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Deflection of battened beams with shear and discrete effects

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Abstract. This paper presents a theoretical analysis for determining the transverse deflection of simply supported battened beams subjected to a uniformly distributed transverse quasi-static load. The analysis considers not only the shear effect but also the discrete effect of battens on the transverse deflection of the battened beam. The analytical solution is obtained using the principle of minimum potential energy. Numerical validation of the present analytical solution is accomplished using finite element methods. The present analytical solution shows that the shear effect on the transverse deflection of battened beams increases with the cross-section area of the main member but decreases with the cross-section area of the batten. The longer the battened beam is, or the larger the moment of inertia of the main member is, the smaller the shear effect will be.

Keywords: analytical method; computational mechanics; finite element method (FEM); frames; quasi-static; steel structures

1. Introduction

Battened beams are widely used in structures such as buildings and bridges. These beams generally consist of two or more parallel main members interconnected by lacing or batten plates (Banerjee and Williams 1983, Aly *et al*. 2010, Chung and Emms 2008). Since the moment of inertia of the built-up cross section increases with the distance between the centroids of the main members, the battened beams can have large bending rigidity. However, compared to the solid beam with the same moment of inertia, the battened beam has weak shear stiffness and thus is more flexible and therefore the deflection induced by shear forces becomes important and cannot be neglected (Greschik 2008, Banerjee and Williams 1994, Rosinger and Ritchie 1977). It is well-known that the shear-induced deflection in a beam with a constant cross-section along the beam length can be calculated using the theory of Timoshenko beams (Noor and Andersen 1979, Noor and Nemeth 1980, Renton 1991, Shooshtari and Khajavi 2010, Adámek and Valeš 2015). Work on the effect of the shear deformation/stiffness on the buckling and free vibration of battened beams has been performed. For instance, Gantes and Kalochairetis proposed an approximate analytical procedure to estimate the shear effect on the strength of axially and transversely loaded Timoshenko and laced built-up columns (Gantes and Kalochairetis 2012). Wang *et al*. (2002) investigated the stability problem of Timoshenko beams/columns using the matrix method. Wu

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and Chang (2013), Aristizabal (2004), Chen and Li (2013), Chen *et al*. (2014) studied the buckling and vibration problems of battened columns using the Hamilton's principle. In addition, experimental work on the buckling and free vibration of laced and battened beams has also been performed by, for example, EI Aghoury *et al*. (2010), Hashemi and Jafari (2009), Kalochairetis *et al* (2014), Bonab *et al.* (2013). The experimental results demonstrated the importance of shear effect in battened beams when considering the buckling and vibration of the beams.

Note that the bending theory of Timoshenko beams cannot be directly applied to analyzing the deformation of beams with varying cross-section or discontinuous web, such as the castellated beam and the battened beam. At the present for such beams one has to use finite element numerical methods to calculate the deflection of the beams induced by both bending and shear loads (Chen and Li 2013, Sahoo and Rai 2007, EI-Sawy *et al*. 2009, Kalochairetis and Gantes 2011). In this paper, an analytical approach is developed to investigate the shear-induced deflection of battened beams subject to a uniformly distributed transverse quasi-static load, which takes into account not only the shear but also the discrete effects of discontinuous battens in the battened beam. By using the principle of minimum potential energy, a closed-form solution for determining the transverse deflection of a specially designed battened beam, is developed. The present analytical solution is validated using the data obtained from the finite element analysis.

2. Principle of minimum potential energy

Consider a battened beam with the length l=na, where a is the distance between two neighboring battens and n is a constant (n+1 represents the total number of battens along the beam length, see Fig. 1), subjected to a uniformly distributed transverse load. Let $u_1(x)$ and $u_2(x)$ be the

