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Optimal design of double layer barrel vaults considering nonlinear behavior

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Abstract. The present paper focuses on size optimization of double layer barrel vaults considering nonlinear behavior. In order to tackle the optimization problem an improved colliding bodies optimization (ICBO) algorithm is proposed. The important task that should be achieved before optimization of structural systems is to determine the best form having the least cost. In this study, an attempt is done to find the best form then it is optimized considering linear and non-linear behaviors. In the optimization process based on nonlinear behavior, the geometrical and material nonlinearity effects are included. A large-scale double layer barrel vault is presented as the numerical example of this study and the obtained results indicate that the proposed ICBO has better computational performance compared with other algorithms.

Keywords: optimization; double layer barrel vault; nonlinear behavior; meta-heuristic

1. Introduction

One of the most popular forms of space structures is barrel vault. A barrel vault consists of one or more layers of elements that are arched in one direction. Barrel vaults can be constructed as single-layer or double-layer structures and with increase of the spans, double-layer systems are often preferred. As the double-layer barrel vaults are employed to cover wide span column free areas, they have a huge number of structural elements and therefore, sufficient attention must be paid to systematic designing of these structures. For this purpose, design of these structures can be conveniently achieved by employing optimization techniques. It is obvious that an optimal design has a great influence on the economy and safety of all types of the structures. In this case, optimization of double-layer barrel vaults results in more efficient structural configurations (Kamyab and Salajegheh 2013). The main aim of the present study is to optimize double-layer barrel vaults considering nonlinear behavior and due to this fact that such problem is computationally complex, application of an efficient optimization algorithm is vital to tackle it.

Many of gradient-based optimization algorithms have difficulties when dealing with complex problems, and they may converge to local optima. To overcome these difficulties, utilizing

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algorithms possessing global search ability is inevitable. In contrast with gradient-based optimization algorithms, meta-heuristics can be efficiently employed to tackle complex optimization problems. Meta-heuristics are designated on the basis of stochastic natural phenomena, and they have attracted a great deal of attention during the last two decades. As the meta-heuristic optimization techniques require no gradient computations, they are simple for computer implementation (Gholizadeh and Fattahi 2014, Nigdeli *et al*. 2015). During the recent years, many successful applications of meta-heuristic algorithms have been reported in the field of structural optimization. In the work of Talatahari *et al*. (2015) a new hybrid eagle strategy with differential evolution (ES-DE) was employed for optimum design of frame structures. Gandomi *et al*. (2011) utilized firefly algorithm (FA) to solve benchmark mixed-variable and non-convex optimization problems. Gholizadeh and Poorhoseini (2015) introduced a modified Dolphin echolocation (MDE) algorithm for design optimization of steel frame structures. Dogan (2014) presented a hunting search algorithm (HSA) based on group hunting process of animals such as wolves, lions, and dolphins for solving engineering optimization problems. One of the newly developed meta-heuristics is colliding bodies algorithm (CBO) which is designed based on the governing laws of one dimensional collision between two bodies from the physics (Kaveh and Mahdavi 2014). An enhanced CBO (ECBO) was proposed by Kaveh and Ilchi Ghazaan (2014) to improve convergence rate and reliability of CBO. In the present work an improved CBO (ICBO) is proposed for optimization of double-layer barrel vaults considering linear and nonlinear behaviors.

Optimization of double-layer space structures based on nonlinear behavior is the subject of few studies. Saka and Ulker (1991) and Saka and Kameshki (1992) optimized space structures considering only geometrical nonlinearity. They employed gradient-based methods for solving the optimization problem. Kamyab and Salajegheh (2013) employed an enhanced particle swarm optimization (EPSO) meta-heuristic for optimization of scallop domes considering material and geometrical nonlinearities. They considered only a uniform load on the top layer of the scallop dome. Gholizadeh and Barati (2014) proposed a serial integration of FA and particle swarm optimization (PSO) for topology optimization of geometrically nonlinear single layer domes. Kaveh and Rezaei (2015) employed ECBO for topology optimization of geometrically nonlinear suspended domes. Gholizadeh (2015) designed double layer grids subject to a uniformly distributed load on the top layer for optimal weight considering material and geometrical nonlinearities by using a sequential grey wolf algorithm.

Finding an appropriate form is prerequisite for performing optimization of structural systems with many members. In this work, prior to optimization, a form finding process is implemented to find the best form for double-layer barrel vaults having the least cost. Afterward, the structure possessing the best form is optimized subject to practical loadings considering linear and nonlinear behaviours using CBO, ECBO and ICBO. Moreover, the fully stressed design (FSD) of the structure is determined. The numerical results reveal that the nonlinear optimization results in a more efficient design compared with those found by linear optimization and FSD process. Furthermore, the superiourity of ICBO over the CBO and ECBO is demonstrated.

