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Dynamic response of curved Timoshenko beams resting on viscoelastic foundation

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Abstract. Curved beams' dynamic behavior on viscoelastic foundation is the subject of the current paper. By rewritten the Timoshenko beams theory formulation for the curved and twisted spatial rods, governing equations are obtained for the circular beams on viscoelastic foundation. Using the complementary functions method (CFM), in Laplace domain, an ordinary differential equation is solved and then those results are transformed to real space by Durbin's algorithm. Verification of the proposed method is illustrated by solving an example by variating foundation parameters.

Keywords: inverse laplace transform; complementary functions method; circular beam; viscoelastic foundation; forced vibration

1. Introduction

Due to the widespread engineering applications of beams which are resting on elastic foundations, numerous models have been proposed for those beams to investigate their static deflection and dynamic responses. In the literature, many researches have been performed to explore the curved beams, which are rested on elastic foundation, free vibration analysis. For instance, natural frequencies of the curved beams were studied by Wang and Brannen (1982) and it was illustrated effects of opening angle and foundation parameter. Natural frequencies of the Winkler and Pasternak foundation based curved beams was examined by Issa (1988) and Issa *et al*. (1990). Tensionless Winkler foundation based elastic circular rings static and vibration analysis problem has been studied by Celep (1990). Arbitrary thick beams, which are rested on Pasternak foundation, have been studied to find out the bending and free vibration behavior by Chen *et al*. (2004). Natural frequencies and vibration modes of thin ring elastic foundation have been studied by Wu and Parker (2006). Natural frequencies of the thin-walled curved beams on elastic foundation were searched by Kim *et al*. (2007). Dynamic response of the Timoshenko beam with variable cross-section on Pasternak foundation was studied by Zhu and Leung (2009). By combining the differential quadrature method and finite element method, dynamic analysis of thick plates on elastic foundation was solved by Dehghan and Baradaran (2011). Straight and circular Timoshenko beams' static free and vibration analysis has been performed by Calim and Akkurt (2011).

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As in elastic foundation researches, a few studies were also performed related to dynamic response of beams resting on viscoelastic foundation. Advantages of the Pasternak foundation model over the other foundation models have demonstrated by Kerr (1964). Dynamic behavior of the railway systems which are constructed on viscoelastic foundation under harmonic moving load have investigated by Chen *et al*. (2001). Viscoelastic foundation based Timoshenko beams' vibration analysis was studied by Verichev and Metrikine (2002). An effective numerical model has been presented to solve the wave propagation problems by Liu and Li (2003). Pasternak type linear and non-linear viscoelastic foundation based Timoshenko beams' under moving load have been studied to find out the dynamic responses by Kargarnovin and Younesian (2004) and Kargarnovin *et al*. (2005). Dynamic behaviors of beams on viscoelastic foundation were studied by Muscolino and Palmeri (2007) and Calim (2009). By subjecting the moving load, dynamic behavior of the six parameters foundation based Timoshenko beams were determined by Yang *et al*. (2013). Non-linear viscoelastic foundation based infinite Timoshenko beams' dynamic behaviors were explored by Ding *et al*. (2013).

As exemplified in the above given paragraphs, the stability, dynamic and static analysis of the plates and beams rested on elastic foundation have been explored in detail, however, to the best knowledge of the author, there is dearth of researches on the transient analysis of curved Timoshenko beams resting on two-parameter viscoelastic foundation. Therefore, in the current paper, forced vibration of the curved beams under impulsive load was studied. In the derivation of the governing equation, effect of shear deformation and rotary inertia, curvature of the axis are taken into account. By applying the CFM, element stiffness matrix in Laplace domain is determined (Çalım 2009 a, b, c, Çalım and Akkurt 2011, Çalım 2012). By using the Durbin's procedure, Laplace domain is transformed to the time domain (Eratlı *et al*. 2014, Celebi *et al*. 2016, Çalım 2016) and the solutions are obtained for the forced vibration of curved beams.

2. The governing equations

To govern the differential equation, a spatial beam which is curved and torsion, is taken into consideration (Fig. 1). The tangent, normal and bi-normal unit vectors (**t**, **n** and **b)** are used to identify moving coordinate system. With the aid of Frenet equation, relationships among the **t, n,** and **b** unit vectors might be attained (Sokolnikoff and Redheffer 1958)

$$\frac{\partial \mathbf{t}}{\partial s} = \chi \mathbf{n}\,, \quad \frac{\partial \mathbf{n}}{\partial s} = \tau \mathbf{b} - \chi \mathbf{t}\,, \quad \frac{\partial \mathbf{b}}{\partial s} = -\tau \mathbf{n} \tag{1}$$

where axis natural twist and curvature are represented by τ and χ, respectively.

Considering the displacement, rotation, inertia moment and force vector of any place on the beam axis be denoted by $\mathbf{U}(s,t)$, $\mathbf{\Omega}(s,t)$, $\mathbf{T}(s,t)$ and $\mathbf{M}(s,t)$. Accepting the deformations are infinitesimal, and an isotropic, linear elastic and homogenous material is used in rod production, for the space rod, in vectorial form, governing equation is proposed as follows

$$\frac{\partial \mathbf{U}}{\partial s} + \mathbf{t} \times \mathbf{\Omega} - \mathbf{C}^{-1}\mathbf{T} = \mathbf{0}, \qquad \frac{\partial \mathbf{\Omega}}{\partial s} - \mathbf{D}^{-1}\mathbf{M} = \mathbf{0} \tag{2}$$

$$\frac{\partial \mathbf{T}}{\partial s} + \mathbf{p}^{(ex)} = \mathbf{p}^{(in)}, \qquad \frac{\partial \mathbf{M}}{\partial s} + \mathbf{t} \times \mathbf{T} + \mathbf{m}^{(ex)} = \mathbf{m}^{(in)} \tag{3}$$

In above given equation, external distributed load and moment vector are symbolized as $\mathbf{p}^{(ex)}$ and $\mathbf{m}^{(ex)}$, respectively.

