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Structural identification based on incomplete measurements with iterative Kalman filter

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Abstract. Structural parameter evaluation and external force estimation are two important parts of structural health monitoring. But the structural parameter identification with limited input information is still a challenging problem. A new simultaneous identification method in time domain is proposed in this study to identify the structural parameters and evaluate the external force. Each sampling point in the time history of external force is taken as the unknowns in force evaluation. To reduce the number of unknowns for force evaluation the time domain measurements are divided into several windows. In each time window the structural excitation is decomposed by orthogonal polynomials. The time-variant excitation can be represented approximately by the linear combination of these orthogonal bases. Structural parameters and the coefficients of decomposition are added to the state variable to be identified. The extended Kalman filter (EKF) is augmented and selected as the mathematical tool for the implementation of state variable evaluation. The proposed method is validated numerically with simulation studies of a time-invariant linear structure, a hysteretic nonlinear structure and a time-variant linear shear frame, respectively. Results from the simulation studies indicate that the proposed method is capable of identifying the dynamic load and structural parameters fairly accurately. This method could also identify the time-variant and nonlinear structural parameter even with contaminated incomplete measurement.

Keywords: simultaneous identification; time-variant structure; nonlinear structure; extended Kalman filter; orthogonal decomposition

1. Introduction

The infrastructures in service may become aging even deteriorated. For the safety and maintenance purpose, it is important to assess the condition of structures in regular service or subjected to severe environmental excitation. Structural parameter identification as the inverse problem is commonly involved in the structural condition assessment, dynamic performance evaluation, damage detection, semi-active and active structural vibration control of infrastructures.

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Solving the inverse problem is also an optimization process. One of the commonly used objectives is to predict the structural response close enough to the measured response. The dynamic structural response is determined by both the structural parameter and external excitation. The exact information of the dynamic excitation actually contributes to the structural design, condition assessment and structural parameter identification. But, in practice, the time history of the external force is always unknown and difficult to record. Although the simultaneous identification of structural parameter and external force are actually performed (Chen and Li 2004, Law and Ding 2011, Feng *et al*. 2015, Sun and Betti 2013, Sun *et al*. 2015) structural identification with limited input information is still a challenging problem for time-variant structures and nonlinear structures.

The evaluation of structural parameter and external force are two of the most important parts in the health monitoring of infrastructures, which is crucial to the disaster prevention and mitigation. Extensive literature views of health monitoring for infrastructures are provided (Ghanem and Shinozuka 1995, Doebling *et al*. 1998, Housner *et al*. 1997). Although there are extensive investigations related to the topics of structural system identification majorities of them focus on the system identification with known input. The input of infrastructures was always supposed to be measured by force transducer. But, it is impossible to measure all the excitations on a structure directly due to the lack of access to the loading position and the limited number of the force transducers. The external excitation estimation methods are increasingly investigated (Steltzner and Kammer 1999, Inoue 2001, Uhl 2007, Asnachindaa *et al.* 2008).

Recently, the investigations in the field of structural parameter identification with unknown external excitation are actively carried out for linear structures. Chen *et al*. (2004) presented methods to identify structural parameters and input time history from output-only measurements iteratively. Lu and Law (2007) identify the physical parameters and the external excitation of linear structures based on the sensitivities of dynamic responses. Law and Ding (2011) proposed sub-structural condition assessment method for structural damage detection and external force identification of linear structural system. Online identification methods for linear substructures have been developed by Hou and Ou (2013) and the local damages can be accurately identified. Sun and Betti (2013) proposed a simultaneous identification method for linear structure with artificial bee colony algorithm. Lei *et al*. (2014) successfully identified the shear building based on partial output measurements with EKF and least-squares estimator. Li *et al*. (2015) proposed a damage assessment method for the shear connector based on power spectral density transmissibility for linear structural system with a good accuracy. The investigations on the parameter identification or damage detection for time-invariant linear structures introduced above supposed the external force to be unknown time history, which is consistent with the practical engineering.

Structures may be time-variant or even nonlinear when subjected to the severe external excitation, such as earthquake, load of blast or strong wind. Parameters evaluation with unknown external excitation is still a challenging problem for time-variant or nonlinear structures. In the past few decades, numerous methods have been developed for the model updating of time-variant or nonlinear structures, e.g., extended Kalman filter (EKF), unscented Kalman filters (UKF) or ensemble techniques with Monte Carlo methods (Chatzi and Smyth 2009, Ching *et al*. 2006). For the infrastructures, EKF is an alternative tool for damage detection although it is not the most accurate tool to deal with severe nonlinear system identification. It is noted that the strong nonlinear cracks or damages in infrastructures can usually be observed by visualization. Therefore, EKF can be used to identify the slight or medium nonlinear cracks. The extended recursive least-squares algorithms were considered for the identification of system parameter and the tracking of a chirped sinusoid with measurement noise (Haykin *et al*. 1997). The normal modes are studied considering the nonlinearity of the structural system with the time-frequency analysis (Kerschen *et al*. 2009). Various types of adaptive tracking techniques have been developed for the online identification of nonlinear hysteretic structures. Sequential weighted least-squares support vector machine technique has been presented to quantify the occurrence of the structural damage (Tang *et al*. 2006). Adaptive EKF is proposed and applied in structural parameter identification with measured ground motion (Yang *et al*. 2006). These methods do not require a priori knowledge of the occurrence time of the anomalies. Hence, these methods could be applied to conduct the online structural condition assessment for time-variant or nonlinear structures. More recently, Monte Carlo methods have been proposed and applied to the nonlinear structural identification. The Monte Carlo methods can identify the state variable with a large number of samples but the identification process are always time consuming. Most existing methods introduced above for parameter identification may require the structural response of displacement or velocity which is difficult to measure in practical engineering. The parameter identification methods for time-variant or nonlinear structures only with acceleration measurements are rarely seen in previous investigation.

In this paper, a new method is proposed to evaluate the structural parameters and external excitation with the orthogonal decomposition iteratively. The structural parameters to be evaluated include stiffness, damping and the parameters of nonlinear model. With this method, the excitation time history can be decomposed with orthogonal polynomials. The coefficients of orthogonal polynomials as well as the structural parameters are taken as state variable to be evaluated. The EKF is augmented and selected as the implementation tool. In case of complicate excitation, it may require a number of orthogonal polynomials to represent the excitation. In this condition, a large number of unknowns including the structural response, decomposition coefficients and the structural parameters cannot be identified efficiently. An improvement with moving time window is proposed for this simultaneous identification method. The measured structural response is divided into several time windows and the length of the discrete data is equal to the order of the orthogonal polynomial. This set is to ensure the polynomial can represent the time history of the excitation accurately. In the identification process, the time window moves from the beginning to the end of the sampled data. In each window, an iteration procedure (Hoshiya and Saito 1984) is applied to identify the structural parameters and decomposition coefficients with better accuracy. The proposed method was validated numerically with the simulation studies of a time-variant linear shear frame, a hysteretic nonlinear shear frame and a time-variant linear shear frame. Results from the numerical simulation studies indicate that the proposed method can be used to identify structural parameters and external excitations effectively based on incomplete contaminated structural responses measurements. This method is available for both linear and nonlinear structures.

