Structural Engineering and Mechanics, *Vol. 59, No. 6 (2016) 1019-1035*
DOI: http://dx.doi.org/10.12989/sem.2016.59.6.1019

A dynamic foundation model for the analysis of plates on foundation to a moving oscillator

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(Received January 28, 2016, Revised May 3, 2016, Accepted May 5, 2016)

Abstract. This paper proposes a new foundation model called "Dynamic foundation model" for the dynamic analysis of plates on foundation subjected to a moving oscillator. This model includes a linear elastic spring, shear layer, viscous damping and the special effects of mass density parameters of foundation during vibration. By using finite element method and the principle of dynamic balance, the governing equation of motion of the plate travelled by the oscillator is derived and solved by the Newmark's time integration procedure. The accuracy of the algorithm is verified by comparing the numerical results with the other numerical results in the literature. Also, the effects of mass and damping ratio of system components, stiffness of suspension system, velocity of moving oscillator, and dynamic foundation parameters on dynamic responses are investigated. A very important role of these factors will be shown in the dynamic behavior of the plate.

Keywords: Winkler foundation; pasternak foundation; dynamic foundation; mass density of foundation; dynamic analysis of plate; moving oscillator; FEM

1. Introduction

The Winkler model suggested quite early is one of the most fundamental elastic foundation models. This model is developed on the assumption that the reaction forces per unit length at each point of the foundation are proportional to the deflection of the foundation itself. The vertical deformation characteristics of the foundation are defined by means of identical, independent, closely spaced, discrete and linearly elastic springs as known as the modulus of subgrade reaction, *k*. Although, there are many studies related to the response analysis of structures on the Winkler model (Kim *et al*. 2006, Abohadima *et al*. 2009, Hsu *et al*. 2009, Akour 2010, Janco 2010, Amiri *et al*. 2010, Wang *et al.* 2011, Mohanty *et al.* 2011, Kim *et al.* 2012, Wang *et al.* 2013, Jang 2013,

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http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

Coşkun *et al.* 2014), they do not accurately represent the characteristics of many practical foundations as the true behavior of soil. In reality, the soil surface does not show any discontinuity while one of the most important deficiencies of the Winkler model is that a displacement discontinuity appears between the loaded and the unloaded part of the foundation surface (Teodoru 2010).

After the Winkler model, the development of foundation models is continuously carried out by several researchers to make the model more realistic such as the model of Hetenyi, Reissener, Kerr, Pasternak and Vlasov. In recent years, the Vlasov model is developed for structural analysis problems. A relationship between the displacement characteristics and the parameter γ of Vlasov model is established and an iterative procedure to determine the parameter γ as a function of the characteristic of the beam and the foundation is used and called a modified Vlasov model (Ozgan *et* al. 2009, Teodoru *et al.* 2010, Ozgan 2012, Ozgan *et al.* 2012). It can be seen that most of foundation models introduced above did not consider the effects of the mass density of foundation on the behavior of structures resting on the foundation.

In reality, the foundation has mass density; therefore the effect of the density of foundation on the dynamic response of structures always exists during the vibration of structures. Hence, the dynamic responses of structures on foundations should be considered with attending of the mass density of foundation, but the researches in the literature did not attend to the effects of it. This paper strongly proposes a new foundation model, called "Dynamic foundation model" including Winkler linear elastic spring, shear layer of Pasternak, viscous damping and mass density parameter of foundation. The effects of parameters such as mass and damping ratio of system components, stiffness of suspension system, velocity of moving oscillator, and dynamic foundation parameters on dynamic responses are discussed.

The organization of the paper is as follows. The next section describes the dynamic foundation model for dynamic analysis of plates on foundation. By using finite element method and the principle of dynamic balance, the governing equation of motion of the plate on the dynamic foundation subjected to a moving oscillator derived and solved by the Newmark's time integration procedure is presented section 3. Numerical validations are given in detail in section 4 including the verified example compared with the results in the literature and responses of the plate numerically investigated. Finally, some concluding remarks are also drawn.

2. The dynamic foundation model

The dynamic foundation model, which fully describes dynamic characteristic parameters for behavior of foundation, is shown in Fig. 1(a). In this model, the elastic stiffness and shear layer parameter of foundation are idealized based on the Winkler foundation modulus *k* and the shear foundation modulus k_s of Pasternak foundation, respectively, the viscous damping *c* and the mass density of foundation ρ_f are respectively replaced by lumped mass *m* at the top of the elastic spring connected between elastic layer and shear layer, shown in Fig. 1(b) and Fig. 1(c).

The lumped mass *m* is given by

$$m = \beta\rho_f \tag{1}$$

where $\beta=\alpha_f H_f$ is dimensionless parameter of foundation mass which describes the effect of both depth of foundation H_f and dimensionless experimental parameter α_f characterized the influence of the mass density of foundation, in special case α_f =1/3 if the spring element is assumed as linear

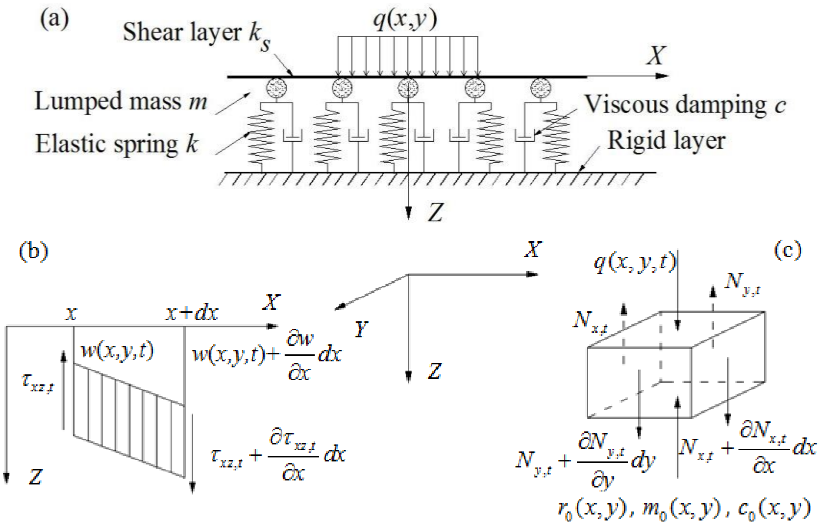


Fig. 1 The dynamic foundation model: (a) The basic model, (b) stress in the shear layer, (c) forces acting on the shear layer

independent elastic spring and contract discontinuity with each others. The pressure-deflection relationship at the time t due to a pressure $q(x,y,t)$ is determined based on the principle of dynamic balance, can be expressed mathematically as follows

$$\frac{\partial N_{x,t}}{\partial x}+\frac{\partial N_{y,t}}{\partial y}+q(x,y,t)-r_0(x,y,t)-m_0(x,y,t)-c_0(x,y,t)=0 \tag{2}$$

where

$$N_{x,t}=\int_o^1 \tau_{xz,t}dz=k_s\frac{\partial w(x,y,t)}{\partial x}\,;\quad N_{y,t}=\int_o^1 \tau_{yz,t}dz=k_s\frac{\partial w(x,y,t)}{\partial y} \tag{3}$$

and $r_0\,(x,\,y,\,t)$, $m_0(x,y,t)$, and $c_0(x,y,t)$ are the reaction of the Winkler foundation, inertia force of the density of foundation and viscous damping resistance at each time t, respectively, given by

$$r_0(x,y,t)=k.w(x,y,t)\,;\quad m_0(x,y,t)=m\frac{\partial^2 w(x,y,t)}{\partial t^2}\,;\quad c_0(x,y,t)=c\frac{\partial w(x,y,t)}{\partial t} \tag{4}$$