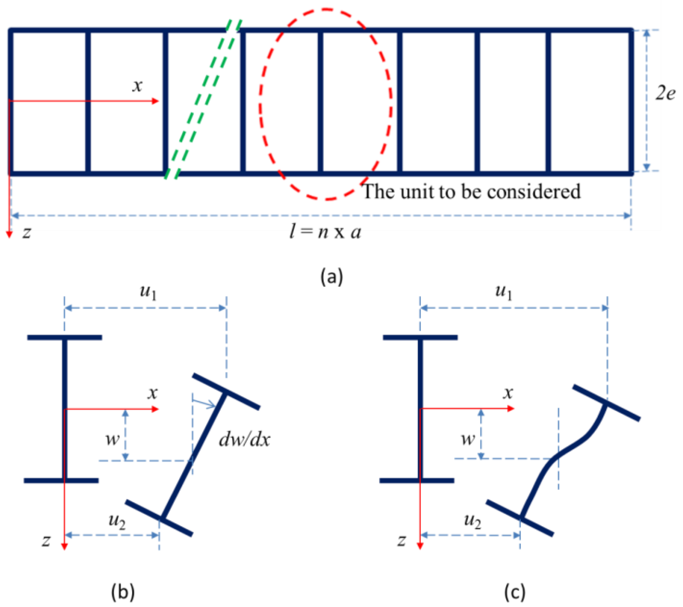

Fig. 1 (a) Analysis model of a battened beam. (b) Displacements of the batten without considering shear effect. (c) Displacements of the batten with considering shear effect axial displacements of the centroids of the upper and lower main members, and $w(x)$ be their transverse displacement (i.e., the two main members have the same transverse displacement). According to the assumption made for displacements shown in Fig. 1(b), the axial displacement at any point with a coordinate z at a section with distance x from the origin can be expressed as follows,

for the upper main member:

$$u_t(x,z)=u_1(x)-(z+e)\frac{\mathrm{d}w}{\mathrm{d}x} \tag{1}$$

for the lower main member:

$$u_b(x,z)=u_2(x)-(z-e)\frac{\mathrm{d}w}{\mathrm{d}x} \tag{2}$$

where $u_t(x, z)$ and $u_b(x, z)$ are the axial displacements of the coordinate point (x, z) defined in the two main members, respectively and e is the half-distance between the centroids of the upper and lower main members.

The strain energy of the two main members due to their axial and transverse displacements thus can be calculated as follows,

$$U_1=\frac{E}{2}\int_0^l\int_A\varepsilon_t^2\mathrm{d}A\mathrm{d}x+\frac{E}{2}\int_0^l\int_A\varepsilon_b^2\mathrm{d}A\mathrm{d}x \tag{3}$$

where E is the Young's modulus, A is the cross-sectional area of the main member, ε_t and ε_b are the axial strains at the coordinate point (x, z) defined in the two main members, which can be expressed in terms of the axial and transverse displacements as follows,

for the upper main member:

$$\varepsilon_t(x,z)=\frac{\partial u_t}{\partial x}=\frac{\mathrm{d}u_1}{\mathrm{d}x}-(z+e)\frac{\mathrm{d}^2w}{\mathrm{d}x^2} \tag{4}$$

for the lower main member:

$$\varepsilon_b(x,z)=\frac{\partial u_b}{\partial x}=\frac{\mathrm{d}u_2}{\mathrm{d}x}-(z-e)\frac{\mathrm{d}^2w}{\mathrm{d}x^2} \tag{5}$$

Substituting Eqs. (4) and (5) into (3), it yields,

$$U_1=\frac{EA}{2}\int_0^l\left[\left(\frac{\mathrm{d}u_1}{\mathrm{d}x}\right)^2+\left(\frac{\mathrm{d}u_2}{\mathrm{d}x}\right)^2\right]\mathrm{d}x+EI\int_0^l\left(\frac{\mathrm{d}^2w}{\mathrm{d}x^2}\right)^2\mathrm{d}x \tag{6}$$

where $I=\int_{A_{upper}}(z+e)^2\,\mathrm{d}A=\int_{A_{lower}}(z-e)^2\,\mathrm{d}A$ is the moment of inertia with respect to local y-axis of the main member, and symbols "A_{upper}" and "A_{lower}" respectively denote the area of upper main member and lower member.

