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Progressive failure of symmetric laminates under in-plane shear : I-positive shear

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Abstract. The objective of this present work is to estimate the failure loads, associated maximum transverse displacements, locations and the modes of failure, including the onset of delamination, of thin, square symmetric laminates under the action in-plane positive (+ *ve*) shear load. Two progressive failure analyses, one using the Hashin criterion and the other using a Tensor polynomial criterion, are used in conjunction with finite element method. First order shear deformation theory along with geometric nonlinearity in the von Karman sense have been employed. Variation of failure loads and failure characteristics with five type of lay-ups and three types of boundary conditions has been investigated in detail. It is observed that the maximum difference between failure loads predieted by various criteria depends strongly on the laminate lay-up and the flexural boundary restraint. Laminates with clamped edges are found to be more susceptible to failure due to transverse shear (ensuing from the out of plane bending) and delamination, while those with simply supported edges undergo total collapse at a load slightly higher than the fiber failure load. The investigation on negative (− *ve*) in-plane shear load is in progress and will be communicated as part-II of the present work.

Key words: progressive failure; laminated plate; failure criteria; in-plane positive shear.

1. Introduction

There have been many investigations in literature which deal with the nonlinear/postbuckling response in terms of the load versus lateral displacement of laminated plates. However, there are not many studies available which deal with the failure of composite plastes subjected to in-plane and/or transverse loadings. Early investigations related to the failure of laminated plates were discussed by Turvey (1980a, b, c, 1981, 1982, 1987) in which analytical solutions for the first-ply failure[1] load are presented for both symmetric and antisymmetric laminates considering simply supported boundary conditions and subjected to transverse loads. The finite element procedure for the prediction of linear first-ply failure loads of composite laminates subjected to transverse and in-plane(tensile) loading was presented by Reddy and Pandey (1987). Another study by Reddy & Reddy (1992) used the first order shear deformation theory in the finite element modeling to present the linear and nonlinear failure analysis. Engelstad, *et al.* (1992) investigated the postbuckling response and failure characteristics of graphite-epoxy panels with and without a circular hole, in axial compression, using a progressive damage failure mechanism in conjunction

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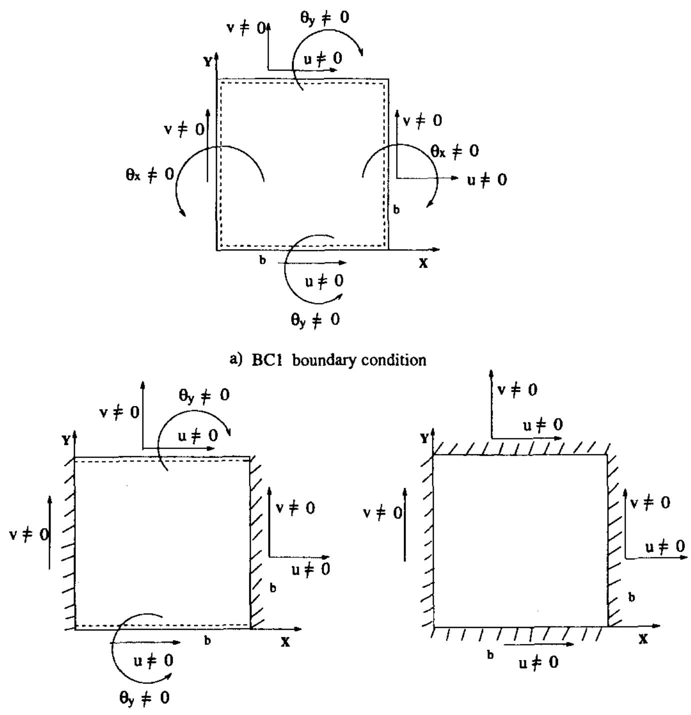
with a 3-D degenerated shell element. Lee and Hyer (1993) studied postbuckling failure characteristics of a square, symmetrically laminated plate with a circular hole, under uni-axial compression, using the maximum stress failure criterion. Kam and Sher (1995) studied the nonlinear behaviour and the first ply failure strength of centrally loaded laminated composite plates with semi-clamped edges using a method developed from the von Karman-Mindlin plate theory in conjunction with the Ritz method. Very recently. Singh, *et al.* (1997) have presented progressive failure results of symmetric laminates subjected to uni-axial compression for various failure criteria.

The present study is, infact, an extension of the work of Singh, *et al.* (1997). It deals with the investigation of the first-ply failure and the subsequent progressive failure (till the ultimate failure) of thin, square and symmetrically laminated composite plates with various lay-ups (Table 1) and boundary conditions(Fig. 1) under the action of positive in-plane shear load. It is to be noted that in-plane restraints at the edges $x=0$, $y=0$, will give rise to normal edge stresses under in-plane shear and, therefore, results should not be compared with those for the pure shear case. However, the results obtained for boundary conditions in Fig. 1 are validated with those of Kosteletos (1992) based on the stress function approach. Two progressive failure procedures are used, one with the Hashin (1980) failure[2] criterion and the other with the Tensor polynomial forms of the maximum stress, maximum strain, Tsai-Hill, Hoffman and Tsai-Wu criteria, with the primary objective to evaluate all these failure criteria. Different material property degradation models for the failed lamina have been considered; the model for the Tensor polynomial criteria is based on Engelstad, *et al.* (1992) whereas the for the Hashin criterion is based on Tsai (1986).

