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Direct design of truss bridges using advanced analysis

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Abstract. This paper presents a new design method of truss bridges using advanced analysis. In this approach, separate member capacity checks encompassed by the specification equations are not required because the stability of separate members and the structure as a whole can be treated rigorously for the determination of the maximum strength of the structures. The method is developed and refined by modifications to the conventional elastic-plastic hinge method. Verification studies are carried out by comparing with the plastic-zone solutions. The load-deflection behavior of the truss shows a good agreement between the plastic-zone analysis. A case study is provided for a truss bridge. Member sizes determined by the proposed method are compared with those determined by the conventional method. It is concluded that the proposed method is suitable for adoption in practice.

Key words: steel design; advanced analysis; *K*-factor; truss bridge; nonlinear analysis.

1. Introduction

In the current engineering practice, the interaction between the structural system and its members is represented by the effective length factor. The effective length method generally provides a good design of framed structures. However, despite its popular use in the past and present as a basis for design, the approach has its major limitations.

The first of these is that it does not give an accurate indication of the factor against failure, because it does not consider the interaction of strength and stability between the member and structural system in a direct manner. It is well-recognized fact that the actual failure mode of the structural system often does not have any resemblance whatsoever to the elastic buckling mode of the structural system that is the basis for the determination of the effective length factor K. The second and perhaps the most serious limitation is probably the rationale of the current two-stage process in design: elastic analysis is used to determine the forces acting on each member of a structural system, whereas inelastic analysis is used to determine the strength of each member treated as an isolated member. There is no verification of the compatibility between the isolated member and the member as part of a frame. The individual member strength equations as specified in specifications are unconcerned with system compatibility. As a result, there is no explicit guarantee that all members will sustain their design loads under the geometric configuration imposed by the frame work. The other limitations of the effective length method include the difficulty to

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compute K-factor, which is not user-friendly for a computer based design, and the inability of the method to predict the actual strength of a framed member, among many others (Chen 1997).

With the development of computer technology, two aspects, the stability of separate members, and the stability of the structure as a whole, can be treated rigorously for the determination of the maximum strength of the structures. This design approach is marked in Fig. 1 as the direct analysis and design method. The development of the direct approach to design is called "Advanced Analysis" or more specifically, "Nonlinear Inelastic Analysis". In this direct approach, there is no need to compute the effective length factor, since separate member capacity checks encompassed by the specification equations are not required. With the current available computing technology with advancement in computer hardware and software, it is feasible to employ advanced analysis techniques for direct frame design. This method has been considered impractical for design office use in the past. Fig. 1 compares the K-factor approach and the direct approach (Chen and Kim 1997). The purpose of this paper is to present a practical, direct method of rigidly jointed truss bridges design, using advanced analysis, that will produce almost identical member sizes as those of the LRFD method.

2. Direct design using advanced analysis

In order to overcome the difficulties of the K-factor approach, "advanced analysis" (nonlinear inelastic analysis) should be carried out in a direct manner. In the plastic-zone method, one of the advanced analyses, frame members are discretized into several finite elements, and the cross-section of each finite element is further subdivided into many fibers. Although the plastic-zone solution is known as the "exact solution", it is not ready yet to be used for practical design purposes. This is because the method is too intensive in computation and costly due to its complexity. The real challenge is making this type of analysis competitive in engineering practice. In the following, a practical solution to the problem by modifying the plastic hinge analysis will be briefly discussed.

