Structural Engineering and Mechanics, Vol. 6, No. 4 (1998) 427-437
DOI: http://dx.doi.org/10.12989/sem.1998.6.4.427

Stiffness values and static analysis of flat plate structures

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Abstract. Flat plate constructions are structural systems which are directly placed on columns without any beams. Various solution methods have been introduced for the solution of flat plate structures under horizontal and vertical loads. In most of these solution methods, models comprising of one column and one plate have been studied. In other solutions, however, co-behavior of two reciprocal columns has been investigated. In this study, interrelations of all the columns on one storey have been examined. At the end of the study structure consisting of nine columns and four plates has been chosen as a model. Then unit moment has been successively applied to each of these columns and unit moments carried over the other columns have been found. By working out solutions for plates and columns varying in ratio, carry-over factors have been found and these factors given in tables. In addition, fixed-end moment factors on the columns arising due to vertical load were also calculated. Then citing slope-deflection equations to which these results could be applied, some examples of moment and horizontal equilibrium equations have been given.

Key words: flat plate; equivalent frame.

1. Introduction

Flat plate structure in which plates are places directly on columns without any beams can generally be applied when a smooth ceiling is intended, and they are preferred because of the large amount of time spent for formwork for the structures with beams (Timeshenko 1959).

For the solutions of flat plate structures under lateral and vertical loads various methods have been applied and codes about that have been introduced. It was thought that it might be appropriate in these solutions to analyze the structure by dividing into frames but it was observed that the frame would not behave as it had been thought because of the fact that the widths of the plates to replace the beams in these frames are larger than the widths of columns (Frazer 1983, 1984). For this reason, some studies have been done in an attempt to determine a width of frame that would show a real frame behavior, in other words, to determine the effective width of plate (Khan 1964, Pecknold 1975, Vanderbild 1979).

In this study, however, attempt was made to investigate carry-over relationships between columns depending on plate and column sizes in flat plate structures, and fixed-end moments occuring due to the vertical loads (Kafrawy and Hartley 1984, Sharan, Clyde and Turcke 1978, Elias and Georgiadis 1979). The solutions of the system are obtained by using a computer program which is based on finite element method technique for plate bending problems (Ünlüoğlu 1986).

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2. Assumptions and choosing a model based on

Fig. 1 illustrates a plan for a flat plate structure resting on corner, side and interior columns. In this system our examinations were based upon four types of columns. In the plan presented in Fig. 1, columns on the corners are nomenclatured as type A, those on the side perpendicular to the lateral force direction as type B, the ones on the side parallel to the direction of force as type C, and the ones inside as type D columns.

In this system, unit moment was applied to columns A, B, C and D successively and the distribution of this moment to adjacent and nonadjacent columns was investigated. In solutions realized by using computer programs, it was observed that significant amount of moment was carried over to all the columns adjacent to the column on which unit moment was applied but ignorable amount of moment was carried over to nonadjacent ones. Only 0.5-1% of the unit moment is carried over to the nonadjacent columns. In this case, if it is accepted that unit moment is not carried over to nonadjacent columns, system can be taken as a model comprising of nine columns and four plates as given in Fig. 2. Making use of symmetry, it is also possible to solve the model as a 1/4 system.

On the other hand, in solution for vertical load, vertical load was successively applied to each plate in flat plate structure in Fig. 1 and fixed-end moments arising from this load has been investigated. Between the fixed-end moments occuring in A, B, C and D type columns due to the load applied to corner, side, and inner plates, a difference of 5% at most has occured. For simplicity, vertical loading was applied only on the corner plates and fixed-end moments occuring on A, B, C and D due to this loading were investigated. Solutions for this application were obtained by taking 1/4 of the structure in Fig. 1 symmetrically and loading the corner plate with vertical load.

Another assumption in these models is the discription of the columns in the structure. Columns have been discribed by increasing the thickness of the elements in areas where the columns are present. Since the rigidity of plates increase proportionally by cubic degree of thickness, enough rigidity is obtained in the column area.

