted. Comparisons are made for the hoop stresses and the stress concentration factors from the present study and those from a rectangular plate with a circular hole under uni-axial and bi-axial uniform tensions and in-plane pure bending moments on two opposite edges.

Keywords: perforated plate; stress concentration factor; non-central hole; in-plane pure shear; hoop stress; airy stress function; exact solution

1. Introduction

Numerous researchers have investigated the mechanical behaviors of perforated plates, with main concerns being classified into four categories; stress concentration (Savin 1961, Muskhelishvili 1963, Miyata 1970, Timoshenko and Goodier 1970, Peterson 1974, Iwaki and Miyao 1980, Broek 1982, Theocaris and Petrou 1987, Mal and Singh 1991, Anderson 1995, Fu 1996, Radi 2001, Yang and He 2002, Zhang *et al.* 2002, Viva *et al*. 2005, She and Guo 2007, Li *et al.* 2008, Yang *et al.* 2008, Yu *et al.* 2008, Kang 2014, Woo *et al.* 2014), vibration, buckling, and fatigue. The various methods have been used to study them. The finite element method (FEM) is the most widely used for this perforated plate problems. Diverse methods other than FEM have been used like the complex variable method, three-dimensional stress analysis, the Ritz method, the boundary element method, the differential quadrature element method, semi-analytical solution method, experimental method, conjugate load/displacement method, and Galerkin averaging method. Most of the shapes of perforated holes have three types of circular, elliptical, and rectangular cutout. Most of the previous researchers have generated approximate solutions, and have dealt with perforated plates subjected to uni-axial or bi-axial uniform tension or compression at the most.

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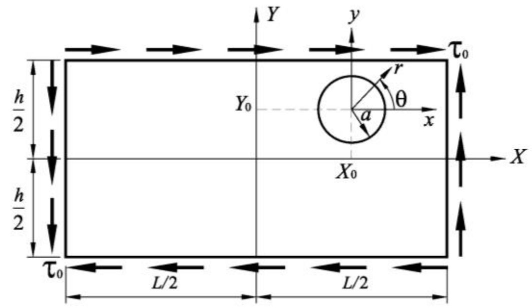

Fig. 1 A perforated rectangular plate with an arbitrarily located circular hole loaded by in-plane pure shear loading τ_0

In the present study, exact solutions for stresses, strains, displacements, and the stress concentration factors of a rectangular plate perforated by an non-central circular hole subjected to in-plan pure shear loading are investigated by two-dimensional (2-D) theory of elasticity using the Airy stress function. The hoop stresses, strains, and displacements occurring at the edge of the circular hole are computed and plotted. Comparisons are made for the hoop stresses and the stress concentration factors from the present study and those from a rectangular plate with a circular hole under uni-axial and bi-axial uniform tensions (Timoshenko and Goodier 1970) and in-plane pure bending moments on two opposite edges (Woo *et al*. 2014). Stress intensity factor (SIF) is often confused with SCF. The SIF is a scaling factor used in fracture mechanics to denote the stress intensity at the tip of a crack of known size and shape.

2. Method of analysis

Fig. 1 represents an infinite plate with an arbitrarily located circular hole of radius of a and subjected to in-plane pure shear loading τ_0. The plate is assumed to be very large compared with the circular hole. The present study comes within the category of the plane stress problem because the plate thickness is assumed to be very thin compared with other dimensions of the plate. The origin of the rectangular coordinate system (X,Y) is located at the center of the plate while the origins of the rectangular (x,y) and the polar (r,θ) coordinate systems coincide with the center of the non-central circular hole. The center of the non-central circular hole is located at $(X,Y)=(X_0,Y_0)$.