2. Form finding

In the case of structures with many members, one of the most important issues that should be addressed prior to optimization is to find the best form. In the process of form finding, different forms are assessed and the most economic one is selected as the best form (Wu *et al*. 2015a,

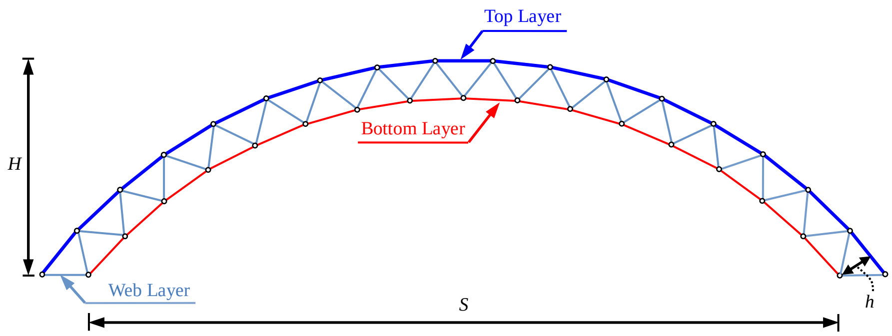

Fig. 1 Vertical section of a typical pin-jointed double layer barrel vault

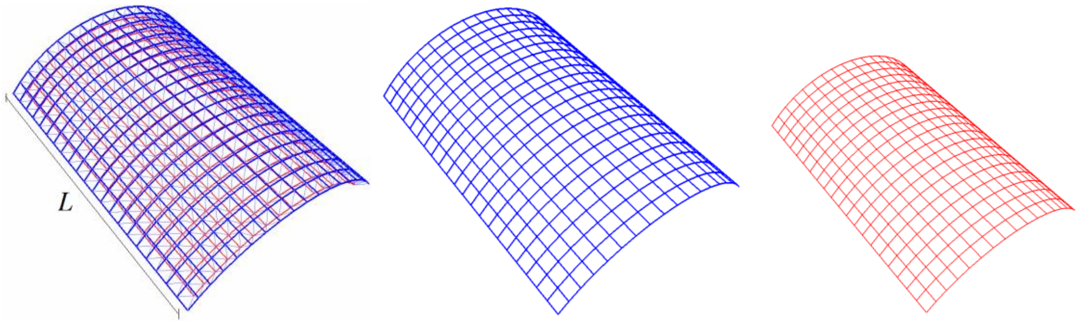

Fig. 2 SS model and its top and bottom layers

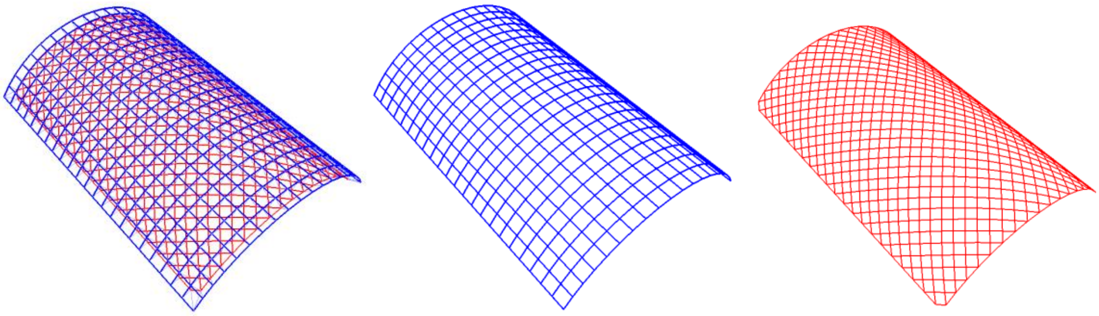

Fig. 3 SD model and its top and bottom layers

b).The vertical section of a typical ball-jointed double layer barrel vault is depicted in Fig. 1 defining its height (*H*), span (*S*), and layer thickness (*h*). As regard *S* is usually predefined, one of the most important issues in the form finding process is to find the best values of *H* and *h*.

In the present study, four basic forms including square-on-square (SS), square-on-diagonal (SD), diagonal-on-square (DS), and diagonal-on-diagonal (DD) with S=42.0 m and length (*L*) equal to 60.0 m, shown in Figs. 2 to 5, are considered.

For the purpose of form finding, 36 different models with 2, 3, and 4 m modulations and H/S ratio in interval [0.2 to 0.5] have been analyzed for the total cost including member, joints and fabrication costs. The results show that a SS model considering H/S=0.28541 and h=2.0798 m which gives a model with 3.0 m modulation is the best form. It should be noted that by using these