Substituting Eq. (3) and Eq. (4) into Eq. (2), it can be expressed as follows

$$q(x,y,t)=k.w(x,y,t)+c\frac{\partial w(x,y,t)}{\partial t}+m\frac{\partial^2 w(x,y,t)}{\partial t^2}-k_s\nabla^2 w(x,y,t) \tag{5}$$

It can be seen that this model is a general foundation model and conform to the true nature of the soil. At the same time, it fully describes dynamic characteristic parameters of foundation such as stiffness, damping and mass. If the influence of the viscous damping and density of foundation neglects, this model will be similar to Filonenko and Pasternak foundation model. Additionally, if the influence of shear foundation modulus overlooks this will be the same with Winkler foundation models. Therefore, it can be said that the dynamic foundation model accurately

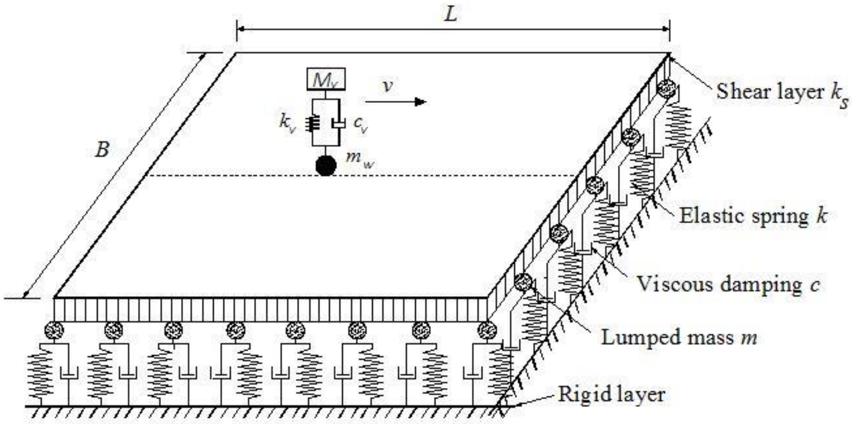


Fig. 2 The plate subjected to a moving oscillator on the dynamic foundation

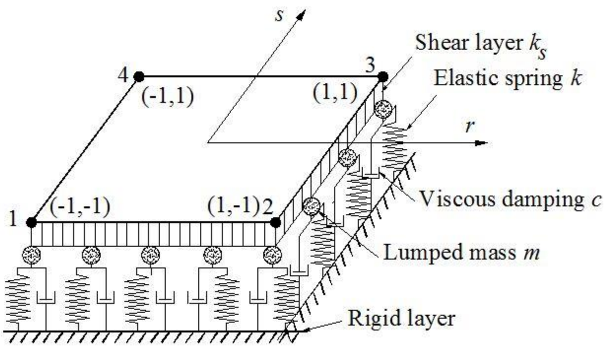


Fig. 3 The plate element on the dynamic foundation

represents the characteristics of the soil in using to analyze dynamic responses of structures resting on the foundation.

3. Formulation

3.1 Finite element procedure

Consider a Mindlin rectangular plate of length *L*, width *B* and thickness *h* on the dynamic foundation subjected to a moving oscillator, shown in Fig. 2. A four-node uniform Minlin plate element, each node having three global degrees of freedom, including vertical displacement and two rotations in global axes, resting on the dynamic foundation is shown in Fig. 3. By using finite element method, the generalized displacements are independently interpolated using the same shape functions

$$w_e = \sum_{1}^{4} N_i w_i \ ; \quad \theta_{ex} = \sum_{1}^{4} N_i \theta_{xi} \ ; \quad \theta_{ey} = \sum_{1}^{4} N_i \theta_{yi} \tag{6}$$

where, N_i is the linear Lagrange interpolation functions, given as follows

$$N_i = \frac{1}{4}(1+rr_i)(1+ss_i) \tag{7}$$

Based on the strain energy of the Minlin plate element, the plate matrices can be found easily in many references related to finite element method, given by

$$[\mathbf{K}]_p = \frac{h^3}{12}\int_{A_e}[\mathbf{B}_b]^T[\mathbf{D}_b][\mathbf{B}_b]\mathrm{dA}_e + \kappa_s \mathrm{h}\int_{A_e}[\mathbf{B}_s]^T[\mathbf{D}_s][\mathbf{B}_s]\mathrm{dA}_e \tag{8}$$

where, κ_s also known as the shear correction factor can be taken as 5/6, D_b and D_s are a matrix of material constants and matrix related to the shear strain, respectively, given as follows

$$[\mathbf{D}_b] = \frac{Eh}{1-\upsilon^2}\begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & \frac{1-\upsilon}{2} \end{bmatrix};\ [\mathbf{D}_s] = \frac{E}{2(1+\upsilon)}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{9}$$

The $[\mathbf{B}_b]$ and $[\mathbf{B}_b]$ are the strain-displacement matrices for bending and shear contributions, respectively, are obtained by derivation of the shape functions by

$$[\mathbf{B}_b] = [\mathbf{B}_{1b} \quad \mathbf{B}_{2b} \quad \mathbf{B}_{3b} \quad \mathbf{B}_{4b}];\ [\mathbf{B}_s] = [\mathbf{B}_{1s} \quad \mathbf{B}_{2s} \quad \mathbf{B}_{3s} \quad \mathbf{B}_{4s}] \tag{10}$$

with

$$\mathbf{B}_{ib} = \begin{bmatrix} 0 & -\partial N_i/\partial x & 0 \\ 0 & 0 & \partial N_i/\partial y \\ 0 & -\partial N_i/\partial y & \partial N_i/\partial x \end{bmatrix};\ \mathbf{B}_{is} = \begin{bmatrix} \partial N_i/\partial x & N_i & 0 \\ \partial N_i/\partial y & 0 & -N_i \end{bmatrix} \tag{11}$$