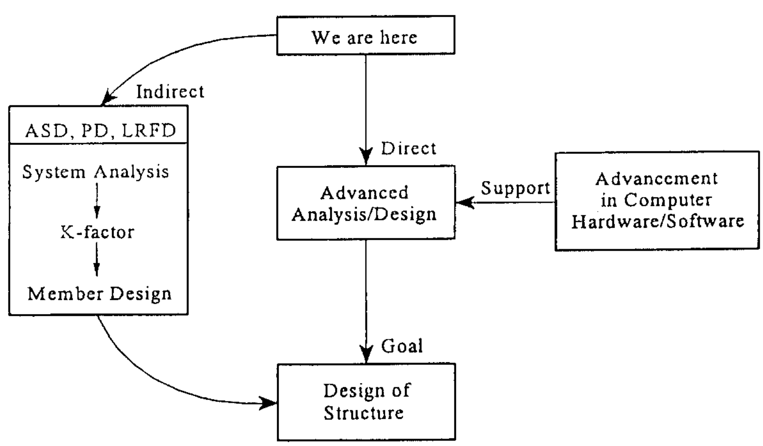


Fig. 1 K-factor and direct approach for steel design

2.1. Geometric nonlinearity

To capture geometric nonlinearity effects, stability functions are recommended since they lead to large savings in modeling and solution efforts by using one or two elements per a member. For a two-dimensional member, the simplified stability functions reported by Chen and Lui (1992) can be used. Considering the prismatic beam-column element, the incremental force-displacement relationship of this element may be written as

$$\begin{bmatrix} \dot{M}_A \\ \dot{M}_B \\ \dot{P} \end{bmatrix} = \frac{EI}{L} \begin{bmatrix} S_1 & S_2 & 0 \\ S_2 & S_1 & 0 \\ 0 & 0 & A/I \end{bmatrix} \begin{bmatrix} \dot{\theta}_A \\ \dot{\theta}_B \\ \dot{e} \end{bmatrix} \tag{1}$$

where S_1, S_2=stability functions, $\dot{M}_A$, $\dot{M}_B$=incremental end moment, $\dot{P}$=incremental axial force, $\dot{\theta}_A$, $\dot{\theta}_B$=incremental joint rotation, $\dot{e}$=incremental axial displacement, A, I, L=area, moment of inertia, and length of beam-column element, and E=modulus of elasticity.

2.2. Cross-section plastic strength

Based on the AASHTO-LRFD bilinear interaction equations (1994), the cross-section plastic strength may be expressed as

$$\frac{P}{\phi_c P_y} + \frac{8}{9}\frac{M}{\phi_f M_p} = 1.0 \qquad \text{for } \frac{P}{\phi_c P_y} \geq 0.2 \tag{2a}$$

$$\frac{P}{2\phi_c P_y} + \frac{M}{\phi_f M_p} = 1.0 \qquad \text{for } \frac{P}{\phi_c P_y} < 0.2 \tag{2b}$$

where P, M=second-order axial force and bending moment, P_y=yield strength, M_p=plastic moment capacity, and ϕ_c, ϕ_f=resistance factors for axial strength and flexural strength. The resistance factors ϕ_c and ϕ_f are built in the analysis program, and are automatically included in the load-carrying capacity of a structural system. The resistance factors are selected as 0.90 for axial strength and 1.0 for flexural strength just as the AASHTO-LRFD Specification (1994) does.

2.3. Residual stresses

The CRC tangent modulus concept is employed to account for the gradual yielding effect due to residual stresses along the length of members under axial loads between two plastic hinges. From Chen and Lui (1992), the CRC E_t is written as (Fig. 2):

$$E_t = 1.0E \quad \text{for } P \leq 0.5P_y \tag{3a}$$

$$E_t = 4\frac{P}{P_y}E\left(1 - \frac{P}{P_y}\right) \quad \text{for } P > 0.5P_y \tag{3b}$$

2.4. Distributed plasticity

The tangent modulus model in Eq. (3a, b) is suitable for $P/P_y > 0.5$, but it is not sufficient to represent the stiffness degradation for cases with small axial forces and large bending moments. A gradual stiffness degradation of a plastic hinge is required to represent the distributed plasticity effects associated with bending actions. The softening plastic hinge model to represent the gradual transition from elastic stiffness to zero stiffness associated with a fully developed plastic hinge can be used. When the softening plastic hinges are present at both ends of an element, the incremental force-displacement relationship may be expressed as (Liew 1992):

$$\begin{bmatrix} \dot{M}_A \\ \dot{M}_B \\ \dot{P} \end{bmatrix} = \frac{E_t I}{L} \begin{bmatrix} \eta_A \left[S_1 - \frac{S_2^2}{S_1}(1-\eta_B)\right] & \eta_A \eta_B S_2 & 0 \\ \eta_A \eta_B S_2 & \eta_B \left[S_1 - \frac{S_2^2}{S_1}(1-\eta_A)\right] & 0 \\ 0 & 0 & A/I \end{bmatrix} \begin{bmatrix} \dot{\theta}_A \\ \dot{\theta}_B \\ \dot{e} \end{bmatrix} \tag{4}$$