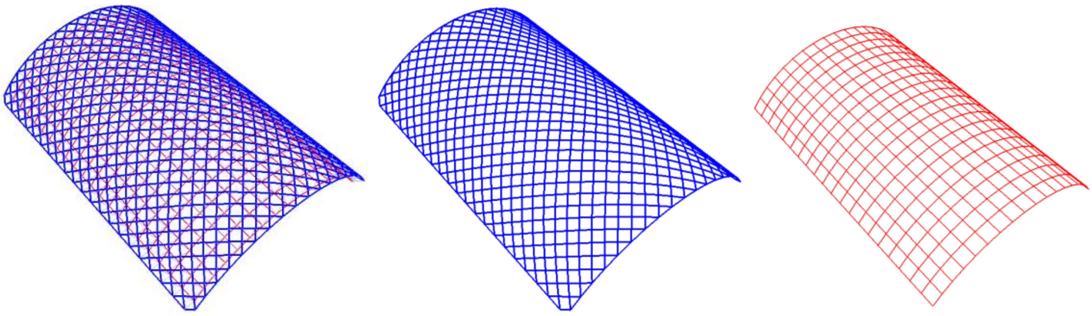

Fig. 4 DS model and its top and bottom layers

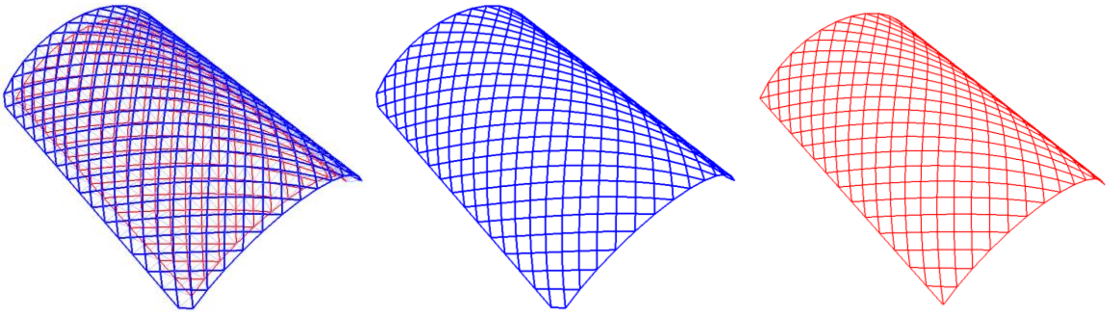

Fig. 5 DD model and its top and bottom layers

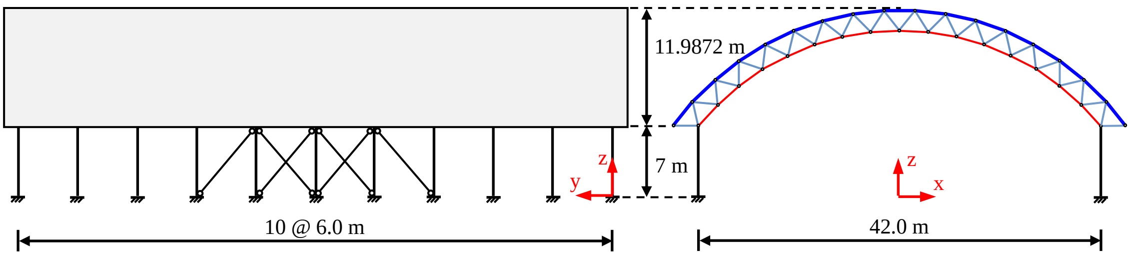

Fig. 6 *x-z* and *y-z* viewes of the best model

parameters, the length of all members in top, web, and bottom layers is equal to 3.0 m.

Details of the best model selected in the present study are given in Fig. 6 together with side columns and bracings.

3. Loading details

In the present work, the load cases listed in Table 1 are taken into account during the design optimization process.

In order to compute the effective loads acting on the double layer barrel vault for FSD and optimization considering linear behavior, 44 load combinations, given in Table 2, are used according to the design codes such as Standard No. 400 (2010), Standard No. 2800 (2014), Eurocode 1, Part 1.3 (2001) and Part 1.4 (2004). For optimal design of double layer barrel vault

Table 1 Applied load cases

No.	Load case	Abbreviation
1	Dead	D
2	Symmetrical snow	S
3	Non-symmetrical snow during wind in the (+ x) direction	S_p
4	Non-symmetrical snow during wind in the (– x) direction	S_n
5	Earthquake in the x direction	E_x
6	Earthquake in the y direction	E_y
7	Earthquake in the z direction	E_z
8	Wind in the x direction	W_x
9	Wind in the y direction	W_y
10	Temperature	T

Table 2 Applied load combination for engineering design

No.	Load combination	No.	Load combination	No.	Load combination	No.	Load combination
1	$1.4D$	12	$1.2D - 0.8W_y$	23	$1.2D + E_v$	34	$1.2D + S - E_y$
2	$1.2D + 1.6S$	13	$1.2D + S + 1.6W_x$	24	$1.2D - E_v$	35	$1.2D + S + E_v$
3	$1.2D + 1.6S_p$	14	$1.2D + S - 1.6W_x$	25	$0.9D + E_x$	36	$1.2D + S - E_v$
4	$1.2D + 1.6S_n$	15	$1.2D + S + 1.6W_y$	26	$0.9D - E_x$	37	$1.2D + 1.6T$
5	$0.9D + 1.6W_x$	16	$1.2D + S - 1.6W_y$	27	$0.9D + E_y$	38	$1.2D - 1.6T$
6	$0.9D - 1.6W_x$	17	$1.2D + S_p + 1.6W_x$	28	$0.9D - E_y$	39	$1.2D + S + T$
7	$0.9D + 1.6W_y$	18	$1.2D + S_n - 1.6W_x$	29	$1.2D + S + E_x$	40	$1.2D + S - T$
8	$0.9D - 1.6W_y$	19	$1.2D + E_x$	30	$1.2D + S - E_x$	41	$1.2D + S_p + T$
9	$1.2D + 0.8W_x$	20	$1.2D - E_x$	31	$1.2D + S_p + E_x$	42	$1.2D + S_p - T$
10	$1.2D - 0.8W_x$	21	$1.2D + E_y$	32	$1.2D + S_n - E_x$	43	$1.2D + S_n + T$
11	$1.2D + 0.8W_y$	22	$1.2D - E_y$	33	$1.2D + S + E_y$	44	$1.2D + S_n - T$

considering nonlinear behavior service load combinations are employed.

The design dead load is determined on the basis of the actual loads that may be expected to act on the structure of constant magnitude. In this study, a uniform dead load of 70 kg/m^2 is considered for estimated weight of sheeting, structural members, and nodes of barrel vault.

The snow load for arched roofs is calculated according to Eurocode 1, Part 1.3 (2001). Snow loads acting on a sloping surface is assumed to act on the horizontal projection of that surface and can be computed as follows

$$S = C_e C_t S_k \mu_i \tag{1}$$

where C_e, C_t, S_k and μ_i are exposure coefficient, thermal coefficient, flat roof snow load and shape coefficient, respectively.