2. Equation of motion for structural system

The equation of motion of an N dofs linear structural system subjected to external excitation is

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=\mathbf{LF} \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices of the structural system respectively. **F** is the external force acting on the structure and **L** is the location matrix of external excitation. $\ddot{\mathbf{x}}$, $\dot{\mathbf{x}}$ and $\mathbf{x}$ are the acceleration, velocity and displacement responses of the structural system, respectively. When the structural system is subjected to the earthquake excitation, the equation of motion can be written as follows.

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = -\mathbf{M}\mathbf{L}\ddot{\mathbf{x}}_g \tag{2}$$

where $\ddot{\mathbf{x}}_g$ denotes the acceleration of ground motion. The equation of motion of the linear structural system shown in Eq. (1) can also be expressed in the state space generally as Eq. (3). The system equation of structure subjected to earthquake excitation is similar to Eq. (3) and is not listed in this paper.

$$\dot{\mathbf{Z}}(t) = \mathbf{A}\mathbf{Z}(t) + \mathbf{B}\mathbf{L}\mathbf{F}(t) \tag{3}$$

where $\mathbf{Z} = \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix}$, $\mathbf{A} = \begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 0 \\ \mathbf{M}^{-1} \end{bmatrix}$.

The equation of motion for a hysteretic nonlinear structure subjected to external force can be written as

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{z}(t) = \mathbf{L}\mathbf{F}(t) \tag{4}$$

where $\mathbf{z}(t) = \begin{bmatrix} \mathbf{z}_1(t) & \mathbf{z}_2(t) & \cdots & \mathbf{z}_i(t) \end{bmatrix}^T$ is the hysteretic component vector. Bouc-Wen model is a typical nonlinear model and one of the usually used nonlinear models. It is selected in this paper for the validation of the simultaneous identification. Other nonlinear models can also be chosen according their detailed cases in the simultaneous identification process if necessary, which is similar to the Bouc-Wen model used in this study. The hysteretic component with Bouc-Wen model can be represented as

$$\dot{z}_i = \dot{x}_i - \beta_i |\dot{x}_i| |z_i|^{n_i - 1} z_i - \gamma_i \dot{x}_i |z_i|^{n_i} \tag{5}$$

where subscript i represent the i^{th} story. $\dot{x}_i$ and z_i are respectively the i^{th} story velocity and hysteretic component, β, γ and n are the nonlinear parameter of Bouc-Wen model. The equation of motion shown in Eq. (5) can also be expressed in the state space generally as following equation

$$\dot{\mathbf{Z}}(t) = \begin{bmatrix} \dot{\mathbf{x}}(t) \\ \mathbf{M}^{-1}[\mathbf{L}\mathbf{F}(t) - (\mathbf{C}\mathbf{X}(t) + \mathbf{K}\mathbf{z}(t))] \\ \dot{\mathbf{x}}(t) - \boldsymbol{\beta}|\dot{\mathbf{x}}(t)||\mathbf{z}(t)|^{n-1}\mathbf{z} - \boldsymbol{\gamma}\dot{\mathbf{x}}(t)|\mathbf{z}(t)|^{n} \end{bmatrix} = f(\mathbf{Z}(t), \mathbf{F}(t)) \tag{6}$$

where $f(\mathbf{Z}(t), \mathbf{F}(t))$ is a nonlinear function of **Z** in state space and **Z** is the state variable as $\mathbf{Z}(t)=[\mathbf{x}(t)^{\mathrm{T}}, \dot{\mathbf{x}}(t)^{\mathrm{T}}, \mathbf{z}(t)]^{\mathrm{T}}$. For the linear and nonlinear structures, responses can be recursively calculated by Eq. (4) or Eq. (6) respectively.

3. EKF for structural system identification

For the general case including the linear structure and nonlinear structure, the equation of motion of a structure subjected to external load can be written as

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{F_d}(\dot{\mathbf{x}}(t))+\mathbf{F_r}(\mathbf{x}(t),\mathbf{z}(t))=\mathbf{LF}(t) \tag{7}$$

where $\mathbf{F_d}(\dot{\mathbf{x}}(t))$ denotes the damping force and $\mathbf{F}_r(\mathbf{x}(t))$ is the resistant force. The state variable can be set as Eq. (8)

$$\mathbf{Z}(t)=[\mathbf{x}(t)^{\mathbf{T}},\dot{\mathbf{x}}(t)^{\mathbf{T}},\mathbf{z}(t)^{\mathbf{T}},\boldsymbol{\theta}^{\mathbf{T}}]^{\mathbf{T}} \tag{8}$$

where $\boldsymbol{\theta}=[\theta_1,\theta_2,\cdots,\theta_m]^{\mathrm{T}}$ denotes the unknown parameters of the structure, including damping, stiffness, or nonlinear parameters and m is the number of the unknown parameters. Eq. (7) can be written in discrete state space as

$$\mathbf{Z}_{k+1}=f(\mathbf{Z}_k,\mathbf{F}_k)+\mathbf{w}_k \tag{9}$$

where **Z**, **F**, **w** are the state variable, the external force and the process noise vectors. The observation equation at discrete time steps $t_k=k\Delta t$ can be written as

$$\mathbf{y}_k=h(\mathbf{Z}_k)+\mathbf{v}_k \tag{10}$$

where $\mathbf{y}_\mathrm{k}$ and $\mathbf{v}_\mathrm{k}$ are the measurement response and measurement noise vectors. The Eqs. (9) and (10) constitute a classical system with discrete time evolution of discrete state variables and measurements. Both process noise and measurement noise are assumed to be uncorrelated zero-mean Gaussian random processes. With the EKF, Eqs. (9) and (10) can be linearized as Eqs. (11) and (12).