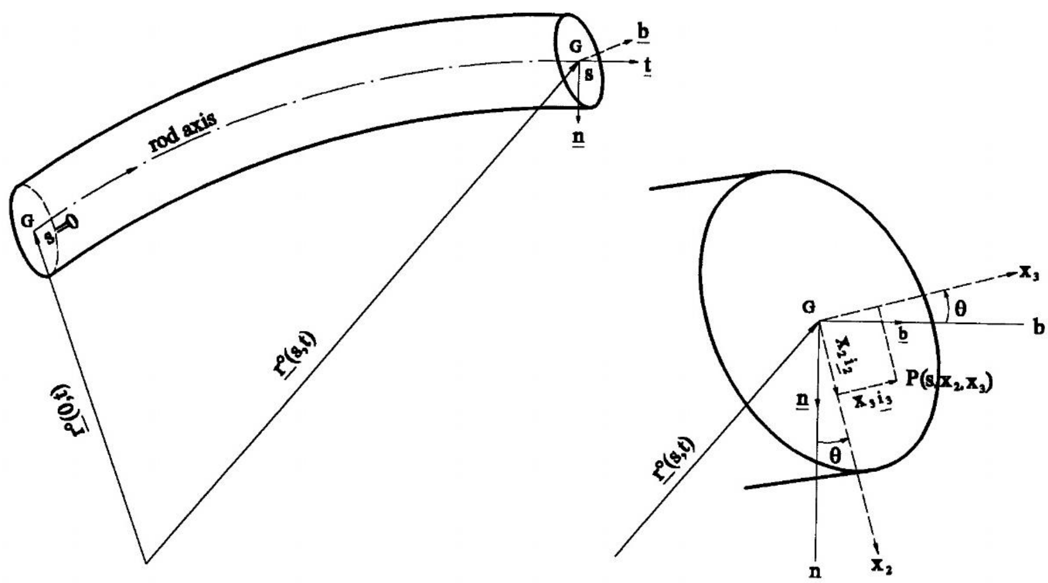


Fig. 1 The rod geometry

Inertia force, moment of the beam and mass density are considered as

$$p_i^{(in)} = -\rho A \frac{\partial U_i^o}{\partial t^2}, \qquad m_i^{(in)} = -\rho I_i \frac{\partial \Omega_i^o}{\partial t^2} \qquad (\, i = t, n, b\,) \tag{4}$$

Matrices of the **C** and **D** are defined as

$$[C] = \begin{bmatrix} EA & 0 & 0 \\ 0 & GA/\alpha_n & 0 \\ 0 & 0 & GA/\alpha_b \end{bmatrix} [D] = \begin{bmatrix} GI_t & 0 & 0 \\ 0 & EI_n & 0 \\ 0 & 0 & EI_b \end{bmatrix} \tag{5}$$

where A is area of cross-section, E is the elastic modules, G is the shear modules, α_n and α_b are shear coefficients, I_t , I_n, I_b are inertia torsional and bending moments.

Let $\mathbf{p}^{(ex)}$ and $\mathbf{m}^{(ex)}$ might be divided into two subsection such as

$$\mathbf{p}^{(ex)} = \mathbf{p}^{\,e} - \mathbf{p}^{f}, \quad \mathbf{m}^{(ex)} = \mathbf{m}^{e} - \mathbf{m}^{f} \tag{6}$$

In above equation, superscripts e and f represents the beam reaction of the loading and foundation, orderly. Response of the foundation force and moment can be calculated by using following equations

$$p_i^f = k_i U_i^o + \eta_i \frac{\partial U_i^o}{\partial t}, \quad m_i^f = \left(k_1\right)_i \Omega_i^o + \mu_i \frac{\partial \Omega_i^o}{\partial t} \quad (i = t, n, b) \tag{7}$$

where k, k_1 , η and μ are the spring constants and viscosity coefficients, respectively.

Considering the congruent of the centroid and shear center, the **n**, **b** axes turn the main axes out. Differential equations in Laplace domain are attained for the dynamic behavior of circular Timoshenko beams regarding to moving coordinate system as follows

$$\frac{d\overline{U}_t}{d\phi} = \overline{U}_n + \frac{R}{EA}\overline{T}_t$$

$$\frac{d\overline{U}_n}{d\phi} = -\overline{U}_t + R\overline{\Omega}_b + R\frac{\alpha_n}{GA}\overline{T}_n$$

$$\frac{d\overline{U}_b}{d\phi} = -R\overline{\Omega}_n + R\frac{\alpha_b}{GA}\overline{T}_b$$

$$\frac{d\overline{\Omega}_t}{d\phi} = \overline{\Omega}_n + \frac{R}{GI_t}\overline{M}_t$$

$$\frac{d\overline{\Omega}_n}{d\phi} = -\overline{\Omega}_t + \frac{R}{EI_n}\overline{M}_n$$

$$\frac{d\overline{\Omega}_b}{d\phi} = \frac{R}{EI_b}\overline{M}_b$$

$$\frac{d\overline{T}_t}{d\phi} = R\left[z^2\rho A + z\eta_t + k_t\right]\overline{U}_t + \overline{T}_n - R\,\overline{p}_t^{(ex)}$$

$$\frac{d\overline{T}_n}{d\phi} = R\left[z^2\rho A + z\eta_n + k_n\right]\overline{U}_n - \overline{T}_t - R\,\overline{p}_n^{(ex)}$$

$$\frac{d\overline{T}_b}{d\phi} = R\left[z^2\rho A + z\eta_b + k_b\right]\overline{U}_b - R\,\overline{p}_b^{(ex)}$$

$$\frac{d\overline{M}_t}{d\phi} = R\left[z^2\rho I_t + z\mu_t + (k_1)_t\right]\overline{\Omega}_t + \overline{M}_n - R\,\overline{m}_t^{(ex)}$$

$$\frac{d\overline{M}_n}{d\phi} = R\left[z^2\rho I_n + z\mu_n + (k_1)_n\right]\overline{\Omega}_n - \overline{M}_t + R\overline{T}_b - R\,\overline{m}_n^{(ex)}$$

$$\frac{d\overline{M}_b}{d\phi} = R\left[z^2\rho I_b + z\mu_b + (k_1)_b\right]\overline{\Omega}_b - R\overline{T}_n - R\,\overline{m}_b^{(ex)} \tag{8}$$