The strain energy of battens can be expressed as follows (Gantes and Kalochairetis 2012, Wang *et al*. 2002),

$$U_2 = \sum_{i=0}^{n} e k_s G A_s \left(\frac{u_1 - u_2}{2e} - \frac{\mathrm{d}w}{\mathrm{d}x} \right)^2 \Bigg|_{x=ia} \tag{7}$$

where G is the shear modulus, A_s is the effective shear area, and k_s is a reduction factor defined as follows,

$$k_s = \frac{3EI_b}{GA_s e^2 + 3EI_b} \tag{8}$$

where I_b is the moment of inertia of the batten. For the simplicity of presentation, the following new notations are used,

$$u_\alpha = \frac{u_1 + u_2}{2} \tag{9}$$

$$u_\beta = \frac{u_1 - u_2}{2} \tag{10}$$

By utilising Eqs. (9) and (10), the total strain energy of the battened beam can be expressed as follows,

$$\begin{aligned} U = U_1 + U_2 = EA \int_0^l \left[\left(\frac{\mathrm{d}u_\alpha}{\mathrm{d}x} \right)^2 + \left(\frac{\mathrm{d}u_\beta}{\mathrm{d}x} \right)^2 \right] \mathrm{d}x + \\ EI \int_0^l \left(\frac{\mathrm{d}^2 w}{\mathrm{d}x^2} \right)^2 \mathrm{d}x + \sum_{i=0}^{n} e k_s G A_s \left(\frac{u_\beta}{e} - \frac{\mathrm{d}w}{\mathrm{d}x} \right)^2 \Bigg|_{x=ia} \end{aligned} \tag{11}$$

The potential of the uniformly distributed load due to the transverse displacement can be expressed as follows,

$$W = q \int_0^l w dx \tag{12}$$

where q is the uniformly distributed load. The use of the principle of minimum potential energy leads,

$$\delta(U - W) = 0 \tag{13}$$

For a simply supported battened beam, $u_\alpha(x)$, $u_\beta(x)$ and $w(x)$ can be assumed as follows,

$$u_\alpha(x) = \sum_{m=1,2,\cdots} A_m \sin \frac{m\pi x}{l} \tag{14}$$

$$u_\beta(x) = \sum_{m=1,2,\cdots} B_m \cos\frac{m\pi x}{l} \tag{15}$$

$$w(x) = \sum_{m=1,2,\cdots} C_m \sin\frac{m\pi x}{l} \tag{16}$$

where A_m, B_m and C_m (m=1, 2, …) are constants. Substituting Eqs. (14)-(16) into (11) and (12), it yields,

$$\begin{gathered} U = \frac{EAl}{2}\sum_{m=1,2,\cdots}\left(\frac{m\pi}{l}\right)^2\left(A_m^2 + B_m^2\right) + \frac{EIl}{2}\sum_{m=1,2,\cdots}\left(\frac{m\pi}{l}\right)^4 C_m^2 + \\ ek_sGA_s\sum_{i=o}^{n}\left[\sum_{m=1,2,\cdots}\left(\frac{B_m}{e} - \frac{m\pi}{l}C_m\right)\cos\frac{mi\pi}{n}\right]^2 \end{gathered} \tag{17}$$

$$W = q\sum_{m=1,2,\cdots}\frac{l}{m\pi}\left[1-(-1)^m\right]C_m \tag{18}$$

Gantes and Kalochairetis (2012) obtained a formula, i.e.,

$$K_{mj} = \sum_{i=o}^{n}\cos\frac{mi\pi}{n}\cos\frac{ji\pi}{n} = \begin{cases} 1+\dfrac{n}{2} & m=j \quad m+j<2n \\ 1+n & m=j \quad m+j=2n \\ \dfrac{1}{2}\left[1+(-1)^{m+j}\right] & m\neq j \quad m+j<2n \\ 1+\dfrac{n}{2} & m\neq j \quad m+j=2n \end{cases} \tag{19}$$

which is helpful for determined A_m, B_m, and C_m. Substituting Eqs. (17) and (18) into (13) and using Eq. (19), it yields,

$$EA\left(\frac{m\pi}{l}\right)^2 A_m = 0 \quad (m=1,2,\cdots) \tag{20}$$

$$EA\left(\frac{m\pi}{l}\right)^2 B_m + \frac{2k_sGA_s}{l}\sum_{j=1}^{n}K_{mj}\left(\frac{B_j}{e} - \frac{j\pi}{l}C_j\right) = 0 \quad (m=1,2,\cdots) \tag{21}$$

$$EI\left(\frac{m\pi}{l}\right)^4 C_m - \frac{2k_sGA_s}{l}\left(\frac{m\pi e}{l}\right)\sum_{j=1}^{n}K_{mj}\left(\frac{B_j}{e} - \frac{j\pi}{l}C_j\right) = \frac{q}{m\pi}\left[1-(-1)^m\right] \quad (m=1,2,\cdots) \tag{22}$$

