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Simplified procedure for seismic demands assessment of structures

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Abstract. Methods for the seismic demands evaluation of structures require iterative procedures. Many studies dealt with the development of different inelastic spectra with the aim to simplify the evaluation of inelastic deformations and performance of structures. Recently, the concept of inelastic spectra has been adopted in the global scheme of the Performance-Based Seismic Design (PBSD) through Capacity-Spectrum Method (CSM). For instance, the Modal Pushover Analysis (MPA) has been proved to provide accurate results for inelastic buildings to a similar degree of accuracy than the Response Spectrum Analysis (RSA) in estimating peak response for elastic buildings. In this paper, a simplified nonlinear procedure for evaluation of the seismic demand of structures is proposed with its applicability to multi-degree-of-freedom (MDOF) systems. The basic concept is to write the equation of motion of (MDOF) system into series of normal modes based on an inelastic modal decomposition in terms of ductility factor. The accuracy of the proposed procedure is verified against the Nonlinear Time History Analysis (NL-THA) results and Uncoupled Modal Response History Analysis (UMRHA) of a 9-story steel building subjected to El-Centro 1940 (N/S) as a first application. The comparison shows that the new theoretical approach is capable to provide accurate peak response with those obtained when using the NL-THA analysis. After that, a simplified nonlinear spectral analysis is proposed and illustrated by examples in order to describe inelastic response spectra and to relate it to the capacity curve (Pushover curve) by a new parameter of control, called normalized yield strength coefficient (η). In the second application, the proposed procedure is verified against the NL-THA analysis results of two buildings for 80 selected real ground motions.

Keywords: nonlinear analysis; capacity; ductility; static; dynamic; response spectrum

1. Introduction

Several simple evaluation methods have been proposed as an alternative to the complex nonlinear dynamic analysis to estimate the seismic demands of structures (Gulkan and Sozen

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1974, Freeman *et al*. 1975, Newmark and Hall 1982, Fajfar and Fischinger 1988, Kowalsky 1994, Sasaki *et al*. 1998, Fajfar 1999, Gupta and Kunnath 2000, Albanesi *et al*. 2000, Priestley and Kowalsky 2000, Miranda 2001, Chopra and Goel 2001, Lin and Chang 2003, Maja and Fajfar 2012, Chikh *et al*. 2014, Zerbin and Aprile 2015, Kazaz 2016). The basic idea of these methods is to relate the structural capacity to the physical basis of elastic or inelastic demand spectra, as the Capacity Spectrum Method (ATC-40 1996) and its different implementations.

The Capacity Spectrum Method (CSM) provides an overview of the inelastic behavior of structures subjected to seismic excitation. The CSM Method was developed for the first time by Freeman *et al*. (1975) and adopted by the Applied Technology Council (ATC-40 1996). Chopra and Goel (1999) proposed an improved Capacity-Demand-Diagram Method, which uses an inelastic design response spectrum as the demand spectrum with an alternative graphical implementation. In another version of the CSM method proposed by Fajfar (1999), highly damped elastic spectra have been used to determine seismic demand. It is implemented in Eurocode 8 (CEN 2005) and it was used in a comparison between traditional non-linear static methods in the evaluation of asymmetric structures by Bosco *et al*. (2013).

The seismic demands assessment methods are generally based on the nonlinear static analysis, where the structure is subjected to lateral loads increasing monotonically over the entire height until a predetermined target displacement. The distribution of these forces and the target displacement are based on the assumption that the response is controlled only by the fundamental mode, knowing that constant distribution of forces will not capture the contribution of higher modes in the overall structural response. Several researchers have proposed adaptive force distributions that attempt to follow more closely the distribution of inertial forces over time (Fajfar and Fischinger 1988, Baracci *et al*. 1997, Gupta and Kunnath 2000). Attempts have also been made to consider more than the fundamental mode of vibration in the Pushover analysis (Paret *et al*. 1996, Sasaki *et al*. 1998, Gupta and Kunnath 2000, Matsumori *et al*. 1999, Chopra and Goel 2001).

The Modal Pushover Analysis (MPA) developed by Chopra and Goel (2001) includes the effects of higher modes of vibration and provides a good estimation of the seismic demand. Later, the modal Pushover analysis undergoes changes by Chopra *et al*. (2004), for which an extension of this analysis was developed under the name of (Modified Modal Pushover Analysis, MMPA). The MMPA method combines the elastic influence of higher modes with inelastic response of the first mode using several combinations such as the Square Root of the Sum of Squares (SRSS). Unlike the MMPA, the Upper-Bound Pushover Analysis (UBPA) (Jan *et al*. 2004) is based on the use of a load vector obtained by combining the vector of the first mode and the second corrected vector mode.

In the same way, Poursha *et al*. (2009) have developed a technique for multi-modal nonlinear analysis, known as (Consecutive Modal Pushover CMP analysis) involving the sequential application of patterns of different loads on the modal basis. An extension of this procedure Extended Consecutive Modal Pushover (ECMP) method was proposed by Timothy *et al*. (2014), indicating that the modal load patterns are varied, encouraging the formation of different inelastic mechanisms in order to reproduce the different mechanisms which can be observed to develop in NL-THA when using a set of ground motion records. Recently, Pushover analysis has undergone modifications to be applicable on asymmetric buildings under bi-directional excitations (Lin and Tsai 2011, Fujii 2013, Bosco *et al*. 2013, Manoukas and Avramidis 2014). It has also been extended to general three-dimensional torsion coupled systems in several studies as (Reyes and Chopra 2011, Camara and Astiz 2012, Kaatsız and Sucuoğlu 2014).

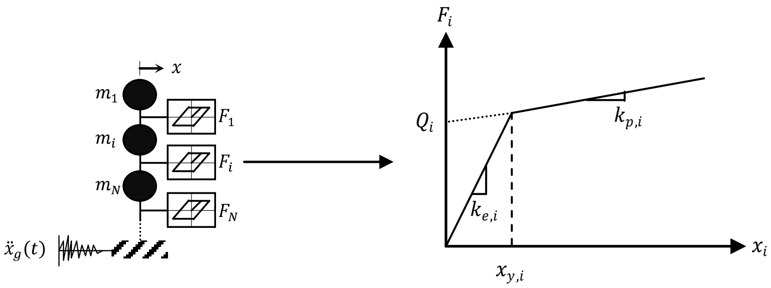


Fig. 1 Example of the resisting force of a MDOF system

This paper is divided into two main sections, an inelastic equation of motion of MDOF system will be rewritten in terms of the ductility to obtain an approximate multimodal dynamic analysis (AMDA) that consider the ductility factor as the inelastic response of the system. In the second section, a developed nonlinear multimodal spectral analysis (NL-MSA) for structures considering contribution of higher modes of vibration is introduced.