where $\dot{M}_A$, $\dot{M}_B$, $\dot{P}$=incremental end moments and axial force, respectively, E_t=tangent modulus, and η_A, η_B=element stiffness parameters. The parameter η represents a gradual stiffness reduction associated with flexure at sections. The partial plastification at cross-sections in the end of elements is denoted by $0 < \eta < 1$. The η may be assumed to vary according to the parabolic expression (Fig. 3):

$$\eta = 1.0 \quad \text{for} \quad \alpha \leq 0.5 \tag{5a}$$

$$\eta = 4\alpha(1-\alpha) \quad \text{for} \quad \alpha > 0.5 \tag{5b}$$

where α is the force-state parameter obtained from the limit state surface corresponding to the element end (Fig. 4).

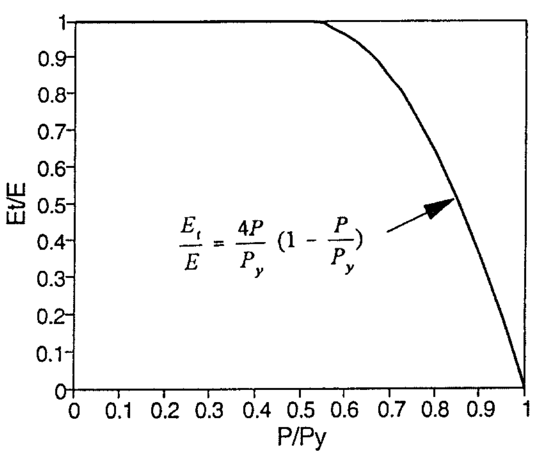


Fig. 2 CRC tangent modulus accounting for residual stresses

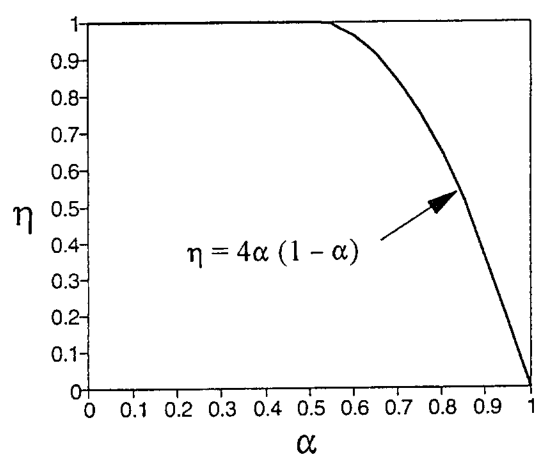


Fig. 3 Parabolic function accounting for gradual yielding due to bending

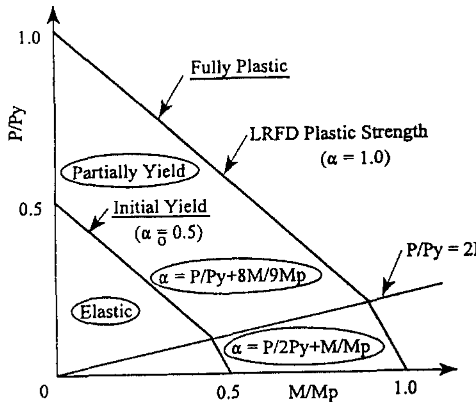


Fig. 4 Force-state parameter

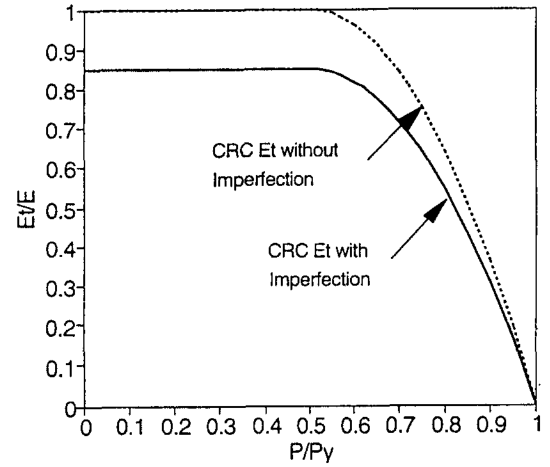


Fig. 5 Reduced tangent modulus

2.5. Geometric imperfection

Geometric imperfection modeling is required to account for fabrication and erection tolerances. The imperfection modeling methods used here are the explicit imperfection, the equivalent notional load, and the reduced tangent modulus models. In the explicit imperfection model, the member out-of-straightness of $L/1{,}000$ is modeled at the mid-length. In the equivalent notional load model, the notional load of 4/1,000 times the axial load is applied transversely on mid-length of the truss member. The reduced tangent modulus method further reduces the tangent modulus E_t (shown in Fig. 5) to account for the degradation of stiffness due to geometric imperfections (Chen and Kim 1997, Kim 1996).

$$E_t' = 4\,\frac{P}{P_y}\,(1-\frac{P}{P_y})\,E\,\xi_i \quad \text{for} \quad P > 0.5P_y \tag{6a}$$

$$E_t' = E\,\xi_i \quad \text{for} \quad P \le 0.5P_y \tag{6b}$$