Based on the strain energy of the dynamic foundation including the effects of both transverse shear deformation and elastic foundation, the elastic foundation stiffness matrix, $[\mathbf{K}]_W$ is defined as

$$[\mathbf{K}]_W = \int_{A_e}[\mathbf{N}_w]^T k[\mathbf{N}_w]dA_e \tag{12}$$

with the shape function $[\mathbf{N}_w]$ and the shear foundation stiffness matrix, $[\mathbf{K}]_s$ given by

$$[\mathbf{N}_w] = [N_1 \quad 0 \quad 0 \quad N_2 \quad 0 \quad 0 \quad N_3 \quad 0 \quad 0 \quad N_4 \quad 0 \quad 0] \tag{13}$$

$$[\mathbf{K}]_S = \int_{A_e}[\mathbf{N}_{s,x}]k_s[\mathbf{N}_{s,x}]dA_e + \int_{A_e}[\mathbf{N}_{s,y}]k_s[\mathbf{N}_{s,y}]dA_e \tag{14}$$

in which

$$[\mathbf{N}_{s,x}] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & 0 & \ldots & \frac{\partial N_4}{\partial x} & 0 & 0 \end{bmatrix},\ [\mathbf{N}_{s,y}] = \begin{bmatrix} \frac{\partial N_1}{\partial y} & 0 & 0 & \ldots & \frac{\partial N_4}{\partial y} & 0 & 0 \end{bmatrix} \tag{15}$$

It can be seen that the overall stiffness matrix of the plate element on the dynamic foundation including the effects of both stiffness matrix of the plate element and foundation stiffness matrix

(Winkler and shear foundation stiffness matrix) can be written as follows

$$[\mathbf{K}]_e = [\mathbf{K}]_P + [\mathbf{K}]_W + [\mathbf{K}]_S \tag{16}$$

The same as above, mass matrix of the plate element on the dynamic foundation including the effects of the mass density of both plate and foundation is determined based on the kinetic energy of the beam element as

$$[\mathbf{M}]_e = [\mathbf{M}]_P + [\mathbf{M}]_F \tag{17}$$

where $[\mathbf{M}]_P$ is the mass matrix of the plate element, can be expressed as follows

$$[\mathbf{M}]_P = \int_{A_e} [\mathbf{N}]^T [\mathbf{H}][\mathbf{N}]\, dA_e \tag{18}$$

with

$$[\mathbf{H}] = \rho \begin{bmatrix} h & 0 & 0 \\ 0 & h^3/12 & 0 \\ 0 & 0 & h^3/12 \end{bmatrix}, \ [\mathbf{N}] = \begin{bmatrix} N_1 & 0 & 0 & & N_4 & 0 & 0 \\ 0 & N_1 & 0 & \dots & 0 & N_4 & 0 \\ 0 & 0 & N_1 & & 0 & 0 & N_4 \end{bmatrix} \tag{19}$$

and $[\mathbf{M}]_F$ is the mass matrix of mass density of the dynamic foundation to be

$$[\mathbf{M}]_F = \int_{A_e} [\mathbf{N}_w]^T m [\mathbf{N}_w]\, dA_e \tag{20}$$

The dashpots system is considered to simulate as the viscous damping property of foundation, the damping matrix of foundation is computed by using the dissipation energy of these dashpots as

$$[\mathbf{C}]_e = \int_{A_e} [\mathbf{N}_w]^T c [\mathbf{N}_w]\, dA_e \tag{21}$$

The Gauss integration scheme is used to evaluate the integrations numerically, but note that when the thickness of the plate is reduced, the element becomes over-stiff, a phenomenon that relates to so-called ‘shear locking’. The simplest and most practical means to solve this problem is to use 2×2 Gauss points for the integration of the first term in Eq. (8), and use only one Gauss point for the rest terms. In addition, non-dimensional foundation parameters K_1 and K_2 used to analysis response of the plate are defined as follows (Zhou *et al.* 2004, Xiang *at al.* 1994, Omurtag *et al.* 1997, Ferreira *et al.* 2010)

$$K_1 = \frac{kB^4}{D}, K_2 = \frac{k_s B^2}{D} \tag{22}$$

where D is the flexural rigidity of the plate.

3.2 Governing equation

The oscillator model is regarded as a two-node system, with one node associated with each of two concentrated masses. The stiffness and damping coefficients of the oscillator are denoted by k_v and c_v, respectively. Also, the mass of the bottom part by m_w and the mass of the upper part by M_v, which are respectively the mass of the wheel and the mass lumped from the car body. In addition,

z_v and z_w denote the vertical displacements of two nodes measured from the static equilibrium position (Mohebpour *et al.* 2011, Phung-Van *et al*. 2014). The equation of motion of the oscillator can be written as follows

$$\begin{bmatrix} M_v & 0 \\ 0 & m_w \end{bmatrix}\begin{Bmatrix} \ddot{z}_v \\ \ddot{z}_w \end{Bmatrix}+\begin{bmatrix} c_v & -c_v \\ -c_v & c_v \end{bmatrix}\begin{Bmatrix} \dot{z}_v \\ \dot{z}_w \end{Bmatrix}+\begin{bmatrix} k_v & -k_v \\ -k_v & k_v \end{bmatrix}\begin{Bmatrix} z_v \\ z_w \end{Bmatrix}=\begin{Bmatrix} 0 \\ f_c-\left(M_v+m_w\right)g \end{Bmatrix} \tag{23}$$

where f_c is the contact force.

Assuming that all information of the system at time t is known and Δt is a small time increment, the first row of Eq. (23) can be expanded in an incremental form at time $t+\Delta t$ as

$$M_v\ddot{z}_{v,t+\Delta t}+c_v\dot{z}_{v,t+\Delta t}+k_v z_{v,t+\Delta t}=q_{vc,t+\Delta t} \tag{24}$$

with

$$q_{vc,t+\Delta t}=-c_v\dot{z}_{w,t+\Delta t}-k_v z_{w,t+\Delta t} \tag{25}$$

Based on Newmark's method, average acceleration method, the displacement z_v and its derivatives at time $t+\Delta t$ can be written as

$$\begin{aligned} \ddot{z}_{v,t+\Delta t}&=\frac{b_0}{\psi_v}\left(-q_{vc,t+\Delta t}+q_{v,t}\right)-b_1\dot{z}_{v,t}-b_2\ddot{z}_{v,t} \\ \dot{z}_{v,t+\Delta t}&=\frac{b_3}{\psi_v}\left(-q_{vc,t+\Delta t}+q_{v,t}\right)-b_4\dot{z}_{v,t}-b_5\ddot{z}_{v,t} \\ z_{v,t+\Delta t}&=z_{v,t}+\frac{1}{\psi_v}\left(-q_{vc,t+\Delta t}+q_{v,t}\right) \end{aligned} \tag{26}$$

with

$$\begin{aligned} \psi_v&=b_0M_v+b_3c_v+k_v \\ q_{v,t}&=M_v\left(b_1\dot{z}_{v,t}+b_2\ddot{z}_{v,t}\right)+c_v\left(b_4\dot{z}_{v,t}+b_5\ddot{z}_{v,t}\right)-k_v z_{v,t} \end{aligned} \tag{27}$$

and coefficients b_i given by

$$b_0=\frac{1}{\beta\Delta t^2},\; b_1=\frac{1}{\beta\Delta t},\; b_2=\frac{1}{2\beta}-1,\; b_3=\frac{\gamma}{\beta\Delta t},\; b_4=\frac{\gamma}{\beta}-1,\; b_5=\frac{\Delta t}{2}\left(\frac{\gamma}{\beta}-2\right) \tag{28}$$