Eq. (20) indicates $A_m \equiv 0$. This result implies that $u_\alpha=0$, i.e., $u_1=-u_2$, which meas u_1 and u_2 are anti-symmetric with respect to x axis. Eqs. (21) and (22) are coupled due to the shear effect and the discontinuity of battens. However, since in practice only the first few terms are required in the series solution of Eqs. (14)-(16), m will be very small and therefore the second case in Eq. (19) can be excluded because $2n$ will be much greater than m. Also, according to Eq. (19) the value of K_{mj} defined in the third case will be much smaller than that defined in the first or fourth case. Hence, from the numerical point of view the mode coupling defined in the third case can also be ignored. Consequently, the mode coupling that need be considered in the analysis is only between wave m and wave $2n$-m. In this case Eqs. (21) and (22) can be simplified as follows,

$$EA\left(\frac{m\pi}{l}\right)^2 B_m + \frac{(2+n)k_s GA_s}{l}\left[\left(\frac{B_m}{e} - \frac{m\pi}{l}C_m\right) + \left(\frac{B_{2n-m}}{e} - \frac{(2n-m)\pi}{l}C_{2n-m}\right)\right] = 0 \tag{23}$$

$$\begin{aligned} &EI\left(\frac{m\pi}{l}\right)^4 C_m - \frac{(2+n)k_s GA_s}{l}\left(\frac{m\pi e}{l}\right)\left[\left(\frac{B_m}{e} - \frac{m\pi}{l}C_m\right) + \left(\frac{B_{2n-m}}{e} - \frac{(2n-m)\pi}{l}C_{2n-m}\right)\right] \\ &= \frac{q}{m\pi}\left[1-(-1)^m\right] \end{aligned} \tag{24}$$

$$\begin{aligned} &EA\left(\frac{(2n-m)\pi}{l}\right)^2 B_{2n-m} + \frac{(2+n)k_s GA_s}{l} \times \\ &\left[\left(\frac{B_{2n-m}}{e} - \frac{(2n-m)\pi}{l}C_{2n-m}\right) + \left(\frac{B_m}{e} - \frac{m\pi}{l}C_m\right)\right] = 0 \end{aligned} \tag{25}$$

$$\begin{aligned} &EI\left(\frac{(2n-m)\pi}{l}\right)^4 C_{2n-m} - \frac{(2+n)k_s GA_s}{l}\left(\frac{(2n-m)\pi e}{l}\right) \times \left[\left(\frac{B_{2n-m}}{e} - \frac{(2n-m)\pi}{l}C_{2n-m}\right) + \right. \\ &\left.\left(\frac{B_m}{e} - \frac{m\pi}{l}C_m\right)\right] = \frac{q}{(2n-m)\pi}\left[1-(-1)^{2n-m}\right] \end{aligned} \tag{26}$$

Eqs. (23)-(26) can be used to solve coefficients B_m and C_m (m=1, 2, …, $2n$-m). Substituting B_m and C_m into Eqs. (15) and (16), we obtain the final solution of the problem.

3. Finite element analysis and validation of analytical solution

In order to validate the analytical solution, the linear finite element analysis of two-dimensional battened beams with lengths ranging from 2 m to 10 m is accomplished using two-dimensional three-node beam elements built in ANSYS software. The main members and battens used to assemble the battened beams are assumed to be the square hollow section steel members with a side length 50 mm and a wall thickness 5 mm. The material properties of the battened beams are Young’s modulus 200 GPa and Poisson’s ratio 1/3. The boundary conditions for the two main members are assumed to have zero transverse displacements at their ends. To eliminate the axial rigid displacement in the global coordinate system, a zero axial displacement boundary condition in the global coordinate system is applied to the mid-point of the first batten.

Fig. 2 shows the comparison of the maximum deflection of the beam at its middle obtained by solving Eqs. (23)-(26) and using Eq. (16), and those obtained from the finite element analysis. It can be seen from the comparison that the present analytical solution taking into account shear and discrete effects agrees excellently with the results obtained from the finite element analysis.