When $\chi=1/R$ *and* $ds=R\,d\phi$, it symbolized the circular beam (Fig. 2). The matrix $\mathbf{Y}(\phi, t)$ is described as

$$\mathbf{Y}(\phi,t) = \{U_t\,, U_n\,, U_b\,, \Omega_t\,, \Omega_n\,, \Omega_b\,, T_t\,, T_n\,, T_b\,, M_t\,, M_n\,, M_b\,,\}^T \tag{9}$$

Laplace transform of Eq. (9) due to time $L[\mathbf{Y}(\phi,t)] = \overline{\mathbf{Y}}(\phi,z)$, for $t>0$ is described as

$$\overline{\mathbf{Y}}(\phi,z) = \int_0^\infty \mathbf{Y}(\phi,t)e^{-zt}\,dt \tag{10}$$

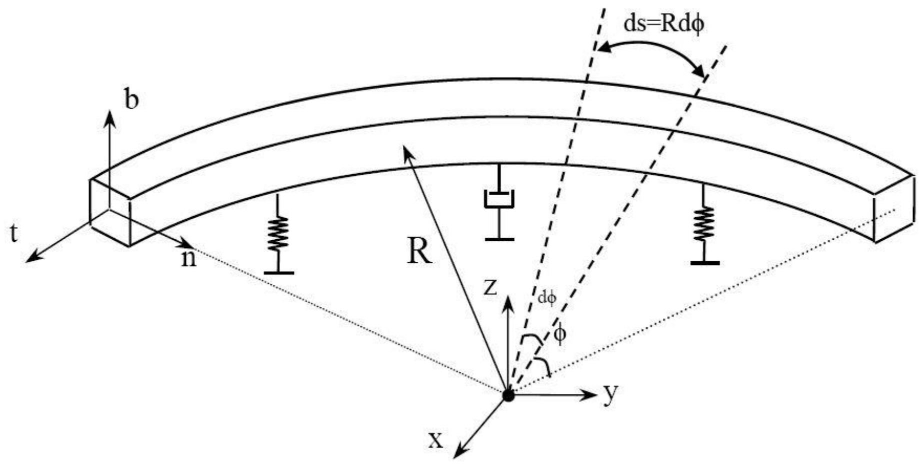


Fig. 2 Circular beam on viscoelastic foundation

In above equation, z is Laplace parameter. By using the CFM in Laplace domain, dynamic stiffness matrix and element forces are examined (please look Refs. (Temel *et al*. 2005, Çalım 2012)). By using Durbin's algorithm, calculations were converted to Laplace domain to the time domain (Temel *et al*. 2005, Eratlı *et al.* 2014).

3. Modified Durbin's procedure

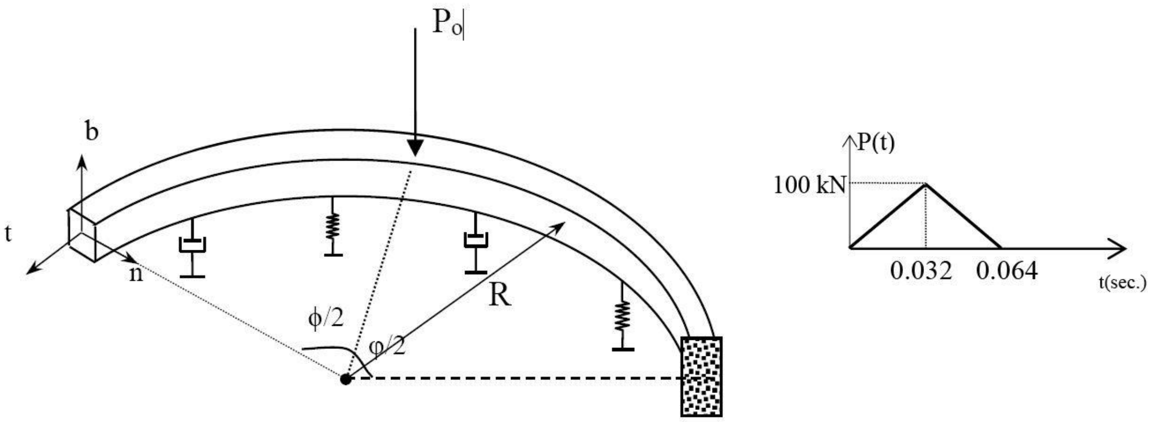
To attain the numerical values in time domain, Laplace transform technique is essential. To perform that mission, Durbin's algorithm constructed on the FFT is applied (Durbin 1974). Durbin's formulation is given as follows

$$f(t_j) \cong \frac{2e^{a\,j\Delta t}}{T}\left[-\frac{1}{2}Re\left\{\overline{F}(a)\right\}+Re\left\{\sum_{k=0}^{N-1}\left(\overline{F}(s_k)L_k\right)e^{\left(i\frac{2\pi}{N}\right)}\right\}\right] \quad (j=0,1,2,\cdots,N-1) \tag{11}$$

in which $s_k = a + ik2\pi/T$, $N = T/\Delta t$ where s_k is the kth Laplace transform parameter, Δt is the time increment and T is the sampling time interval. In the literature (Durbin 1974), to get a good results, aT values must be accepted in the range of 5 to 10. Therefore, in our case, when solving the numerical examples, that value accepted as 6. Moreover, Narayanan (1979) stated that if the each value of the obtained results are modified with Lanczos (L_k) factors, considerably better results might be gathered.

$$L_k = \begin{cases} 0\,, & k = 0 \\ \frac{Sin\,(k\pi\,/N)}{k\pi/N}, & k > 0 \end{cases} \tag{12}$$