Substituting Eq. (27) into incremental form of the second row of Eq. (23), the contact force f_c in time $t+\Delta t$ is determined by

$$f_{c,t+\Delta t}=m_w\ddot{z}_{w,t+\Delta t}+c_c\dot{z}_{w,t+\Delta t}+k_c z_{w,t+\Delta t}+p_{c,t+\Delta t}+q_{c,t} \tag{29}$$

in which

$$\begin{aligned} c_c&=c_v\left(1+\frac{\psi_{vw}}{\psi_v}\right),\; k_c=k_v\left(1+\frac{\psi_{vw}}{\psi_v}\right) \\ p_{c,t+\Delta t}&=\left(M_v+m_w\right)g,\; p_{c,t}=\frac{\psi_{vw}}{\psi_v}q_{v,t}-q_{w,t} \end{aligned} \tag{30}$$

with

$$\psi_{vw} = -b_3 c_v - k_v,\ q_{w,t} = -c_v\left(b_4 \dot{z}_{v,t} + b_5 \ddot{z}_{v,t}\right) + k_v z_{v,t} \tag{31}$$

Note that in Eqs. (26) and (29), it assumes that there is no loss of contact between the tire and the upper surface of the plate, and hence, the displacement of the wheel (z_w) equals to the deflection of the plate at the contact position of vehicle and plate, can be expressed in terms of the nodal displacement vector $\{\mathbf{u}_e\}$as

$$z_w = [\mathbf{N}_w]\{\mathbf{u}_e\} \tag{32}$$

At each time step, the governing differential equation for the displacement of the plate element resting on the dynamic foundation subjected to moving oscillator at time $t+\Delta t$ can be expressed as

$$[\mathbf{M}]_e\{\ddot{\mathbf{u}}_e\}_{t+\Delta t} + [\mathbf{C}]_e\{\dot{\mathbf{u}}_e\}_{t+\Delta t} + [\mathbf{K}]_e\{\mathbf{u}_e\}_{t+\Delta t} = \{\mathrm{F}_e\}_{t+\Delta t} \tag{33}$$

where $[\mathbf{M}]_e$, $[\mathbf{M}]_e$ and $[\mathbf{M}]_e$ are mass, damping and stiffness matrices of the i^{th} plate element, respectively, and $\{\mathbf{F}_e\}$ the vector of consistent nodal forces resulting from the contact force is

$$\{\mathbf{F}_e\}_{t+\Delta t} = -[\mathbf{N}_w]\, f_{c,t+\Delta t} \tag{34}$$

By using the finite element theory, the corresponding degrees of freedom of the stiffness matrix and the mass of the plate element on dynamic foundation are connected in the global coordinate, the equation of motion of the plate on the dynamic foundation subjected to a moving oscillator in each time step is defined as follows

$$[\mathbf{M}]\{\ddot{\mathbf{U}}\} + [\mathbf{C}]\{\dot{\mathbf{U}}\} + [\mathbf{K}]\{\mathbf{U}\} = \{\mathbf{F}\} \tag{35}$$

where, [**M**], [**C**], and [**K**] are the overall mass, damping and stiffness matrices of the system, respectively; $\{\boldsymbol{U}\}$ is the nodal displacement vector, and $\{\mathbf{F}\}$ is the external force vector. It can be seen that symbols [**M**], [**C**], and [**K**] in Eq. (35) are called instantaneous matrices because they are time-dependent matrices which constant matrices are due to the structure itself and time-dependent matrices are due to the moving oscillator. The Eq. (35) is used for studying the dynamic response of the plate subjected to a moving oscillator on the dynamic foundation and is solved by means of the direct step-by-step integration method based on Newmark algorithm.

4. Numerical results

4.1 Verified examples

This first example considers fully simply supported (SSSS) and fully clamped (CCCC) plates, with thickness-to-side ratio *h/a*. The non-dimensional natural frequencies for CCCC and SSSS plate expressed in Table 1 are obtained, show that quite good agreement with the solution given in literature (Ferreira *et al.* 2010).

The next comparative study is investigated to the natural frequencies of square isotropic plates (*a*x*a*) on Pasternak foundation. The demonstrate natural frequency of the plates with fully simply supported and clamped edges are plotted in Table 2. As demonstrated in these tables, discrepancies between the result of the present work and those of other researches are in excellent agreement.

Table 1 The natural frequencies of the square isotropic plate without foundation

No. Mode	SSSS, κ=0.833, ν=0.3				CCCC, κ=0.8601, ν=0.3			
	h/a=0.1		h/a=0.01		h/a=0.1		h/a=0.01	
	Present (15×15)	Ferreira (15×15)	Present (20×20)	Ferreira (20×20)	Present (20×20)	Ferreira (20×20)	Present (20×20)	Ferreira (20×20)
1st	0.9345	0.9346	0.0965	0.0965	1.5996	1.5955	0.1765	0.1750
2nd	2.2544	2.2545	0.2430	0.2430	3.0784	3.0662	0.3635	0.3635
3rd	2.2544	2.2545	0.2430	0.2430	3.0784	3.0662	0.3635	0.3635
4th	3.4591	3.4592	0.3890	0.3890	4.3129	4.2924	0.5358	0.5358
5th	4.3029	4.3031	0.4928	0.4928	5.1513	5.1232	0.6634	0.6634

Table 2 The natural frequencies of the square plate on Pasternak foundation with non-dimensional parameter $\varpi = \frac{\omega a^2}{\pi^2}\sqrt{\frac{\rho h}{D}}$