In this study, C_e=1.0, C_t=1.0 and S_k=1.5 kN/m^2. The shape coefficient μ_i is computed for symmetrical and non-symmetrical snow load based on the values of α and δ, shown in Fig. 7, as follows

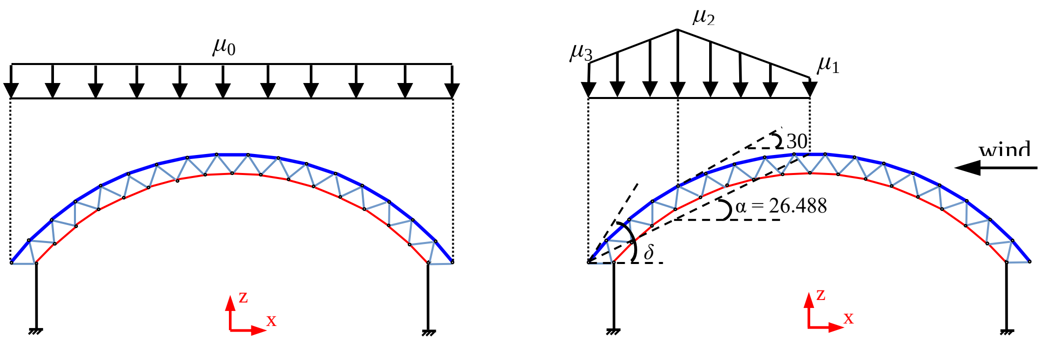

Fig. 7 Distribution of symmetrical and non-symmetrical snow loads

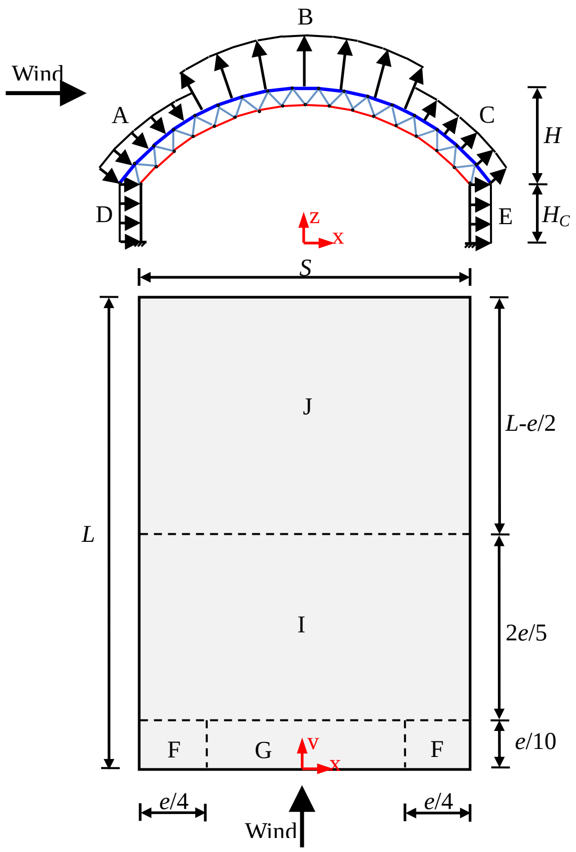

Fig. 8 Distribution of wind load in x and y directions

$$\begin{cases} \mu_0 = 0.8 \\ \mu_1 = 0.4 \\ \mu_2 = 0.8 + 0.4[(\alpha - 15)/15] \\ \mu_3 = \mu_2 (60 - \delta)/30 \end{cases} \tag{2}$$

Table 3 Values of C_{pe} in different zones shown in Fig. 8

Zone	A	B	C	D	E	F	G	I	J
C_{pe}	-0.45	0.55	0.40	-0.75	0.50	1.1	1.2	0.8	0.5

In the case of wind load in arched roofs, different loads are applied in the windward quarter, center half and leeward quarter of the roof as depicted in Fig. 8. Wind induced loads are computed according to Eurocode 1, Part 1.4 (2014) as follows

$$W = C_e q_b C_{pe} \tag{3}$$

C_e and q_b are exposure coefficient and basic wind pressure, respectively and C_{pe} is external pressure coefficient which determines the distribution of wind load.

In the present work, C_e=2.0, q_b=0.6 kN/m^2. Based on Fig. 8 values of C_{pe} in different zones are given in Table 3 for x and y directions. The value of e in Fig. 8 is determined as follows

$$e = \min\left(S, 2(H + H_C)\right) \tag{4}$$

Earthquake equivalent static loads in x and y directions are applied according to Standard No. 2800 (2014). Vertical earthquake load acting on a sloping surface shall be assumed to act on the horizontal projection of that surface. The vertical earthquake load is calculated using the following equation in agreement with Standard No. 400 (2010) requirements.

$$E_v = 0.2(D + 0.5S) \tag{5}$$

One of the most important loads acting on space structures is temperature load. In the present study, temperature loads are applied on all structural members as a temperature gradient of 25℃.

4. Nonlinear behaviour

The nonlinear behavior of columns is modeled by a bilinear constitutive law with pure strain hardening slope equal to 3% of the elastic modulus. In order to model nonlinear behavior of bracings and members of barrel vault an element with plasticity and large deflection capabilities is utilized. In elasto-plastic analysis the von-mises yield function is used as yield criterion, flow rule in this model is associative and the hardening rule is Bi-linear kinematics hardening in tension. In compression, according to FEMA-274 (1997), it is assumed that the element buckles at its corresponding buckling stress state and its residual stress is about 20% of the buckling stress. In this case, the stress-strain relation is shown in Fig. 9.