$$\mathbf{Z}_{k+1}=f(\hat{\mathbf{Z}}_{k|k},\mathbf{F}_k)+\mathbf{A}_{k|k}(\mathbf{Z}-\hat{\mathbf{Z}}_{k|k})+\mathbf{w}_k \tag{11}$$

$$\mathbf{y}_{k+1}=h(\hat{\mathbf{Z}}_{k+1|k})+\mathbf{H}_{k+1|k}(\mathbf{Z}_{k+1}-\hat{\mathbf{Z}}_{k+1|k})+\mathbf{v}_k \tag{12}$$

where $\mathbf{A}_{k|k}$ is the linearized matrix of $f(\hat{\mathbf{Z}}_{k|k},\mathbf{F}_k)$, $\hat{\mathbf{Z}}_{k|k}$ denotes the state estimation at kth time step given $h(\mathbf{Z}_{k+1})$ and $\hat{\mathbf{Z}}_{k+1|k}$ is the state estimation given $h(\mathbf{Z}_k)$, $\mathbf{H}_{k+1|k}$ is the linearized matrix of $h(\hat{\mathbf{Z}}_{k+1|k})$. $\mathbf{A}_{k|k}$ and $\mathbf{H}_{k+1|k}$ are defined as follows

$$\mathbf{A}_{k|k}=[\frac{\partial f}{\partial \mathbf{Z}}]_{\hat{Z}_{k|k}} \tag{13}$$

$$\mathbf{H}_{k+1|k}=[\frac{\partial h}{\partial \mathbf{Z}}]_{\hat{Z}_{k+1|k}} \tag{14}$$

The recursive optimal evaluation for $\mathbf{Z}_k$ at the k th time step is implemented as follows

$$\hat{\mathbf{Z}}_{k+1|k+1}=\hat{\mathbf{Z}}_{k+1|k}+\mathbf{K}_{k+1}[\mathbf{y}_{k+1}-h(\hat{\mathbf{Z}}_{k+1|k})] \tag{15}$$

$$\mathbf{K}_{k+1}=\mathbf{P}_{k+1|k}\mathbf{H}^T_{k+1|k}[\mathbf{H}_{k+1|k}\mathbf{P}_{k+1|k}\mathbf{H}^T_{k+1|k}+\mathbf{R}_{k+1}]^{-1} \tag{16}$$

$$\mathbf{P}_{k+1|k}=\boldsymbol{\Phi}_{k+1|k}\mathbf{P}_{k|k}\boldsymbol{\Phi}^T_{k+1|k}+\mathbf{Q}_k \tag{17}$$

$$\boldsymbol{\Phi}_{k+1|k}\approx\mathbf{I}+\Delta t\times\mathbf{A}_{k|k} \tag{18}$$

$$\mathbf{P}_{k|k}=[\mathbf{I}-\mathbf{K}_k\mathbf{H}_{k|k-1}]\mathbf{P}_{k|k-1}[\mathbf{I}-\mathbf{K}_k\mathbf{H}_{k|k-1}]^T+\mathbf{K}_k\mathbf{R}_k\mathbf{K}_k{}^T \tag{19}$$

where $\mathbf{K}_{k+1}$ is the Kalman gain matrix, $\mathbf{P}_{k+1|k}$ is the error covariance matrix of posteriori state vector $\hat{\mathbf{Z}}_{k+1|k}$, $\mathbf{Q}_k$ is the covariance matrix covariance matrix of process noise at $t=(k+1)\Delta t$. $\mathbf{\Phi}_{k+1|k}$ is the state transition matrix of the linearized system and $\mathbf{I}$ is the identity matrix. $\mathbf{P}_{k|k}$ in Eq. (17) is the error covariance matrix of priori state vector $\hat{\mathbf{Z}}_{k+1|k}$ given by $\mathbf{\Phi}_{k+1|k} = \mathbf{I} + \Delta t \times \mathbf{A}_{k|k}$.

4. Orthogonal decomposition of excitation

The dynamic external force is always difficult to be directly identified with EKF since the force is non-stationary and the amplitude is time-variant. The input is a kind of random process which can be decomposed by standard orthogonal polynomial if the order of the polynomial is high enough (Zhang and Zhu 1996, Zhu and Law 2001). Since the orthogonal polynomials are determined, the input history will be re-constructed as if the orthogonal parameters can be identified. Therefore, the input identification transforms to the polynomial parameters identification based on input orthogonal decomposition method. The total external excitation can be decomposed as follows

$$F(t)=\sum_{i=1}^{N_f}\sum_{m=1}^{N_m} w_m{}^i T^i{}_m(t) \tag{20}$$

where $w_m{}^i$ is the polynomial coefficients of the i^{th} input. $T_m{}^i$ is the m^{th} orthogonal polynomial of the i^{th} input. N_m is the order of input decomposition. N_f is the number of inputs. The orthogonal polynomial $T_m{}^i$ and order of input decomposition N_m can affect the accuracy of input approximation. The order of input decomposition N_m is closely related to input history length and complexity. The orthogonal polynomial $T_m{}^i$ can be selected based on different decomposition methods. Chebyshev decomposition method is one of the generally used decomposition methods. The m th order polynomial T^i_m for the i th external excitation can be written as follows

$$T_m^i = \begin{cases} \dfrac{1}{\sqrt{\pi}} & (m=0) \\ \sqrt{\dfrac{2}{\pi}}\left(\dfrac{2t}{L_{inp}}-1\right) & (m=1) \\ 2\left(\dfrac{2t}{L_{inp}}-1\right)T_m^i(t)-T_{m-1}^i(t) & (m=2,3....,N_m) \end{cases} \tag{21}$$

where L_{inp} is the length of the input history and N_m is the order of decomposition. Based on the Chebyshev standard orthogonal polynomial decomposition, Eq. (7) can be written as follows

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{F}_c(\dot{\mathbf{x}}(t))+\mathbf{F}_s(\mathbf{x}(t))=\sum_{i=1}^{N_f}\sum_{m=1}^{N_m} w_m{}^i T^i{}_m(t) \tag{22}$$

The input can be re-constructed if the polynomial parameters $w_m{}^i$ can be identified.

5. Simultaneous identification of structural parameter and decomposition coefficients

A new time domain simultaneous identification method is proposed in this section based on orthogonal decomposition of excitation and EKF. The time history of the input is firstly decomposed by Chebyshev standard orthogonal polynomial as Eq. (21). Then, the structural parameters and polynomial parameters will be identified by extended Kalman estimator based on structural measurements, as Eq. (15) to Eq. (19).