First of all, considering a plate with no hole subjected to in-plane pure shear loading τ_0, the stress components are below

$$\sigma_{XX}^0=\frac{\partial^2\phi^0}{\partial Y^2}=0\,,\quad \sigma_{YY}^0=\frac{\partial^2\phi^0}{\partial X^2}=0\,,\quad \sigma_{XY}^0=-\frac{\partial^2\phi^0}{\partial X\partial Y}=\tau_0 \tag{1}$$

where ϕ^0 is a fundamental Airy stress function. The Airy stress function ϕ satisfies the governing equation $\nabla^4\phi=\nabla^2(\nabla^2\phi)=0$ for 2-D plane problems in elasticity with no body forces, where the Laplacian operator ∇^2 is expressed as

$$\nabla^2=\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Y^2} \tag{2}$$

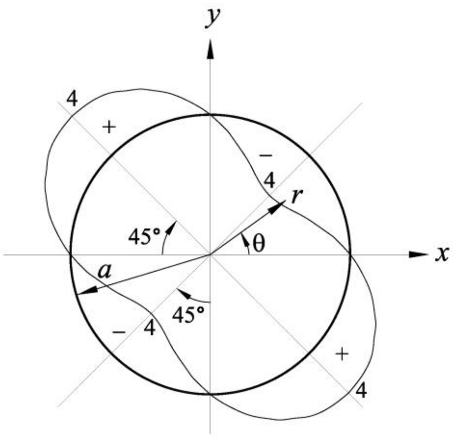

Fig. 2 The non-dimensional hoop stress $\sigma_{\theta\theta}/\tau_0$ occurring at the edge of the circular hole on $r=a$

and ∇^4 is the bi-harmonic differential operator defined by

$$\nabla^4 = \nabla^2(\nabla^2) = \frac{\partial^4}{\partial X^4} + 2\frac{\partial^4}{\partial X^2 \partial Y^2} + \frac{\partial^4}{\partial Y^4} \tag{3}$$

in the rectangular coordinates. From the relation of the Airy stress function and the stress components in rectangular coordinates in Eqs. (1), the fundamental Airy stress function ϕ^0 can be assumed as

$$\phi^0 = -\tau_0 XY + C_1 X + C_2 Y + C_3 \tag{4}$$

where C_1, C_2 and C_3 are arbitrary integration constants. Since the relations of $X=x+X_0$ and $Y=y+Y_0$, Eq. (4) becomes

$$\phi^0 = -\tau_0(x+X_0)(y+Y_0) + C_1(x+X_0) + C_2(y+Y_0) + C_3 \tag{5}$$

A linear function of X or Y and a constant in the Airy stress function in rectangular coordinates are trivial terms which do not give rise to any stresses and strains. Dropping the trivial terms in Eq. (5), the fundamental Airy stress function ϕ^0 becomes finally

$$\phi^0 = -\tau_0 xy \tag{6}$$

Using the relations of

$$x = r\cos\theta \tag{7}$$

$$y = r\sin\theta \tag{8}$$

the fundamental Airy stress function ϕ^0 in Eq. (6) can be transformed into the bi-harmonic functions as

$$\phi^0 = -\frac{\tau_0}{2} r^2 \sin 2\theta \tag{9}$$

Table 1 Stresses and displacements of potential candidates of bi-harmonic functions ϕ

ϕ	σ_{rr}	$\sigma_{r\theta}$	$\sigma_{\theta\theta}$
$r^2\sin 2\theta$	$-2\sin 2\theta$	$-2\cos 2\theta$	$2\sin 2\theta$
$\sin 2\theta$	$-4\sin 2\theta/r^2$	$2\cos 2\theta/r^2$	0
$r^4\sin 2\theta$	0	$-6r^2\cos 2\theta$	$12r^2\sin 2\theta$
$\sin 2\theta\, r^2$	$-6\sin 2\theta/r^4$	$6\cos 2\theta/r^4$	$6\sin 2\theta/r^4$

ϕ	$2\mu u_r$	$2\mu u_\theta$
$r^2\sin 2\theta$	$-2r^2\sin 2\theta$	$-2r^2\cos 2\theta$
$\sin 2\theta$	$(\kappa+1)\sin 2\theta/r$	$(\kappa+1)\cos 2\theta/r$
$r^4\sin 2\theta$	$-(3-\kappa)r^3\sin 2\theta$	$-(3-\kappa)r^3\cos 2\theta$
$\sin 2\theta\, r^2$	$2\sin 2\theta/r^3$	$-2\cos 2\theta/r^3$

which satisfies the governing equation $\nabla^4\phi^0=\nabla^2(\nabla^2\phi^0)=0$, where ∇^2 is defined by