4. Numerical example



(a) (b)

Fig. 3 (a) Circular beam at fixed ends on viscoelastic foundation (b) A triangular impulsive load

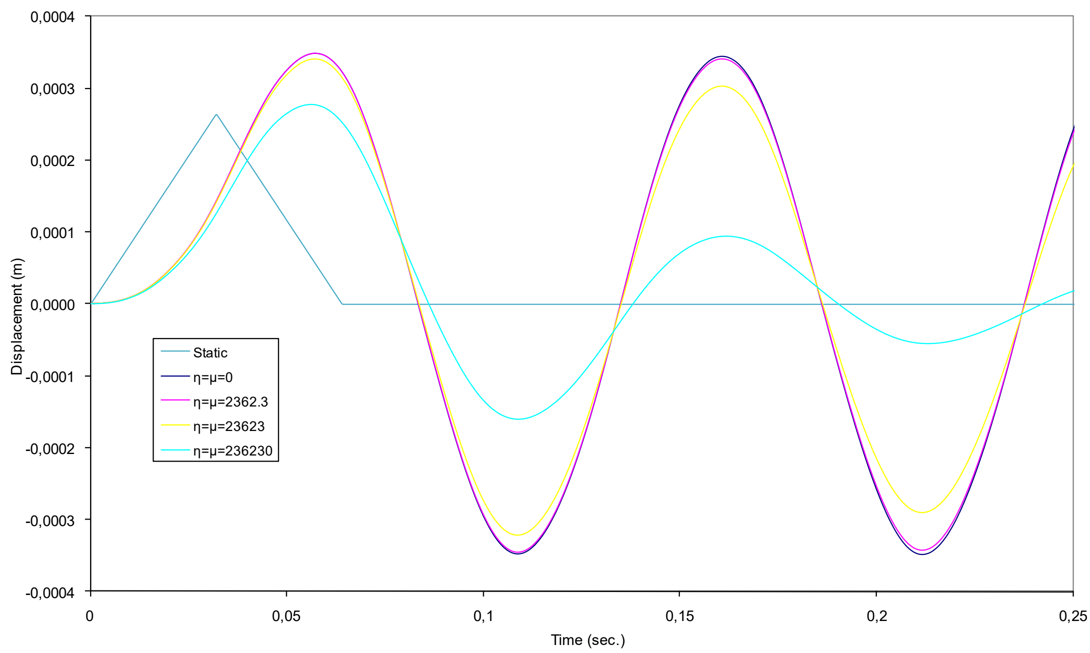


Fig. 4 Displacement versus time midpoint of the semi-circular beam (R/h=5)

In present research, a Fortran based computer program is developed to examine dynamic analysis of the curved beams which is rested on viscoelastic foundation. Using the CFM, in Laplace domain, an ordinary differential equation is solved. Initial value problem lean on CFM is determined by using the Runge-Kutta procedure.

Example. A clamped ends semi-circular beam on viscoelastic foundation is taken into account as seen in Fig. 3(a). The beam has Young's modulus E=47.24 GPa, shear modulus G=19.68 GPa,

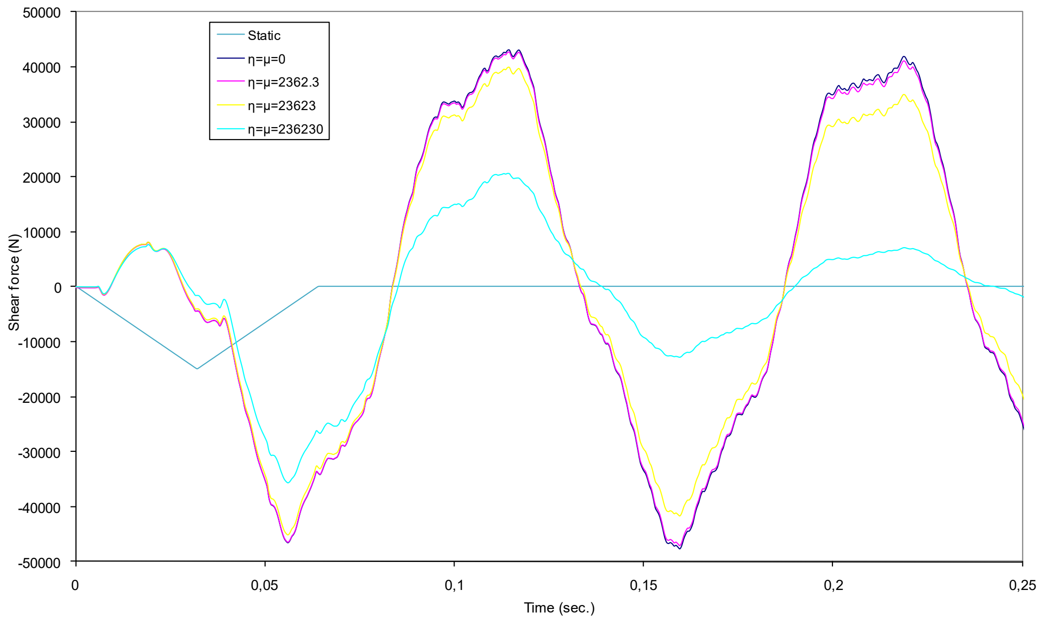


Fig. 5 Shear force versus time at the fixed end of the semi-circular beam (R/h=5)