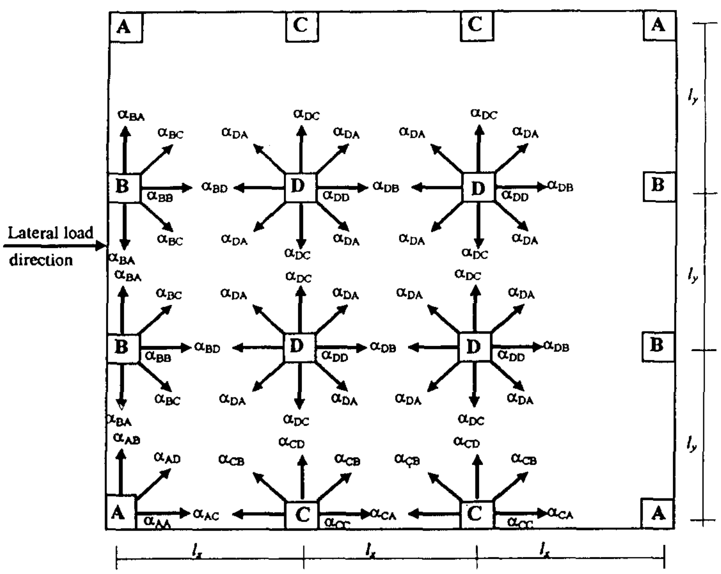

Fig. 1 Plan of the flat plate system

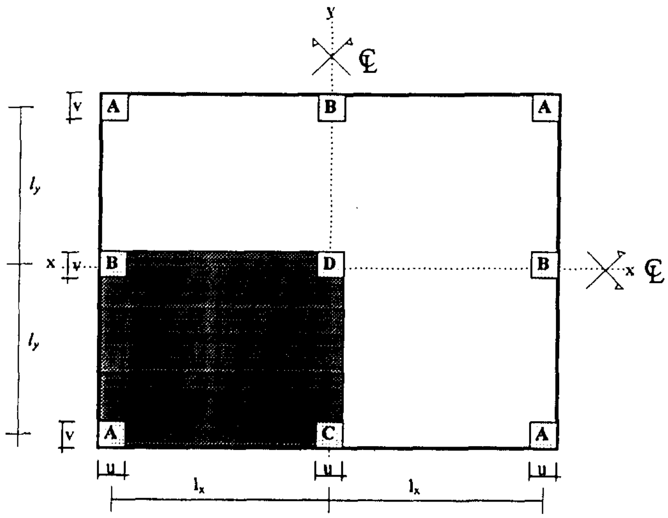

Fig. 2 Flat plate model which is applied to the solutions

3. Investigation with different sizes of plate and column

Making use of symmetry, solutions for both structure with unit moment application and for the structure with vertical load application have been realized on a 1/4 structure. In these solutions, l_x length of plates was held constant while l_y length was changed. This change was taken as l_y / l_x=0.5-0.75-1.00-1.25-1.50 and for each l_y / l_x ratio nine different solutions were obtained as the combination 0.05-0.10-0.15 values of u / l_x and v / l_y, which are the ratios of column lengths to the sides of plate in parallel direction to it. For each l_y / l_x ratio, the ratio of column length to plate lengths are shown on Table 1.

4. Unit moment application on columns and the evaluation of the moments

Making use of the moment carried over from a column on to which unit moment was applied to another and let for free rotation, α moment carry-over factors were found as follows.

If in a fixed-end beam with uniform cross section, as seen in Fig. 3, end moments occuring at the ends 1 and 2 due to the rotation of end 1 as much as ψ_1 are taken as follows:

$$M_{12} = 4 \frac{EJ}{l} \psi_1 \tag{1}$$

Table 1 Various ratios for column and plate lengths

	l_y / l_x=0.50-0.75-1.00-1.25-1.50								
u / l_x	0.05	0.05	0.05	0.10	0.10	0.10	0.15	0.15	0.15
v / l_y	0.05	0.10	0.15	0.05	0.10	0.15	0.05	0.10	0.15