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} \tag{10}$$

and ∇^4 is expressed as

$$\nabla^4 = \nabla^2(\nabla^2) = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right) \tag{11}$$

in the polar coordinates. From the below relations between the stress components and the Airy stress function in polar coordinates, the stresses in the plate with no hole subjected to in-plane pure shear loading τ_0 can be calculated as below

$$\sigma_{rr}^0 = \frac{1}{r}\frac{\partial\phi^0}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi^0}{\partial\theta^2} = \tau_0 \sin 2\theta$$

$$\sigma_{r\theta}^0 = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\phi^0}{\partial\theta}\right) = \tau_0 \cos 2\theta$$

$$\sigma_{\theta\theta}^0 = \frac{\partial^2\phi^0}{\partial r^2} = -\tau_0 \sin 2\theta \tag{12}$$

Let us return to the original problem of a perforated plate by a non-central circular hole. The total Airy function ϕ becomes

$$\phi = \phi^0 + \phi^* \tag{13}$$

where ϕ^* is an Airy stress function to cancel unwanted traction due to ϕ^0 on $r=a$. The normal (σ_{rr}) and shear ($\sigma_{r\theta}$) stresses on $r=a$ must be free as below

$$\sigma_{rr}\Big|_{r=a} = [\sigma_{rr}^0 + \sigma_{rr}^*]_{r=a} = 0$$

$$\sigma_{r\theta}\Big|_{r=a} = [\sigma_{r\theta}^0 + \sigma_{r\theta}^*]_{r=a} = 0 \tag{14}$$

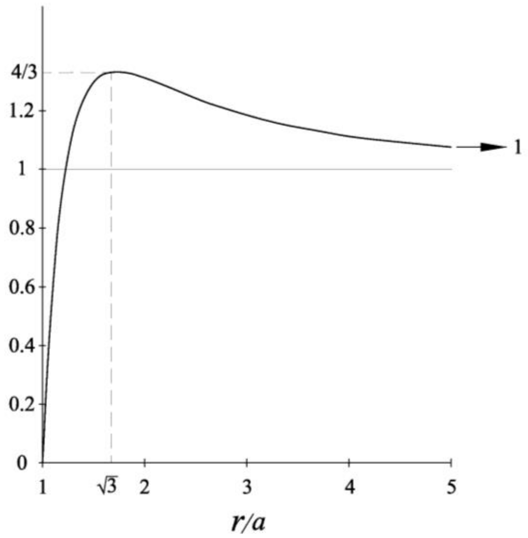

Fig. 3 The non-dimensional shear stress $\sigma_{r\theta}/\tau_0$ at θ=0° and 180°

Therefore, σ^*_{rr} and $\sigma^*_{r\theta}$ on r=a must have terms of sin 2θ and cos 2θ, respectively, in order to eliminate the stresses on r=a due to ϕ^0 in Eqs. (12). Table 1 shows the potential candidates of the bi-harmonic functions for the present problem identified as $r^2\sin 2\theta$, $\sin 2\theta$, $r^4\sin 2\theta$, and $\sin 2\theta/r^2$ from the tables by Dundurs (Fu 1996), which contain stresses and displacements of certain bi-harmonic functions in the polar coordinates. However, the term of $r^2\sin 2\theta$ in the fundamental Airy stress function ϕ^0 in Eq. (9) must not be included in ϕ^0 in order not to disturb the traction boundary conditions at the boundaries of the plate in Eq. (1). It is observed in Table 1 that the singularity at infinity occurs in stresses and displacements because of the term of $r^4\sin 2\theta$. Omitting these inadequate terms, the total Airy stress function ϕ in Eq. (13) becomes

$$\phi = -\frac{\tau_0}{2}\left(r^2\sin 2\theta + Aa^2\sin 2\theta + Ba^4\frac{\sin 2\theta}{r^2}\right) \tag{15}$$