In this figure, σ_b, and σ_y are buckling and yield stresses, respectively and ε_b, and ε_y are their corresponding strains. For bracings and elements of barrel vault the buckling stress is computed as follows (AISC-LRFD 2001)

$$\sigma_b = \begin{cases} (0.658^{\lambda_c^2})\sigma_y & \lambda_c \le 1.5 \\ \left(\dfrac{0.877}{\lambda_c^2}\right)\sigma_y & \lambda_c > 1.5 \end{cases} \quad , \quad \lambda_c = \frac{KL}{r\pi}\sqrt{\frac{\sigma_y}{E}} \tag{6}$$

where λ_c is slenderness parameter; E is modulus of elasticity; and K is effective length factor.

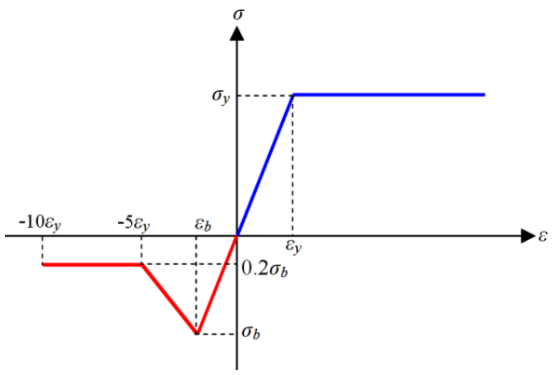

Fig. 9 Stress-strain relation of bracing and elements of barrel vault

5. Optimization problem

The main aim of the optimization problem of space structures such as double layer barrel vaults considering linear and nonlinear behaviors is to minimize the weight of the structure, subject to some constraints. In the case of both linear and nonlinear optimization processes the first constraint limits the maximum deflection of the structure. In the case of linear optimization process, the induced stresses in all structural members are limited to the allowable stresses according to the AISC-LRFD (2001) design code. In the case of nonlinear optimization process, the second constraint is checked to ensure the overall stability of the structure during the optimization process. For a double layer barrel vault with *ne* members collected in *ng* groups, if the design variables associated with each design group are selected from a given profile list, the linear and nonlinear optimization problems can be formulated as follows

Minimize

$$w(X) = \sum_{i=1}^{ne} \rho_i \, A_i \sum_{j=1}^{nm} L_j \tag{7}$$

Subject to

$$g_{\varDelta}(X) = \frac{\varDelta_{\max}}{\varDelta_{\text{all}}} - 1 \le 0 \tag{8}$$

For linear optimization

$$g_{\sigma}^{i}(X) = \frac{\sigma_i}{\sigma_{all}} - 1 \le 0 \, , i = 1, \ldots, ne \tag{9}$$

For nonlinear optimization

$$g_S(X) = \frac{f_{\text{app}}}{f_u} - 1 \le 0 \tag{10}$$

$$X = \{X_1 \quad X_2 \quad \ldots \quad X_i \quad \ldots \quad X_{ng}\}^{\mathrm{T}} \tag{11}$$

where w represents the weight of the frame; ρ_i and A_i are weight of unit volume and cross-sectional area of the ith group section, respectively; nm is the number of elements collected in the ith group; L_j is the length of the jth element in the ith group; $g_{\Delta}(X)$ is the maximum deflection constraint; $\Delta_{\max}$ is the maximum deflection of the structure and Δ_{all} is its allowable value; $g_{\sigma}^{i}(X)$ is the stress constraints of ith member; σ_i and σ_{all} are induced stress (tension or compression) and its allowable value for the ith member; $g_S(X)$ is the stability constraint; f_{app} is applied load and f_u is ultimate load of the structure which can be determined by incremental nonlinear analysis; X_i is an integer value expressing the sequence numbers of steel sections assigned to the ith group.

In this study, the constraints of the optimization problems are handled using the concept of exterior penalty function method (EPFM). In this case, the pseudo unconstrained objective function for linear and nonlinear optimization processes are expressed as follows

$$\Phi_L(X) = w(X)\left(1 + r\left(\max\{0,\ g_{\Delta}(X)\}\right)^2 + r\sum_{i=1}^{ne}\left(\max\{0,\ g_{\sigma}^{i}(X)\}\right)^2\right) \tag{12}$$

$$\Phi_{NL}(X) = w(X)\left(1 + r\left(\max\{0,\ g_{\Delta}(X)\}\right)^2 + r\left(\max\{0,\ g_S(X)\}\right)^2\right) \tag{13}$$

where $\Phi_L(X)$ and $\Phi_{NL}(X)$ are pseudo objective functions of linear and nonlinear optimization processes, respectively; r is the penalty parameter.

In the present study, the pseudo objective functions of linear and nonlinear optimization processes are minimized using CBO, ECBO, and ICBO meta-heuristics. The theoretical backgrounds of these meta-heuristics are explained below.

6. Meta-heuristics

Meta-heuristics are the recent generation of the stochastic optimization methods which inspired by nature. Besides the high computational performance, their programming and implementation is simple and therefore they can be easily applied to solve complex optimization problems (Gandomi *et al*. 2013). In the present study, CBO as a newly developed meta-heuristic is focused and an improved version of this algorithm termed as improved CBO (ICBO) is proposed.

6.1 Colliding bodies optimization

Colliding bodies optimization (CBO) is a new meta-heuristic search algorithm that is developed by Kaveh and Mahdavi (2014). In this technique, one object collides with other object and they move towards a minimum energy level. The CBO is simple in concept and does not depend on any internal parameter. Each colliding body (CB), has a specified mass defined as follows

$$m_i = \frac{1}{F(X_i)} \tag{14}$$

where $F(X_i)$ is the objective function value of the ith CB.