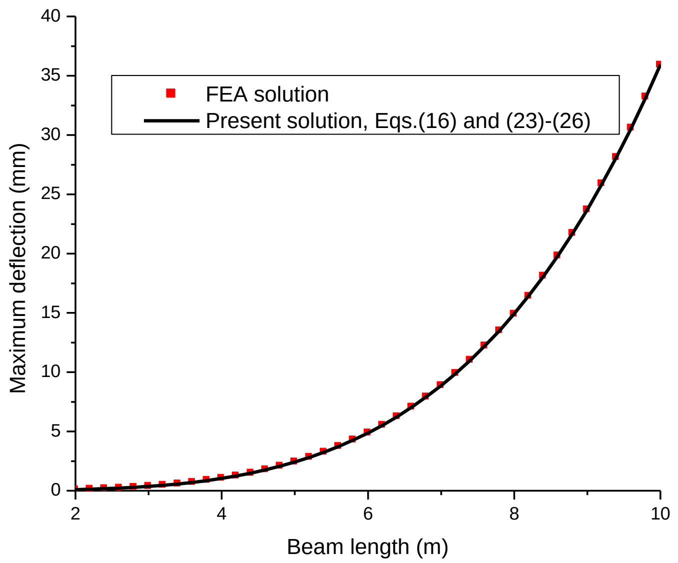

Fig. 2 Comparison of the maximum deflections for a 2-D battened beam (a=200 mm, e=100 mm)

4. Shear and discrete effects of battens

If the shear effect is ignored then $G\to\infty$ and $B_m=(m\pi e/l)C_m$. In this case Eqs. (23)-(26) can be simplified as follows,

$$E(I+e^2A)\left(\frac{m\pi}{l}\right)^4 C_m=\frac{q}{m\pi}\left[1-(-1)^m\right]\ m=1,2,3,\cdots,2n \tag{27}$$

Solve Eq. (27) for C_m, yielding,

$$C_m=\frac{q}{m\pi}\frac{\left[1-(-1)^m\right]}{E(I+e^2A)\left(\dfrac{m\pi}{l}\right)^4}\ m=1,2,3,\cdots,2n \tag{28}$$

Thus, the deflection of the battened beam can be expressed as follows,

$$w(x)=\frac{ql^4}{E(I+e^2A)}\sum_{m=1,3,\cdots}\frac{2}{(m\pi)^5}\sin\frac{m\pi x}{l} \tag{29}$$

The maximum deflection occurs at the middle of the beam and is given by

$$w_{\max o}=w\big|_{x=l/2}=\frac{ql^4}{E(I+e^2A)}\sum_{k=1,2,\cdots}\frac{2}{\pi^5}\frac{(-1)^{k+1}}{(2k-1)^5}=\frac{5ql^4}{384E(2I+2e^2A)} \tag{30}$$

If only the discrete effect is neglected, then Eqs. (23)-(26) are simplified as follows

$$EA\left(\frac{m\pi}{l}\right)^2 B_m + \frac{(2+n)k_s GA_s}{l}\left(\frac{B_m}{e} - \frac{m\pi}{l}C_m\right) = 0 \tag{31}$$

$$EI\left(\frac{m\pi}{l}\right)^4 C_m - \frac{(2+n)k_s GA_s}{l}\left(\frac{m\pi e}{l}\right)\left(\frac{B_m}{e} - \frac{m\pi}{l}C_m\right) = \frac{q}{m\pi}\left[1-(-1)^m\right] \tag{32}$$

Solve Eqs. (31) and (32) for B_m and C_m, yielding,

$$B_m = \frac{(m\pi)(2+n)k_s eGA_s}{eEA(m\pi)^2 + (2+n)k_s lGA_s} C_m \quad m = 1, 2, \cdots, 2n \tag{33}$$

$$C_m = \frac{ql^4}{EI + e^2 EA} \times \frac{1-(-1)^m}{(m\pi)^5} \times \frac{I + e^2 A}{I + \dfrac{e^2 A}{1 + \dfrac{eEA(m\pi)^2}{(n+2)k_s lGA_s}}} \quad m = 1, 2, \cdots, 2n \tag{34}$$