where A and B are arbitrary integration constants to be determined by the traction boundary conditions the edge of the circular hole in Eq. (14). In order to make the constants A and B dimensionless, a constant a^2 or a^4 is multiplied by the constants. Using the relations between the stress components and the Airy stress function ϕ in polar coordinates in Eq. (12) and applying the stress free boundary conditions the edge of the circular hole on r=a in Eq. (14)

$$\sigma_{rr}\Big|_{r=a} = \left[\frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}\right]_{r=a} = 0 \tag{16}$$

$$\sigma_{r\theta}\Big|_{r=a} = \left[-\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\phi}{\partial\theta}\right)\right]_{r=a} = 0 \tag{17}$$

the unknown constants are computed as A=−2 and B=1. Thus the total Airy stress function in Eq. (15) becomes finally

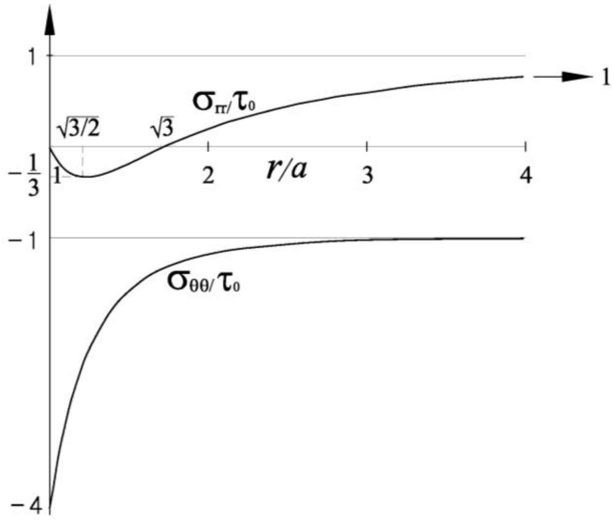

Fig. 4 The non-dimensional normal stresses σ_{rr}/τ_0 and $\sigma_{\theta\theta}/\tau_0$ at θ=45° and 225°

$$\phi = -\frac{\tau_0}{2}\left(r^2 \sin 2\theta - 2a^2 \sin 2\theta + \frac{a^4 \sin 2\theta}{r^2}\right) \tag{18}$$

It is noticed from Eq. (18) that the Airy stress function ϕ is not dependent on the location of the circular hole at $(X,Y)=(X_0,Y_0)$.

From Eqs. (12) and (18), the stress components can be easily calculated as below

$$\sigma_{rr} = \tau_0\left(1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4}\right)\sin 2\theta$$

$$\sigma_{r\theta} = \tau_0\left(1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4}\right)\cos 2\theta$$

$$\sigma_{\theta\theta} = -\tau_0\left(1 + \frac{3a^4}{r^4}\right)\sin 2\theta \tag{19}$$

From Table 1 by Dundurs (Fu 1996), displacements can be easily obtained as selecting and summing the displacement corresponding to each term of the total Airy stress functions ϕ in Eq. (18) as below

$$u_r = \frac{\tau_0}{2\mu}\left[r + \frac{a^2(\kappa+1)}{r} - \frac{a^4}{r^3}\right]\sin 2\theta$$

$$u_\theta = \frac{\tau_0}{2\mu}\left[r + \frac{a^2(\kappa-1)}{r} + \frac{a^4}{r^3}\right]\cos 2\theta \tag{20}$$