5.1 Equations in state space for the n-storey structures

For *n*-storey linear shear frame, the equation of motion can be written in state space as Eq. (23)

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \ddot{\mathbf{x}} \\ \dot{\mathbf{k}} \\ \dot{\mathbf{c}} \\ \dot{\mathbf{w}} \end{bmatrix} = \begin{bmatrix} \dot{\mathbf{x}} \\ \mathbf{M}^{-1}[\mathbf{L}\sum_{i=1}^{Nf}\sum_{m=1}^{Nm} w_m{}^i T^i{}_m - (\mathbf{c}\dot{\mathbf{x}} + \mathbf{k}\mathbf{x})] \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \tag{23}$$

where **k** and **c** denote the stiffness and damping of each storey of the shear frame, respectively. And the state vector is defined as

$$\mathbf{Z}(t) = [\mathbf{x}^{\mathbf{T}}, \dot{\mathbf{x}}^{\mathbf{T}}, \mathbf{k}^{\mathbf{T}}, \mathbf{c}^{\mathbf{T}}, \mathbf{w}^{\mathbf{T}}]^{\mathbf{T}} \tag{24}$$

For *n*-storey hysteretic structure the equation of motion are given by

$$\mathbf{M}\ddot{\mathbf{x}}(T) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{z}(t) = \mathbf{L}\left(\sum_{i=1}^{N_f}\sum_{m=1}^{N_m} w_m{}^i T^i{}_m(t)\right) \tag{25}$$

where

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 & 0 & 0 \\ m_1 & m_2 & 0 & 0 \\ \vdots & \vdots & \ddots & 0 \\ m_1 & m_2 & \dots & m_n \end{bmatrix}, \mathbf{C} = \begin{bmatrix} c_1 & -c_2 & 0 & 0 & 0 \\ 0 & c_2 & -c_3 & 0 & 0 \\ 0 & 0 & \ddots & \ddots & 0 \\ 0 & 0 & 0 & c_{n-1} & -c_n \\ 0 & 0 & 0 & 0 & c_n \end{bmatrix}, \mathbf{K} = \begin{bmatrix} k_1 & -k_2 & 0 & 0 & 0 \\ 0 & k_2 & -k_3 & 0 & 0 \\ 0 & 0 & \ddots & \ddots & 0 \\ 0 & 0 & 0 & k_{n-1} & -k_n \\ 0 & 0 & 0 & 0 & k_n \end{bmatrix}$$

$\mathbf{w} = [w_1, w_2 ... w_i]^T$, i=(1,2…N_f) is the polynomial parameter vector. The responses $x_i(t)$ and $z_i(t)$ in Eq. (25) are the inter-storey displacement and hysteretic component of *i* th storey unit respectively. $T^i{}_m$ is the orthogonal polynomial which is given by Eq. (20). The state vector is augmented as

$$\mathbf{Z}(t) = [\mathbf{x}_{1-nst}, \dot{\mathbf{x}}_{1-nst}, \mathbf{z}_{1-nst}, k_{1-nst}, c_{1-nst}, \beta_{1-nst}, \gamma_{1-nst}, w_{1-N_m}]^{\mathbf{T}} \tag{26}$$

where the subscript 1 to *nst* denotes the first storey to the *n*-th storey of the structure and the subscript of 1~N_m represent the order of the orthogonal decomposition coefficients from the first tothe N_m th. Consider the $\dot{k}_{1-m} = \dot{c}_{1-n} = \dot{w}_{1-Nm} = 0$, Eq. (25) can be written in state space as follows

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \ddot{\mathbf{x}} \\ \dot{\mathbf{z}} \\ \dot{\mathbf{k}} \\ \dot{\mathbf{c}} \\ \dot{\boldsymbol{\beta}} \\ \dot{\boldsymbol{\gamma}} \\ \dot{\mathbf{w}} \end{bmatrix} = \begin{pmatrix} \dot{\mathbf{x}} \\ \mathbf{M^{-1}}[\mathbf{L}\sum_{i=1}^{Nf}\sum_{m=1}^{Nm} w_m{}^i T^i{}_m - (\mathbf{c}\dot{\mathbf{x}} + \mathbf{kz})]] \\ \dot{\mathbf{x}}_\mathbf{i} - \beta_i |\dot{x}_i| |z_i|^{\alpha_i - 1} z_i - \gamma_i \dot{x}_i |z_i|^{\alpha_i} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{27}$$

When the acceleration response is taken as measurement, the observation equation is nonlinear in state space. The discrete linearized observation matrix can be calculated by Eq. (12).

In general, excitation may be complicate and contain high order frequency component, which needs more coefficients and higher order polynomial to represent. But large number of coefficients in state variable will affect the identification accuracy. An improvement is proposed for this case. The sampled structural response would be divided into several time windows without overlapping. In the first time window, the initial guess of structural parameter and coefficients for decomposition are provided by users while the initial guess for other time windows are set as the identified value of the last previous time window. The order of the orthogonal decomposition is taken as the length of the time history window for accurate identification result. Therefore, there are enough pre-determined polynomials to represent the excitation in each time window. The proposed simultaneous identification method then can be used iteratively for the identification. Convergence is considered achieved in each time window if the following criterion is met

$$\left\| \frac{\mathbf{X}_{i+1} - \mathbf{X}_i}{\mathbf{X}_{i+1}} \right\| \le Tol \tag{28}$$

where i is the number of iteration and *Tol* is set as 10^{-4} in the following study, and $\mathbf{X}$ denotes the all the structural parameter included in the state variable. In the first time window, the initial value can be set as the initial guess while in the following time window the initial value are set as the identification results of the previous time window.

5.2 Implementation procedure of the simultaneous identification

The identification procedure can be implemented as follows.

Step 1: Obtain the mass, damping and stiffness matrices of the initial structural model, which may be inaccurate with model errors or initial structural damage.

Step 2: Determine the order of the Chebyshev standard orthogonal polynomials and decompose structural excitation by Chebyshev standard orthogonal polynomials.

Step 3: Conduct measurement on the structure. The “measured” data for the simulation studies is obtained from the solution of Eq. (4) or Eq. (6).

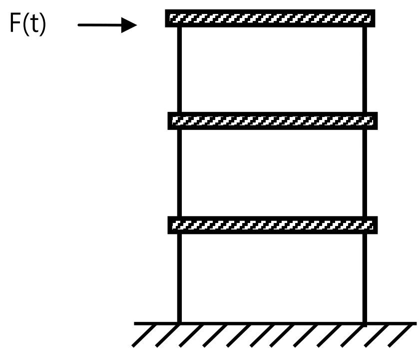
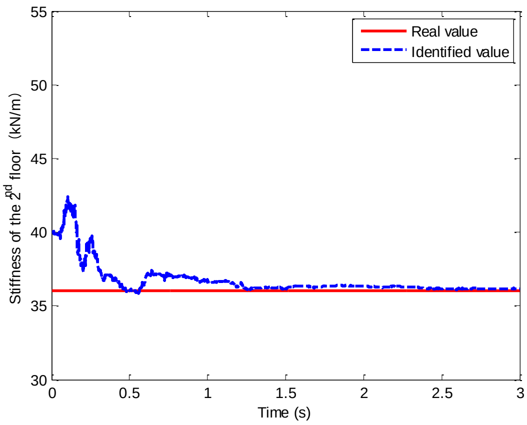
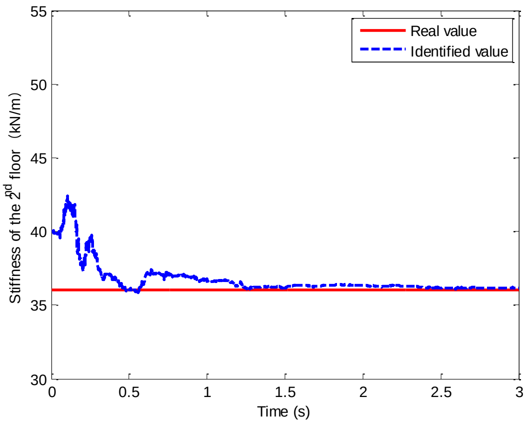