Boundary condition	h/a	K_1	K_2	Methods	ϖ_1	ϖ_2	ϖ_3
SSSS (ν=0.3)	0.01	100	10	Zhou *et al.* (2004)	2.6551	5.5717	8.5406
				Xiang *et al.*(1994)	2.6551	5.5718	8.5405
				Ferreira *et al.* (2010)	2.6559	5.5718	8.5384
				Present	2.6570	5.5924	8.5771
		500	10	Zhou *et al.* (2004)	3.3398	5.9285	8.7775
				Xiang *et al.* (1994)	3.3400	5.9287	8.7775
				Ferreira *et al.* (2010)	3.3406	5.9285	8.7754
				Present	3.3414	5.9481	8.8131
	0.1	200	10	Zhou *et al.* (2004)	2.7756	5.2954	7.7279
				Xiang *et al.* (1994)	2.7842	5.3043	7.7287
				Ferreira *et al.* (2010)	2.7902	5.3452	7.8255
				Present	2.7857	5.3207	7.7539
		1000	10	Zhou *et al.* (2004)	3.9566	5.9757	8.1954
				Xiang *et al.* (1994)	3.9805	6.0078	8.2214
				Ferreira *et al.* (2010)	3.9844	6.0403	8.3112
				Present	3.9816	6.0222	8.2451
CCCC (ν=0.15)	0.015	1390.2	166.83	Zhou *et al.* (2004)	8.1673	12.8229	16.8332
				Omurtag *et al.* (1997)	8.1375	12.8980	16.9320
				Ferreira *et al.* (2010)	8.1669	12.8210	16.8420
				Present	8.1718	12.8620	16.8890

This final example is conducted to verify the present algorithm for the problem of plates traversed by a moving sprung mass for the isotropic plate. The obtained results are compared with those of the semi-analytical solution (Ghafoori *et al*. 2010). The displacements of the car body of a vehicle with various damping coefficients of the suspension system are plotted in Fig. 4. It can be that the results of the present method are in good agreements with the results of the semi-analytical

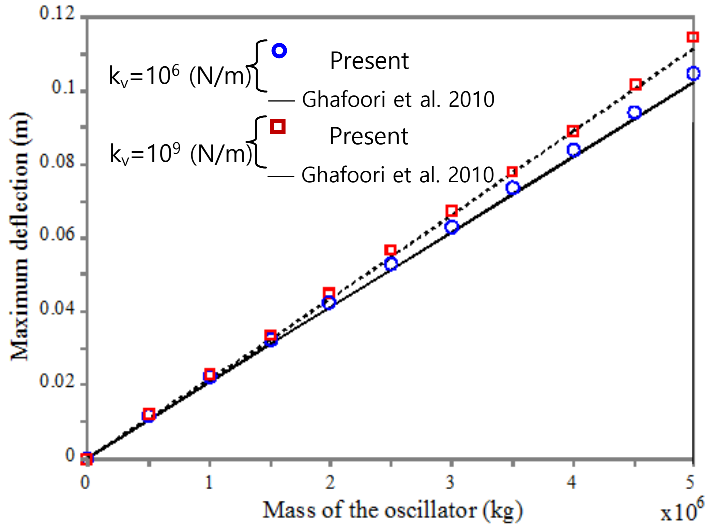


Fig. 4 Displacements of the plate traversed by a moving sprung mass versus the mass of oscillator

solution. This example will demonstrate excellent performance of the proposed algorithm to illustrate the dynamic response of the plate subjected to the moving objects.

Through above examples, the numerical results from the program based on the suggested formulation show quite good agreement with numerical results in the literature. Therefore, the program which will analyze the influence of many parameters to dynamic response of the plate on the dynamic foundation subjected to a moving oscillator is reliable.

4.2 Numerical investigation

In the first investigation, the effects of mass density parameter of the dynamic foundation on free vibration of the square plate (t/a=0.01) for various foundation stiffness parameters are studied with ν=0.2 and μ=0.5. The natural frequencies of the plate are shown in Table 3. It can be seen that the natural frequency parameters decrease with an increase in the experimental parameter β and ratio of mass density μ which is defined as the ratio of the mass density of dynamic foundation to the mass density of the plate. At the same time, an increase in the stiffness parameters of dynamic foundation cause the increase in fundamental frequencies of the plate.

In this next section now consider a moving oscillator on the middle line along the longitudinal longer side direction of a rectangular plate with the simply supported boundary along the two shorter sides. The material parameters of the plate are given by Young's modulus $E=3.1\times10^{10}$ N/m^2, Poisson's ratio ν=0.2, length L=20 m, width B=10 m, thickness t=0.3 m, and density mass ρ=2500 kg/m^3. The dynamic foundation parameter are given by ratio mass density μ=0.75, the foundation coefficient K_1=50 and K_2=1, and the damping coefficient $c=10^2$ Ns/m^2. The dimensionless parameters of the moving oscillator used to analyze the dynamic response of the considered system are given as κ=0.5 and γ=0.5, in which the mass parameter κ, the ratio of the total mass of the oscillator to the total mass of the plate M_p and the frequency parameter γ, the ratio of the natural vibration frequency of the oscillator ω_v to the first fundamental natural frequency of the plate on dynamic foundation ω_p. The plate is discretized by a mesh of 10×20 rectangular

Table 3 The influence of mass density of the dynamic foundation on the natural non-dimension frequencies of the plate for various stiffness foundation parameter

K_1	K_2	β	SSSS			CCCC		
			ϖ_1	ϖ_2	ϖ_3	ϖ_1	ϖ_2	ϖ_3
10^2	50	0	3.8957	7.2044	10.334	5.1522	9.3363	13.058
		0.25	1.0604	1.9612	2.8134	1.4024	2.5416	3.5551
		0.5	0.7641	1.4132	2.0273	1.0105	1.8314	2.5618
		0.75	0.6279	1.1613	1.6660	0.8304	1.505	2.1052
	100	0	5.0306	8.793	12.136	6.1892	10.766	14.686
		0.25	1.3693	2.3936	3.3041	1.6846	2.9308	3.9983
		0.5	0.9867	1.7248	2.3809	1.2139	2.1119	2.8811
		0.75	0.8108	1.4174	1.9566	0.9976	1.7355	2.3677
10^3	50	0	4.9411	7.8192	10.771	5.9818	9.8184	13.407
		0.25	1.3449	2.1285	2.9325	1.6282	2.6728	3.6501
		0.5	0.9691	1.5338	2.1131	1.1732	1.9260	2.6302
		0.75	0.7964	1.2604	1.7365	0.9642	1.5827	2.1615
	100	0	5.8775	9.3033	12.511	6.8952	11.187	14.997
		0.25	1.5998	2.5325	3.4060	1.8768	3.0454	4.0830
		0.5	1.1528	1.8249	2.4543	1.3524	2.1944	2.9422
		0.75	0.9473	1.4997	2.0169	1.1114	1.8033	2.4178