where E_t'=reduced E_t, and ξ_i=reduction factor for geometric imperfection (=0.85).

Users may choose one of these three models in the advanced analysis. The advantage of the reduced tangent modulus models over the other two models is its convenience and simplicity. It eliminates the inconveniences of both the explicit imperfection and the equivalent notional load methods. Another advantage of this technique is that it does not require guessing what the direction of the geometric imperfections are. Of the three geometric imperfection models, the reduced tangent modulus one has been demonstrated as the simplest, and is, therefore, the one recommended for general use.

3. Verification study

Clarke *et al.* (1992 and 1993) studied a rigidly jointed roof truss subjected to gravity loads between and at the joints of the top chord members as shown in Fig. 6. The chord members of the truss have both axial and flexural forces since it is rigidly jointed. Therefore, this frame

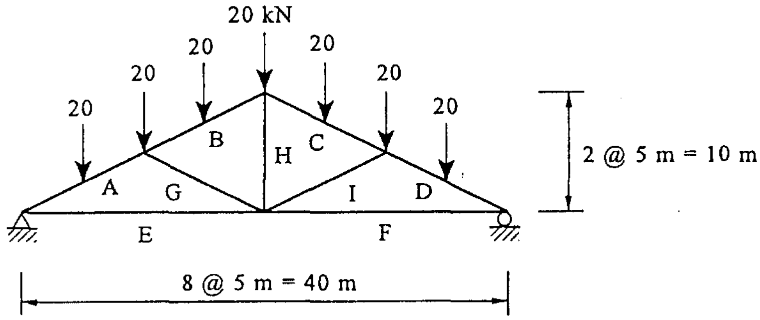


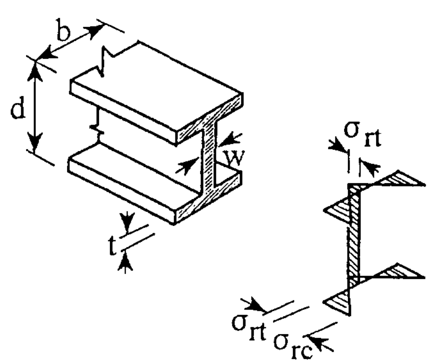
Fig. 6 Configuration of rigidly jointed truss for verification study

Table 1 Section properties of rigid-jointed truss members

Member	Location	Section	A (mm^2)	I_y (mm^4)	S_y (mm^3)	F_y (MPa)
A, D	Top Chord	250UC72.9	9290	38.7×10^6	462×10^3	250
B, C	Top Chord	200UC52.2	6640	17.7×10^6	264×10^3	250
E, F	Bot. Chord	100UC14.8	1890	1.14×10^6	35.3×10^3	260
G, I	Web Chord	150UC23.4	2980	4.03×10^6	80.9×10^3	260
H	Web Chord	100UC14.8	1890	1.14×10^6	35.3×10^3	260

is an atypical frame compared to the usual rectangular portal or multistory frame subjected to gravity loads. Another feature of this truss is that the *W*-sections are oriented such that the weak-axis is subjected to the primary bending actions. The section properties are summarized in Table 1. Residual stresses have greater effect on *W*-section members bent about their weak- than their strong-axis. The residual stress pattern is modeled as shown in Fig. 7.