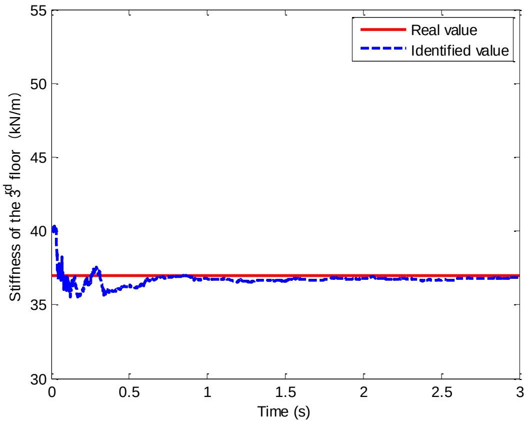
Fig. 1 A three-storey shear frame



(a) Stiffness of the 1st floor



(b) Stiffness of the 2nd floor



(c) Stiffness of the 3rd floor

Fig. 2 Identified stiffness of linear structural system

Step 4: Simultaneously identify structural parameter and polynomial parameters with the iterative EKF algorithm from time window to time window with Eq. (23) to Eq. (28).

6. Numerical simulation studies

In this Section the proposed system identification method is validated numerically by the investigations of three structures: a linear shear frame, a nonlinear 3-storey shear frame and a time-variant linear shear frame as shown in Fig. 1. The structural parameters and external force are simultaneously identified with measured accelerations. The sampling rate is 1000Hz and 3 seconds measurements are collected for the identification in the first two cases. The sampling rate is set as 100Hz and 30 seconds acceleration are measured for the identification in the third case. The length of the time window is set with 30 sampled points in all these three cases.

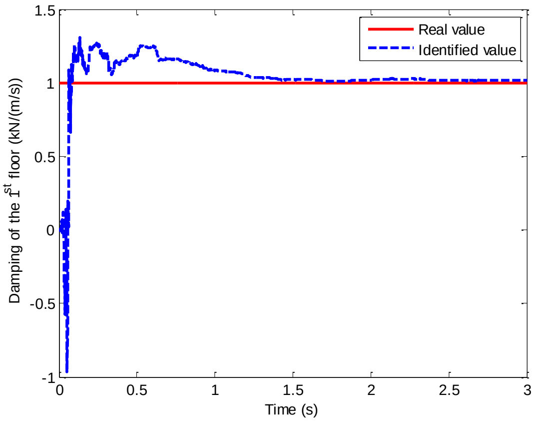
6.1 Case 1-Three-storey linear structure subjected to periodic excitation

A linear three-storey shear frame subjected to external force as $F(t)=\sin(4\pi t)+2\cos(2\pi t)+\sin(5\pi t)$ kN on the top of the floors is studied in this case. The equation of motion with excitation decomposition is shown as Eq. (20). The value of real parameters are $m_1=m_2=m_3=500$kg, $c_1=c_2=c_3=1$ kNs/m, $k_1=38$ kN/m, $k_2=36$ kN/m and $k_3=37$ kN/m. In the identification process, the mass is taken as known constant. The initial guess of the stiffness is 4 0kN/m and the initial damping is taken as zeros. The horizontal accelerations on the first and third floors are measured. The "measured" accelerations for this simulation are obtained from the solution to Eq. (1) with 5% RMS noise.

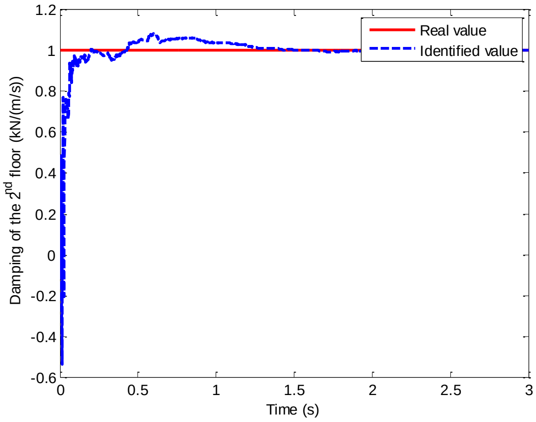
The excitation history is decomposed with thirty-order orthogonal polynomial. The unknown parameters are k_i, c_i, and the decomposition coefficients w_j, (i=1,2,3 and j=1,2…,30). The augmented state vector is expressed as $\mathbf{Z}(t)=[x_{1-3},\dot{x}_{1-3},k_{1-3},c_{1-3},w_{1-30}]^{\mathbf{T}}$. The initial guess of displacement and velocity in state variable are supposed to be zeros. Fig. 2 compares the identified structural stiffness to the real values and the identified damping is shown in Fig. 3. In the beginning of the identification results of stiffness and damping, the fluctuations are a little large partially due to the influence of measurement noise. But the identified results converge to the real value efficiently as shown in Figs. 2 and 3. The identified force for the target linear structural system is shown as Fig. 4. The external force is identified accurately and the fluctuations in the force identification result are not very large as shown in Fig. 4. It is demonstrated that for the linear case, structural parameter and external force can be identified accurately based on the proposed method in this paper.

6.2 Case 2 - Three -storey hysteretic nonlinear structure subjected to periodic force

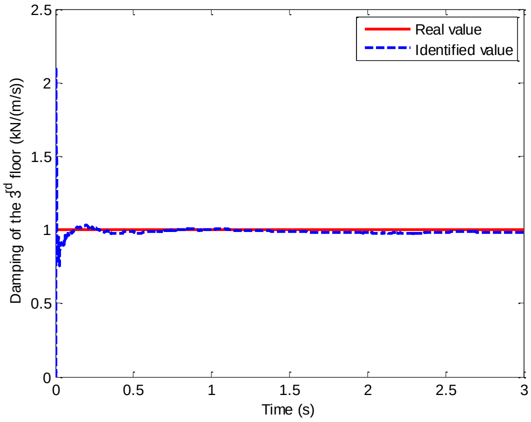
A three-storey hysteretic nonlinear shear frame subject to $F(t)=4\sin(6\pi t)+2\cos(2\pi t)+\sin(4\pi t)$ kN on the top of the floors is investigated. The real value of the parameters are $m_1=m_2=m_3=500$kg, $c_1=c_2=c_3=1$ kNs/m, $k_1=48$ kN/m, $k_2=43$ kN/m, $k_3=40$ kN/m, $\beta_1=\beta_2=\beta_3=4$, $\gamma_1=\gamma_2=\gamma_3=2$ and $n_1=n_2=n_3=2$. The mass and parameter n is also taken as known constant. The other parameters including stiffness, damping and the parameters of β and γ are supposed as the unknowns to be identified. Only the horizontal accelerations on the first and third storey are measured for the structural identification in this case study. The "measured" accelerations for this simulation are obtained from the solution of Eq. (3) with 5% RMS noise. The excitation history is also decomposed with thirty-order orthogonal polynomial. The unknown parameters are k_i, c_i, w_j, (i=1,2,3 and j=1,2…,30), the extended state vector is $\mathbf{Z}(t)=[\mathbf{x}_{1-3},\dot{\mathbf{x}}_{1-3},\mathbf{z}_{1-3},k_{1-3},c_{1-3},\beta_{1-3},\gamma_{1-3},w_{1-30}]^{\mathbf{T}}$,