For the optimization problem of double layer barrel vaults, mass of each CB is defined as

$$\text{For linear optimization:}\quad m_i = \frac{1}{\Phi_L(X_i)} \tag{15}$$

$$\text{For nonlinear optimization:}\quad m_i = \frac{1}{\Phi_{NL}(X_i)} \tag{16}$$

In order to select pairs of objects for collision, CBs are divided into two equal groups:

(a) Stationary group; $i_S = 1,2,...,\frac{n}{2}$ and (b) Moving group; $i_M = \frac{n}{2}+1,\frac{n}{2}+2,...,n$

The velocities of stationary and moving bodies before collision are evaluated as follows

$$V_{i_S} = 0 \tag{17}$$

$$V_{i_M} = X_{i_S} - X_{i_M} \tag{18}$$

The velocities of stationary and moving bodies after collision are evaluated as follows

$$V'_{i_S} = \left(\frac{(1+\varepsilon)\,m_{i_M}}{m_{i_S} + m_{i_M}}\right)V_{i_M} \tag{19}$$

$$V'_{i_M} = \left(\frac{(m_{i_M} - \varepsilon\, m_{i_S})}{m_{i_S} + m_{i_M}}\right)V_{i_M} \tag{20}$$

$$\varepsilon = 1 - \frac{iter}{iter_{\max}} \tag{21}$$

where *iter* and $iter_{\max}$ are the current iteration number and the total number of iterations for optimization process, respectively; ε is the coefficient of restitution (COR).

The new position of each CB is calculated as follows

$$X_{i_S}^{\text{new}} = X_{i_S} + \overline{R}_{i_S}.V'_{i_S} \tag{22}$$

$$X_{i_M}^{\text{new}} = X_{i_M} + \overline{R}_{i_M}.V'_{i_M} \tag{23}$$

where $\overline{R}_{i_S}$ and $\overline{R}_{i_M}$ are random vectors uniformly distributed in the range of [-1,1].

6.2 Enhanced colliding bodies optimization

Enhanced CBO (ECBO) has been proposed to improve convergence rate and reliability of CBO by adding a memory to save some of the best solutions during the optimization process and also utilizing a mutation operator to decrease the probability of trapping into local optima. The basic steps of ECBO are summarized as follows (Kaveh and Ilchi Ghazaan 2014):

1. The initial positions of all colliding bodies (CBs) are determined randomly in an *m*-dimensional search space using Eq. (24).

$$X_i^0 = X_{\min} + R.(X_{\max} - X_{\min}),\, i = 1,2,...,n \tag{24}$$

in which X_i^0 is the initial solution vector of the *i*th CB. Here, $X_{\min}$ and $X_{\max}$ are respectively the lower and upper bounds of design variables; *R* is a random vector in the interval [0, 1]; *n* is the number of CBs.

2. The value of mass for each CB is evaluated using Eq. (15) or Eq. (16).

3. Colliding memory (CM) is utilized to save a number of historically best CB vectors and their related mass and objective function values. Solution vectors which are saved in CM are added to the population and the same number of current worst CBs are deleted. Finally, CBs are

sorted according to their masses in a decreasing order.
4. CBs are divided into Stationary and Moving groups.
5. The velocities of CBs before collision are evaluated using Eqs. (17) and (18).
6. The velocities of CBs after collision are evaluated using Eqs. (19) to (21).
7. The new position of each CB is calculated using Eqs. (22) and (23).
8. A parameter like *pro* within (0, 1) is introduced and it is specified whether a component of each CB must be changed or not. For each CB, *pro* is compared with rn_i (i=1,…,n) which is a random number uniformly distributed within (0, 1). If $rn_i < pro$, one dimension of the ith CB is selected randomly and its value is regenerated in interval [$X_{\min}$, $X_{\max}$]. In order to protect the structures of CBs, only one dimension is changed. In the framework of ECBO, the value of *pro* is considered to be 0.3.

When a stopping criterion is satisfied, the optimization process is terminated.