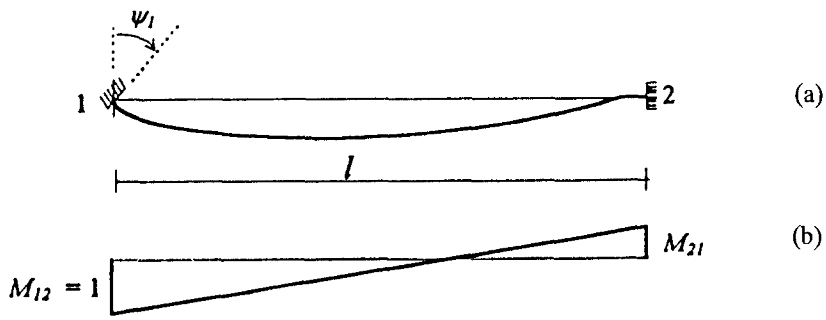

Fig. 3 Fixed-end moments of the beam

$$M_{21} = 2 \frac{EJ}{l} \psi_1 \tag{2}$$

α carry-over factors of fixed-end beam shown in Fig. 3 are the factors of 4 and 2 in the expression (1) and (2).

Solutions of the flat plate structures observed in the current study were done by using the same beam analogy, and moment carry-over factors (α) were determined. This was done by applying a unit moment to the axis of the column, and then the angle of rotation, the moments carried to the columns around this column were established, finally carry-over factors of the moment transfer to the adjacent columns were calculated.

Similar to the expression (1) and (2), carry-over factor can be given with the expression:

$$\alpha = \frac{M \cdot l_x}{E \cdot J \cdot \psi} \tag{3}$$

as equivalent of l_x plate length, and l beam span. Where J is the moment of inertia of the plate with the width of l_y.

If unit moment (M_B=1) is applied to column B in Fig. 4, moment as much as M_A, M_C, and M_D would occur in columns A, C and D, and a rotation of ψ would occur in column B. Here making use of the expression (3), α_{BB} factor of column B can be calculated as follows:

$$\alpha_{BB} = \frac{M_B \cdot l_x}{E \dfrac{l_y \cdot h^3}{12} \psi_B} = \frac{12 \cdot M_B}{E \cdot h^3 \cdot \psi_B \cdot l_y / l_x} \tag{4}$$

where h is the thickness of plate, M_B is the unit moment applied to column B, ψ_B is the rotation in column B due to unit moment, l_x and l_y are the dimensions of the flat plate in the direction of x and y axes respectively, E is the modulus of elasticity of the material.

Similarly using moments of M_A, M_C, and M_D carry-over factors α_{BA}, α_{BC} and α_{BD} can be obtained as follows:

$$\alpha_{BA} = \frac{12 \cdot M_A}{E \cdot h^3 \cdot \psi_B \cdot l_y / l_x}, \qquad \alpha_{BC} = \frac{12 \cdot M_C}{E \cdot h^3 \cdot \psi_B \cdot l_y / l_x}, \qquad \alpha_{BD} = \frac{12 \cdot M_D}{E \cdot h^3 \cdot \psi_B \cdot l_y / l_x} \tag{5}$$

Unit moments were applied to columns A, C, and D and using similarly the carry-over moments passing to other columns, α carry-over moments were calculated (Ünlüoğlu 1986). α

carry-over factors for ratios of l_y/l_x=0.50, 1.00, and 1.50 are given in Tables 2, 3, and 4.

5. Vertical load application on a plate and evaluation of fixed-end moments

In this system chosen for vertical loads, all the displacements were locked and vertical loading was applied on the corner plate. Under this vertical load, fixed-end moments occuring in columns A, B, C, and D were found as ($\overline{M}_{Ax}$, $\overline{M}_{Ay}$, $\overline{M}_{Bx}$, $\overline{M}_{By}$, $\overline{M}_{Cx}$, $\overline{M}_{Cy}$, $\overline{M}_{Dx}$, $\overline{M}_{Dy}$). Using these values and considering distributed vertical plate loads for widths of $l_x/2$ for $\overline{M}_x$ moments, and $l_y/2$ for $\overline{M}_y$ moments, the following equations were written:

$$\overline{M}_x = \frac{q \frac{l_x}{2} l_y^2}{\beta_x}, \qquad \overline{M}_y = \frac{q \frac{l_y}{2} l_x^2}{\beta_y} \tag{6}$$