2. Methodology

A special purpose computer program is developed to carry out the present study which is based on the finite element formulation using the first order shear deformation theory with a nine noded Lagrangian element having five degrees of freedom per node. Geometric nonlinearity based on von Karman's assumptions (Fung 1965), which imply that derivatives of in-plane displacements u and v with respect to x, y and z are small, has been incorporated. The nonlinear algebraic equations are solved using the Newton-Raphson technique. The calculation of stresses is done on the nodal points. Due to connectivity of a particular node to various elements, nodal point stresses are calculated taking the average value of stresses at that node from various elements associated with that node. All the six stress components are calculated at each node point. However, to predict the failure of a lamina only five stress components (three in-plane stress and two transverse shear stress) are used in the selected failure criterion. To prediet the onset of delamination, transverse stresses (two shear stress components and one normal stress component) are used in the maximum stress failure criterion. Delamination at any interface is said have occurred when any of the transverse stress components in any of the two layers adjacent to interface becomes equal to or greater than its corresponding strength. The ply failure is said to have occurred when the state of stress at any point within the lamina satisfies the selected failure criterion. The first-ply failure refers to the situation at which one or more than one plies fail first as the load is increased. After the first-ply failure, the progressive failure analysis is car-

[1] The definition of failure is stated in the methodology
[2] Failure criteria are presented, in brief, in the appendix for the sake of ready reference.



a) BC1 boundary condition

b) BC2 boundary condition

c) BC3 boundary condition

Fig. 1 Details of various boundary conditions for full plate.

ried out using two progressive failure procedures described below:

2.1. Tensor polynomial progressive failure procedure

At each load step, nodal point stresses are used in the selected tensor polynomial failure criterion. If failure occurs at a node point in a layer, a reduction in the lamina stiffness is introduced in accordance with the mode of failure which causes the changes in the overall laminated stiffness. Following failure indices are used to determine failure modes.

$$H_1 = F_1 \sigma_1 + F_{11} \sigma_1^2; \qquad H_2 = F_2 \sigma_2 + F_{22} \sigma_2^2$$

$$H_4 = F_{44} \sigma_4^2; \qquad H_5 = F_{55} \sigma_5^2; \qquad H_6 = F_{66} \sigma_6^2$$

Notations in above expressions are defined in the appendix. Failure indices (H_1, $H_2 \cdots$) represent the weightage of various principal stress terms in the failure index (L.H.S. of the failure criterion) of tensor polynomial failure criterion and the largest H_i term is selected to represent the dominant failure causing stress and the corresponding mode of failure. For example, if H_1 is the largest then σ_1 is the failure causing stress and the corresponding failure mode is fiber failure. Similarly, H_2 corresponds to the transverse mode of failure (failure due to in-plane normal stresses transverse to the fiber direction). H_4 to τ_{23} (transverse shear) mode of failure. H_5 to τ_{13} (transverse shear) mode of failure and H_6 to τ_{12} (in-plane shear) mode of failure. After the identification of the mode of failure, the corresponding elastic moduli of the failed lamina is reduced to a negligible value. The fiber mode of failure corresponds to moduli

E_1, ν_{12}; the transverse mode of failure corresponds to E_2, ν_{21}; the transverse shear (τ_{23}) mode of failure corresponds to G_{23}; the transverse shear (τ_{13}) mode of failure corresponds to G_{13} and the in-plane shear mode of failure corresponds to G_{12}, An outline of the steps required is as follows:

(1) After nonlinear iterative convergence is achieved, calculate the stresses at the middle of the each layer and at its interfaces with the adjacent layers at each of the nodal point.
(2) Transform the stresses to that in planes of the material property symmetry.
(3) Compute failure indices, H_1, $H_2 \cdots$
(4) If failure occurs reduce the appropriate lamina moduli and recompute laminate stiffness and restart nonlinear analysis at the same load step.
(5) If no failure occurs, proceed to the next load step.
(6) Final failure is said to have occurred when delamination occurs or when the plate is no longer able to carry any further increase in load due to large transverse deflection.

2.2. Hashin progressive failure procedure

As per the Hashin criterion, failure of the lamina occurs if any of a set of four failure criteria is satisfied (the fiber/the matrix fail in tension/compression, see appendix) at any point in a lamina of the laminate and the corresponding mode of failure is also determined with the possibility of occurrence of two modes (fiber and matrix) of failure, simultaneosly. An outline of the steps required in this procedure is as follows:

- Steps (1) and (2) are the same as with Tensor polynomial criteria.
- If matrix failure occurs, reduce the lamina moduli as per recommendations in (Tsai 1986) which are based on the logic that the equivalent properties of the damaged element will lie somewhere between the properties of the original undamaged element and a property value of zero. Hence the properties of the equivalent damaged element are assumed to be a constant multiple of the properties before degradation. Also, this scheme of stiffness reduction results in the gradual and partial unloading of an element and allow repeated failures of the same element (accumulation of damage in the element) until it is unloaded sufficiently (no more failure occurs).

The stiffness reduction scheme based on the above recommendations is given below:

1. Reduce E_2 to 45% of its original value.
2. Reduce shear modulus to 35% of its original value.
3. Reduce major Poisson's ratio to 30% of its original value.

- If fiber failure occurs reduce E_1 to zero.
- Recompute the laminate stiffness and restart nonlinear analysis at the same load step.
- Steps (5) and (6) are the same as with the Tensor polynomial criteria.

A total of five symmetric lamination schemes are employed to understand the progressive failure. The individual laminates are designated from A to E for identification. The details of the lamination schemes are shown in Table 1. The ply and the interface numbering scheme within the laminate is shown in Fig. 2. Properties of the material of the laminate (Reddy and Reddy, 1992) are presented in Table 2.

In the Table E_1, E_2, E_3 are the principal Young's moduli while G_{12}, G_{13}, G_{23} are the shear moduli corresponding to the planes 1-2, 1-3, and 2-3 respectively and ν_{12}, ν_{13}, ν_{23} are the corresponding Poisson's ratios. In this study a full square plate of width b is used with 25 element mesh, the details of which are shown in Fig. 3a. Three types of flexural boundary conditions,

Table 1 Lamination schemes of symmetric laminates

Lamination scheme	*$(\pm 45/0/90)_{2s}$	$(\pm 45/0_2)_{2s}$	$(\pm 45)_{4s}$	$(\pm 45/0_6)_s$	$(0/90)_{4s}$
Type	A	B	C	D	E

*The terms within the parenthesis are the fiber orientations of the ply-group and digit in the subscript represents the repetition of the ply-group on one side of the mid-plane of the laminate while s represents ths symmetry of the laminate about the mid-plane

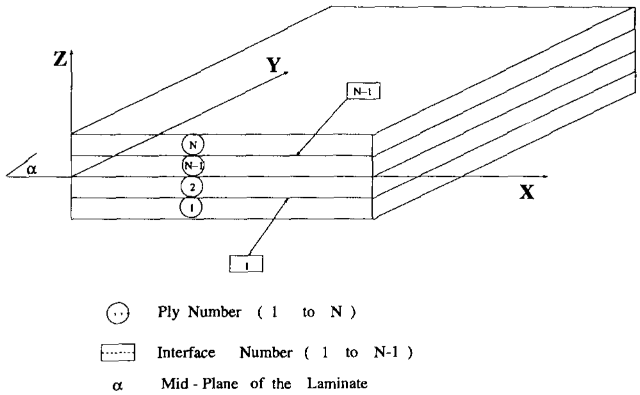


Fig. 2 Ply and interface numbering within the laminate.