(a) Damping of the 1st floor



(b) Damping of the 2nd floor



(c) Damping of the 3rd floor

Fig. 3 Identified damping of linear structural system

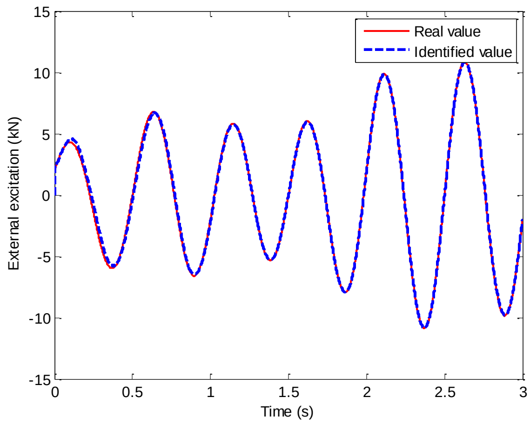
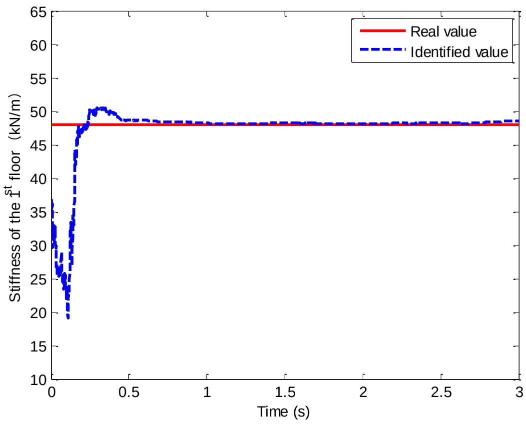
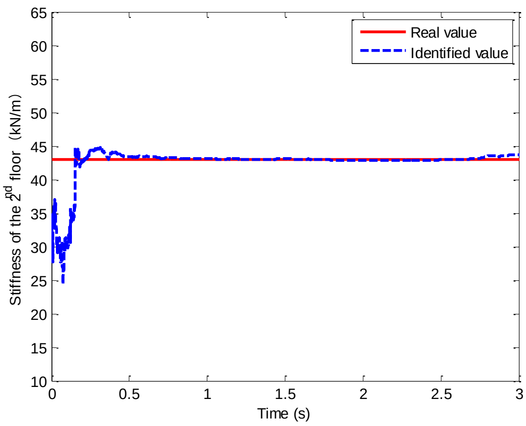


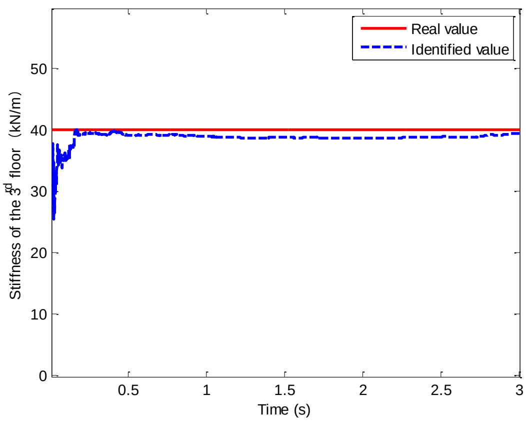
Fig. 4 Identified force of linear structural system



(a) Stiffness of the 1st floor



(b) Stiffness of the 2nd floor

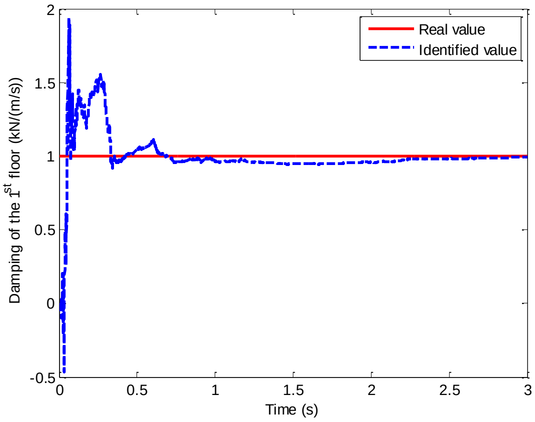


(c) Stiffness of the 3rd floor

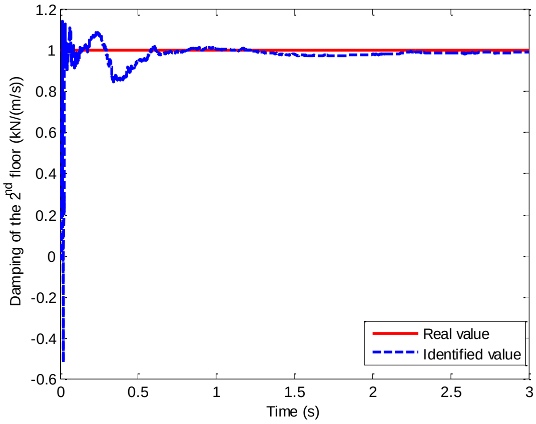
Fig. 5 Identified stiffness of nonlinear structural system

the initial guess for k_i, c_i, w_m are k_i=35 kN/m, c_i=0 kN s/m, w_m=0. Initial guess of nonlinear parameter β and γ are taken as 3 and the initial values of structural response are zero.