Clarke *et al.* (1992 and 1993) performed plastic-zone analysis of the truss. The chords were assumed to have a sinusoidal out-of-straightness with a maximum in-plane imperfection of $\delta_0 = L/1000$ at the center. Fig. 8 shows the assumed orientation of the imperfections. Strain hardening was included to the stress-strain curve although its contribution to the frame strength is negligible. Twenty elements were used along each chord length between the joints. The number of the cross-sectional subdivisions used along the flange width was 17, along the web half-depth was 9, through the flange thickness was 3, and through the web thickness was



$$\sigma_{rc} = 0.3\sigma_y$$

$$\sigma_{rt} = \left[\frac{bt}{bt + w(d-2t)}\right]\sigma_{rc}$$

Fig. 7 Residual stress distribution for a wide flange section

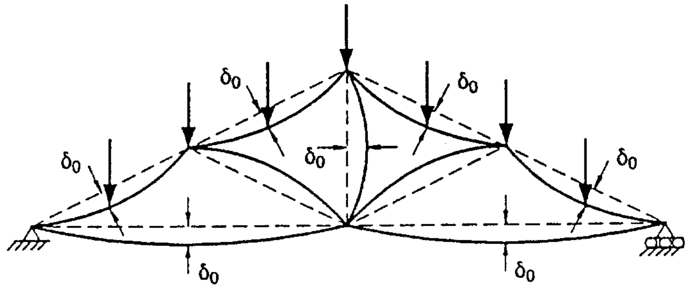


Fig. 8 Geometric imperfection for Clarkes rigidly jointed truss

3, respectively. The load factor determined by the plastic-zone method was 1.93.

In explicit imperfection modeling for the proposed analysis, each chord is divided into two elements with an initial out-of-straightness of $\delta_0=L/1{,}000$ at mid-length as shown in Fig. 9. In the equivalent notional load approach, the notional loads are applied at mid-length of the chords normal to the axis of the chords. In the further reduced tangent modulus approach, the reduced $E_t'=0.85E_t$ is used for all members except the bottom chords subjected to tensile forces. The explicit modeling and the notional load methods predict the same load factor (1.88), and the further reduced tangent modulus method predicts a load factor of 1.84. The proposed methods predict the ultimate load with conservative errors of less than 5%. The load-deflection behavior of the truss shows a good agreement between the plastic zone analysis and the practical advanced analysis as shown in Fig. 10.

4. Analysis and design principles

Fig. 11 shows a flow chart of analysis and design procedure in the use of advanced analysis. Detail principles and procedures are presented as follows.

4.1. Design format

Advanced analysis follows the format of Load and Resistance Factor Design. In AASHTO-