Table 2 Material properties of T300/5208 (*pre-peg*)$^{\oplus}$ graphite-epoxy

Mechanical properties	Values	Strength properties	Values
E_1	132.58 Gpa	X_t	1.515 Gpa
E_2	10.8 Gpa	X_c	1.697 Gpa
E_3	10.8 Gpa	$Y_t=Z_t$	43.8 Mpa
$G_{13}=G_{13}$	5.7 Gpa	$Y_c=Z_c$	43.8 Mpa
$\nu_{12}=\nu_{13}$	0.24	R	67.6 Mpa
ν_{23}	0.49	$S=T$	86.9 Mpa

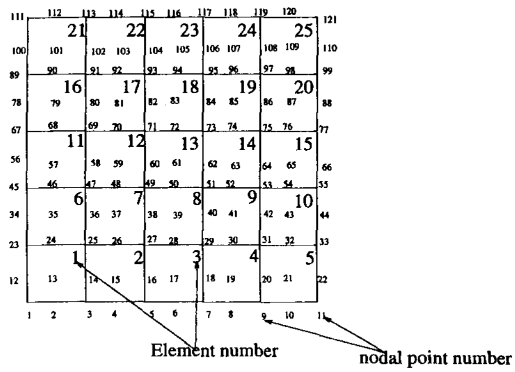
$\oplus$ Pre-Peg refers to the graphite fibers impregnated with epoxy resin and available in tape form

namely BC1, BC2, BC3, have been considered; BC1- refers to a plate with all edges simply supported. BC2- refers to a plate with two logitudinal edges (y=0 and y=b) simply supported and the other two edges clamped and BC3- refers to a plate with all edges clamped. In all the three cases the in-plane boundary conditions (Fig. 1) are identical and the shear load is applied on all the four edges as shown in Fig. 3b. Results for failure loads and corresponding displacements are presented in the following nondimensionalized forms:

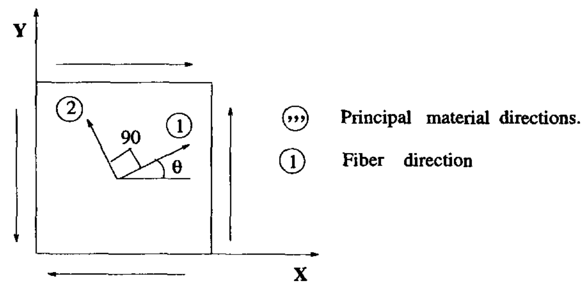
In-plane shear load=$N_{xy}\, b^2/E_2h^3$

Maximum transverse displacement=w_{max}/h

where h is the total thickness of the laminate and N_{xy} is the applied in-plane shear load per unit length.



a) Finite element mesh for full plate.



b) Positive shear

Fig. 3 Finite element mesh for full plate and the sign convention for applied shear load.

3. Results and discussion

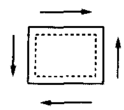
3.1. Laminates with BC1 boundary condition

Progressive failure results are presented in Tables 3a-3e. The first-ply failure loads predicted by various failure criteria are found to differ from one another by a maximum of 8.7 percent for laminate A, 10.5 percent for laminate B, 7.6 percent for laminate C, 11 percent for laminate D, and 30 percent for laminate E, while the ultimate loads are found to differ by a maximum of 24. 4 percent for laminate A, 21.8 percent for laminate B, 22.4 percent for laminate C, 32.6 percent for laminate D, 58 percent for laminate E. The first-ply failure locations and the modes of failure predicted by various failure criteria in the case of laminate A are found to be identical and the same holds good for laminates B, C, and D as well. However, in the case of laminate E, the Hashin criterion and the Tensor polynomial criterion prediet different first failed ply number. Moreover, all criteria prediet the same first failed nodal point number in all laminates. It is observed that the progressive failure starts primarily due to in-plane normal stresses transverse[3] to the fiber direction in all the laminates followed by fiber failure in case of laminates A and B; wide spread in-plane shear mode of failure in case laminate C; transverse shear mode of failure in case of laminate D; and the in-plne shear mode of failure in case of laminate E. It is further observed that the fiber failure occurs at a load closer to the ultimate load in all laminates except

[3] In table transverse mode of failure refers to the matrix failure due to in-plane normal stresses transverse to the fiber direction

Table 3a Progressive failure results of $(\pm 45/0/90)_{2s}$, laminate with BC1 boundary condition

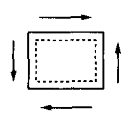
Failure criteria	First-ply failure load	Ultimate failure load	$\left(\frac{w_{man}}{h}\right)^{\oplus}$	$FL^{\dagger}$	$FP^{\ddagger}$	Mode of first-ply failure
Maximum stress	59.38 (0.0)★	116.18 (0.0)	1.31	1	1	Transverse
Maximum strain	56.30 (−5.2)	87.78 (−24.4)	1.16	1	1	Transverse
Tsai-Hill	59.38 (0.0)	116.18 (0.0)	1.31	1	1	Transverse
Tsai-Wu	59.38 (0.0)	116.18 (0.0)	1.31	1	1	Transverse
Hoffman	59.38 (0.0)	116.18 (0.0)	1.31	1	1	Transverse
Hashin	54.21 (−8.7)	102.84 (−11.5)	1.04	1	1	Compressive matrix



⊖ Non-dimensionalized maximum transverse displacement in the plate at the first-ply failure.
† First failed layer number; ‡ First failed nodal point number
★ Percentage difference based on Tsai-Wu criterion

Table 3b Progressive failure results of $(\pm 45/0_2)_{2s}$, laminate with BC1 boundary condition