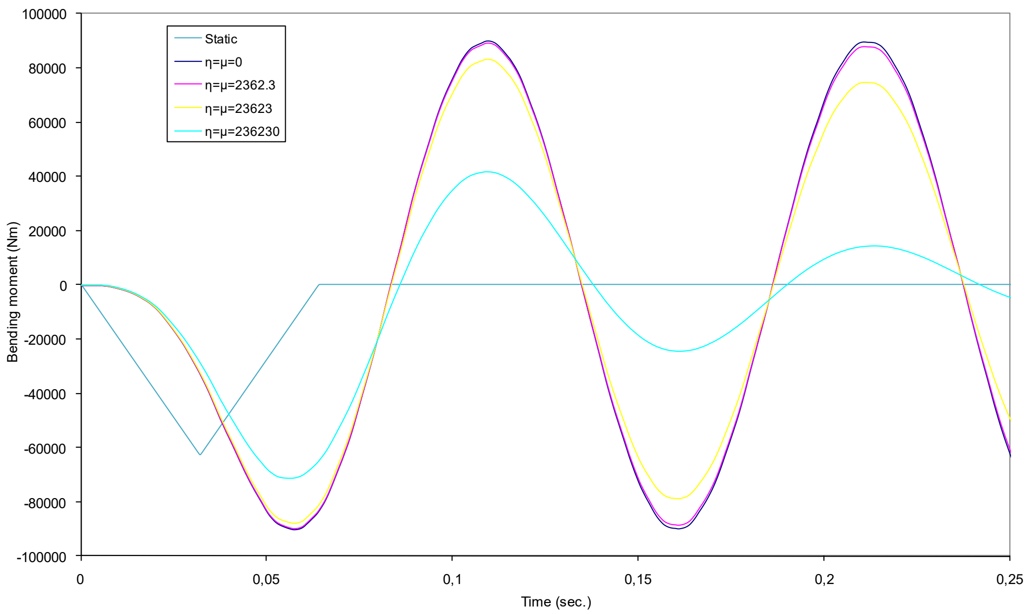


Fig. 6 Bending moment versus time at the fixed end of the semi-circular beam (R/h=5)

material density ρ=5000 kg/m^3, R=7.63 m and b=h=0.762 m. In this case, the stiffness of foundation is k_b=23.623 MPa and $(k_1)_t$=1143 kNm/m, viscosity coefficients of η=0, 2362.3, 23623, 236230 Ns/m^2 and μ= 0, 2362.3, 23623, 236230 Ns.

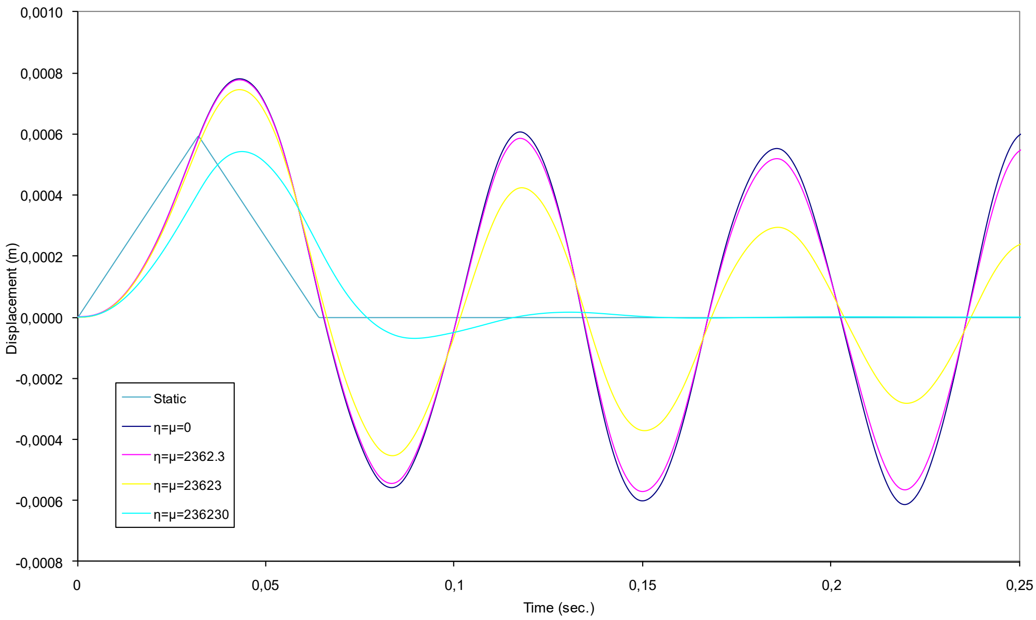


Fig. 7 Displacement versus time midpoint of the semi-circular beam (*R*/*h*=10)

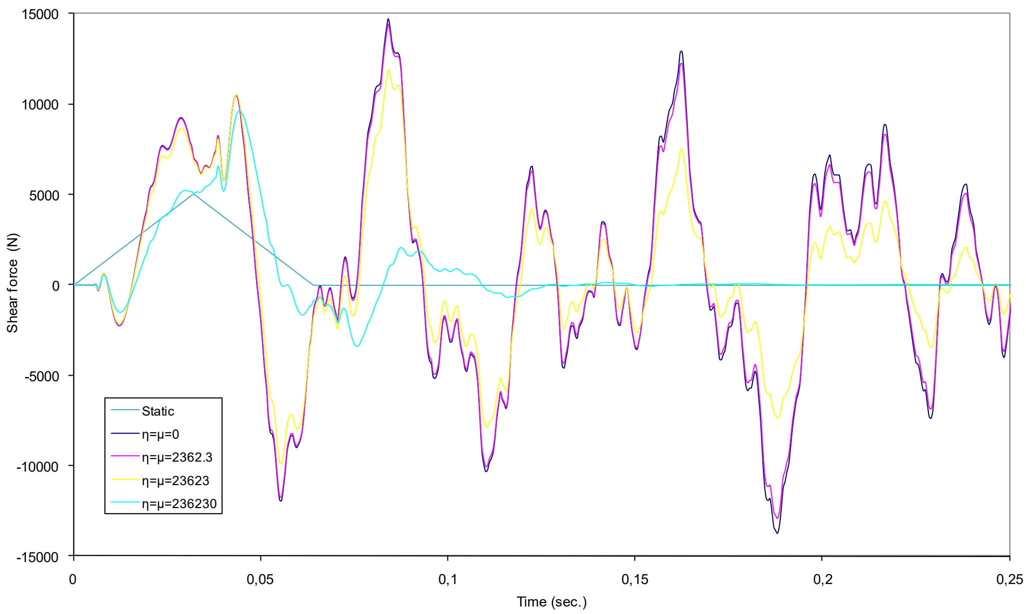


Fig. 8 Shear force versus time at the fixed end of the semi-circular beam (*R*/*h*=10)