For middle and long beams where $(n+2)k_s lGA_s > eEA(m\pi)^2$, Eq. (34) can be further simplified into

$$C_m = \frac{ql^4}{EI + e^2 EA} \times \frac{1-(-1)^m}{(m\pi)^5} \times \left(1 + \frac{e^2 A}{I + e^2 A} \times \frac{eEA(m\pi)^2}{(n+2)k_s lGA_s}\right) \quad m = 1, 2, \cdots, 2n \tag{35}$$

Thus, the deflection of the battened beam can be expressed as follows,

$$w(x) = \frac{ql^4}{E(I + e^2 A)} \sum_{m=1,3,\cdots} \frac{2}{(m\pi)^5}\left(1 + \frac{e^2 A}{I + e^2 A} \times \frac{eEA(m\pi)^2}{(n+2)k_s lGA_s}\right) \sin\frac{m\pi x}{l} \tag{36}$$

The maximum deflection occurs at the middle of the beam and is given by

$$\begin{aligned} w\big|_{x=l/2} &= \frac{ql^4}{E(I + e^2 A)} \sum_{k=1,2,\cdots}\left[\frac{2}{\pi^5}\frac{(-1)^{k+1}}{(2k-1)^5} + \frac{e^2 A}{I + e^2 A}\frac{eEA}{(n+2)k_s lGA_s}\frac{2}{\pi^3}\frac{(-1)^{k+1}}{(2k-1)^3}\right] \\ &= \frac{5ql^4}{384E(2I + 2e^2 A)}\left[1 + \frac{e^2 A}{I + e^2 A}\frac{48eEA}{5(n+2)k_s lGA_s}\right] \end{aligned} \tag{37}$$

It is obvious from Eq. (37) that, the first term in the brackets is the deflection generated by the bending load, whereas the second term in the brackets represents the deflection generated by the shear load. Eq. (37) indicates that the shear effect increases with the cross-section area of the main member but decreases with the increase of the cross-section area of the batten. Also, the longer the battened beam or the larger the moment of inertia of the main member, the smaller the shear effect. It is obvious from Eq. (37) that, if $G\to\infty$ then Eq. (37) tends to Eq. (30) in which case the shear effect is excluded.

Figs. 3-5 show the shear and discrete effects on the maximum deflections of the battened beams

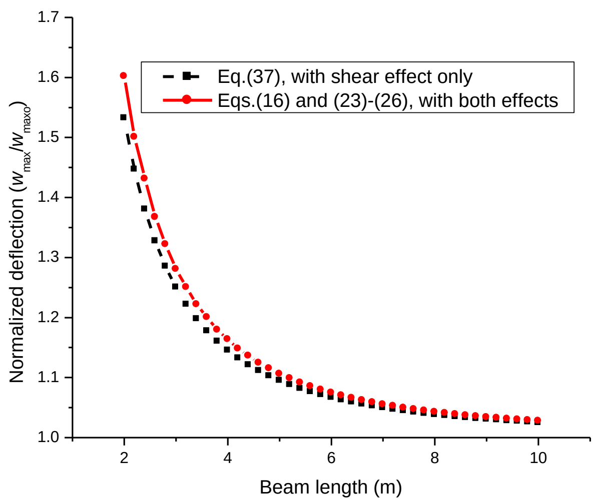

Fig. 3 Maximum deflections of a battened beam (a=200 mm, e=100 mm)