Failure criteria	First-ply failure load	Ultimate failure load	$\left(\frac{w_{man}}{h}\right)^{\oplus}$	$FL^{\dagger}$	$FP^{\ddagger}$	Mode of first-ply failure
Maximum stress	58.82 (−0.73)★	99.40 (0.43)	1.30	1	1	Transverse
Maximum strain	55.94 (−5.1)	79.60 (−19.6)	1.14	1	1	Transverse
Tsai-Hill	58.52 (−0.73)	105.85 (6.95)	1.30	1	1	Transverse
Tsai-Wu	58.95 (0.0)	98.97 (0.0)	1.33	1	1	Transverse
Hoffman	58.52 (−0.73)	98.97 (0.0)	1.30	1	1	Transverse
Hashin	53.36 (−10.5)	101.123 (2.2)	0.96	1	1	Compressive matrix

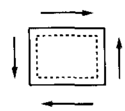


⊕ Non-dimensionalized maximum transverse displacement in the plate at the first-ply failure.
† First failed layer number; ‡ First failed nodal point number
★ Percentage difference based on Tsai-Wu criterion

for the laminate C in which wide spread in-plane shear modes of failure after the first-ply failure leads to the total collapse. The On-set of delamination is not predicted in any laminate except the laminate B in which it takes place just after the fiber failure. Average values of the first-ply failure loads predicted by the various failure criteria are found to be 1.3 times the buckling load for laminate A, 1.24 times for laminate B, 1.5 times for laminate C, 1.03 times for laminate D and 1.2 times for laminate E; the corresponding values for ultimate loads are found to be 2.5 times the buckling load for laminate A, 2.1 times for laminate B, 1.9 times for laminate C, 1.64 times for laminate D and 2.7 times for laminate E. It is to be noted that the maximum strain cri-

Table 3c Progressive failure results of $(\pm 45)_{4s}$, laminate with BC1 boundary condition

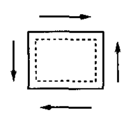
Failure criteria	First-ply failure load	Ultimate failure load	$\left(\frac{w_{man}}{h}\right)^{\oplus}$	$FL^{\dagger}$	$FP^{\ddagger}$	Mode of first-ply failure
Maximum stress	70.14 (3.2)★	89.50 (−10.3)	2.73	1	1	Transverse
Maximum strain	65.40 (−3.8)	86.06 (−13.8)	2.43	1	1	Transverse
Tsai-Hill	70.57 (3.8)	86.92 (−12.9)	2.75	1	1	Transverse
Tsai-Wu	67.99 (0.0)	99.83 (0.0)	2.60	1	1	Transverse
Hoffman	70.14 (3.1)	86.49 (−13.4)	2.73	1	1	Transverse
Hashin	67.13 (−1.3)	77.45 (−22.4)	2.54	1	1	Compressive matrix



⊕ Non-dimensionalized maximum transverse displacement in the plate at the first-ply failure.
† First failed layer number; ‡ First failed nodal point number
★ Percentage difference based on Tsai-Wu criterion

Table 3d Progressive failure results of $(\pm 45/0_6)_s$, laminate with BC1 boundary condition

Failure criteria	First-ply failure load	Ultimate failure load	$\left(\frac{w_{man}}{h}\right)^{\oplus}$	$FL^{\dagger}$	$FP^{\ddagger}$	Mode of first-ply failure
Maximum stress	49.06 (−3.4)★	76.59 (0.0)	0.47	3	1	Transverse
Maximum strain	46.04 (−9.3)	64.54 (−15.7)	0.0°	3	1	Transverse
Tsai-Hill	48.63 (−4.2)	77.03 (0.57)	0.41	3	1	Transverse
Tsai-Wu	50.78 (0.0)	76.59 (0.0)	0.71	3	1	Transverse
Hoffman	49.06 (−3.4)	77.45 (1.12)	0.47	3	1	Transverse
Hashin	45.18 (−11.0)	89.50 (16.9)	0.0•	3	1	Compressive matrix



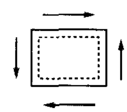
⊕ Non-dimensionalized maximum transverse displacement in the plate at the first-ply failure.
† First failed layer number; ‡ First failed nodal point number
★ Percentage difference based on Tsai-Wu criterion; ○ First-ply failure occurs before buckling
● No transverse deflection occurs till the ultimate load is reached

terion predicts the first-ply failure load of laminate D before the buckling load and no transverse deformation is predicted till the ultimate failure for Hashin criterion. The absolute maximum value of transverse deflections (w_{max}/h) obtained for various failure criteria just before the ultimate load is found to be for laminate B and is equal to 4.39.

The progressive failure responses for three typical laminates are shown in Figs. 4 and 5 using the Tsai-Wu and the Hashin criteria, respectively. In general, the responses with these two criteria are quite different. A drastic difference in the response is observed in the case of cross-ply

Table 3e Progressive failure results of $(0/90)_{4s}$, laminate with BC1 boundary condition

Failure criteria	First-ply failure load	Ultimate failure load	$\left(\frac{w_{man}}{h}\right)^{\oplus}$	$FL^{\dagger}$	$FP^{\ddagger}$	Mode of first-ply failure
Maximum stress	42.17 (−10.9)★	89.07 (0.0)	1.03	1	1	Transverse
Maximum strain	40.44 (−14.6)	76.16 (−14.4)	0.96	1	1	Transverse
Tsai-Hill	41.74 (−11.8)	88.21 (−0.97)	1.01	1	1	Transverse
Tsai-Wu	47.33 (0.0)	89.07 (0.0)	1.22	1	1	Transverse
Hoffman	42.17 (−10.9)	89.07 (0.0)	1.07	1	1	Transverse
Hashin	33.14 (−30.0)	37.43 (−58.0)	0.58	2	1	Compressive matrix



⊕ Non-dimensionalized maximum transverse displacement in the plate at the first-ply failure.
† First failed layer number; ‡ First failed nodal point number
★ Percentage difference based on Tsai-Wu criterion

laminate (E). It is worth noting that he Tsai-Wu criterion predicts a change in configuration in case of laminates C and E before ultimate failure, whereas the same dose not hold good with Hashin criterion. In these figures, kinks in the load-deflection curve represents the failure points during progressive failure. The first kink refers to the first-ply failure of laminates.