An impulsive load in triangular distributed form (Fig. 3(b)) with the amplitude P_o=100 kN is implemented at the mid position of the beam. As an increment rate of time Δt, is accepted as 0.0005 sec in the numerical solutions. Figs. 4-12 elucidate the shear force and bending moment

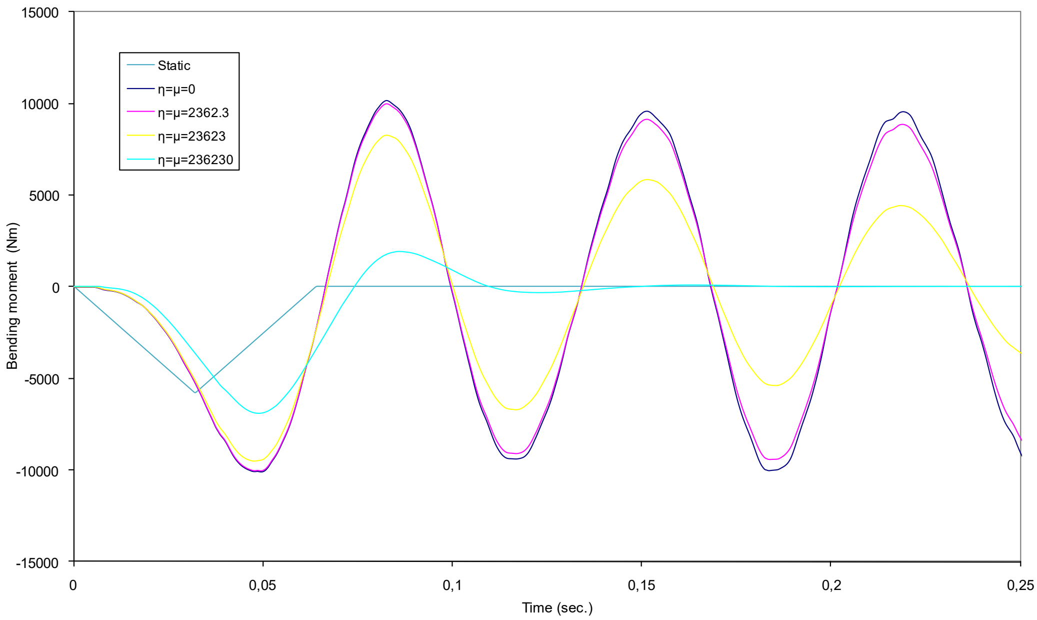


Fig. 9 Bending moment versus time at the fixed end of the semi-circular beam (*R/h*=10)

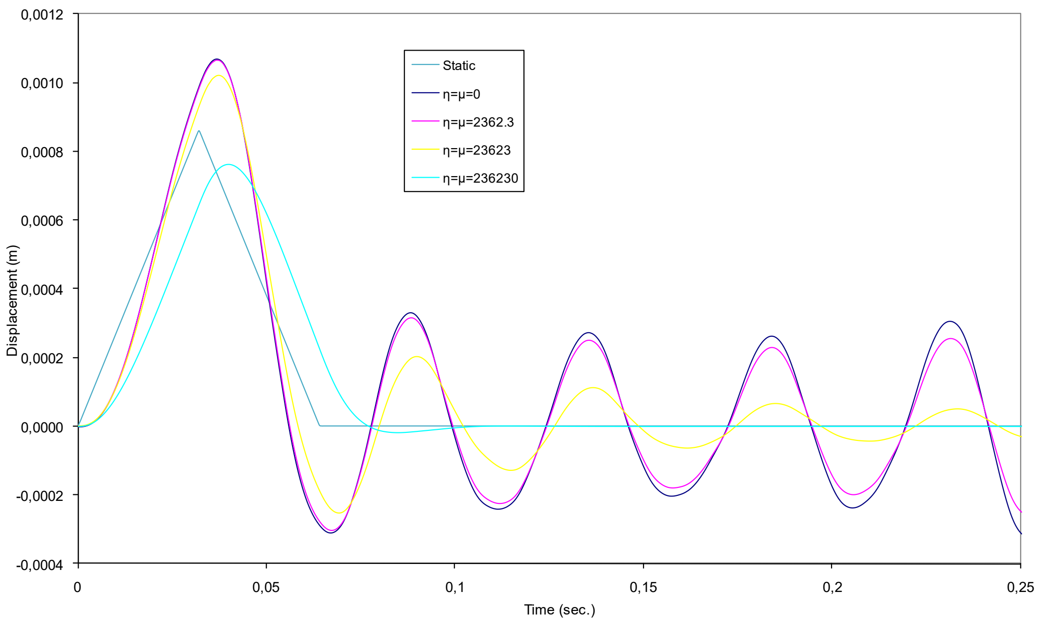


Fig. 10 Displacement versus time midpoint of the semi-circular beam (*R/h*=15)

clamped end and displacement at the mid position of the beam.

The effects of the viscosity coefficients and the ratio *R/h* on forced vibration of the circular beam resting on viscoelastic foundation are investigated. It is observed that the viscosity