Table 2 α carry-over factors for ratio of l_y/l_x=0.50

l_y/l_x	0.50								
u/l_x	0.05			0.10			0.15		
v/l_y	0.05	0.10	0.15	0.05	0.10	0.15	0.05	0.10	0.15
α_{AA}	2.35	2.70	3.03	3.17	3.61	4.03	4.06	4.61	5.16
α_{BB}	5.04	5.46	5.87	6.86	7.42	7.95	8.78	9.53	10.23
α_{CC}	4.29	4.98	5.64	5.55	6.40	7.21	6.92	7.97	8.98
α_{DD}	9.44	10.23	10.99	12.36	13.34	14.29	15.38	16.63	17.84
$\alpha_{AB}=\alpha_{BA}$	−0.68	−0.80	−0.94	−1.14	−1.34	−1.54	−1.70	−1.97	−2.26
$\alpha_{AC}=\alpha_{CA}$	0.59	0.70	0.81	0.85	0.98	1.11	1.33	1.30	1.46
$\alpha_{AD}=\alpha_{DA}$	0.24	0.27	0.29	0.30	0.33	0.35	0.34	0.36	0.39
$\alpha_{BC}=\alpha_{CB}$	0.24	0.27	0.29	0.30	0.33	0.35	0.34	0.36	0.38
$\alpha_{BD}=\alpha_{DB}$	0.89	0.96	1.03	1.31	1.40	1.46	1.78	1.88	1.96
$\alpha_{CD}=\alpha_{DC}$	−1.04	−1.27	−1.50	−1.60	−1.93	−2.27	−2.25	−2.71	−3.18

Table 3 α carry-over factors for ratio of l_y/l_x=1.00

l_y/l_x	1.00								
u/l_x	0.05			0.10			0.15		
v/l_y	0.05	0.10	0.15	0.05	0.10	0.15	0.05	0.10	0.15
α_{AA}	1.34	1.61	1.88	1.70	2.02	2.32	2.08	2.45	2.81
α_{BB}	2.64	2.92	3.18	3.37	3.69	3.99	4.11	4.49	4.85
α_{CC}	2.55	3.10	3.63	3.13	3.77	4.39	3.77	4.53	5.25
α_{DD}	5.17	5.69	6.17	6.36	6.97	7.54	7.63	8.34	9.01
$\alpha_{AB}=\alpha_{BA}$	−0.09	−0.12	−0.15	−0.14	−0.18	−0.22	−0.20	−0.25	−0.30
$\alpha_{AC}=\alpha_{CA}$	0.43	0.55	0.67	0.60	0.76	0.91	0.81	1.00	1.18
$\alpha_{AD}=\alpha_{DA}$	0.07	0.08	0.10	0.10	0.12	0.13	0.12	0.14	0.16
$\alpha_{BC}=\alpha_{CB}$	0.07	0.08	0.10	0.10	0.12	0.13	0.12	0.14	0.16
$\alpha_{BD}=\alpha_{DB}$	0.69	0.80	0.90	1.01	1.14	1.27	1.37	1.54	1.70
$\alpha_{CD}=\alpha_{DC}$	−0.13	−0.17	−0.21	−0.17	−0.22	−0.28	−0.22	−0.28	−0.36