2. Approximate multimodal dynamic analysis

2.1 Inelastic modal decomposition in terms of ductility

The matrix form of differential equations governing the response of a MDOF system to earthquake induced ground motion can be written as

$$M\ddot{x}(t) + C\dot{x}(t) + F\ (x, sign\dot{x}) = -M\iota\ddot{x}_g(t) \tag{1}$$

Where M and C are the mass and damping matrices respectively, F denotes the resisting force vector, ι is the vector of earthquake influence coefficients and $\ddot{x}_g(t)$ denotes the earthquake acceleration. The damping matrix C would not be needed in this analysis of earthquake response; instead modal damping ratios suffice.

The resisting force vector F is defined as the sum of the linear and the hysteretic parts as represented in Fig. 1.

$$F = K_p x + Qz(x, \dot{x}) \tag{2}$$

Where, $m_i, F_i, k_{e,i}, k_{p,i}, Q_i$ and $x_{y,i}$ are the mass, resisting force, elastic stiffness, postyield stiffness, yield strength and yield displacement of the ith level, respectively.

In Eq. (2), the resisting force is a vector for MDOF systems, K_p is the postyield stiffness matrix, Q the yield strength vector, and z a dimensionless variable that characterizes the Bouc-Wen model of hysteresis (Wen 1976). It is given by

$$\dot{z} = \frac{\dot{x}}{x_y}\left[A - |z|^{\lambda}(Bsign(\dot{x}z) + \beta)\right] \tag{3}$$

Where, x_y is the yield displacement vector; A, B, λ and β are the parameters that control the shape of the hysteresis loop which are taken as: $A = 1$, $B = 0.1$, $\lambda = 0.9$ and $\beta = 6$ for bilinear system, $sign(.)$ is the sign function (Wen 1976).

Using Eq. (1) and Eq. (2) we get

$$M\ddot{x}(t) + C\dot{x}(t) + K_p x(t) + Qz(x, \dot{x}) = -M\iota\ddot{x}_g(t) \tag{4}$$

The decomposition of the MDOF system as a series of normal modes is reasonable. Eq. (5) is used to involve the influence of higher modes in the peak and overall response of the structure (Chopra 2007).

$$x(t) = \sum_n x_n(t) = \sum_n \phi_n \gamma_n(t) \tag{5}$$

Where: $\gamma_n(t)$ is the modal coordinate and ϕ_n is the nth natural vibration mode of the structure.

Substituting Eq. (4) into Eq. (5), using the mass, stiffness and classical damping orthogonality mode properties, we obtain the following differential equation for the single-degree-of-freedom (SDOF) system response

$$\ddot{\gamma}_n(t) + 2\xi_n\omega_n\dot{\gamma}_n(t) + \alpha_n\omega_n^2\gamma(t) + \frac{Q_n z_n(\gamma, \dot{\gamma})}{M_n^*} = -\Gamma_n\ddot{x}_g(t) \tag{6}$$

Where, ω_n is the natural vibration frequency, ξ_n the damping ratio, α_n the post-to-preyield stiffness ratio, $Q_n = \phi_n^t Q$ the yield strength, $M_n^* = \frac{L_n}{\Gamma_n}$, the effective mass, $\Gamma_n = \phi_n^T m\, \iota / \phi_n^T m \phi_n$ the modal participation factor and $L_n = \phi_n^T m\, \iota$ for the nth natural vibration mode.

The solution, γ_n, of Eq. (6) is given by (Chopra 2007)

$$\gamma_n(t) = \Gamma_n D_n(t) \tag{7}$$

With this approximation, the solution of Eq. (6) can be expressed by Eq. (7), where the displacement $D_n(t)$ of the SDOF system can be assessed by the following equation

$$\ddot{D}_n(t) + 2\xi_n\omega_n\dot{D}_n(t) + \alpha_n\omega_n^2 D(t) + \frac{Q_n z_n(D, \dot{D})}{\Gamma_n M_n^*} = -\ddot{x}_g(t) \tag{8}$$

This ductility demand (or ductility factor) for the SDOF bilinear system is expressed as

$$\mu_n = \frac{D_{n,m}}{D_{n,y}} \tag{9}$$

Where: $D_{n,m}$ is the peak displacement and $D_{n,y}$ is the yield displacement.

It seems worth to associate for each instantaneous inelastic displacement $D_n(t)$ an instantaneous ductility factor $\mu_n(t)$ defined as

$$\begin{cases} D_n(t) = \mu_n(t) \times D_{n,y} \\ \dot{D}_n(t) = \dot{\mu}_n(t) \times D_{n,y} \\ \ddot{D}_n(t) = \ddot{\mu}_n(t) \times D_{n,y} \end{cases} \tag{10}$$

Eq. (8) can be rewritten in terms of ductility factor μ_n, by substituting Eq. (10) in Eq. (8) and dividing by $D_{n,y}$, which gives

$$\ddot{\mu}_n + 2\xi_n\omega_n\dot{\mu}_n + \alpha_n\omega_n^2\mu_n + \frac{q_n g z_n(\mu, \dot{\mu})}{D_{n,y}} = -\frac{1}{D_{n,y}}\ddot{x}_g(t) \tag{11}$$

q_n is the yield strength coefficient for the nth natural vibration mode of the structure (defined as yield strength divided by L_n).