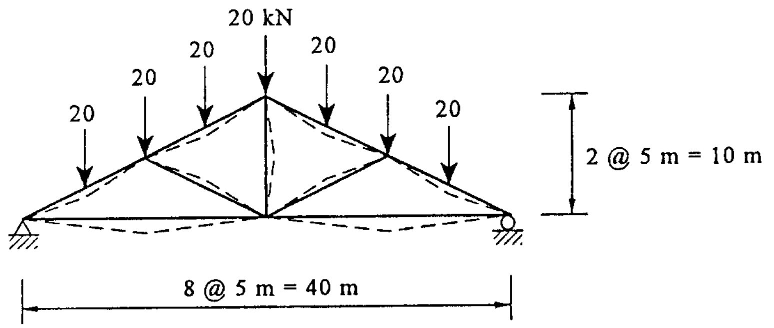


Fig. 9 Explicit imperfection modeling of the rigidly jointed truss

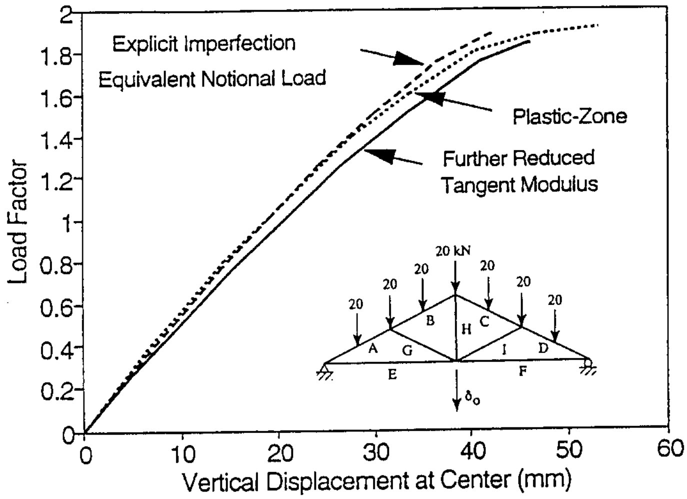


Fig. 10 Comparison of load-displacement relationships of the rigidly jointed truss

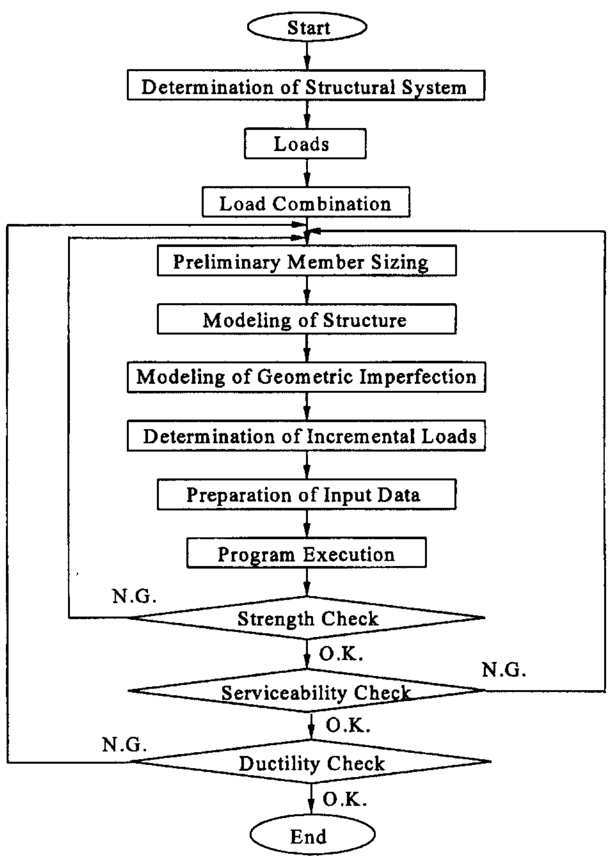


Fig. 11 Analysis and design procedure

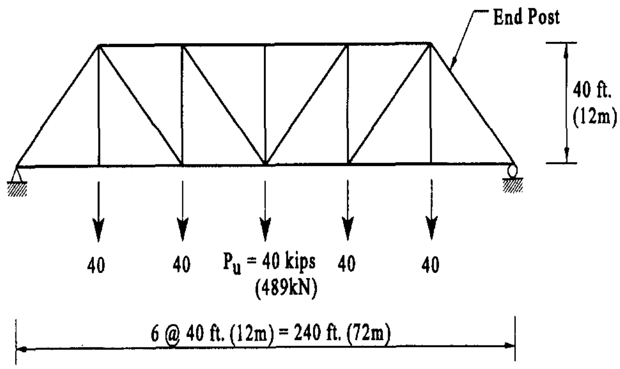


Fig. 12 Configuration and load condition of Pratt truss

LRFD, the factored load effect does not exceed the factored nominal resistance of structure. Two kinds of factors are used: one is applied to loads, the other to resistances. The load and resistance factor design has the format

$$\eta\sum\gamma_i Q_i \leq \phi R_n \tag{7}$$

where R_n=nominal resistance of the structural member, Q_i=force effect, ϕ=resistance factor, γ_i= load factor corresponding to Q_i, η=a factor relating to ductility, redundancy, and operational importance.