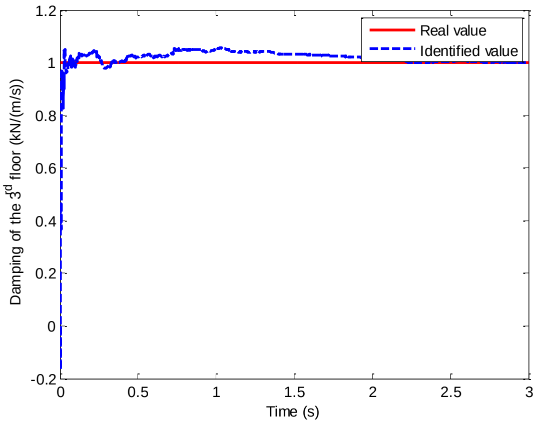
Figs. 5 and 6 show the identification results of stiffness and damping for the nonlinear structural system, respectively. Similar to Case 1, there is a large fluctuation in the beginning of the identified time history but the stiffness and damping are fairly accurately identified with contaminated measurement. Fig. 7 shows the identified parameter of β and γ. Although the fluctuation at the beginning is a little large in the identified parameters of β and γ, the nonlinear parameters of Bouc-Wen model are identified with acceptable accuracy as shown from Fig. 7. The identified external force is also identified accurately as shown in Fig. 8. It is indicated that the proposed system identification method with EKF and excitation decomposition are suitable for the hysteretic nonlinear structures subjected to unknown external force. It is also demonstrated that the proposed method is also robust to the measurement noise.



(a) Damping of the 1st floor

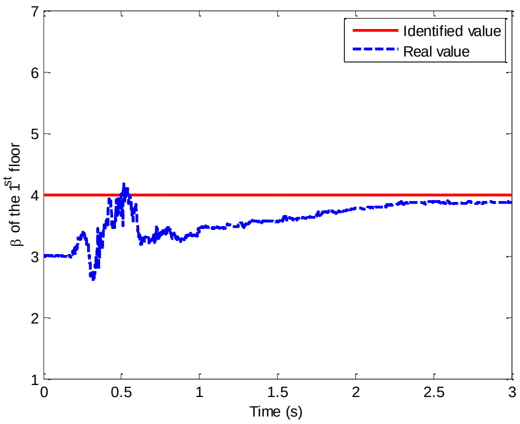


(b) Damping of the 2nd floor

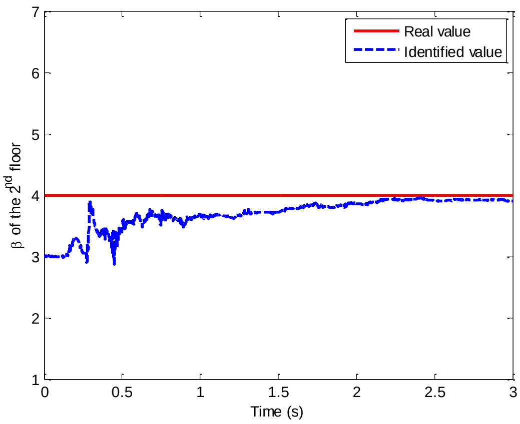


(c) Damping of the 3rd floor

Fig. 6 Identified damping of nonlinear structural system

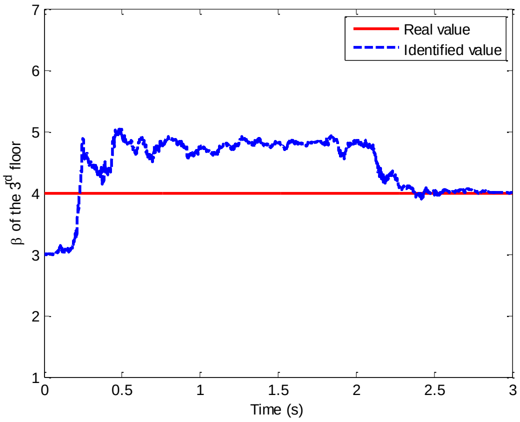


(a) β of the 1st floor

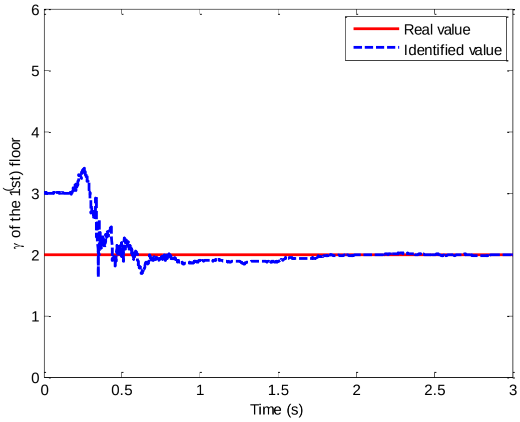


(b) β of the 2nd floor

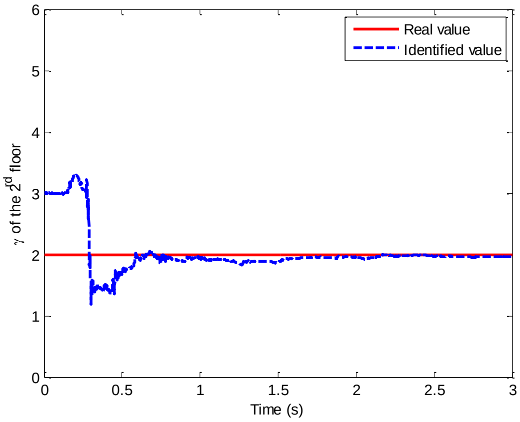
Fig. 7 Identified nonlinear parameters of the nonlinear structural system