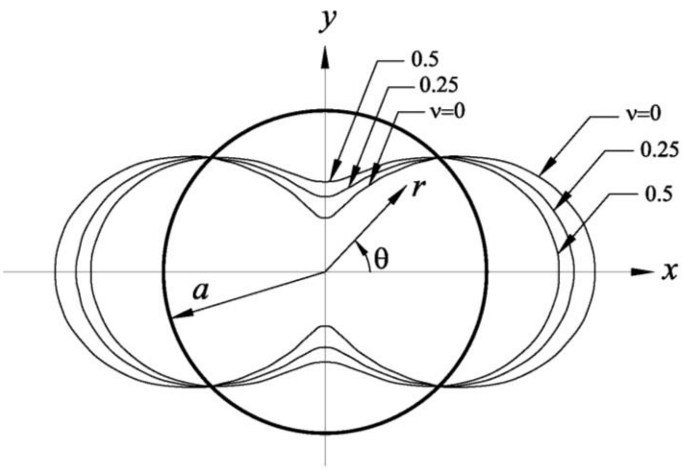

Fig. 5 The non-dimensional hoop displacements $u_\theta \mu/\tau_0 a$

where u_r and u_θ are the radial and circumferential displacements, respectively, μ is shear modulus and κ is a secondary elastic constant defined by

$$\kappa = \frac{3-\nu}{1+\nu} \tag{21}$$

for the plane stress problems of the present study, and ν is Poisson's ratio.

Substituting the displacements in Eq. (20) into the relations of strain-displacement in polar coordinates

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}, \quad \varepsilon_{r\theta} = \frac{1}{2}\left[r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right) + \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right], \quad \varepsilon_{\theta\theta} = \frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r\right) \tag{22}$$

where ε_{rr}, $\varepsilon_{\theta\theta}$, and $\varepsilon_{r\theta}$ are the radial, circumferential, and shear strains, respectively, the strain components can be calculated as below

$$\varepsilon_{rr} = \frac{\tau_0}{2\mu}\left[1-(1+\kappa)\frac{a^2}{r^2}+\frac{3a^4}{r^4}\right]\sin 2\theta$$

$$\varepsilon_{r\theta} = \frac{\tau_0}{2\mu}\left[1+\frac{2a^2}{r^2}-\frac{3a^4}{r^4}\right]\cos 2\theta$$

$$\varepsilon_{\theta\theta} = -\frac{\tau_0}{2\mu}\left[1-(3-\kappa)\frac{a^2}{r^2}+\frac{3a^4}{r^4}\right]\sin 2\theta \tag{23}$$

3. Stress concentration factor

By using of Eq. (19) the non-dimensional hoop stress $\sigma_{\theta\theta}$ occurring at the edge of the circular

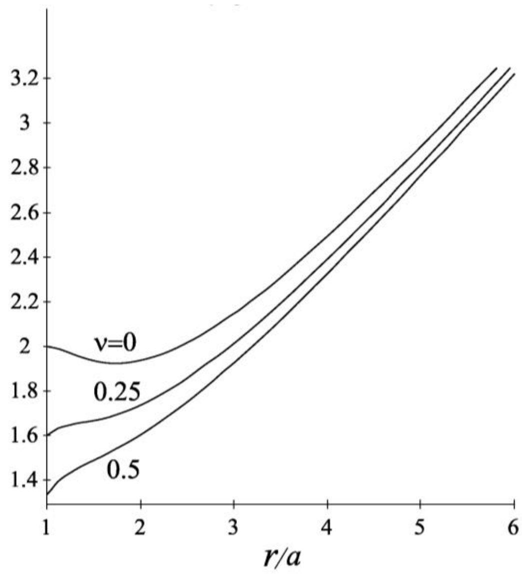

Fig. 6 The non-dimensional displacements $u_r\mu/\tau_0 a$ on θ=45° and 225°

hole on r=a becomes

$$\left.\frac{\sigma_{\theta\theta}}{\tau_0}\right|_{r=a} = -4\sin 2\theta \tag{24}$$

and is zero at $\theta=n\pi/2$ (n=0,1,2,3) irrespective of Poisson's ratio as shown in Fig. 2. The non-dimensional maximum hoop stresses $\sigma_{\theta\theta}/\tau_0$ occur at θ=45°, 135°, 225°, and 315°, as expected, and are calculated as ±4 which is called the stress concentration factor (S.C.F.) (=$\sigma_{\theta\theta\max}/\tau_0$), defined as the ratio between the maximum normal stress and the nominal stress.