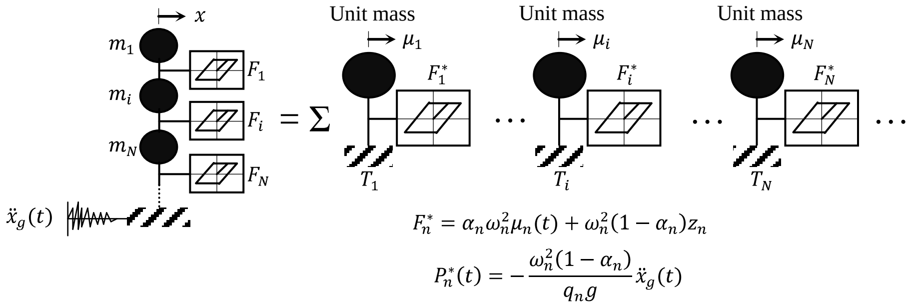


Fig. 2 Approximate multimodal dynamic procedure for MDOF structures

$$q_n = \frac{Q_n}{L_n} \tag{12}$$

Also, Eq. (3) may be expressed in terms of ductility factor μ_n as

$$\dot{z} = \dot{\mu}_n\left[A - |z|^{\lambda}(B\, sign(\dot{x}z) + \beta)\right] \tag{13}$$

The term $\frac{q_n g}{D_{n,y}}$ in Eq. (11) is rewritten as

$$\frac{q_n g}{D_{n,y}} = \omega_n^2(1-\alpha_n) \tag{14}$$

Substituting Eq. (14) into Eq. (11) gives

$$\ddot{\mu}_n + 2\xi_n\omega_n\dot{\mu}_n + \alpha_n\omega_n^2\mu_n + \omega_n^2(1-\alpha_n)z_n(\mu,\dot{\mu}) = -\frac{\omega_n^2(1-\alpha_n)}{q_n g}\ddot{x}_g(t) \tag{15}$$

It can be observed from Eq. (15) that for a given ground acceleration, $\mu_n(t)$ depends on $\xi_n, \omega_n, \alpha_n$ and q_n of the nth natural vibration mode.

Based on Eq. (5) and (7) and dividing by $D_{n,y}$ give the ductility demand and the displacement of the original structure

$$\mu(t) = \sum_n \phi_n \Gamma_n \mu_n(t) \qquad x(t) = \sum_n \phi_n \Gamma_n D_n(t) \tag{16}$$

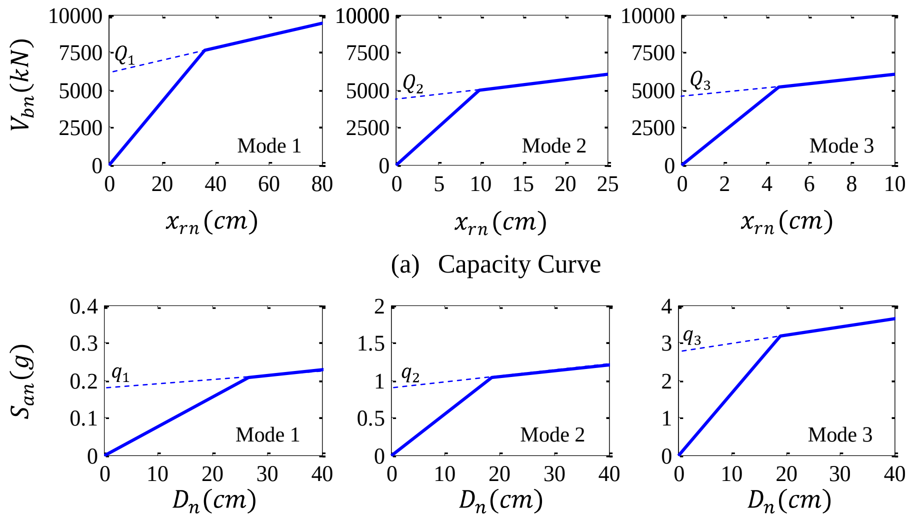
Fig. 2 illustrates the technique of uncoupling the equation of motion in terms of ductility factor characterizing the MDOF system. The response of a MDOF system to earthquake ground motion can be computed as a function of time by the procedure just developed the approximate multimodal dynamic analysis (AMDA), which is detailed in the next application. The proposed approximate analysis consists to solve Eq. (15) for $\ddot{x}_g(t)$ that will be multiplied by a new factor $-\omega_n^2(1-\alpha_n)/q_n g$ to constitute a new excitation for the structure to determine finally the total response quantities of interest by using Eq. (16).

2.2 Application

In recent years Chopra and Goel (2002) assessed the strength variation of several procedures including the modal Pushover analysis (MPA), that they developed. The MPA analysis is based on structural dynamics theory. Its accuracy and reliability in estimating the peak response of inelastic MDOF systems has been evaluated extensively by the authors. Goel and Chopra (2004) analyzed

Table 1 Properties of modal inelastic SDOF systems

Properties	Mode 1	Mode 2	Mode 3
$L_n(kg)$	2736789	-920860	696400
Γ_n	1.36	-0.5309	0.2406
M_n^* (kg)	3740189	488839.1	167531.5
$D_{n,y}(cm)$	26.51	18.65	19.12
T_n (sec)	2.2671	0.8525	0.4927
α_n	0.19	0.13	0.14
k_n (kN/cm)	210.3867	500.2020	1132.6086
ξ_n $(\%)$	1.948	1.103	1.136
$Q_n(kN)$	6168.977	4374.343	4414.347
$q_n(g)$	0.168	0.912	2.685



(b) $S_{an} - D_n$ Relationship Curve

Fig. 3 Modal Pushover curves and capacity diagrams for the first three modes

and evaluated the response of several procedures for nonlinear static analysis, including Pushover analysis where only fundamental mode was taken into account.

The accurate of the proposed procedure is evaluated for a 9-story SAC steel building (Chopra and Goel 2001). The ‘exact’ response of a rigorous nonlinear time history analysis (NL-THA) is compared with the response obtained by the approximate multimodal dynamic analysis (AMDA).

The 9-story structure meets the seismic code requirements and represents typical medium-rise buildings designed for the Los Angeles, California region. The Pushover curves of this structure presented in (Chopra and Goel 2001) are sufficient for the objectives of this study. The selected structure is tested and detailed in this section when subjected to one time and half (1.5) El Centro 1940 ground motion. The properties of the first three modes of vibration are summarized in Table 1.

The capacity curves of the three first modes are shown in Fig. 3. Next, the Pushover curves are transformed to equivalent SDOF systems (see Fig. 3). The conversion of the idealized Pushover curve to the force-displacement, (see Fig. 3(b)) for the nth–mode of inelastic SDOF system is obtained by using $(F_n^* - D_{n,y})$

$$S_{an} = \frac{V_{bn}}{M_n^*} = F_n^*, \quad D_n = \frac{x_{rn}}{\Gamma_n \phi_{rn}} \tag{17}$$

In which S_{an} is the spectral acceleration, V_{bn} the base shear, ϕ_{rn} is the amplitude of ϕ_n and x_{rn} the roof displacement.