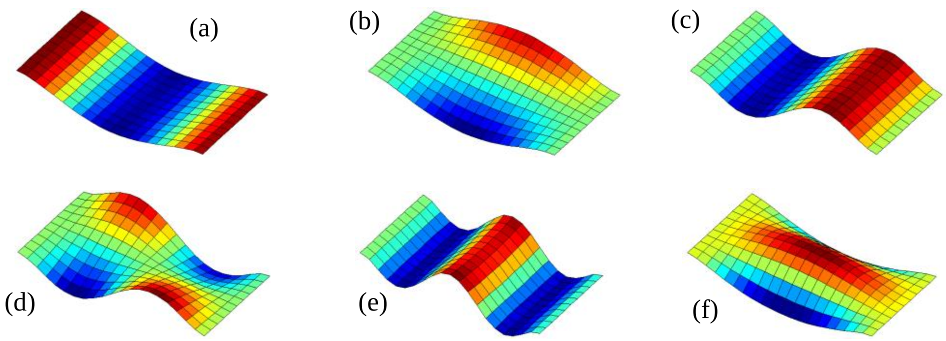


Fig. 5 Shapes of the six lowest eigenmodes of the plate on dynamic foundation: (a) 1st Mode; (b) 2nd mode; (c) 3rd mode 3; (d) 4th mode; (e) 5th mode; (f) 6th mode

elements and 100 time steps are used in the time domain solution.

In this first investigation presented free vibration of the plate with the above material and foundation parameters, Fig. 5 plots the shape of the six lowest eigenmodes of the plate on the dynamic foundation. It is seen that the shapes of eigenmodes reveal the real physical modes.

Next, the effects of the mass density of the dynamic foundation on dynamic response of the plate travelled by the oscillator are investigated. The dynamic magnification factor (DMF) which is defined as the ratio of maximum dynamic deflection to maximum static deflection at the center

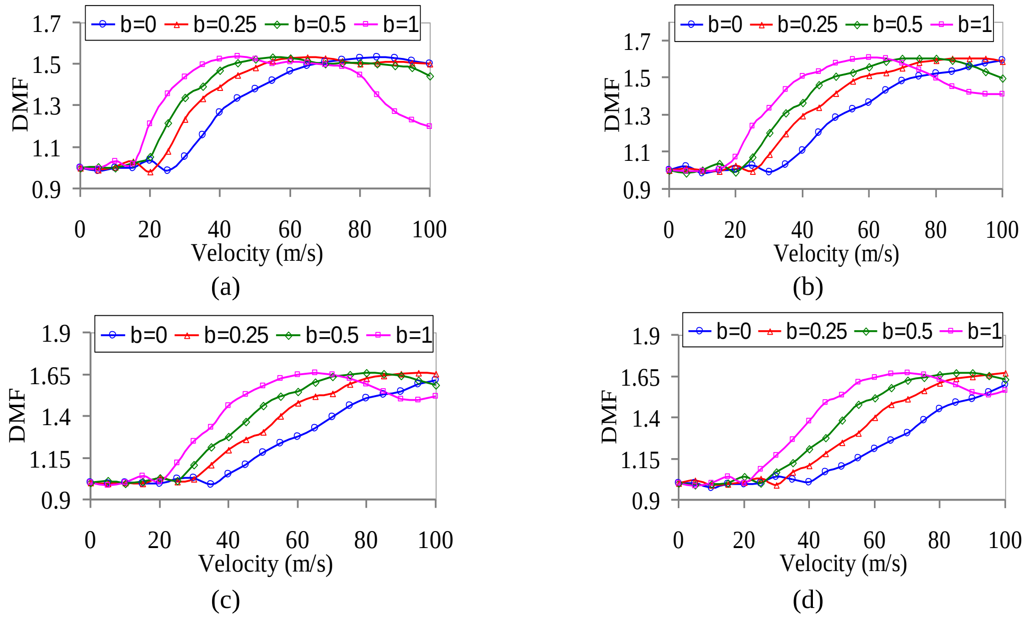


Fig. 6 The DMF for various elastic stiffness parameters of the dynamic foundation: (a) K_1=25; (b) K_1=50; (c) K_1=75; (d) K_1=100

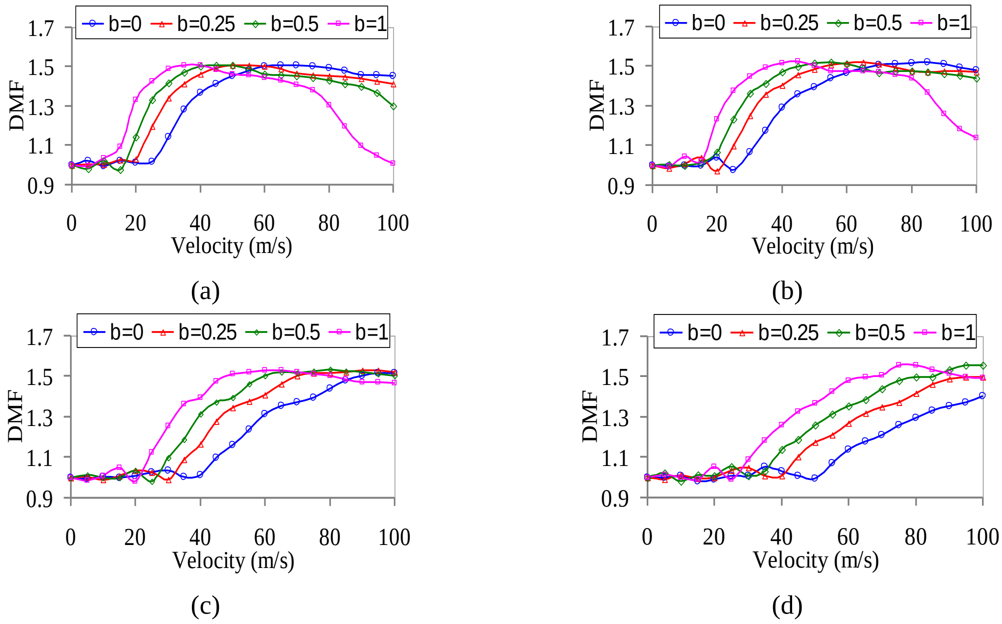


Fig. 7 The DMF for various shear layer parameters of the dynamic foundation with K_1=10 (a) K_2=1; (b) K_2=5; (c) K_2=25; (d) K_2=50

point of the plate. Figs. 6 and 7 plot the variation of DMF for different values of the elastic stiffness and shear layer parameter of the dynamic foundation, respectively. The effects of the