For a large plate with a central circular hole under uni-axial (x-direction) uniform tensile load σ_0, the Airy stress function is given by (Timoshenko and Goodier 1970)

$$\phi = \frac{\sigma_0}{4}\left(r^2 - r^2\cos 2\theta - 2a^2\ln r - a^4\frac{\cos 2\theta}{r^2} + 2a^2\cos 2\theta\right) \tag{25}$$

The normal stress $\sigma_{\theta\theta}$ becomes

$$\sigma_{\theta\theta} = \frac{\partial^2\phi}{\partial r^2} = \frac{\sigma_0}{2}\left[1 + \frac{a^2}{r^2} - \left(1 + \frac{3a^4}{r^4}\right)\cos 2\theta\right] \tag{26}$$

and so the non-dimensional hoop stress $\sigma_{\theta\theta}/\sigma_0$ on r=a is

$$\left.\frac{\sigma_{\theta\theta}}{\sigma_0}\right|_{r=a} = 1 - 2\cos 2\theta \tag{27}$$

This stress is independent of the radius of the circular hole for a large plate compared with the circular hole. On $\theta=\pi/2$ or $\theta=3\pi/2$, we find the S.C.F $(\sigma_{\theta\theta})_{\max}/\sigma_0$ becomes 3.

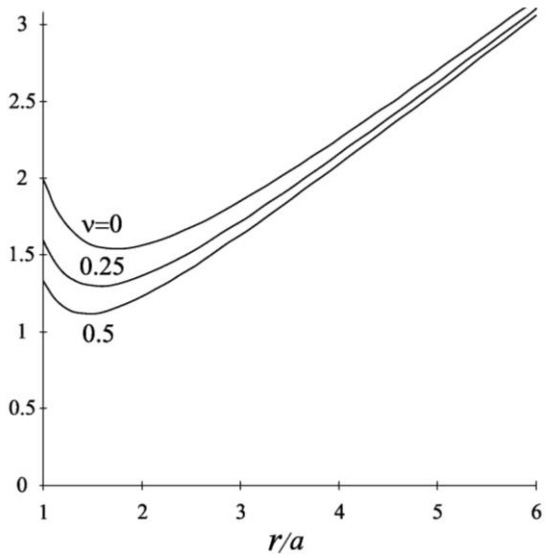

Fig. 7 The non-dimensional displacements $u_\theta \mu/\tau_0 a$ on θ=0° and 180°

For uniform tension applied in both *x*- and *y*-directions, we can find the solution by using the above solution for uni-axial uniform tension in Eq. (27) and the principle of superposition as below

$$\left.\frac{\sigma_{\theta\theta}}{\sigma_0}\right|_{r=a} = 1-2\cos 2\theta + 1 - 2\cos[2(\theta+\pi/2)] = 2 \tag{28}$$

It is of interest to note that the S.C.F decreases to 2 from 3.

By taking a tensile stress σ_0 in the *X*-direction and a compressive stress $-\sigma_0$ in the *Y*-direction, we obtain the case of pure shear (Timoshenko and Goodier 1970). By using the solution for uni-axial uniform tension in Eq. (27) and the principle of superposition, the hoop stress becomes

$$\left.\frac{\sigma_{\theta\theta}}{\sigma_0}\right|_{r=a} = 1-2\cos 2\theta - 1 + 2\cos[2(\theta+\pi/2)] = -4\cos 2\theta \tag{29}$$

We find that the S.C.F. ($=\sigma_{\theta\theta}/\sigma_0$) is 4 on $\theta=\pi/2$ or $\theta=3\pi/2$, and the S.C.F. is -4 on $\theta=0$ or $\theta=\pi$, which is exactly same with the S.C.F for the present study.

The hoop stress for the plate under in-plane pure bending moment M_0 is given by (Fu 1996)

$$\left.\frac{\sigma_{\theta\theta}}{\sigma_0}\right|_{r=a} = -4\left(\frac{a}{h}\right)\sin\theta\,(1-2\cos^2\theta) \tag{30}$$

in which assuming $\sigma_{XX}=\sigma_0$ at $X=\pm L/2$ and $Y=-h/2$, one obtains $\sigma_0=6M_0/bh^2$, where b is a plate thickness. The hoop stress is depend upon the size of the circular hole.