The approximate multimodal dynamic analysis of the structure starts with obtaining the multimodal Pushover curves of the MDOF system subjected to lateral forces distributed over the building height. In the proposed procedure, the movements will be decomposed in the form of a series of normal modes in terms of the ductility demand. Eq. (15) is solved, and the resulting ductility demand history is decomposed into its "modal" components. The obtained response histories of ductility demand and roof displacements for the three first modes of the selected building subject to 1.5 times El Centro ground motion (N/S) component ($PGA = 0.32g$, $PGV = 36.14\ cm/sec$, and $PGD = 21.34cm$) are shown in Fig. 4.

The proposed procedure is evaluated by comparing the computed displacements histories according to Eqs. (15)-(16), considering three modes with those estimated by the NL-THA analysis and the Uncoupled Modal Response History Analysis (UMRHA) that was developed by Chopra and Goel (2001) (see Figs. 4 and 5).

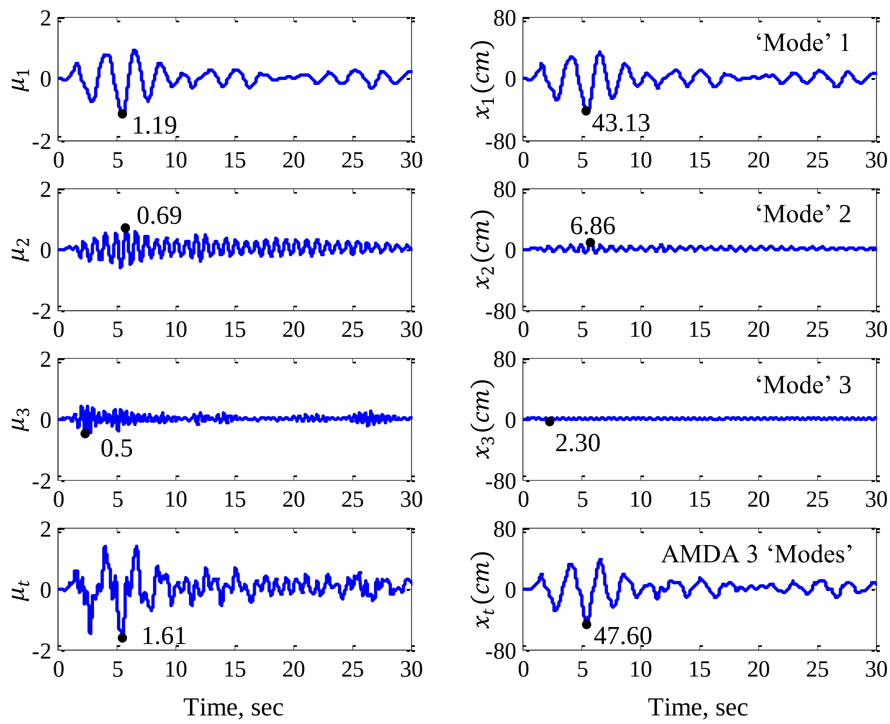


Fig. 4 Response histories of ductility demand and roof displacement from the proposed procedure for 1.5×El Centro ground motion: first three modal responses and total (all modes) response

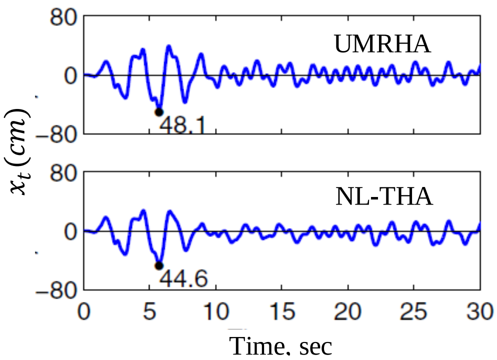


Fig. 5 Total response histories of roof displacement for 1.5×El Centro ground motion from the UMRHA and NL-THA (Chopra and Goel 2001)

Following the AMDA procedure aforementioned, the total response is determined using the UMRHA and NL-THA ("exact"). Fig. 4 shows the ductility demand, also is shown in the same figure the roof displacements time histories. It is clear from the comparison shown in Figs. 4 and 5 that the AMDA gives results in good agreement with the NL-THA.

3. Nonlinear multimodal spectral analysis (NL-MSA)

The Approximate Multimodal Dynamic Analysis (AMDA) developed in the precedent part provides structural responses as a function of time and evaluated specifically for the n^{th} mode, but design and verification of different structural elements and the seismic demand is usually based on peak values of forces and deformations over the duration of the earthquake-induced response. An approximate spectral method is proposed in this paper. This method is suitable for estimating the inelastic seismic response of MDOF systems. However, it is based on the inelastic modal decomposition, and will be simplified when using an inelastic response spectrum in terms of the ductility demand. In this procedure the maximum ductility demand for n^{th} mode are generally not attained at the same time t. Therefore the value of ductility demand of the structure should be calculated as a combination. A summary of the implementation of this procedure is presented below. In this method, the peak responses of inelastic MDOF systems can be calculated from the inelastic response spectrum. The results of this procedure are quite accurate for structural design applications.

3.1 Design spectra

The seismic demand in this paper is determined by using the inelastic design spectra developed by the authors (Benazouz *et al*. 2012), which have been called ductility demand response spectrum (DDRS). These spectra depend on the normalized yield strength coefficient (η) (defined as the yield strength coefficient divided by the Peak Ground Acceleration of the earthquake) that is fixed. The ductility demand quantity is calculated directly by drawing a vertical line passing by the natural period of the n^{th} mode of the structure (see Fig. 6). On the other hand, DDRS spectra are obtained when maximum ductility demands are assessed for a constant value of normalized yield