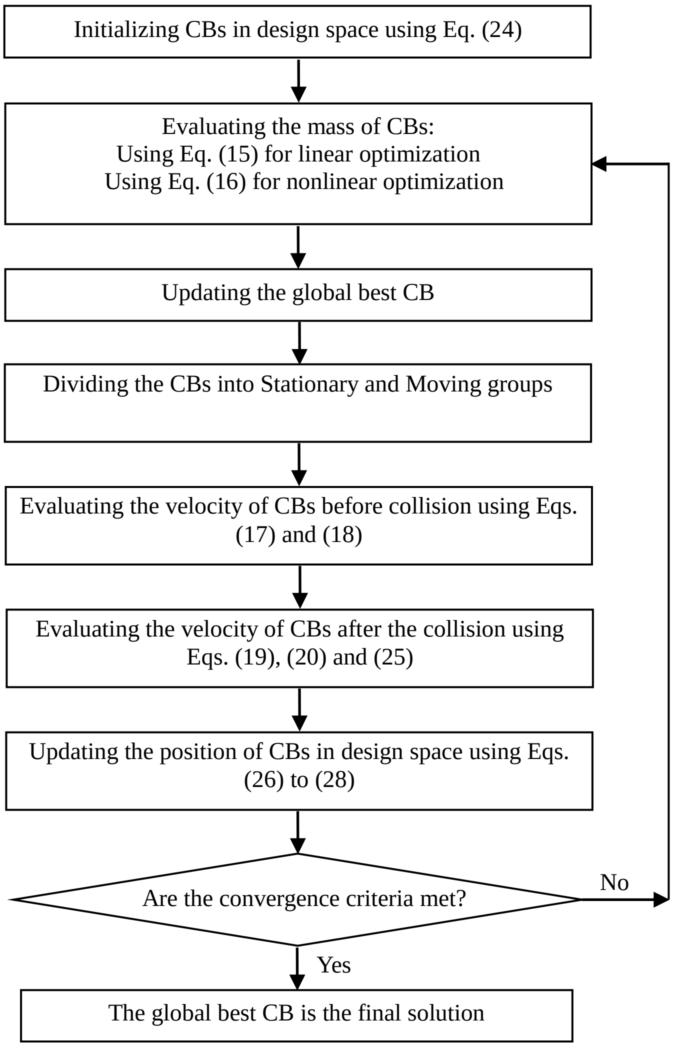

Fig. 10 Flowchart of ICBO

6.3 Improved colliding bodies optimization

In order to improve the convergence rate of CBO a different computational strategy in comparison with ECBO is proposed in the present study and the resulted algorithm is termed as improved colliding bodies optimization (ICBO). In the framework of ICBO the global best body up to current iteration is saved based on this important point that during the optimization process the best solutions should not be lost and should be passed onto the next generations. Furthermore, in order to escape from local optima a simple mechanism is proposed.

A number of CBs are randomly selected from design space using Eq. (24). Mass definition and dividing the CBs into Stationary and Moving groups are achieved same as the standard CBO. The velocities of stationary and moving bodies before collision are evaluated using Eqs. (17) and (18). After the collision the velocities of stationary and moving bodies are determined by Eqs. (19) and (20) and for updating ε the following expression is proposed in this study

$$\varepsilon = C^0 - \frac{iter}{iter_{\max}} \tag{25}$$

in which the best value of C^0 should be determined by performing sensitivity analysis.

The following equations are proposed in the present work to update the position of CBs in design space

$$X_{i_S}^{iter+1} = X_{i_S}^{iter} + \overline{R}_{i_S}.V'_{i_S} + \alpha^{iter}.\overline{R}_i \tag{26}$$

$$X_{i_M}^{iter+1} = X_{i_M}^{iter} + \overline{R}_{i_M}.V'_{i_M} + \alpha^{iter}.\overline{R}_i \tag{27}$$

$$\alpha^{iter} = \alpha^{iter-1}.\alpha_{damp} \tag{28}$$

where $\overline{R}_{i_S}$ and $\overline{R}_{i_M}$ are random vectors uniformly distributed in the range of [-1,1]; $\overline{R}_i$ is a random vector in the interval [-0.5, 0.5]. In this work α_{damp} is considered to be 0.995 and in order to find the best value of α^0 sensitivity analysis should be conducted.

Flowchart of the ICBO meta-heuristic to tackle the optimization problems of double layer barrel vaults considering both linear and nonlinear behaviors is depicted in Fig. 10.

Table 4 The available list of standard Pipe profiles

No.	Profile	A (cm^2)	r (cm)
1	D48×2.9	4.1089	1.5978
2	D60×3.0	5.3721	2.0180
3	D76×3.0	6.8801	2.5831
4	D89×3.0	8.1053	3.0424
5	D114×4.0	13.823	3.8917
6	D114×5.0	17.121	3.8578
7	D140×4.0	17.090	4.8104
8	D140×5.0	21.206	4.7762
9	D168×5.0	25.604	5.7656
10	D168×6.0	30.536	5.7315