Fig. 3 shows the comparisons of the non-dimensional hoop stress on $r=a$ for the plates with a circular hole under in-plane pure shear loading, uni-axial and bi-axial uniform tensile loads, and in-plane bending moment. The maximum hoop stress for in-plane pure shear loading is much

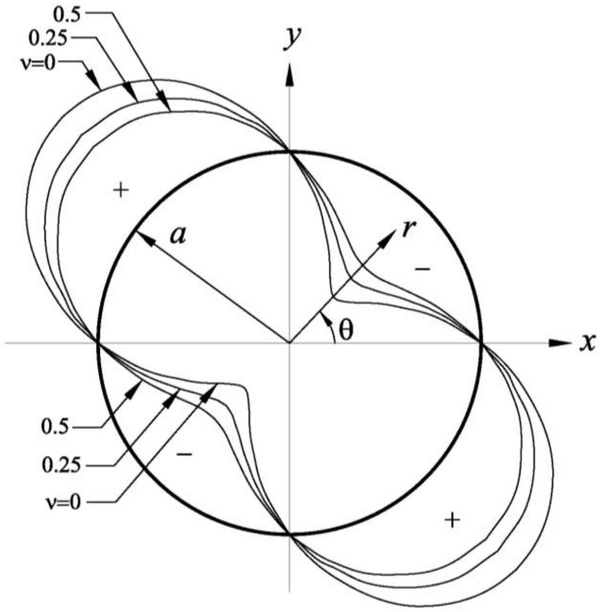

Fig. 8 Hoop strains $\varepsilon_{\theta\theta}\mu/\tau_0$

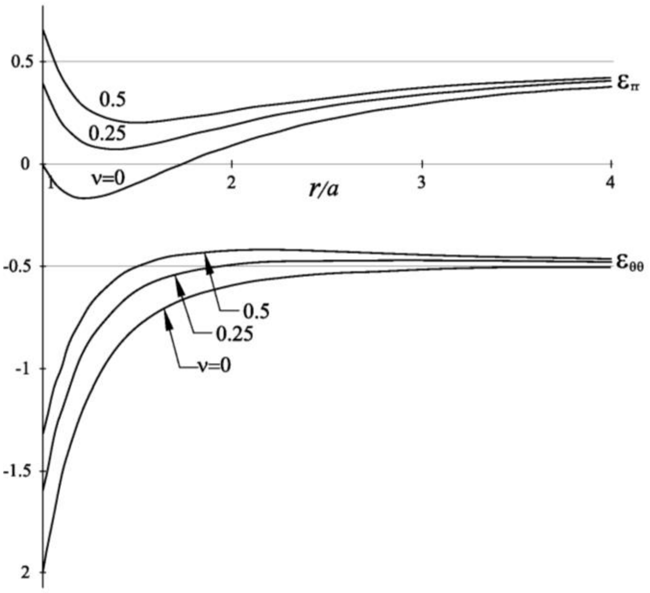

Fig. 9 Normal strains $\varepsilon_{rr}\mu/\tau_0$ and $\varepsilon_{\theta\theta}\mu/\tau_0$ on θ=45° and 225°

larger than the other three cases.

Table 2 shows comparisons of the stress concentration factors (S.C.F) for the present problem and the plates with a circular hole subjected to uni-axial and bi-axial uniform tensions and in-plane bending moments. In the case of in-plane pure shear loading, the S.C.F is 4, which is the largest in the table. For the plate under in-plane bending moments, the S.C.F increases