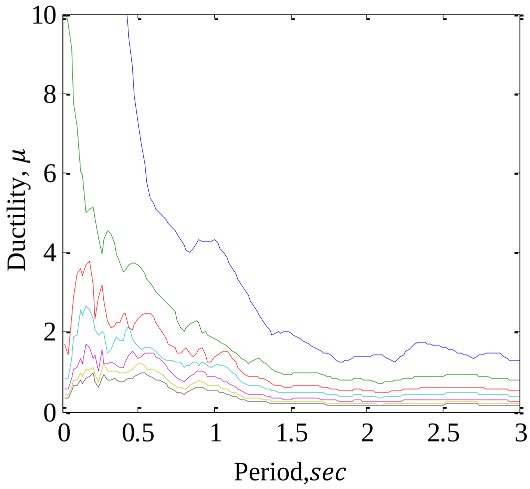


Fig. 6 DDRS for inelastic system computed for El Centro 1940 (N/S) component (η=0.25, 0.5, 0.75, 1, 1.5, 2, 2.5 from top line to bottom line)

strength coefficient (η). To calculate peak meaningful system response to an ensemble of ground motions, Benazouz *et al*. (2012) have used the normalized yield strength coefficient (η) to construct DDRS spectra. The utilization of this parameter has to be defined relative to the intensity of individual ground motions. Using the parameter η_n as (Benazouz *et al*. 2012, Mahin and Lin 1984)

$$\eta_n = \frac{q_n g}{PGA} \tag{18}$$

Where, PGA stands for the Peak Ground Acceleration.
Incorporating η_n into Eq. (15) results:

$$\ddot{\mu}_n + 2\xi_n\omega_n\dot{\mu}_n + \alpha\omega_n^2\mu_n + \omega_n^2(1-\alpha_n)z_n(\mu,\dot{\mu}) = -\frac{\omega_n^2(1-\alpha_n)}{\eta_n}\overline{\ddot{x}_g}(t) \tag{19}$$

In which, $\overline{\ddot{x}_g}(t)$ represents the ground acceleration normalized with respect to the PGA.

The ground acceleration has been normalized such that its value varies from -1 to 1. Eq. (19) implies that for a MDOF inelastic system, if α_n and η_n are fixed, the intensity of the ground motion has no effect on the peak normalized ductility factor. This permits the construction of the ductility response spectrum for an ensemble of ground motions with common frequency content.

The DDRS is constructed also for El Centro 1940 ground motion (N/S) component and is shown in Fig. 6 in function of the parameter (η).

Simplified equations for ductility demands μ would obviously facilitate the estimation of the ductility and deformation of inelastic SDOF system has been developed by (Benazouz *et al*. 2012). Such an equation for μ could be used to determine the ductility demand for a new or rehabilitated structure with known normalized yield strength coefficient (η) and post-to-preyield stiffness ratio (α).

The spectrum is divided into three period regions according to the procedure described in (Benazouz *et al*. 2012), where $T = 0.6sec$ marks the transition from the acceleration-sensitive region to the velocity sensitive region which ends at $T \sim 3\ s$. Such an equation for μ has been

Table 2 Parameters in Eq. (20) for each value of η and α

Parameters	$\eta < 1$				$\eta \geq 1$ (all values of α)
	$\alpha(\%)$				
	0	3	5	10	
a	1.24	1.12	1.08	1.04	1.23
b	0.98	0.94	0.92	0.88	0.85
c	1.69	1.65	1.68	1.68	$1.21 + \alpha$

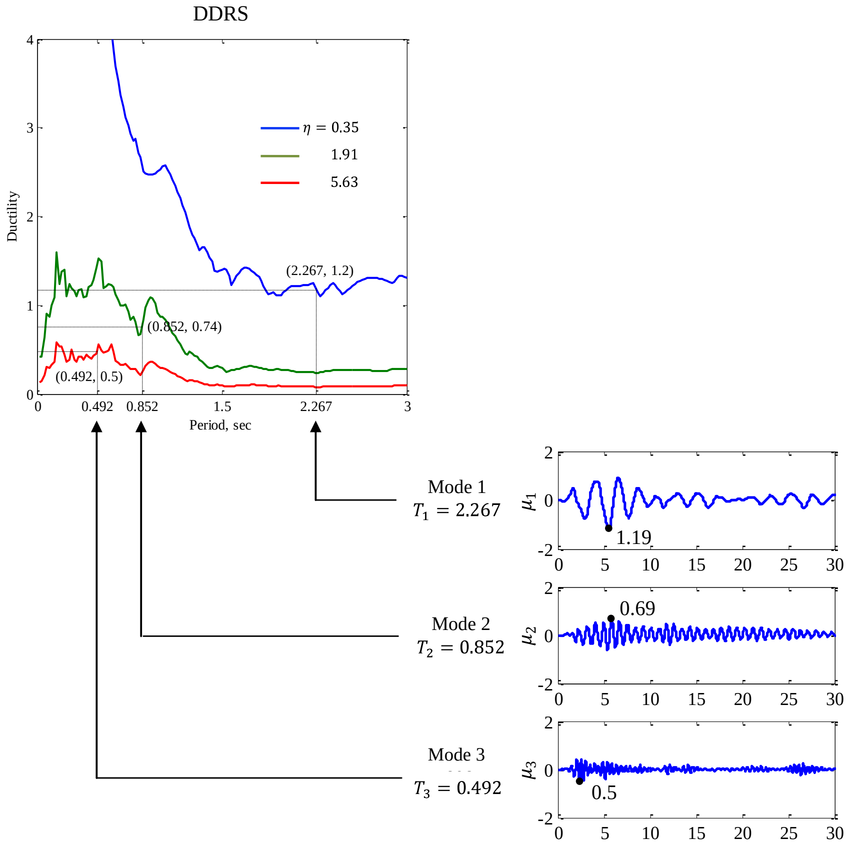


Fig. 7 Peak response of ductility demand from the DDRS spectra and the AMDA procedure for 1.5 ×El Centro ground motion: first three modal responses

derived in terms of η, α, and T, and they are tabulated in Table 2 for different ranges of η and α.

$$\mu = \frac{a\,(b)^{1/T}(T)^{\eta-c}}{\eta} \qquad \textit{for All values of } T \tag{20}$$

$$\mu = \frac{\omega^2(1-\alpha)}{\eta} 0.027\, T^{0.84} \qquad T \geq 0.6 sec \tag{21}$$

Fig. 7 provides a comparative study of the AMDA and the DDRS spectra (Benazouz *et al.*

2012), in the case of the first application under 1.5* El-Centro ground motion. However, it is challenging to develop a simple procedure able to include the higher vibration modes in the total response.