The main difference between current LRFD methods and advanced analysis methods is that the right side of Eq. (7), (ϕR_n) in the LRFD method is the resistance or strength of the component of a structural system, but in the advanced analysis method, it represents the resistance or the load-carrying capacity of the whole structural system. In the advanced analysis method, the load-carrying capacity is obtained from applying incremental loads until a structural system reaches its strength limit state such as yielding or buckling. The left-hand side of Eq. (7), ($\eta\Sigma\gamma_i Q_i$) represents the member forces in the LRFD method, whereas the applied load on the structural system in the advanced analysis method.

4.2. Modeling consideration

4.2.1. Sections

The AASHTO-LRFD Specification uses only one column curve for rolled and welded sections of W, WT, and HP shapes, pipe, and structural tubing. The Specification also uses same interaction equations for doubly and singly symmetric members including W, WT, and HP shapes, pipe and structural tubing, even though the interaction equations were developed on the basis of W shapes by Kanchanalai (1977).

The proposed analysis was developed by calibration with the LRFD column curve and interaction equations. To this end, it is concluded that the proposed methods can be used for various rolled and welded sections including W, WT, and HP shapes, pipe, and structural tubing without further modifications.

4.2.2. Structural members

An important consideration in making this advanced analysis practical is the required number of elements for a member in order to predict realistically the behavior of frames. A sensitivity study of advanced analysis has been performed on the required number of elements (Chen and Kim 1997). Two-element model adequately predicts the strength of a truss member. To model a parabolic out-of-straightness in the member, two-element model with a maximum initial deflection at the mid-height of a member adequately captures imperfection effects. It is concluded that practical advanced analysis is computationally efficient.

4.2.3. Geometric imperfection

The reduced tangent modulus model is recommended and the reduction factor of 0.85 can be coded in the computer program.

4.2.4. Load

1) Proportional loading

In the advanced analysis, the gravity and lateral loads should be applied simultaneously, since it does not account for unloading. As a result, the method under-predicts the strength of frames subjected to sequential loads, large gravity loads first and then lateral loads. It is, however, justified for the practical design since the development of the LRFD interaction equations was also based on strength curves subjected to simultaneous loading (AASHTO 1994) and the current LRFD elastic analysis uses the proportional loading rather than the sequential loading.

2) Incremental loading

It is necessary, in an advanced analysis, to input each increment load (not the total loads) to trace nonlinear load-displacement behavior. The incremental loading process can be achieved by scaling down the combined factored loads by a number between 10 and 50. For a highly redundant structure, dividing by about 10 is recommended and for a nearly statically determinate structure, the incremental load may be factored down by 50. One may choose a number between 10 and 50 to reflect the redundancy of a particular structure. Since a highly redundant structure has the potential to form many plastic hinges and the applied load increment is automatically reduced as new plastic hinges form, the larger incremental load (i.e., the smaller scaling number) may be used.

The simple incremental solution method is implemented for practical use. The errors in using the simple incremental solution method are minimized by automatically scaling the incremental loads, when changes in the element stiffness parameter exceed a predefined tolerance.

4.3. Design consideration

4.3.1. Load-carrying capacity

The elastic analysis method does not capture the inelastic redistribution of internal forces throughout a structural system, since the first-order forces, even with the δ_b and δ_s factors, account for the second-order geometric effect but not the inelastic redistributions of internal forces. The method may provide a conservative estimation of the ultimate load-carrying capacity, but advanced analysis directly considers moment redistribution due to material yielding and thus allows smaller member sizes to be selected. This is particularly beneficial in highly indeterminate steel frames. Because consideration at moment redistribution may not always be desirable, the two approaches (including and excluding inelastic moment redistribution) can be used. First, the load-carrying capacity, including the effect of inelastic moment redistribution, is obtained from the final loading step (limit state) given by the computer program. Secondly, the load-carrying capacity without the inelastic moment redistribution is obtained by extracting that force sustained when the first plastic hinge formed. Generally, advanced analysis predicts the same member size as the LRFD method when moment redistribution is not considered.

4.3.2. Serviceability limit

The most common parameter affecting the design serviceability of steel bridge is the deflection. The deflection limits are specified by the code. The advanced analysis does not set any specific limitations of serviceability, but the usual serviceability requirements are used. Service live load deflections may be limited to $L/800$ where L is the span length of a truss bridge. At service load state, no plastic hinges are permitted anywhere in the structure to avoid permanent deformation under service loads.