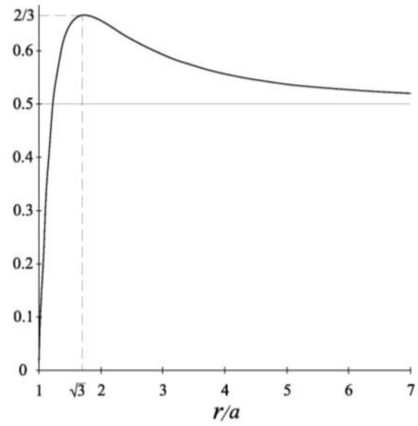

Fig. 10 Shear strain $\varepsilon_{r\theta}\mu/\tau_0$ on θ=0° and 180°

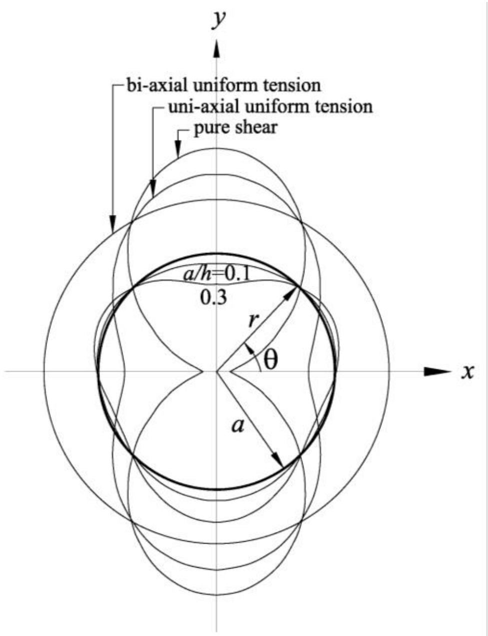

Fig. 11 Comparisons of the non-dimensional hoop stresses $\sigma_{\theta\theta}/\sigma_0$ for the rectangular plates with a circular hole under in-plane pure shear loading, uni- and bi-axial uniform tensile stresses, and in-plane bending moment for a/h=0.1 and 0.3

Table 2 Comparisons of the stress concentration factors S.C.F.

	In-plane bending moment						Bi-axial uniform tension	Uni-axial uniform tension	Pure shear
a/h	0.01	0.05	0.1	0.2	0.3	0.4			
S.C.F.	0.04	0.2	0.4	0.8	1.2	1.6	2	3	4

linearly with a/h, while for the other three problems under in-plane pure shear loading and uni-axial and bi-axial uniform tensions, the S.C.F's are irrespective of the size of the circular hole.

4. Conclusions

Exact solutions for stresses, strains, displacements, and the stress concentration factors of a perforated plate with an arbitrarily located circular hole subjected to in-plane pure shear loading are investigated by two-dimensional theory of elasticity using the Airy stress function. The hoop stresses occurring at the edge of the circular hole are plotted and the stress concentration factors are calculated. It is noticed that the solutions is not related to the location of the non-central circular hole.

The bi-harmonic functions ϕ to satisfy the governing equation $\nabla^4\phi$=0 for plane problems with no body force in elasticity are called the Airy stress functions. Considering multi-valueness and singularity in stresses and displacements, once proper bi-harmonic functions for a certain problem are decided from the table presented by Dundurs (Fu 1996), the stresses and displacements according to the Airy stress functions can be easily selected from the table, and then strains can be calculated from the relations of strain-displacement in Eqs. (22). Also the integration constants could be calculated applying the stress free boundary conditions at the edge of the circular hole. The obtained Airy stress function $\phi(r, \theta)$ satisfies both the governing equation $\nabla^4\phi$=0 and the stress boundary conditions, therefore the solutions are exact.

Comparisons are made for the non-dimensional hoop stresses $\sigma_{\theta\theta}$ on the edge of the circular hole and the stress concentration factors from the present study and plates with a circular hole under uni-axial and bi-axial uniform tensions, and in-plane bending moments on two opposite edges (Kang *et al*. 2014). It is seen that the stress concentration factor for the present problem under in-plane pure shear loading is the largest. The stress concentration factors from the case of pure shear obtained by taking a uniform tension on two opposite edges and a uniform compression on the other two edges by a previous researcher (Timoshenko and Goodier 1970) is exactly same as those from the present study.

The exact solutions for stresses, displacements, and strains in Eqs. (19), (20), and (23), respectively, can be used in the case of a very long cuboid with a central circular cylindrical hole, which is a sort of plane strain problem. In plane strain problems, the secondary elastic constant κ in Eq. (21) must be change to 3−4v from (3−v)/(1+v).

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