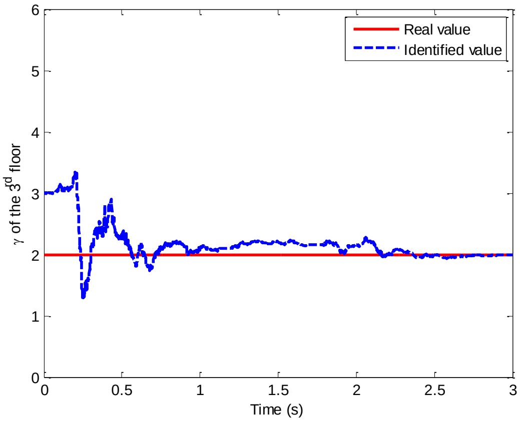
(c) β of the 3[rd] floor



(d) γ of the 1[st] floor



(e) γ of the 2[nd] floor



(f) γ of the 3[rd] floor

Fig. 7 Continued

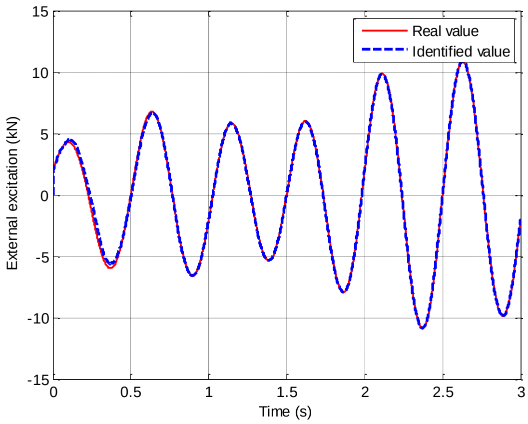
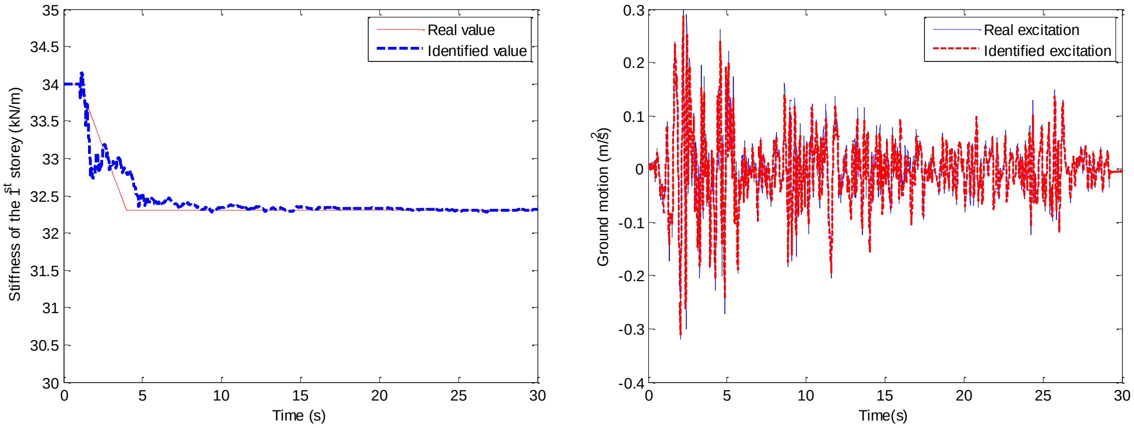
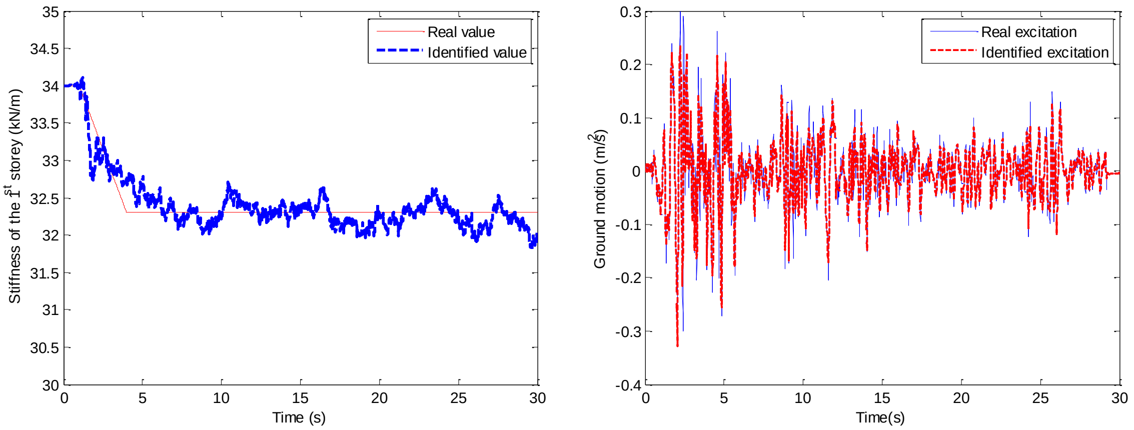


Fig. 8 Identified external force of nonlinear structural system



(a) Identified stiffness of the 1st floor (b) Earthquake excitation Identification

Fig. 9 Simultaneous identification of structural parameter and earthquake excitation without noise



(a) Identified stiffness of the 1st floor (b) Earthquake excitation Identification

Fig. 10 Simultaneous identification of structural parameter and earthquake excitation with 5% noise

6.3 Case 3 - Three-storey time-variant linear structure subjected to earthquake

For the first two cases the excitations consist of low frequency component and can be decomposed with a few order of orthogonal basis. In this case, a 3-storey linear structure subject to scaled El-Centro (1940, NS) earthquake excitation is investigated with the improvements of time window and iteration. The initial guesses of structural parameters are the same as the real value. In the simulation study, a ramped stiffness reduction 20% reduction in the stiffness of the 1st storey occurred due to the peak ground motion. The peak ground acceleration is taken as 0.3g. Sampling rate is 100 Hz. The length of the time window is set as 0.3 second with 30 sampling points. The time history in each time window is decomposed with thirty orthogonal polynomials. In the first scenario the measurement noise is not considered. The identified stiffness of the 1th storey is shown in Fig. 9(a) and the excitation identification result is shown in Fig. 9(b). The identified stiffness can track the real one accurately when the damage occurred. After the damage the identified stiffness can also converge quickly to the decreased stiffness. In this process, identified excitation coincides with the real excitation well. When the 5% measurement noise is considered the parameter and excitation identification results are shown in Fig. 10(a) and 10(b). The structural parameter can still identify the reduction of the stiffness of the first storey and converge to the decreased stiffness after the process of damage, but with larger fluctuations as shown in Fig. 10(a) than the identification result in Fig. 9(a). The earthquake excitation can also be identified but with large error as the comparison of Fig. 9(b) and Fig. 10(b).

In this simulation study, the order of the polynomial is taken as the length of the window to obtain an accurate identification result. For the civil infrastructures with unknown base excitation, the order can be reduced but further studies should be conducted. The length of the time window is recommended to be less than 100 sampled points with the sampling rate no more than 1000 Hz. This is because civil infrastructure is always low-frequency structure and they are not very sensitive to the high frequency excitation. It should be noted that the number of the iterations will be reduced in the time windows when the identified parameter converges to the real one in case of the linear time-invariant structures. Fewer coefficients of the polynomials and shorter length of time window can promote the computational effect. The identification results of structural parameters in this study with fluctuations are due to the measurement noise as shown in the comparison between Fig. 9(a) and Fig. 10(a). But it is illustrated that the identification results still converge to the real value in the end. The results in this paper are consistent to the results of Yang *et al*. (2006).

7. Conclusions

A new method in time domain was proposed in this paper for the inverse identification of structural parameter and external excitation with EKF and orthogonal polynomials decomposition. The structural excitation is decomposed by orthogonal approximation. Then the structural parameters and coefficients of orthogonal polynomial are simultaneously identified with EKF. This method is also improved with time window and iteration process for better identification accuracy. Numerical simulations of time-invariant linear structure, time-variant linear structure and hysteretic nonlinear structure are utilized to study the effectiveness of the proposed method. From the simulation results, the proposed system identification method can conduct the structural parameter identification and force evaluation accurately and effectively even with contaminated measurement. However, the severe nonlinear parameter, such as the nonlinear parameter n of Bouc-Wen model is taken as known in this paper. New identification method dealing with the severe nonlinear system identification will be developed in the future research.

Acknowledgements

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