4.3.3. Ductility requirement

Adequate rotation capacity is required for members to develop their full plastic moment capacity. This is achieved when members are adequately braced and their cross-sections are compact. The limits for lateral unbraced lengths and compact sections are explicitly defined in AASHTO-LRFD (1994).

5. Design example

Fig. 12 shows a Pratt truss used for medium spans. A square structural tube of A46 will be used for all compression members such as the upper chords and vertical members, and a *W*-section of A36 will be used for all tension members such as the lower chords and diagonals. Two elements for a truss member and the reduced tangent modulus model are used. The increment loads of 5.5 kips (24.5kN) are applied on joints using the scaling factor 20.

The load-carrying capacity of the truss bridge with the tube ($10\times10\times5/8$") and the W-section (W10$\times$49) is 116.5 kips (518kN) at joints, and is slightly larger than the applied loads of 110 kips (489kN). The important advantage of the advanced analysis is that the separate member capacity checks are not necessary. The vertical deflection at mid-span is 2. 14" (54.4mm) corresponding to the service live load of 50 kips (222kN) at joints. The deflection slope is $L/1{,}343$ satisfying the limit L/800. The advanced analysis predicts the smaller tube ($10\times10\times5/8$") than the AASHTO-LRFD method ($12\times12\times1/2$") because the proposed one takes advantage of the inelastic moment redistribution. The two methods predict the same W-section (W10$\times$49).

6. Conclusions

A new design method of truss bridges using advanced analysis is first summarized, and some concluding remarks are then made.

1) The proposed method can practically account for all key factors influencing behavior of a truss bridge: gradual yielding associated with flexure; residual stresses; geometric nonlinearity;

and geometric imperfections.

2) The proposed method is adequate in assessing the strengths when compared with the exact plastic-zone solutions. The maximum unconservative errors are no more than 5%.

3) The proposed method does not require tedious separate member capacity checks, including the calculations of K-factor, and thus it is time-effective in design of truss bridges.

4) The proposed method overcomes the difficulties due to incompatibility between the elastic global analysis and the limit state member design in the conventional LRFD method.

5) The proposed method can account for an inelastic moment redistribution and thus may allow some reduction of steel weight, especially for highly indeterminate truss bridges.

Since the proposed method strikes a balance between the requirement for realistic representation of actual behavior of a truss bridge and the requirement for simplicity in use, it is recommended for general use.

Acknowledgements

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References

AASHTO (1994), *LRFD Bridge Design Specifications*, American Association of State Highway and Transportation Officials, First Edition, Washington D.C.

Chen, W.F. (1997), "Structural stability: from theory to practice", *Structural Engineering Report No. CE-STR-97-6*, School of Civil Engineering, Purdue University, West Lafayette, IN.

Chen, W.F. and Kim, S.E. (1997), *LRFD Steel Design Using Advanced Analysis*, CRC Press, Boca Raton, FL, 441.

Chen, W.F. and Lui, E.M. (1992), *Stability Design of Steel Frames*, CRC Press, Boca Raton, FL, 380.

Clarke, M.J., Bridge, R.Q., Hancock, G.J., and Trahair, N.S. (1992), "Advanced analysis of steel building frames", *J. Constr. Steel Res.*, **23**(1-3), 1-29.

Clarke, M.J., Bridge, R.Q., Hancock, G.J., and Trahair, N.S. (1993), "Australian trends in the plastic analysis and design of steel building frames, In plastic hinge based methods for advanced analysis and design of steel frames: An assessment of the state-of-the-art, White", D. W. And Chen, W. F., Eds., Structural Stability Research Council, Lehigh University, Bethlehem, PA, 65-93.

Kanchannalai, T. (1977), "The design and behavior of beam-columns in unbraced steel frames", AISI Project No. 189, Report No. 2, Civil Engineering/Structures Research Lab., University of Texas, Austin, TX, 300.

Kim, S.E. (1996), "Practical advanced analysis for steel frame design", Ph.D. Thesis, School of Civil Engineering, Purdue University, West Lafayette, IN, 271.

Liew, J.Y.R. (1992), "Advanced analysis for frame design", Ph.D. Thesis, School of Civil Engineering, Purdue University, West Lafayette, IN, 392.