Table 4 α carry-over factors for ratio of l_y / l_x=1.50

l_y / l_x	1.50								
u / l_x	0.05			0.10			0.15		
v / l_y	0.05	0.10	0.15	0.05	0.10	0.15	0.05	0.10	0.15
α_{AA}	1.04	1.29	1.54	1.28	1.58	1.86	1.55	1.89	2.22
α_{BB}	1.94	2.18	2.41	2.41	2.69	2.96	2.91	3.23	3.55
α_{CC}	2.01	2.52	3.02	2.44	3.01	3.58	2.87	3.57	4.23
α_{DD}	3.85	4.29	4.72	4.62	5.15	5.66	5.46	6.09	6.69
$\alpha_{AB}=\alpha_{BA}$	−0.02	−0.03	−0.04	−0.03	−0.04	−0.05	−0.05	−0.06	−0.07
$\alpha_{AC}=\alpha_{CA}$	0.37	0.49	0.61	0.50	0.66	0.81	0.66	0.85	1.04
$\alpha_{AD}=\alpha_{DA}$	0.02	0.02	0.03	0.02	0.03	0.04	0.04	0.05	0.06
$\alpha_{BC}=\alpha_{CB}$	0.02	0.02	0.03	0.02	0.03	0.04	0.04	0.05	0.06
$\alpha_{BD}=\alpha_{DB}$	0.59	0.70	0.81	0.82	0.96	1.10	1.10	1.28	1.45
$\alpha_{CD}=\alpha_{DC}$	−0.02	−0.03	−0.04	−0.03	−0.04	−0.05	−0.03	−0.05	−0.07

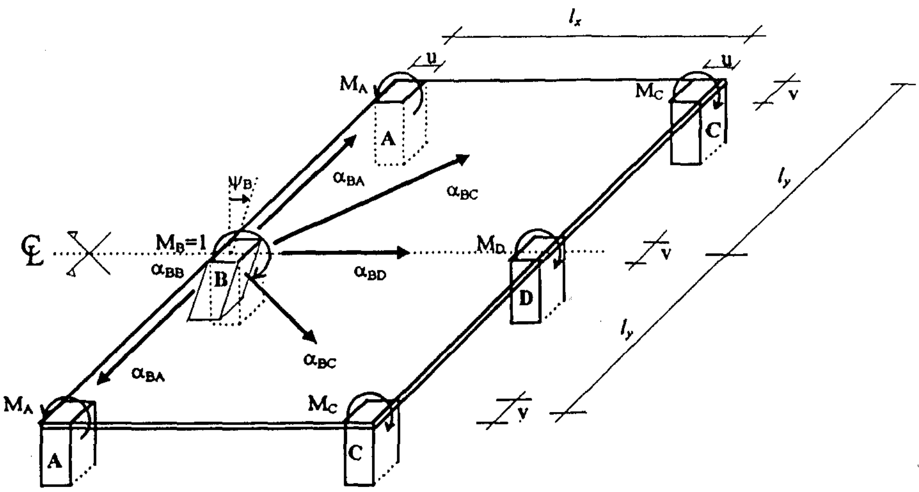

Fig. 4 Applied unit moment and its distribution to the other columns

where q is the distributed vertical load effecting the plate. Also by including l_y / l_x ratios into the expressions (6), the following fixed-end expressions of moment were found:

$$\overline{M}_x = \frac{q \cdot l_y^3}{\beta_x}, \qquad \overline{M}_y = \frac{q \cdot l_x^3}{\beta_y} \tag{7}$$

where β_x and β_y are factors belonging to $\overline{M}_x$, $\overline{M}_y$ fixed-end moments. These factors were given for values of l_y / l_x=0.50, 1.00, and 1.50 in the Tables 5, 6, and 7. So $\overline{M}_x$ and $\overline{M}_y$ moments to occur in columns A, B, C, and D because of the vertical loading on a plate are found.

6. Solution of flat plate structures by using α and β factors

By using α moment carry-over factors and β fixed-end moment factors obtained in this study

Table 5 β fixed-end moment factors for ratio of l_y/l_x=0.50

l_y/l_x	0.50								
u/l_x	0.05			0.10			0.15		
v/l_y	0.05	0.10	0.15	0.05	0.10	0.15	0.05	0.10	0.15
β_{Ax}	16.4	15.3	14.6	14.4	13.5	12.9	13.1	12.4	11.9
β_{Ay}	60.4	55.2	51.1	54.7	50.6	47.4	51.6	48.1	45.4
β_{Bx}	22.9	21.7	20.5	18.8	17.9	17.1	16.3	15.6	15.0
β_{By}	50.2	49.8	49.4	47.0	46.9	47.0	45.6	45.9	46.2
β_{Cx}	14.7	13.9	13.4	14.9	14.0	13.5	14.9	14.1	13.6
β_{Cy}	59.4	54.3	50.4	54.6	50.4	47.1	51.0	47.5	44.7
β_{Dx}	21.4	20.4	19.5	20.5	19.5	18.7	19.5	18.6	17.8
β_{Dy}	48.1	47.9	47.8	46.3	46.2	46.3	44.8	45.0	45.2