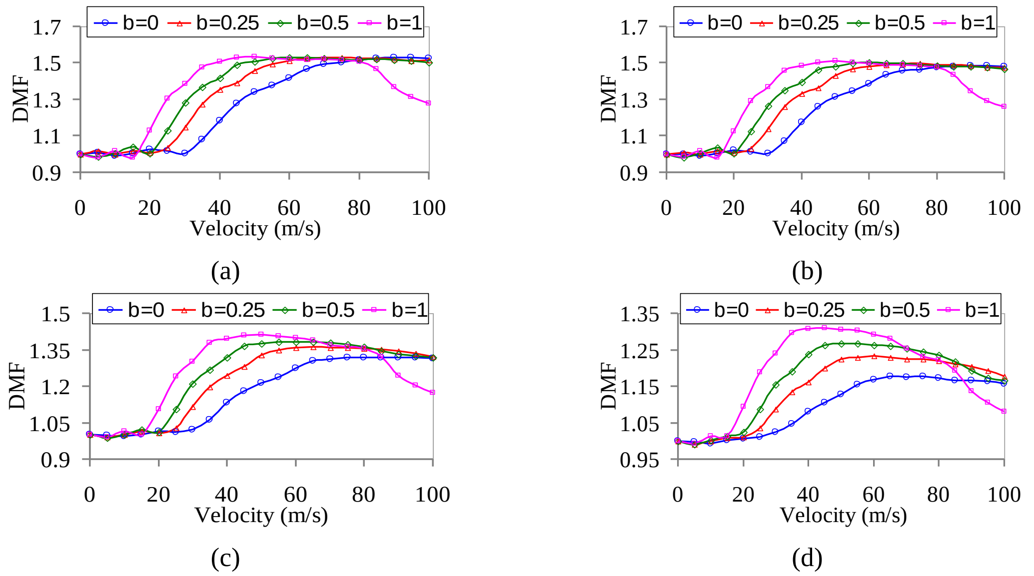


Fig. 8 The DMF for various damping coefficient of the dynamic foundation with K_1=25 and K_2=5: (a) $c=10^2$ Ns/m^2; (b) $c=10^3$ Ns/m^2; (c) $c=5\times10^3$ Ns/m^2; (d) $c=10^4$ Ns/m^2

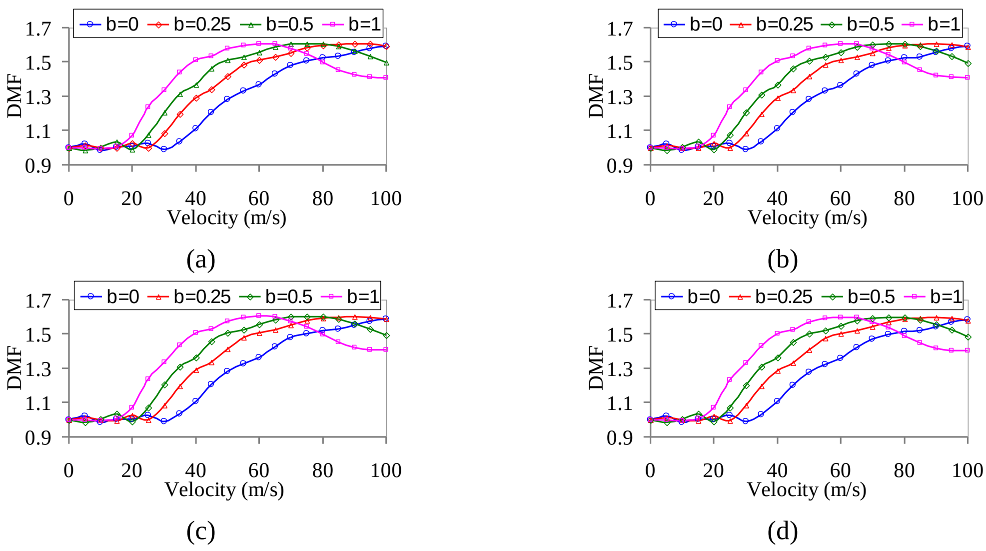


Fig. 9 The DMF for various masses of vehicle body and wheel with K_1=50 and K_2=1: (a) κ=0.25; (b) κ=0.5; (c) κ=1; (d) κ=2

mass density of the dynamic foundation on DMF for various damping viscous are presented in Fig. 8.

It can be seen that the mass density of dynamic foundation affects significantly on the dynamic response of the plate for various values of the stiffness and damping coefficient parameter of the dynamic foundation. However, in the range of high velocity V>20 m/s, the effect of the mass density of dynamic foundation on the DMF is quite clear, and the comparisons show that the mass

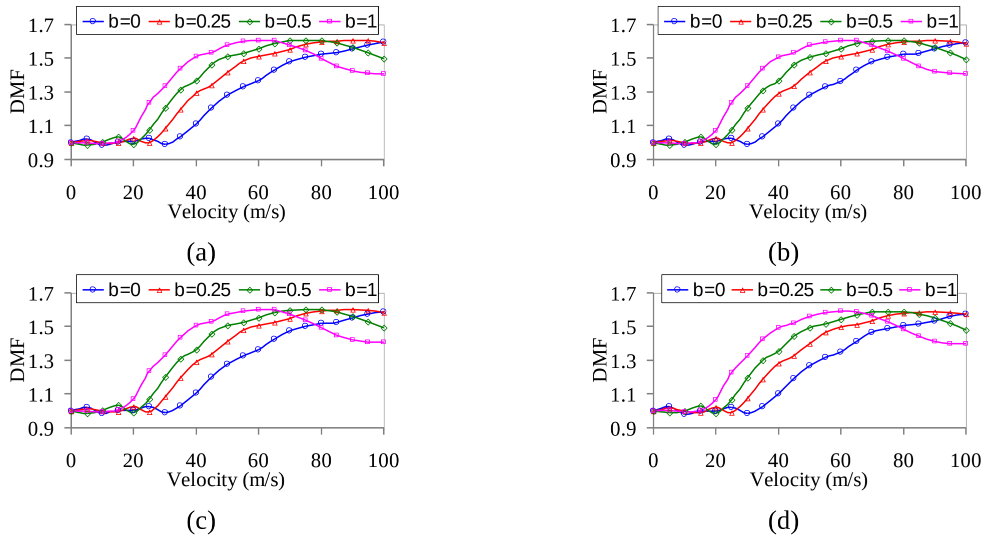


Fig. 10 The DMF for various stiffness coefficients of the suspension system with $K_1 = 50$ and $K_2 = 1$: (a) γ=0.25; (b) γ=0.5; (c) γ=1; (d) γ=2