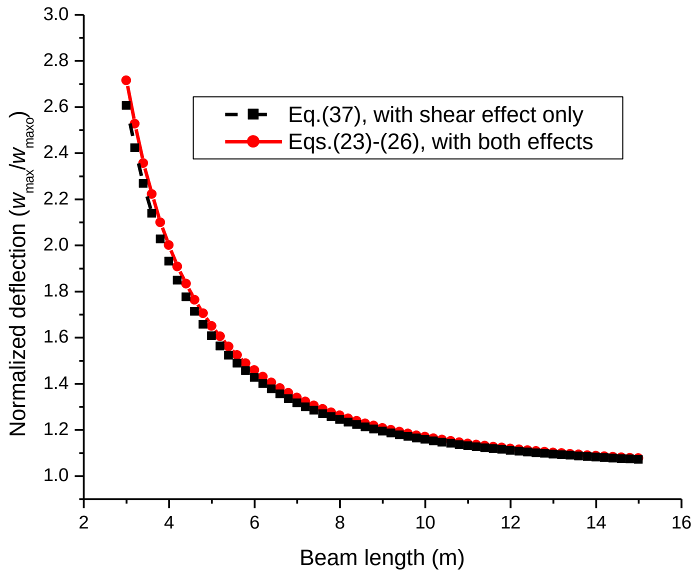

Fig. 4 Maximum deflections of a battened beam (a=200 mm, e=200 mm)

of different batten spacings (a) and batten lengths ($2e$). The section dimensions of the two main members and battens employed herein are identical to those used in the finite element analysis; so do the material properties. The deflection shown in the figures is normalized using the maximum deflection without considering the shear and discrete effects given by Eq. (30). It is evident from these figures that the shear effect of battens on the deflection of battened beams is important, particularly for short and medium length beams. Ignoring shear effect could lead to a significant under-prediction of the deflection. Also it can be observed from the comparison of the three figures that the shear effect increases with the length of batten members. Compared to the shear

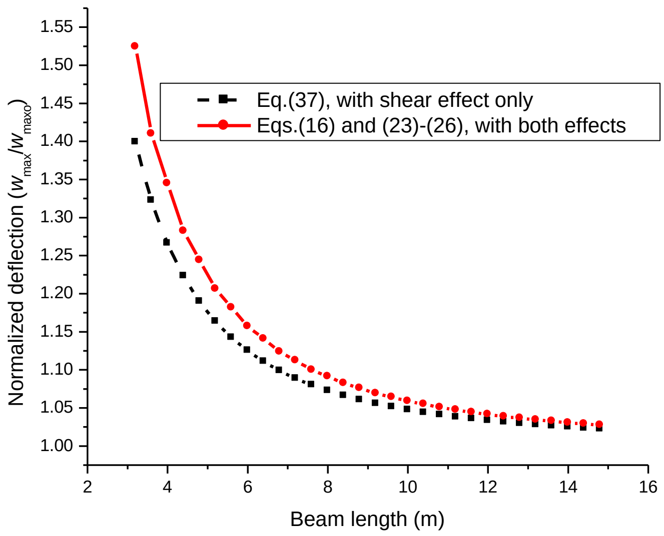

Fig. 5 Maximum deflections of a battened beam (a=400 mm, e=100 mm)

effect, the discrete effect seems not so significant. The remarkable discrete effect is found when the batten spacing is large as is demonstrated in Fig. 5.

5. Conclusions

This paper has presented the theoretical and numerical analyses for determining the deflection of simply supported battened beams subjected to a uniformly distributed transverse load. The analyses consider not only the shear effect but also the discrete effect of battens on the transverse deflection of battened beams. From the present study the following conclusions can be drawn:

- The present analytical results are in excellent agreement with those obtained from the finite element analysis, which demonstrates the appropriateness of the proposed approach.
- Shear effect on the deflection of battened beams is very important, particularly for short and medium length beams. Ignoring the shear effect could lead to an under-estimation of the deflection. In contrast, the discrete effect on the deflection is much small. The noticeable discrete effect is found only when the beam has large batten spacing.
- The shear effect on the transverse deflection of battened beams increases with the cross-section area of the main member but decreases with the cross-section area of the batten. The longer the battened beam or the larger the moment of inertia of the main member, the smaller the shear effect.

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References

Adámek, V. and Valeš, F. (2015), “Analytical solution for a heterogeneous Timoshenko beam subjected to an arbitrary dynamic transverse load“, *Eur. J. Mech. A/Solid.*, **49**, 373-381.

Aly, T., Elchalakani, M., Thayalan, P. and Patnaikuni, I. (2010), “Incremental collapse threshold for pushout resistance of circular concrete filled steel tubular columns”, *J. Constr. Steel Res.*, **66**, 11-18.

Aristizabal-Ochoa, J.D. (2004), “Column stability and minimum lateral bracing: effects of shear deformations”, *J. Eng. Mech.*, **130**(10), 1223-1232.