Table 6 β fixed-end moment factors for ratio of l_y/l_x=1.00

l_y/l_x	1.00								
u/l_x	0.05			0.10			0.15		
v/l_y	0.05	0.10	0.15	0.05	0.10	0.15	0.05	0.10	0.15
β_{Ax}	32.2	29.0	27.2	28.6	26.2	24.8	26.1	24.2	23.0
β_{Ay}	32.2	28.6	26.1	29.0	26.2	24.2	27.2	24.8	23.0
β_{Bx}	33.9	31.3	29.3	29.8	27.8	26.3	26.9	25.3	24.0
β_{By}	29.5	29.3	29.0	26.9	26.9	26.7	25.4	25.5	25.5
β_{Cx}	29.5	26.9	25.4	29.3	26.9	25.5	29.0	26.7	25.5
β_{Cy}	33.9	29.8	26.9	31.3	27.8	25.3	29.3	26.3	24.0
β_{Dx}	31.2	29.1	27.5	30.4	28.6	27.1	29.7	28.0	26.7
β_{Dy}	31.2	30.4	29.7	29.1	28.6	28.0	27.5	27.1	26.7

Table 7 β fixed-end moment factors for ratio of l_y/l_x=1.50

l_y/l_x	1.00								
u/l_x	0.05			0.10			0.15		
v/l_y	0.05	0.10	0.15	0.05	0.10	0.15	0.05	0.10	0.15
β_{Ax}	46.2	41.6	39.1	41.8	38.1	36.2	38.4	35.5	33.9
β_{Ay}	22.2	19.5	17.7	20.3	18.0	16.5	19.2	17.1	15.8
β_{Bx}	46.0	42.3	39.5	41.5	38.6	36.4	38.1	35.7	33.9
β_{By}	20.1	20.1	20.0	18.6	18.7	18.7	17.7	17.8	17.7
β_{Cx}	40.8	37.4	35.9	40.4	37.5	36.1	40.0	37.4	36.2
β_{Cy}	26.7	22.6	19.9	24.9	21.3	18.9	23.4	20.2	18.0
β_{Dx}	39.8	37.6	36.0	39.4	37.5	36.1	39.0	37.3	36.0
β_{Dy}	25.0	24.0	23.0	23.4	22.6	21.8	22.1	21.4	20.7

for equations of slope-deflection, solutions for flat plate structures under lateral and vertical load can be obtained (Kennedy and Madugule 1990).

As in frame structures, using moments in joints whi^c^h are the junction points of column and