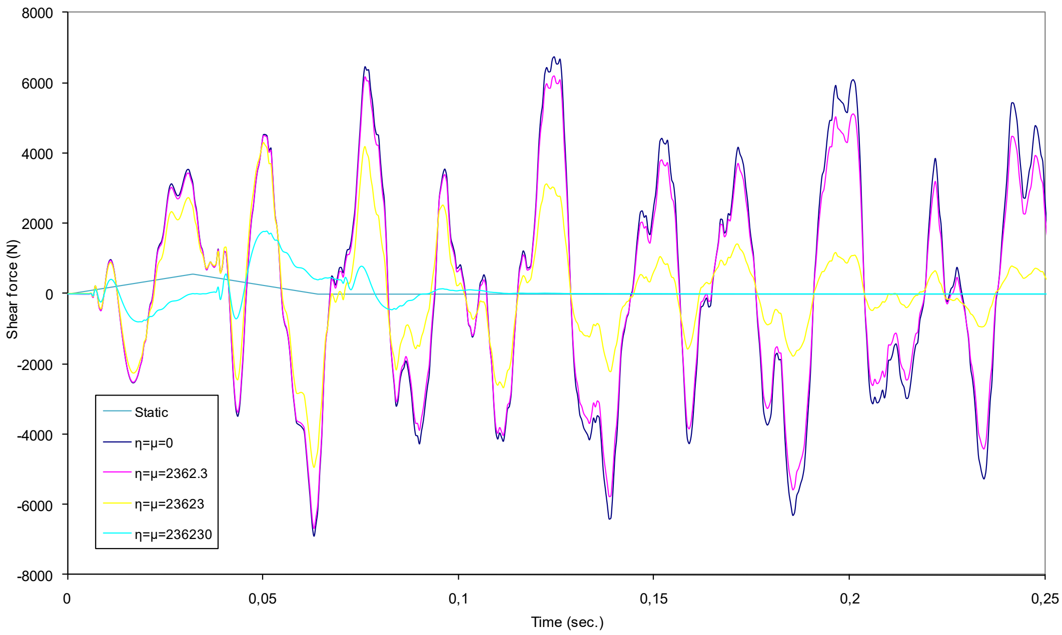


Fig. 11 Shear force versus time at the fixed end of the semi-circular beam (*R/h*=15)

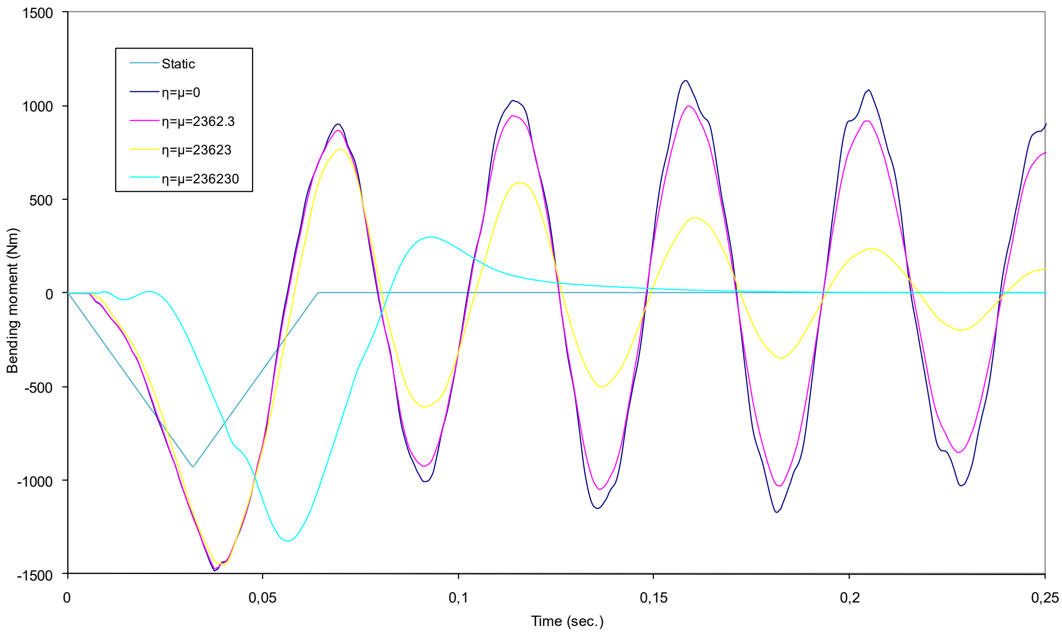


Fig. 12 Bending moment versus time at the fixed end of the semi-circular beam (*R/h*=15)

coefficients increases the displacement decreases. Furthermore, the effect of the ratio *R/h* on forced vibration of semicircular beam clamped ends is examined. It is monitored that increase of

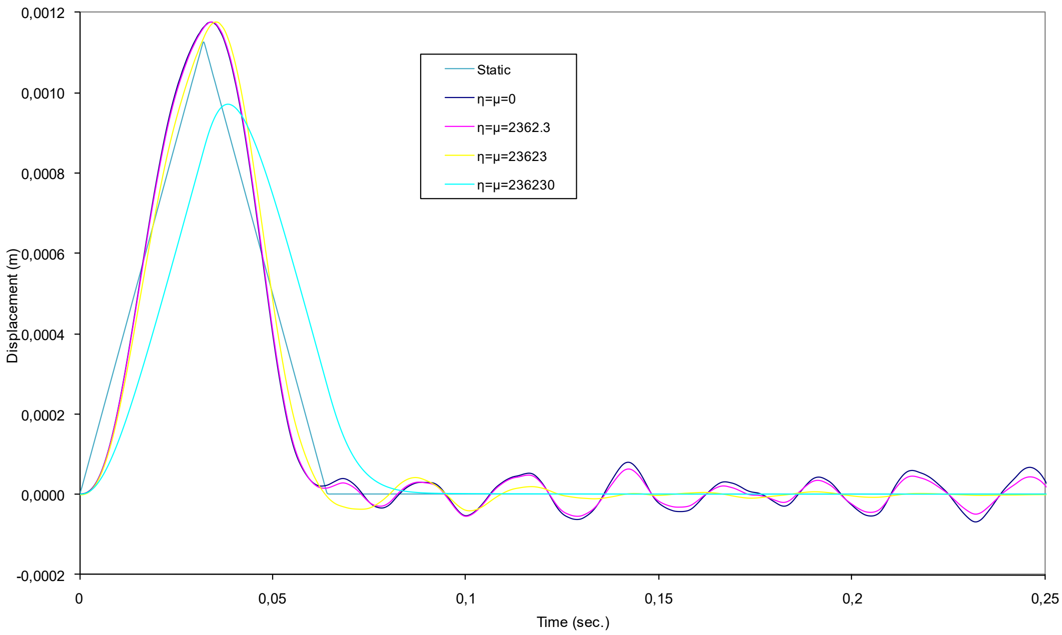


Fig. 13 Displacement versus time midpoint of the semi-circular beam (*R/h*=20)