3.2. $(\pm 45/0/90)_{2s}$ laminate with different boundary conditions

Progressive failure results of this laminate for three boundary conditions are presented in Tables 4a-4c. First-ply failure loads predicted by various failure criteria differ from one another by a maximum of about 8.7 percent for BC1, 10.1 percent for BC2 and 20.6 percent for BC3, while the corresponding values for ultimate loads are found to be about 24.4 percent for BC1,

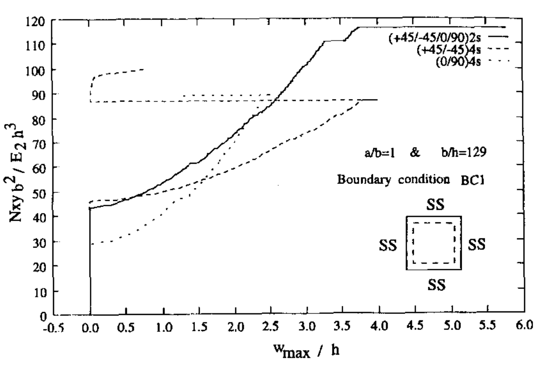


Fig. 4 Progressive failure response using Tsai-Wu criterion for different lay-ups with BC1 boundary.

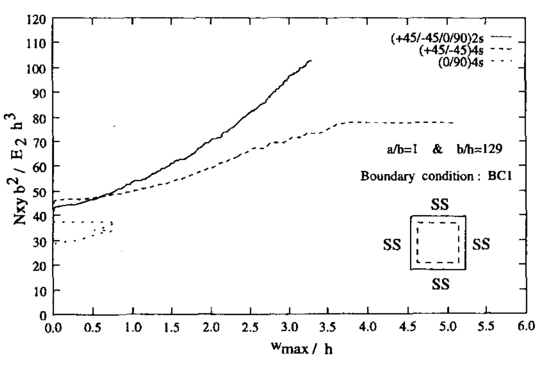
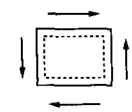


Fig. 5 Progressive failure response using Hashin criterion for different lay-ups with BC1 boundary.

Table 4a Progressive failure results of $(\pm 45/0/90)_{2s}$, laminate with BC1 boundary condition

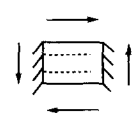
Failure criteria	First-ply failure load	Ultimate failure load	$\left(\frac{w_{man}}{h}\right)^{\oplus}$	$FL^{\dagger}$	$FP^{\ddagger}$	Mode of first-ply failure
Maximum stress	59.38 (0.0)★	116.18 (0.0)	1.31	1	1	Transverse
Maximum strain	56.30 (−5.2)	87.78 (−24.4)	1.16	1	1	Transverse
Tsai-Hill	59.38 (0.0)	116.18 (0.0)	1.31	1	1	Transverse
Tsai-Wu	59.38 (0.0)	116.18 (0.0)	1.31	1	1	Transverse
Hoffman	59.38 (0.0)	116.18 (0.0)	1.31	1	1	Transverse
Hashin	54.21 (−8.7)	102.84 (−11.5)	1.04	1	1	Compressive matrix



⊕ Non-dimensionalized maximum transverse displacement in the plate at the first-ply failure.
† First failed layer number; ‡ First failed nodal point number
★ Percentage difference based on Tsai-Wu criterion

Table 4b Progressive failure results of $(\pm 45/0/90)_{2s}$, laminate with BC2 boundary condition

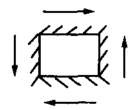
Failure criteria	First-ply failure load	Ultimate failure load	$\left(\frac{w_{man}}{h}\right)^{\oplus}$	$FL^{\dagger}$	$FP^{\ddagger}$	Mode of first-ply failure
Maximum stress	82.19 (2.1)★	87.78 (0.0)	1.28	16	23	Transverse
Maximum strain	78.30 (−2.7)	80.90 (−7.9)	1.12	16	12	Transverse
Tsai-Hill	80.03 (−0.5)	86.49 (−1.5)	1.19	16	12	Transverse
Tsai-Wu	80.47 (0.0)	87.78 (0.0)	1.21	16	23	Transverse
Hoffman	80.03 (−0.5)	84.77 (−3.6)	1.13	16	12	Transverse
Hashin	74.01 (−8.0)	111.88 (27.5)	0.84	16	12	Compressive matrix



⊕ Non-dimensionalized maximum transverse displacement in the plate at the first-ply failure.
† First failed layer number; ‡ First failed nodal point number
★ Percentage difference based on Tsai-Wu criterion

35.4 percent for BC2 and 84.6 percent for BC3. Such a high difference of ultimate loads for BC 3 boundary condition is attributed to the fact that for the Hashin criterion no transverse deflection is observed (see Fig. 7). Figs. 6 and 7 represent the progressive failure response of the laminate using the Tsai-Wu and the Hashin criteria, respectively for different boundary conditions. It is observed the there is a drastic difference in the response of laminate with clamped boundary conditions (BC3), for two failure criteria. Moreover, it is worth noting that the Hashin criterion predicts the first-ply failure before the buckling load is reached and no transverse deflection is observed until the ultimate failure for BC3 boundary condition. Average values of the

Table 4c Progressive failure results of $(\pm 45/0/90)_{2s}$, laminate with BC1 boundary condition



Failure criteria	First-ply failure load	Ultimate failure load	$\left(\frac{w_{man}}{h}\right)^{\oplus}$	$FL^{\dagger}$	$FP^{\ddagger}$	Mode of first-ply failure
Maximum stress	98.11 (1.8)★	100.26 (−0.4)	1.0	16	3	Transverse
Maximum strain	94.24 (−2.2)	95.10 (−5.6)	0.91	16	3	Transverse
Tsai-Hill	95.96 (−0.5)	98.54 (−2.1)	0.98	16	3	Transverse
Tsai-Wu	96.39 (0.0)	100.7 (0.0)	1.0	16	3	Transverse
Hoffman	95.96 (−0.5)	99.40 (−1.3)	0.98	16	3	Transverse
Hashin	78.32 (−18.8)	180.30 (79.0)	0.0°	4	1	Compressive matrix

⊕ Non-dimensionalized maximum transverse displacement in the plate at the first-ply failure.
† First failed layer number; ‡ First failed nodal point number
★ Percentage difference based on Tsai-Wu criterion
○ First-ply failure occurs before buckling load is reached and no transverse deflection is observed till the ultimate load is reached

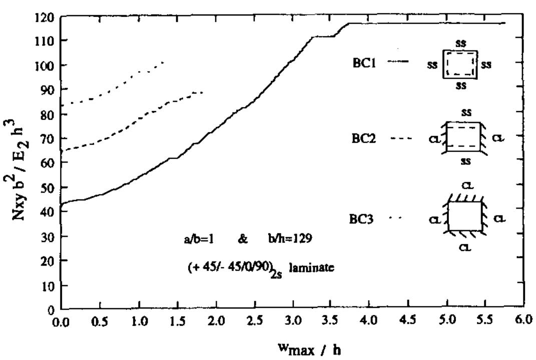


Fig. 6 Progressive failure response of $(\pm 45/0/90)_{2s}$ quasi-isotropic laminate with Tsai-Wu criterion for different boundary conditions.