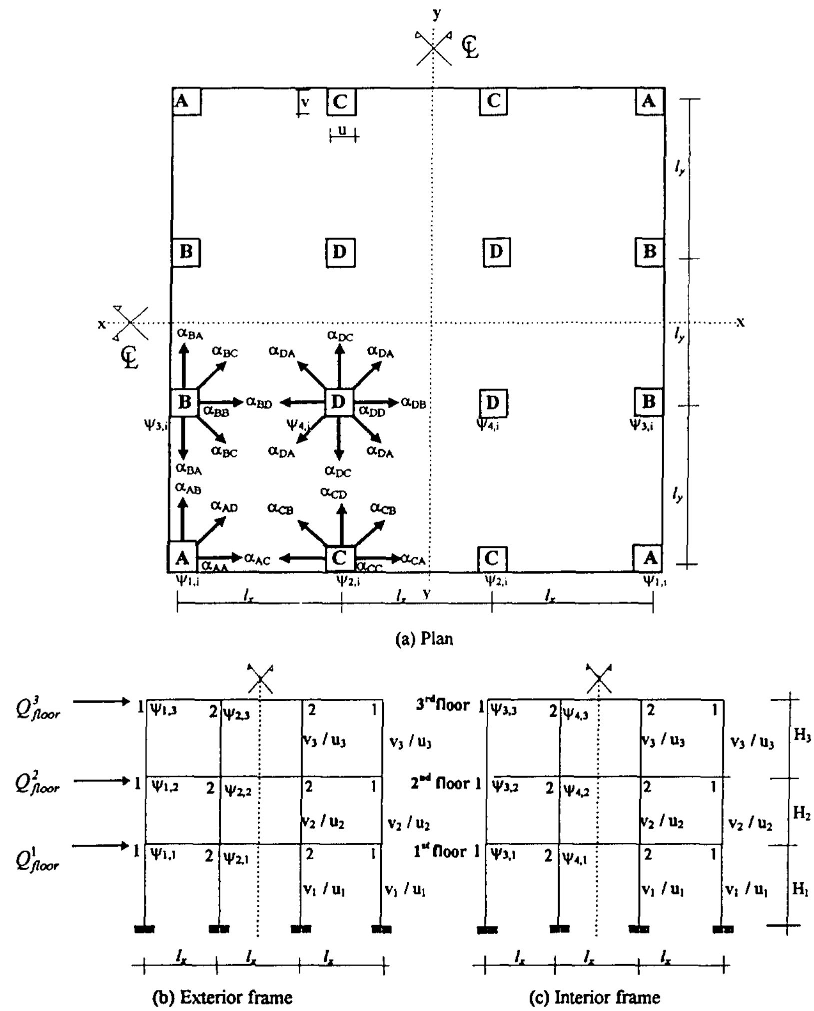

Fig. 5 Flat plate system

plate in flat plate structures, on the other hand, making use of lateral equilibrium equation, α moment carry-over factor and β fixed-end moment factor can be used. These equations were explained on an example.

For this, a structure given as an example shown in Fig. 5, we tried to explain how moment and lateral equilibrium equations are used. Because of the symmetry and antimetry in the system, except for lateral displacement, 4 unknown (rotation of joints) instead of 16 unknown (rotation of joints) can be taken. Rotation angles of 1st floor $\psi_{1,i}$, $\psi_{2,i}$, $\psi_{3,i}$ and $\psi_{4,i}$, where the second subscript shows the number of floors, while the first subscript shows the number of joints of floor i (Fig. 5a and b).

As an example, we can take the equation of the moment equilibrium at joints where column A and plate on floor i meet. The notation to be used in equations are given below:

h_i : slab thickness of floor i

H_i : heigth of floor i

l_x, l_y : side lengths parallel and perpendicular to the lateral force of the plate i

u_i, v_i : column size of floor i

δ_i : relative displacement of floor i
J_i : inertia moment of the plate on floor i
J_{Ci} : inertia moment of the column on floor i
q : uniformly distributed vertical load per unit area of plate
From this, the following equations can be written:

$$J_i = \frac{l_y \cdot h_i^3}{12} \tag{8}$$

$$J_{ci} = \frac{v_i \cdot u_i^3}{12} \tag{9}$$

In equation to be written signs of moment, joint rotation, axial rotation angles and shear forces are accepted as clockwise positive.

If moment equilibrium equation at the point column A on floor i meets the plate:
moments of plate:

$$\alpha_{AA} \frac{E \cdot J_i}{l_x} \psi_{1,i} + \alpha_{CA} \frac{E \cdot J_i}{l_x} \psi_{2,i} + \alpha_{BA} \frac{E \cdot J_i}{l_x} \psi_{3,i} + \alpha_{DA} \frac{E \cdot J_i}{l_x} \psi_{4,i} \tag{10}$$

moments of column (due to the angles of rotation):

$$\frac{4 \cdot E \cdot J_{ci}}{H_i} \psi_{1,i} + \frac{4 \cdot E \cdot J_{ci+1}}{H_{i+1}} \psi_{1,i} + \frac{2 \cdot E \cdot J_{ci}}{H_i} \psi_{1,i-1} + \frac{2 \cdot E \cdot J_{ci+1}}{H_{i+1}} \psi_{1,i+1} \tag{11}$$