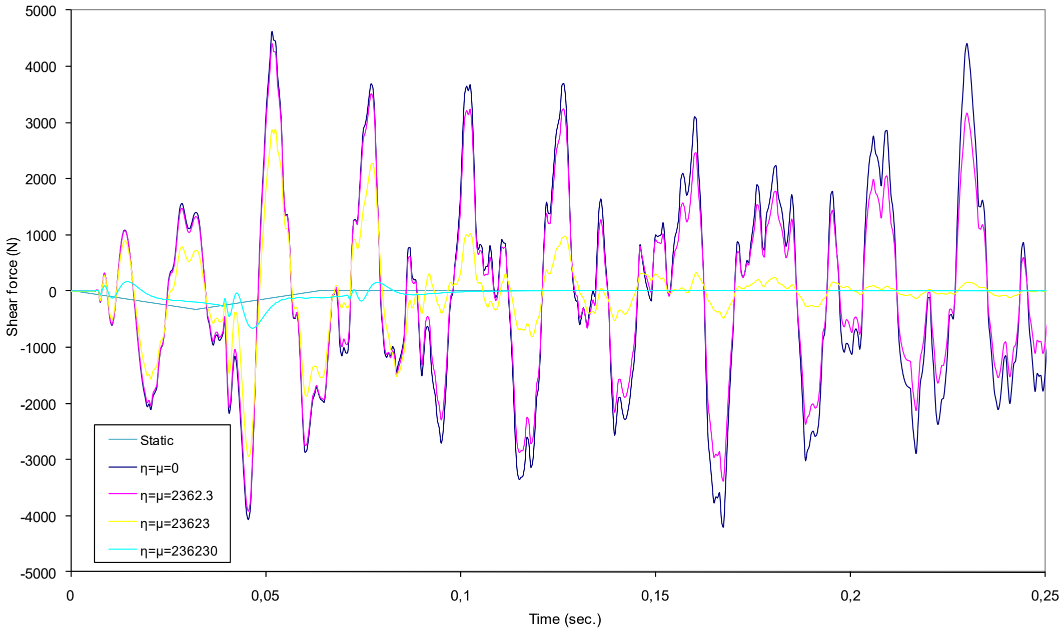


Fig. 14 Shear force versus time at the fixed end of the semi-circular beam (*R/h*=20)

R/h of semicircular beam on viscoelastic foundation lead to a significant increase in the displacement amplitude, however caused to decrease in vibration period.

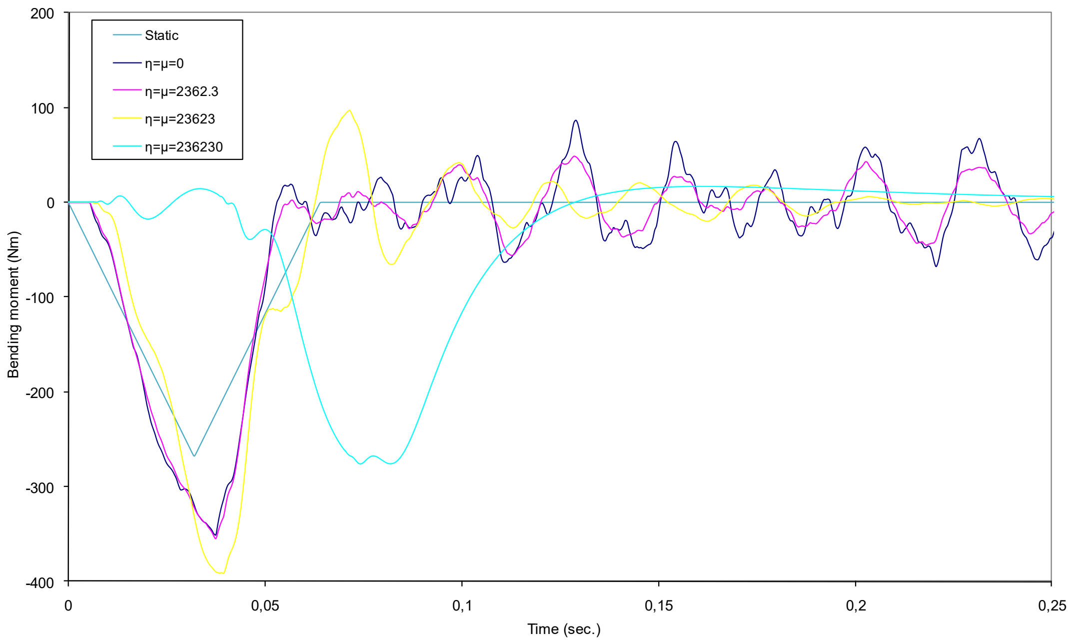


Fig. 15 Bending moment versus time at the fixed end of the semi-circular beam (*R/h*=20)

5. Conclusions

In present study, forced vibration of curved beams on viscoelastic foundation which is exposed to time dependent loads is investigated and an efficacious procedure is proposed. In proposed procedure, by rewritten the Timoshenko beams theory formulation for the curved and naturally twisted spatial rods, governing equations are obtained for the circular beams on viscoelastic foundation. The effect of shear deformation and rotary inertia, curvature of the axis are taken into account in the formulation. By applying the CFM, dynamic stiffness matrix is computed in the Laplace domain. By using CFM, in Laplace domain, ordinary differential equations with altering coefficients might be determined exactly.

The viscosity coefficients and *R/h* ratio affect the forced vibration of curved Timoshenko beam on viscoelastic foundation. As the viscosity coefficients increases, the displacement amplitude decreases. Moreover, it is monitored that increase of *R/h* of semicircular beam on viscoelastic foundation lead to a significant increase in the displacement amplitude, however caused to decrease in vibration period.

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