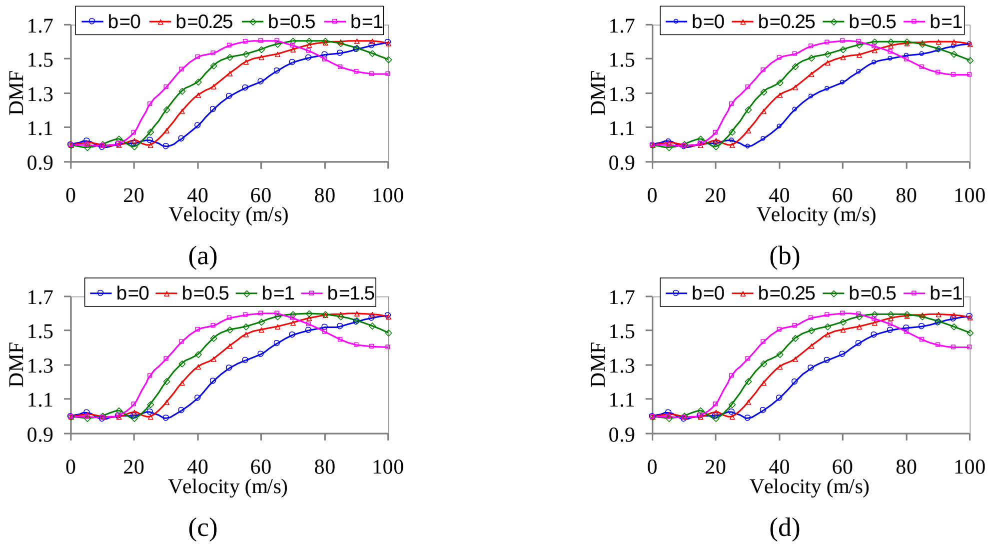
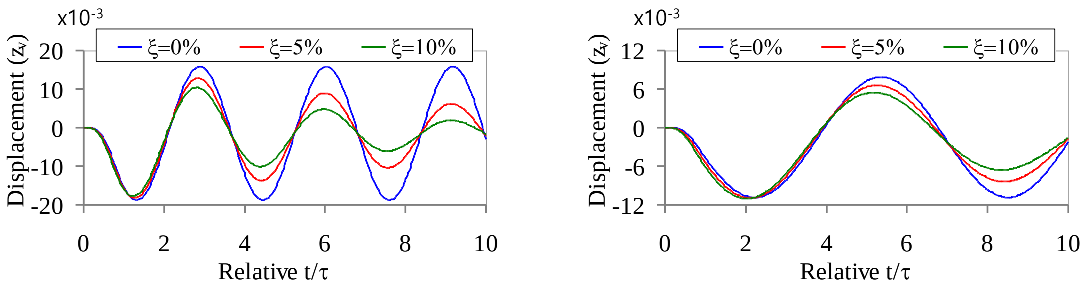


Fig. 11 The DMF for various damping coefficients of the moving oscillator with K_1=50 and K_2=1: (a) ξ=1%; (b) ξ=10%; (c) ξ=15%; (d) ξ=20%

density parameters of dynamic foundation are an increase the DMF of the plate than the foundation model without the influence of mass density (β=0) which increase with an increase of values of its. At the same time, it can be expected that the influence of mass density of the dynamic foundation on dynamic response of the plate is significantly and clearly with a decrease of values of stiffness parameter and damping coefficient.

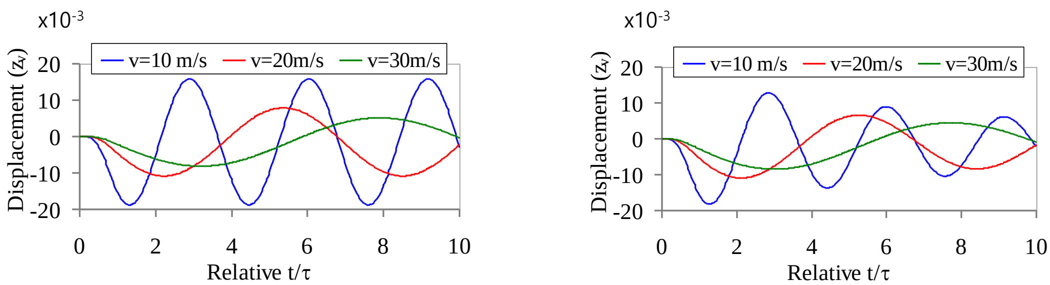
In continuation, the effects of moving oscillator parameters on the DMF of the plate are also studied for different value of the mass density parameter of the dynamic foundation and velocity of moving oscillator. Fig. 9 presents the DMF of the plate for various masses of a vehicle body and wheel. The influence of stiffness and damping coefficients of the suspension system between the vehicle body and the wheels on the DMF of the plate for various velocities of moving oscillator are plotted in Figs. 10 and 11. It can be seen that the mass density of dynamic foundation affects significantly on dynamic response of the plate for various values of the moving oscillator. The comparisons show that the mass density parameters of dynamic foundation are an increase the DMF of the plate than the foundation model without the influence of mass density (β=0) which increase with an increase of values of its. At the same time, it can be expected that the influence of mass density of the dynamic foundation on dynamic response of the plate is significantly and clearly in the range of high velocity V >20 m/s.

At last, the dynamic response of the moving oscillator is also investigated. The effect of various damping coefficients of the suspension system to the vibration of the vehicle body is plotted in Fig. 12. It can be seen that when the damping coefficient of the suspension system increases, the displacement vibration of the vehicle body is also damped strongly, and when the damping coefficient equals zero (without damping), the vibrations of the vehicle body are harmonic, as expected. Fig. 13 shows the displacement vibration of the vehicle body versus various velocities of the moving oscillator. The results show that when the velocity of the vehicle is faster, the vibration period of the vehicle also becomes larger.



(a) (b)

Fig. 12 Vertical displacement of the car body of a vehicle for various damping coefficients of the suspension system with K_1 =100, K_2 =50, κ =1, γ =1: (a) V =10 m/s; (b) V =20 m/s



(a) (b)

Fig. 13 Vertical displacement of the car body of a vehicle for various velocities with K_1=100, K_2=50, κ =1, γ =1: (a) ξ =0 %; (b) ξ =5 %

5. Conclusions

A dynamic foundation model which fully describes the dynamic parameters of foundation, including viscous elastic and mass parameters has been proposed for dynamic analysis of the plate traversed by a moving oscillator. The contact force resulted by the moving oscillator has been obtained in terms of the contact displacement and its derivatives using the Newmark method, and the governing equation of motion of the plate is derived by using finite element method and the principle of dynamic balance. The accuracy and reliability of the results of free and forced vibration analysis are verified by comparing its numerical solutions with those of other available numerical results. The parametric analysis has been performed to investigate the effects of stiffness parameter, damping viscous and mass density parameter of the dynamic foundation, motion velocity, the mass and stiffness parameter of suspension system, and the damping ratio of the oscillator on the dynamic response of the plate. A comparison shows that the mass density parameter of the dynamic foundation increases significantly dynamic response of the plate in the range of high velocity of the moving oscillator than foundation model without the influence of mass density. The presented numerical can be employed to perform the parametric studies about various dynamic and structural properties of the vehicle–road systems and high speed train, which are useful for the practical design problems.

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