moments of column (due to the relative displacements δ):

$$-\frac{6 \cdot E \cdot J_{ci}}{H_i^2} \delta_i - \frac{6 \cdot E \cdot J_{ci+1}}{H_{i+1}^2} \delta_{i+1} \tag{12}$$

fixed-end moments due to vertical loads:

$$\sum \overline{M}_y^A = \frac{q \cdot l_x^3}{\beta_y} \tag{13}$$

expressions of (10), (11), (12), and (13) are added and corrections made, the following equation can be written:

$$E\left[\frac{J_i}{l_x} \alpha_{AA} + 4\left(\frac{J_{ci}}{H_i} + \frac{J_{ci+1}}{H_{i+1}}\right)\right] \psi_{1,i} + \frac{E \cdot J_i}{l_x} (\alpha_{CA} \cdot \psi_{2,i} + \alpha_{BA} \cdot \psi_{3,i} + \alpha_{DA} \cdot \psi_{4,i}) +$$

$$+2 \cdot E\left(\frac{J_{ci}}{H_i} \psi_{1,i-1} + \frac{J_{ci+1}}{H_{i+1}} \psi_{1,i+1}\right) - 6 \cdot E\left(\frac{J_{ci}}{H_i^2} \delta_i + \frac{J_{ci+1}}{H_{i+1}^2} \delta_{i+1}\right) + \sum \overline{M}_y^A = 0 \tag{14}$$

Equation of lateral equilibrium on floor i:
shear forces due to the rotations at the end of the columns on floor i:

$$-6 \frac{E \cdot J_{ci}}{H_i^2} (\psi_{1,i-1} + \psi_{1,i} + \psi_{2,i-1} + \psi_{2,i} + \psi_{3,i-1} + \psi_{3,i} + \psi_{4,i-1} + \psi_{4,i}) \tag{15}$$

Shear forces due to δ displacements of the columns on floor i is:

$$4\left(12\frac{E\cdot J_{ci}}{H_i^3}\delta_i\right) \tag{16}$$

Thus, lateral equilibrium equations of the floor i for 1/4 of the structure can be written as:

$$-6\frac{E\cdot J_{ci}}{H_i^2}(\psi_{1,i-1}+\psi_{1,i}+\psi_{2,i-1}+\psi_{2,i}+\psi_{3,i-1}+\psi_{3,i}+\psi_{4,i-1}+\psi_{4,i})+4\left(12\frac{E\cdot J_{ci}}{H_i^3}\delta_i\right)=\frac{Q_{floor}^i}{4} \tag{17}$$

where Q_{floor}^i is the total of all lateral forces on the floor i.

All moment and lateral equilibrium equations can be formed similarly. From the solution of these equations all the displacements can be obtained and the moments of the plates and columns on joints can be found.

When the moment equilibrium equations are formed, the symmetricity of the system can be used by giving attention to the following points.

If the axis of symmetry is passing through the middle of the plate, the equilibrium equation of the moment can be written as it is in Eq. (14). If the axis of symmetry is passing through the axes of columns, the equation can be formed by taking half values of inertia moments of the columns and plates in the 1/2 structure. In this case, double values of the results of the columns should be taken into consideration as the end moment.

7. Conclusions

From the solutions realized in this study, slab stiffness were determined and from the moments carried over to the other columns due to the unit moment applied column α carry-over factors were obtained. In addition, from vertical loads accumulating on column areas β fixed-end moment factors were determined.

In most of the previous studies, the plates determining the effective width were accepted as a beam. As it is known, when one end of a beam element is let free a moment is applied to this end only half of the moment is carried over to the other end. Whereas, it can be seen from the α carry-over factors obtained from the current study, when the rotation of a column is let free and unit moment is applied to this point, half of the moment is not carried over to the other columns through the plate. Since the applied unit moment is distributed to the other columns through the plate on the sides and across it, the moment carried to the opposing column is more less than half of the applied moment. For this reason it is not convenient to accept the plate as a beam (Saether 1994).

α moment carry-over factors, especially when l_y/l_x ratios are small, are more important for lateral and cross translations.

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