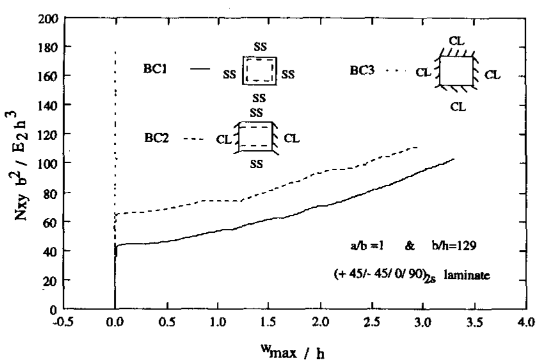


Fig. 7 Progressive failure response of $(\pm 45/0/90)_{2s}$ quasi-isotropic laminate with Hashin criterion for different boundary conditions.

first-ply failure loads predicted by various criteria are found to be 1.3 times the buckling load for BC1, 1.2 times for BC2 and 1.1 times for BC3. Similarly, the average value of ultimate loads are found to be 2.5 times the buckling load for BC1, 1.4 times for BC2 and 1.3 times for BC3. It is observed that the progressive failure gets initiated by martix cracking primarily due to in-plane normal stresses transverse to the fiber direction for all boundary conditions followed by transverse shear mode of failure leading to the onset of delamination for BC2 and BC3 and the fiber failure in the case of BC1 boundary condition.

4. Concluding remarks

Based on the results obtained following conclusions can be made:

1) The maximum percent difference in first-ply failure loads and ultimate loads predieted by various failure criteria occurs for the cross-ply laminate while the corresponding minimum values occur for $(\pm 45)_{4s}$, and $(\pm 45/0_2)_{2s}$ laminates, respectively.
2) The maximum percent difference in first-ply failure loads and ultimate failure loads of $(\pm 45/0/90)_{2s}$ laminate predicted by various failure criteria occurs in the case of clamped boundary condition and the minimum is observed for the simply supported boundary condition.
3) Among all the tensor polynomial criteria, the maximum strain criterion is found to give inconsistent results. Hashin criterion predicts even more inconsistent failure loads, especially for cross-ply laminates.
4) Failure mode at the first-ply failure is associated with localised matrix cracking and occurs primarily due to in-plane normal stresses transverse to the fiber directions irrespective of the laminate lay-ups and boundary conditions.
5) Laminates with two opposite edges or all edges clamped are more susceptible to ultimate failure due to transverse shear and delamination.
6) Maximum value of the transverse displacement (w_{max}/h) just before the ultimate failure is found to be 5.0 irrespective of boundary conditions and types of laminate. Hence, the use of non-linear theory in the von Karman sense is justified for laminates under consideration.
7) It is observed that the fiber breakage precedes very closely the ultimate loads for simply supported laminates and this mode of failure is not predicted in laminates with clamped edges.
8) The first-ply failure loads and the ultimate failure loads for $(+45/-45/0/90)_{2s}$ laminates (with respect to the buckling load) are found to be largest for simply supported laminates.

Appendix

Hashin criterion (1980)

In this criterion four distinect failure modes-tensile matrix, tensile fiber, compressive matrix and compressive fiber are modelled separately, resulting in a piece-wise smooth failure surface. Another unique feature of this failure criterion is that it avoids prediction of multi-axial tensile (compressive) modes in terms of compressive (tensile) failure stresses. The four criteria corresponding to the different failure modes are:

(i) Tensile fiber mode $\sigma_1 > 0.0$

$$\left(\frac{\sigma_1}{X_t}\right)^2 + \frac{1}{T^2}(\sigma_6^2 + \sigma_5^2) = 1 \tag{1}$$

(ii) Tensile matrix mode $\sigma_2 + \sigma_3 > 0.0$

$$\frac{1}{Y_t^2}(\sigma_2 + \sigma_3)^2 + \frac{1}{R^2}(\sigma_4^2 - \sigma_2\sigma_3) + \frac{1}{T^2}(\sigma_6^2 + \sigma_5 \quad = 1 \tag{2}$$

(iii) Compressive fiber mode $\sigma_1 < 0.0$

$$\sigma_1 = X_c \tag{3}$$

(iv) Compressive matrix mode $\sigma_2 + \sigma_3 < 0.0$

$$\frac{1}{Y_c}\left[\left(\frac{Y_c}{2R}\right)^2-1\right](\sigma_2+\sigma_3)+\frac{1}{4R^2}(\sigma_2+\sigma_3)^2+\frac{1}{R^2}(\sigma_4^2-\sigma_2\sigma_3)+\frac{1}{T^2}(\sigma_6^2+\sigma_5^2)=1 \qquad (4)$$

In above expressions σ_1, σ_2, σ_3 are the normal stress components; σ_4, σ_5, σ_6 are the shear stress components in the principal material directions (the subscript 1 refering to the fiber direction); X_t, Y_t are the tensile strengths of the lamina along and transverse to the fiber directions; X_c, Y_c are the corresponding compressive strengths. R and T are the shear strengths of lamina in planes 2-3 and 1-2 respectively. The shear strength in plane 1-3 will be designated by S in the expressions to follow.