3.2 Estimation of the ductility and base shear of structures

To obtain the peak response from the NL-MSA procedure, the ductility demand when estimated by Eqs. (20)-(21) is used to read and compute the roof displacement and base shear using the simple equation repeated here (see Eq. (10) and (16)):

For the nth mode, the maximum displacement of the original structure based on the ductility demand is given by

$$D_n = \mu_n \, D_{n,y} \tag{22}$$

$$\mu_n = \phi_{n,roof} \, \Gamma_n \, \mu_{n,max} \tag{23}$$

Eqs. (5) and (7) lead to estimate the maximum floor displacements relative to the nth mode by the following equation

$$x_n = \phi_n \Gamma_n D_n \tag{24}$$

By using the second derivation of the precedent equation and multiplied by the mass vector of the structure, the equivalent static lateral forces f_i are given by the following equation

$$f_{jn} = \Gamma_n m_i \phi_n \ddot{D}_n \tag{25}$$

In which m_i is the mass of the ith level and $\ddot{D}_n$ the acceleration of the SDOF system.

In the nonlinear static analysis, for a SDOF equivalent system, the Push of the mass M_n^* with a positive displacement D_n implies that $\dot{D}z \geq 0$, in other words (see Eq. (13))

$$\dot{D}z > 0 \;\text{ if } \begin{cases} \mathrm{z} > 0 & and\ D\ is\ increasing \\ \mathrm{z} < 0 & and\ D\ is\ decreasing \end{cases} \tag{26}$$

In this case, Eq. (13) can be rewritten in the following form

$$\dot{z} = \dot{\mu}_n \left[1 - |z|^{\lambda}\right] \tag{27}$$

Solving Eq. (27) (determination of homogeneous and particular solutions) leads to the following equation

$$z = 1 - e^{-\mu_n} \tag{28}$$

Eq. (28) is valid only if the mass of the system is pushed by a positive displacement and $z > 0$. Then, to determine the coordinates of the capacity curve for a SDOF system, one determine the dimensioless z for a range of displacements. The base shear $V_{b,n}$ is given by the following equation

$$V_{b,n} \approx \alpha_n \omega_n^2 \mu_n + \omega_n^2 (1 - \alpha_n)(1 - exp(-\mu_n)) \tag{29}$$

Combining those modal responses and using any of the proved modal combination methods (SRSS, square root of the sum of squares), we obtain a good estimation of the peak value of the total structural response. Eventhought this technique lacks theoretical basis, however it is proved to provide meaningful results.

$$r = (\sum r_n^2)^{1/2} \tag{30}$$

At this level, the NL-MSA provides a good estimation of the maxima (peak modal response) of any response (displacement profile, story drifts, joint rotations, etc.)

3.3 Step by step NL-MSA procedure

For convenience we summarize the nonlinear multimodal spectral analysis (NL-MSA) as a series of steps used to estimate the peak inelastic response of MDOF systems:

1- Calculate the natural vibrating period T_n, the natural frequency ω_n and the mode shapes ϕ_n, for linearly elastic vibration of the structure.

2- For the nth mode, develop the diagram of force-deformation relationship between the base shear and the top displacement $V_{bn} - x_{rn}$ (capacity curve).

3- Idealize this curve as a bilinear curve.

4- Transform the idealized Pushover curve to the capacity diagram $S_{an} - D_n$ (Acceleration – Displacement format) using Eq (17).

5- Define the ground motion $\ddot{x}_g(t)$.

6- Compute the post-to-preyield stiffness ratio α_n and the normalized yield strength η_n with the known q_n and PGA, and select the damping ratio ξ_n of the design structure.

7- Calculate the ductility demand for the nth mode by using Eqs. (20)-(21).

8- Calculate peak roof displacement and base shear of the original structure by using Eq. (24) and Eq. (29), respectively.

9- Calculate the floor displacement, story drifts, etc, from the capacity curve database obtained in the second step of the procedure.

10- Repeat Steps 3 to 8 many times (for different modes) as required for sufficient accuracy.

11- Determine the total response (demand) by combining the peak “modal” responses using the SRSS rule (Eq. (30)).

Table 3 presents the results of the combined response of the first application (9-story SAC steel building) obtained by the DDRS spectra considering one, two, and three “modes”, respectively, and the errors in these estimates relative to the exact response from NL-THA (Chopra and Goel 2001).

4. Application and discussion

Two examples of eight and three stories reinforced concrete plane frame structures (regular structures) are considered in order to evaluate the efficiency of the proposed method. They have been chosen for representing the behavior of low and medium-rise buildings. These examples are

Table 3 Peak values of floor displacements from the NL-MSA and NL-THA for 1.5 ×El Centro ground motion

Procedure	Combined (NL-MSA)			NL-THA	Errors (%)		
	1 Mode	2 Modes	3 Modes		1 Mode	2 Modes	3 Modes
DDRS Spectra	44.12	44.38	44.45	44.6	-1.08	-0.49	-0.33
DDRS Design Spectra	34.54	34.90	34.91		-29.12	-27.79	-21.39

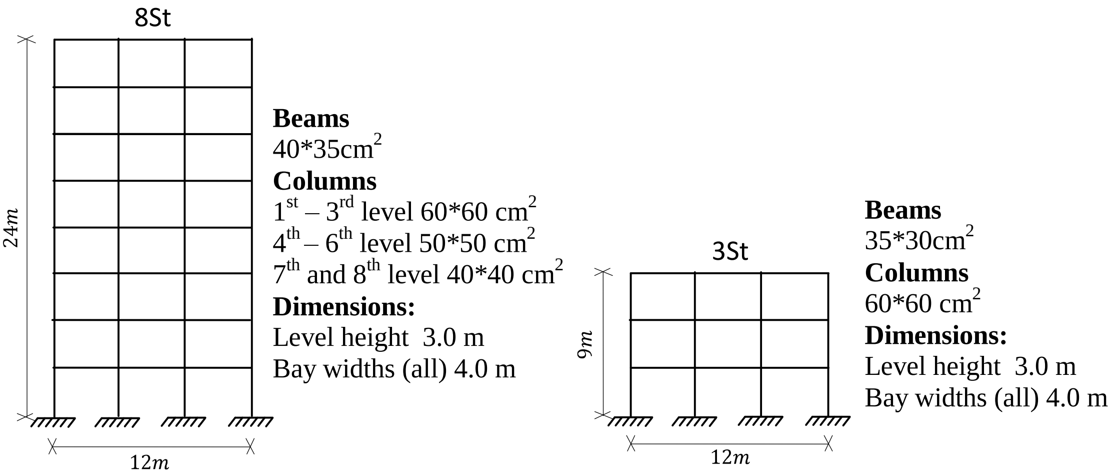


Fig. 8 Elevation view of the 8St and 3St structures

Table 4 Dynamic characteristics

	Mode	$f_n\ (Hz)$	$T_n\ (sec)$	Γ_n	$L_n(\%)$
8St	1	1,22	0,81	1,558	77,10
	2	3,60	0,27	-0,611	11,87
	3	6,40	0,15	-0,386	4,74
3St	1	3.13	0.32	0.461	85.63
	2	10.54	0.094	0.168	11.38
	3	20.07	0.05	0.086	2.3