Banerjee, J.R. and Williams, F.W. (1994), “The effect of shear deformation on the critical buckling of columns”, *J. Sound Vib*., **174**(5), 607-616.

Banerjee, J.R. and Williams, FW. (1983), “Vibration characteristics of self-expanding stayed columns for use in space”, *J. Sound Vib*., **90**(2), 245-261.

Bonab, A.P., Hashemi, B.H. and Hosseini, M. (2013), “Experimental evaluation of the elastic buckling and compressive capacity of laced columns”, *J. Constr. Steel Res.*, **86**, 66-73.

Chen, J.K. and Li, L.Y. (2013), “Elastic axially compressed buckling of battened columns”, *Int. J. Mech. Sci.*, **77**, 1-7.

Chen, J.K., Kim, B. and Li, L.Y. (2014), “Analytical approach for transverse vibration analysis of castellated beams”, *Int. J. Struct. Stab. Dyn*., **14(3)**, 1-17.

Chung, H. and Emms, G. (2008), “Fourier series solutions to the vibration of rectangular lightweight floor/ceiling structures”, *ACTA Acust. United AC*., **94**(3), 401-409.

EI-Sawy, K.M., Sweedan, A.M.I. and Martini, M.I. (2009), “Major-axis elastic bucking of axially loaded castellated steel columns”, *Thin Wall Struct*., **47**(11), 1295-1304.

EI Aghoury, M.A., Salem, A.H., Hanna, M.T. and Amoush, E.A. (2010), “Experimental investigation for the behaviour of battened beam-columns composed of four equal slender angles”, *Thin Wall. Struct*., **48**(9), 669-683.

Gantes, C.J. and Kalochairetis, K.E. (2012), “Axially and transversely loaded Timoshenko and laced built-up columns with arbitrary supports”, *J. Constr. Steel Res*., **77**, 95-106.

Greschik, G. (2008), “Truss beam with tendon diagonals: mechanics and designs”, *AIAA J*., **46**(3), 557-567.

Hashemi, B.H. and Jafari, M.A. (2009), “Experimental evaluation of elastic critical load in batten colums”, *J. Constr. Steel Res.*, **65**(1), 125-131.

Kalochairetis, K.E. and Gantes, C.J. (2011), “Numerical and analytical investigation of collapse loads of laced built-up columns”, *Comput. Struct.*, **89**(11-12), 1166-1176.

Kalochairetis, K.E., Gantes, C.J. and Lignos, X.A. (2014), “Experimental and numerical investigation of eccentrically loaded laced built-up steel columns”, *J. Constr. Steel Res.*, **101**, 66-81.

Noor, A.K. and Andersen, C.M. (1979), “Analysis of beam-like lattice trusses”, *Comput. Meth. Appl*., **20**(1), 53-70.

Noor, A.K. and Nemeth, M.P. (1980), “Analysis of spatial beamlike lattices with rigid joints”, *Comput. Meth. Appl*., **24**(1), 35-59.

Renton, J.D. (1991), “Generalized beam theory applied to shear stiffness”, *Int. J. Solid. Struct.*, **27**(15), 1955-1967.

Rosinger, H.E. and Ritchie, I.G. (1977), “On Timoshenko's correction for shear in vibrating isotropic beams”, *J. Phys. D. Appl. Phys.*, **10**(11), 1461-1466.

Sahoo, D.R. and Rai, D.C. (2007), “Built-up battened columns under lateral cyclic loading”, *Thin Wall Struct*., **45**(5), 552-562.

Shooshtari, A. and Khajavi, R. (2010), “An efficient procedure to find shape functions and stiffness matrices of nonprismatic Euler-Bernoulli and Timoshenko beam element“, *Eur. J. Mech. A/Solid.*, **29**(5), 826-836.

Wang, C.M., Ng, K.H. and Kitipornchai, S. (2002), “Stability criteria for Timoshenko columns with intermediate and end concentrated axial load”, *J. Constr. Steel Res.*, **58**, 1177-1193.

Wu, J.S. and Chang, B.H. (2013), "Free vibration of axial-loaded multi-step Timoshenko beam carrying arbitrary concentrated elements using continuous-mass transfer matrix method", *Eur. J. Mech. A-Solid.*, **38**, 20-37.

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