Tensor polynomial failure criteria

The most general polynomial failure criterion, as proposed by Tsai (1984), contains linear, quadratic and higher order terms of stresses and is expressed as

$$F_1\sigma_1+F_2\sigma_2+F_3\sigma_3+2F_{12}\sigma_1\sigma_2+2F_{13}\sigma_1\sigma_3+2F_{23}\sigma_2\sigma_3+$$
$$F_{11}\sigma_1^2+F_{22}\sigma_2^2+F_{33}\sigma_3^2+F_{44}\sigma_4^2+F_{55}\sigma_5^2+F_{66}\sigma_6^2+\cdots\geq 1 \qquad (5)$$

wherein F_i, F_{ij} are the strength tensors of the second and fourth rank. It is a phenomenological failure criterion which predicts the imminence of failure but says nothing about the mode of failure. It is the simplest presentation of the failure criterion which can fit the data reasonably well and in view of the significant scatter of the failure test data, cubic of higher order approximations are not employed. Particular cases of the above criterion differ from one another by their strength tensors F_i and F_{ij}. Various degenerate cases of this criterion are given below;

(a) Maximum stress criterion

This criterion is an independent failure criterion. This is based on the fact that there is no interaction between modes of the failure. As per this criterion, the failure is said to have occured when stresses in principal material directions are greater than or equal to the respective strengths. Tensor polynomial form of this criterion can be obtained by using following tensor strength factors in Eq. (5).

$$F_1=\frac{1}{X_t}-\frac{1}{X_c};\quad F_2=\frac{1}{Y_t}-\frac{1}{Y_c};\quad F_3=\frac{1}{Z_t}-\frac{1}{Z_c}$$
$$F_{11}=\frac{1}{X_tX_c};\quad F_{22}=\frac{1}{Y_tY_c};\quad F_{33}=\frac{1}{Z_tZ_c}$$
$$F_{44}=\frac{1}{R^2};\quad F_{55}=\frac{1}{S^2};\quad F_{66}=\frac{1}{T^2}$$
$$F_{12}=-\frac{F_1F_2}{2};\quad F_{13}=-\frac{F_1F_3}{2};\quad F_{23}=-\frac{F_2F_3}{2} \qquad (6)$$

The remaining strength tensor terms are zero.

In the above expressions Z_t, and Z_c are the tensile and the compressive strength. respectively, in the principal direction 3 of the lamina and the other strength terms are the same as described in the Hashin criterion.

(b) Maximum strain criterion

This criterion is similar to the maximum stress criterion except that the strain quantities in the principal material directions are used in the failure criterion inplace of stresses. Tensor polynomial form of this criterion can be obtained by using the following strength tensors in Eq. (5.)

$$F_1 = F_1^A + \frac{S_{12}}{S_{22}} F_2^A + \frac{S_{13}}{S_{33}} F_3^A$$

$$F_2 = \frac{S_{12}}{S_{11}} F_1^A + F_2^A + \frac{S_{23}}{S_{33}} F_3^A$$

$$F_3 = \frac{S_{13}}{S_{11}} F_1^A + \frac{S_{23}}{S_{22}} F_2^A + F_3^A$$

$$F_{11} = \frac{1}{X_t X_c} + \left(\frac{S_{12}}{S_{22}}\right)^2 \frac{1}{Y_t Y_c} + \left(\frac{S_{13}}{S_{33}}\right)^2 \frac{1}{Z_t Z_c} - \frac{S_{13}}{S_{33}} F_1^A F_3^A - \frac{S_{12}}{S_{22}} F_1^A F_2^A - \frac{S_{12} S_{13}}{S_{22} S_{23}} F_2^A F_3^A$$

$$F_{22} = \frac{1}{Y_t Y_c} + \left(\frac{S_{12}}{S_{11}}\right)^2 \frac{1}{X_t X_c} + \left(\frac{S_{23}}{S_{33}}\right)^2 \frac{1}{Z_t Z_c} - \frac{S_{12}}{S_{11}} F_1^A F_2^A - \frac{S_{23}}{S_{33}} F_2^A F_3^A - \frac{S_{12} S_{13}}{S_{11} S_{33}} F_1^A F_3^A$$

$$F_{33} = \frac{1}{Z_t Z_c} + \left(\frac{S_{13}}{S_{11}}\right)^2 \frac{1}{X_t X_c} + \left(\frac{S_{23}}{S_{22}}\right)^2 \frac{1}{Y_t Y_c} - \frac{S_{13}}{S_{11}} F_1^A F_3^A - \frac{S_{23}}{S_{22}} F_2^A F_3^A - \frac{S_{13} S_{23}}{S_{11} S_{22}} F_1^A F_2^A$$

$$F_{44} = \frac{1}{R_2}; \qquad F_{55} = \frac{1}{S^2}; \qquad F_{66} = \frac{1}{T^2}$$

$$F_{12} = \frac{S_{12}}{S_{11}} \frac{1}{X_t X_c} + \frac{S_{12}}{S_{22}} \frac{1}{Y_t Y_c} + \frac{S_{13} S_{23}}{S_{33}^2} \frac{1}{Z_t Z_c} - \frac{1}{2}\left(\frac{S_{12}^2}{S_{11} S_{22}} + 1\right) F_1^A F_2^A - \frac{1}{2}\left(\frac{S_{13} S_{12}}{S_{11} S_{33}} + \frac{S_{23}}{S_{33}}\right) F_1^A F_3^A - \frac{1}{2}\left(\frac{S_{13} S_{23}}{S_{22} S_{33}} + \frac{S_{13}}{S_{33}}\right) F_2^A F_3^A$$

$$F_{13} = \frac{S_{13}}{S_{11}} \frac{1}{X_t X_c} + \frac{S_{13}}{S_{33}} \frac{1}{Z_t Z_c} + \frac{S_{12} S_{23}}{S_{22}^2} \frac{1}{Y_t Y_c} - \frac{1}{2}\left(\frac{S_{13}^2}{S_{11} S_{33}} + 1\right) F_1^A F_2^A - \frac{1}{2}\left(\frac{S_{12} S_{13}}{S_{11} S_{22}} + \frac{S_{23}}{S_{22}}\right) F_1^A F_2^A - \frac{1}{2}\left(\frac{S_{13} S_{23}}{S_{23} S_{33}} + \frac{S_{12}}{S_{22}}\right) F_2^A F_3^A$$

$$F_{23} = \frac{S_{23}}{S_{22}} \frac{1}{Y_t Y_c} + \frac{S_{23}}{S_{33}} \frac{1}{Z_t Z_c} + \frac{S_{12} S_{13}}{S_{11}^2} \frac{1}{X_t X_c} - \frac{1}{2}\left(\frac{S_{23}^2}{S_{22} S_{33}} + 1\right) F_1^A F_2^A - \frac{1}{2}\left(\frac{S_{12} S_{23}}{S_{11} S_{22}} + \frac{S_{13}}{S_{11}}\right) F_1^A F_2^A - \frac{1}{2}\left(\frac{S_{23} S_{13}}{S_{11} S_{33}} + \frac{S_{12}}{S_{11}}\right) F_2^A F_3^A \tag{7}$$

In the above expressions S_{11}, S_{12} etc, are the components of the complicance matrix and F_1^A, F_2^A, F_3^A are the expressions given for F_1, F_2, F_3 in the maximum stress criterion.