an eight (8St) and three-story (3St) reinforced concrete buildings as illustrated in Fig. 8. These structures are designed according to the Algerian code RPA-2003 (CGS 2003). Each structure consists of three bays frame, spaced at 4m and a uniform story height of 3m with no significant height irregularities. The purpose of this study is to confirm the application of the proposed method for each frame structure under a design earthquake. The geometric properties of the components are presented in Fig. 8. Vibration modes and periods of these structures for linearly elastic case are summarized in Table 4. The capacity curves were obtained by applying the Adaptive Pushover Analysis (Reinhorn *et al*. 2006). These curves were transformed to the capacity diagram ($S_{an} - D_n$) format by using Eq. (17), and gave the curves shown in Fig. 9. The numerical models were simulated using IDARC program (Reinhorn 1997). The supports were modeled as infinitely rigid to avoid the soil-structure interaction. The periods of the selected structures were: $T = 0.81$ sec for the 8St building, in the velocity-sensitive spectral region, and $T = 0.32$ sec for the 3St building, in the acceleration-sensitive spectral region. Results obtained show that the proposed procedure give peak displacements close to those that obtained by NL-THA. Concrete compressive strength was 25 MPa. Concrete density is $2.5\ \text{t/m}^3$, concrete Young modulus is $3.21\ 10^7$ kN/m² and reinforcements yield strength is 400 MPa.

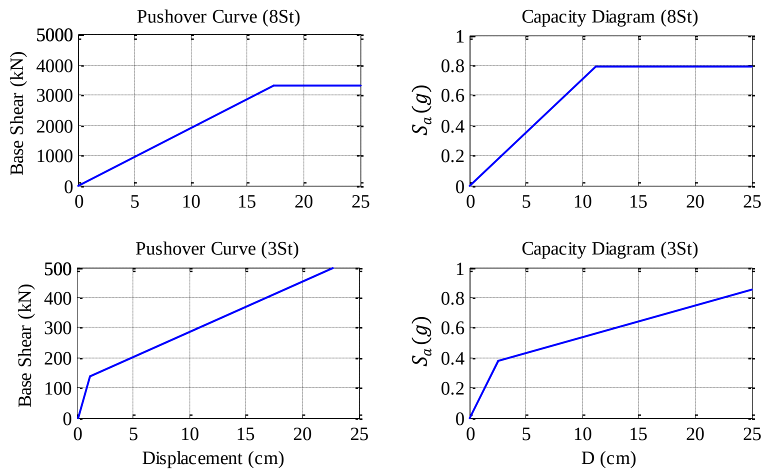


Fig. 9 Pushover curve and capacity diagram of the 1st mode for the 8St and the 3St structures

4.1 Ground motion records

Four ensembles of ground motions, each with 20 records were considered in this application. The ensembles, denoted by LMSR, LMLR, SMSR, and SMLR represent four combinations of large (M=6.6-6.9) or small (M=5.8-6.5) magnitude and short (R=13-30 km) or long (R=30-60 km) epicentral distance. These motions were selected from the PEER (Pacific Earthquake Engineering Research Center) Strong Motion Database. More details about this database can be found elsewhere (Chopra *et al*. 2003). These values were used for both horizontal directions of ground motion in the proposed procedure in order to be able to compare its results with those obtained by NL-THA.

4.2 Capacity curve

First, the Pushover curves of the two structures were transformed to the equivalent SDOF system (see Fig. 9). For example the Pushover curve of the eight-story structure is transformed to the equivalent SDOF capacity curve. In this case, the transformation factor amounts of $\Gamma_1 = 1.558$, and the effective mass is equal to $L_1 = 40912454.4$ kg. The capacity diagram shown in Fig. 9 was obtained by dividing the forces corresponding to the idealized SDOF system by the equivalent mass. For 8St structure, the yield strength ratio is equal to $q_1 = Q_1/L_1 = 0.795$.

4.3 Seismic demand

The AMDA was evaluated by comparing the computed displacements (displacement divided by the building height), and the inter-story drift ratio (relative drift between two consecutive stories normalized by story height). Since the time-history results were based on a set of 80 simulations, both the mean and the dispersion (standard deviation σ) about the mean value are shown in the plots.

Peak displacement profiles and inter-story drift ratio profiles estimated by the NL-THA analyses and those obtained by the AMDA procedure for the buildings are shown in Figs. 10-11.

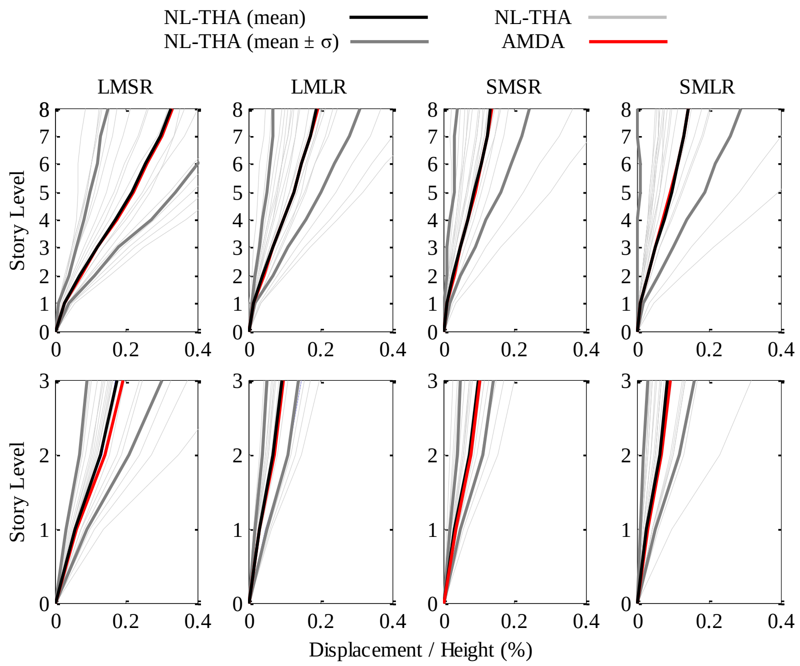


Fig. 10 Displacements obtained by the AMDA and NL-THA for the four ensembles

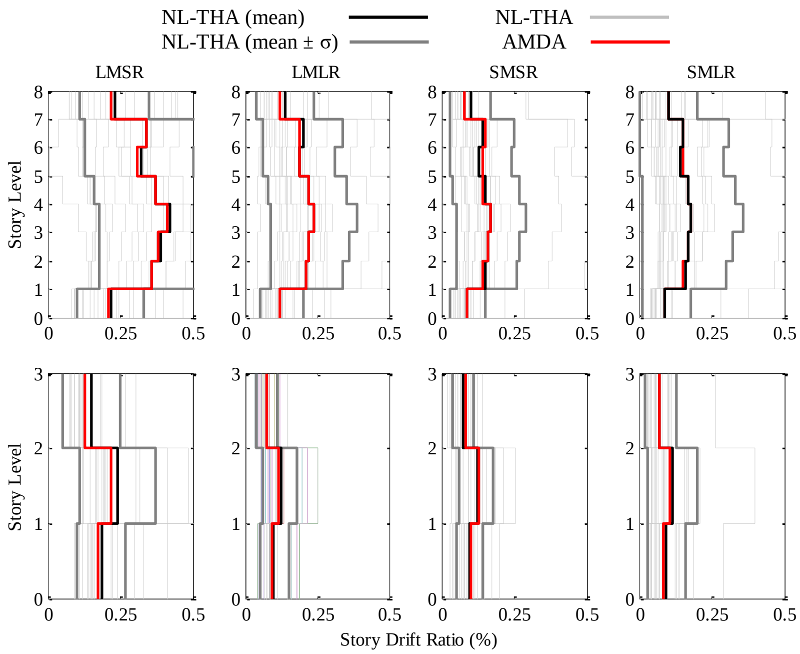


Fig. 11 Inter-story drift ratio obtained by the AMDA and NL-THA for the four ensembles

These figures show the means and the standard deviations of the peak displacement profile estimated by NL-THA analyses and the predictions using the AMDA method for each building

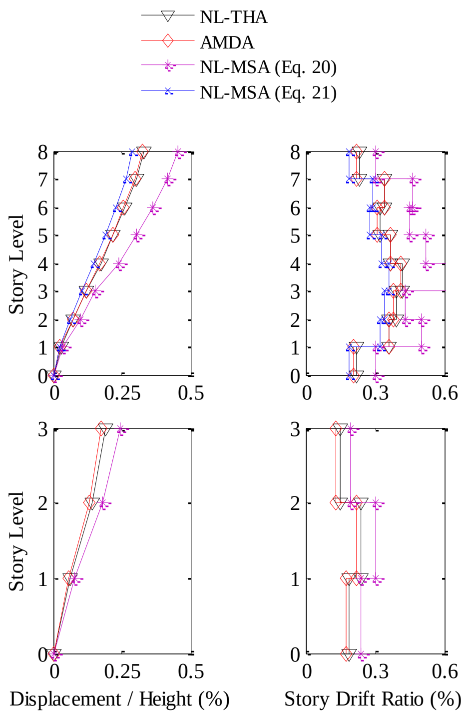


Fig. 12 Displacements and inter-story drifts ratio along the elevation for the 8St and the 3St buildings, obtained by the NL-THA, AMDA, NL-MSA (Eq.20) and NL-MSA (Eq.21) for LMSR ensemble

sorted by type of record.

The displacement demands along the heights of the buildings showed that the AMDA method gives good results and generally yield to better estimates of the peak displacement profiles particularly for the 8St building. Comparing the time-history responses for the different accelerations indicates that the difference between the ground motion of far field records and near field records generally produce more variability in the demands. It is interesting to mention that the story displacement demands from the AMDA method are always conservative.

Presented in Fig. 12 are the displacements profile and inter-story drift ratios of the two structures determined using two different equations for all periods (see Eq. (20)), and for the structure that its period less than 0.6 sec for the sensitive acceleration region by using Eq. (21). The displacements profile and inter-story drift ratio estimates by the two equations are compared in Fig. 12 with those from the AMDA and the NL-THA. These results shown in Fig. 12 illustrate that the approximate equations leads to significant error. The approximate Eq. (20) significantly overestimates the displacements and the inter-story drift demands; Eq. (21) significantly underestimates the seismic demands and provides excellent estimations.

5. Conclusions

An approximate procedure for seismic demands assessment of MDOF system has been developed and its accuracy was verified by examples. An inelastic modal decomposition in terms of ductility has been developed to construct the Approximate Multimodal Dynamic Analysis. That was verified using the seismic response of an example steel frame structure for which capacity curve data is available. The results indicated that more reliable displacement predictions are obtained from the proposed method.

Also, as presented in this paper, a nonlinear multimodal spectral analysis was developed. The base shear–roof displacement $(V_{bn} - x_{rn})$ curve is developed from a Pushover analysis. This Pushover curve is idealized as a bilinear force–deformation relation for the n^{th} mode of inelastic SDOF system. This idealization is used to determine the normalized yield strength coefficient η_n and the post-to-preyield stiffness ratio α_n to estimate the ductility demand. The peak deformation of this SDOF system, determined by the design spectra DDRS, is used to determine the target value of roof displacement at which the seismic response is determined by the Pushover analysis. The total demand is determined by combining the responses of the first three modes by the SRSS combination rule.

The efficiency of the NL-MSA is evident; the designer needs only to have the Pushover curve of the structure and the design earthquake(s) to determine peak response of any structure, namely, base displacement and base shear. This method is applicable to a variety of uses such as a rapid evaluation technique for a large inventory of buildings, a design verification procedure for new construction, an evaluation procedure for an existing structure to identify damage states. The ductility demand is given by the direct estimation where the ductility calculated from the design spectra diagram matches the value associated with the period of the system. This method gives the deformation value consistent with the selected DDRS inelastic response spectrum, while retaining the attraction of graphical implementation of other methods.

The results of the NL-MSA method are acceptable while compared to the nonlinear dynamic analysis results. For structures that the first mode contributes significantly to the response, the NL-MSA method will generally give good estimates of demand for global deformations (application 2). However, inelastic dynamic response may differ significantly from a selected or from the response distributions with constant lateral loads. For example, we can expect significant differences in structural response due to the influence of vibration modes at higher frequencies.

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