(c) Tsai-Hill criterion

Tsai (1965) modified the failure surface equation of Hill (1948) as an interaction criterion and so the modified criterion is called Tsai-Hill criterion. In this criterion a considerable interaction exists among failure strengths of the lamina as against the non-interactive criterion. Tensor polynomial form of this criterion can be obtained by using the following strength tensors in Eq. (5).

$$F_1 = F_2 = F_3 = 0; \quad F_{11} = \frac{1}{X^2}; \quad F_{22} = \frac{1}{Y^2}; \quad F_{33} = \frac{1}{Z^2}$$
$$F_{44} = \frac{1}{R^2}; \quad F_{55} = \frac{1}{S^2}; \quad F_{66} = \frac{1}{T^2}; \quad F_{12} = -\frac{1}{2}\left(\frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}\right)$$
$$F_{13} = -\frac{1}{2}\left(\frac{1}{Z^2} + \frac{1}{X^2} - \frac{1}{Y^2}\right); \quad F_{23} = -\frac{1}{2}\left(\frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2}\right) \tag{8}$$

The values of X, Y, Z are taken as either X_t, Y_t, Z_t or as X_c, Y_c, Z_c, depending upon the sign of σ_1, σ_2, σ_3.

(d) Hoffman criterion

Hoffman (1967) modified the failure criterion of Hill (1948) by adding linear stress terms so as to account for the unequal failure stress in tension and in compression with a single quadratic expression. Tensor polynomial form of this criterion can by obtained by using following strength tensors in Eq. (5).

$$F_1 = \frac{1}{X_t} - \frac{1}{X_c}; \quad F_2 = \frac{1}{Y_t} - \frac{1}{Y_c}; \quad F_3 = \frac{1}{Z_t} - \frac{1}{Z_c}$$
$$F_{11} = \frac{1}{X_t X_c}; \quad F_{22} = \frac{1}{Y_t Y_c}; \quad F_{33} = \frac{1}{Z_t Z_c}$$
$$F_{44} = \frac{1}{R^2}; \quad F_{55} = \frac{1}{S^2}; \quad F_{66} = \frac{1}{T^2}$$
$$F_{12} = -\frac{1}{2}\left(\frac{1}{X_t X_c} + \frac{1}{Y_t Y_c} - \frac{1}{Z_t Z_c}\right)$$
$$F_{13} = -\frac{1}{2}\left(\frac{1}{X_t X_c} + \frac{1}{Z_t Z_c} - \frac{1}{Y_t Y_c}\right)$$
$$F_{23} = -\frac{1}{2}\left(\frac{1}{Z_t Z_c} + \frac{1}{Y_t Y_c} - \frac{1}{X_t X_c}\right) \tag{9}$$

The other strength tensor terms are zero.

(e) Tsai-Wu criterion

This criterion is based on the thesis that the correlation between theory and experiment can be improved by increasing the number of terms in the failure prediction equation. This increase in the curve fitting ability plus the added feature of representing the various strengths in tensor form was presented by Tsai and Wu (1971). In this criterion also considerable interaction exists between various failure strengths and linear tensors. Tensor polynomial form of this criterion can be obtained by using following strength tensors in Eq. (5).

$$
\begin{gathered}
F_1=\frac{1}{X_t}-\frac{1}{X_c}; \quad F_2=\frac{1}{Y_t}-\frac{1}{Y_c}; \quad F_3=\frac{1}{Z_t}-\frac{1}{Z_c} \\
F_{11}=\frac{1}{X_t X_c}; \quad F_{22}=\frac{1}{Y_t Y_c}; \quad F_{33}=\frac{1}{Z_t Z_c} \\
F_{44}=\frac{1}{R^2}; \quad F_{55}=\frac{1}{S^2}; \quad F_{66}=\frac{1}{T^2} \\
F_{12}=-\frac{1}{2}\left(\frac{1}{\sqrt{X_t X_c Y_t Y_c}}\right) \\
F_{13}=-\frac{1}{2}\left(\frac{1}{\sqrt{X_t X_c Z_t Z_c}}\right) \\
F_{23}=-\frac{1}{2}\left(\frac{1}{\sqrt{Y_t Y_c Z_t Z_c}}\right)
\end{gathered}
\tag{10}
$$

All other strength tensor components are zero.

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Notations

E_1	young's modulus of elasticity in the principal material direction-1 (fiber direction)
E_2	young's modulus of elasticity in direction transverse to the fiber direction
E_3	young's modulus of elasticity in principal material direction-3
G_{12}, G_{13}, G_{23}	shear moduli in the planes 1-2, 1-3 and 2-3, respectively
ν_{12}, ν_{13}, ν_{23}	major Poissons's ratios in the plantes 1-2.1-3 and 2-3. respetively
σ_1, σ_2, σ_3	normal stress components in principal material directions 1, 2, and 3 respectively
τ_{12}, τ_{13}, τ_{23}	shear stress components in planes 1-2, 1-3 and 2-3, respectively
X_t, X_c	tensile and compressive strength of lamina in fiber direction, respectively
Y_t, Y_c	tensile and compressive strength of lamina in direction transverse to the fiber direction, respectively
Z_t, Z_c	tensile and compressive strength of lamina in principal material direction-3, respectively
R	shear strength of lamina in plane 2-3
S	shear strength of lamina in plane 1-3
T	shear strength of lamina in plane 1-2
F_i, F_{ij}	strength tensors
S_{ij}	components of compliance matrix