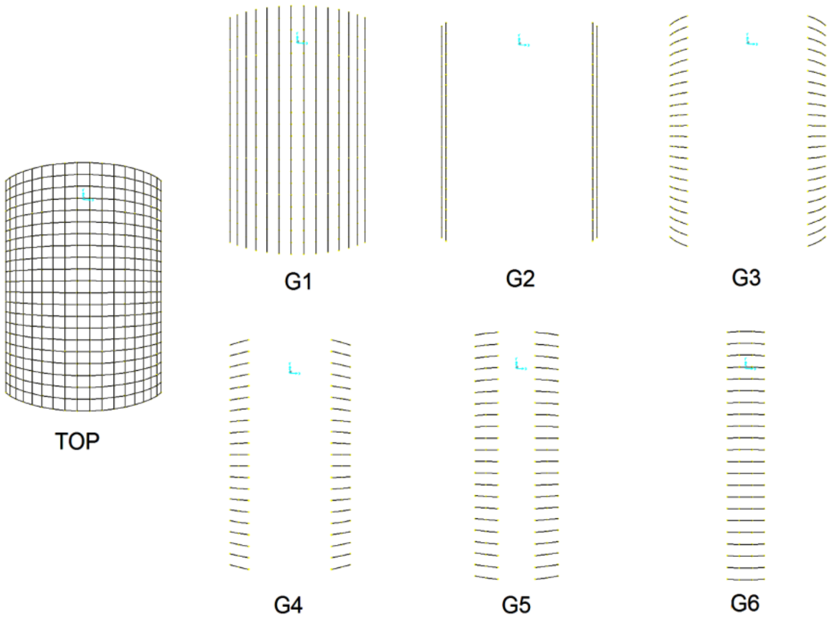

Fig. 11 Member grouping details of the top layer

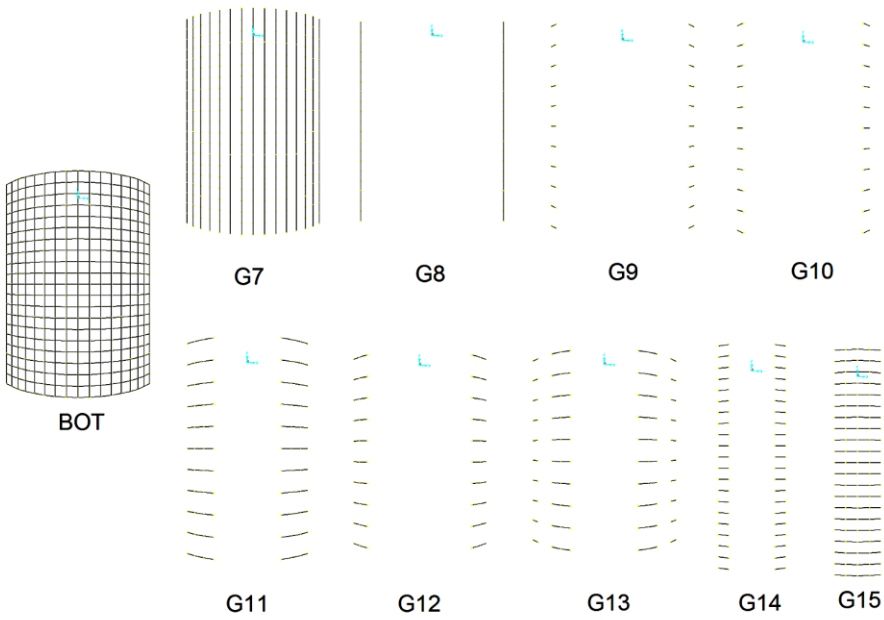

Fig. 12 Member grouping details of the Bottom layer

7. Numerical results

The best model determined by implementation of form finding process in section 2 is optimized using CBO, ECBO and ICBO considering linear and nonlinear behaviours. Young's modulus, mass density and yield stress are 2.1×10^{10} kg/m^2, 7850 kg/m^3, and 2.4×10^7 kg/m^2, respectively. The design variables are selected from a set of available Pipe profiles in Iran listed in Table 4. Figs. 11, 12, and 13 present the grouping details of the structural members in top, bottom and web layers, respectively.

For performing optimization process, the number of CBs for CBO, ECBO and ICBO is chosen

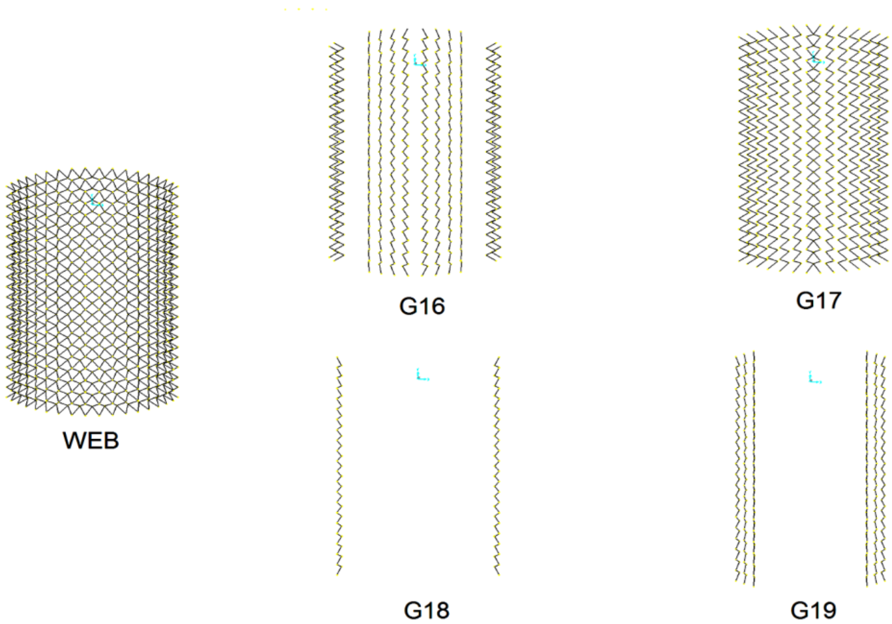

Fig. 13 Member grouping details of the web layer

Table 5 Results of sensitivity analysis carry out for ICBO

No.	C^0	α^0	Best weight (kg)	
			Linear optimization	Nonlinear optimization
1	1.0	1.5	52431.64	43941.39
2	1.0	2.0	51841.24	42823.17
3	1.0	2.5	53293.12	43641.96
4	2.0	1.5	46467.93	39924.62
5	2.0	2.0	46013.28	39530.37
6	2.0	2.5	47127.03	40313.61
7	3.0	1.5	45984.09	39643.94
8	3.0	2.0	44791.49	39277.88
9	3.0	2.5	46048.34	39754.32
10	4.0	1.5	49817.06	42136.48
11	4.0	2.0	49034.61	41729.13
12	4.0	2.5	50419.35	42457.07

Table 6 Comparison of FSD with linear and nonlinear optimal solutions

Design variables	FSD	Linear Optimal Designs			Nonlinear Optimal Designs		
		CBO	ECBO	ICBO	CBO	ECBO	ICBO
G1	D48×2.9	D48×2.9	D48×2.9	D48×2.9	D48×2.9	D48×2.9	D48×2.9
G2	D60×3.0	D60×3.0	D60×3.0	D60×3.0	D48×2.9	D48×2.9	D48×2.9
G3	D89×3.0	D89×3.0	D89×3.0	D89×3.0	D60×3.0	D60×3.0	D60×3.0
G4	D114×4.0	D114×4.0	D114×4.0	D114×4.0	D89×3.0	D89×3.0	D89×3.0
G5	D140×4.0	D140×4.0	D114×4.0	D114×4.0	D114×4.0	D114×4.0	D114×4.0
G6	D140×4.0	D140×4.0	D140×4.0	D140×4.0	D114×4.0	D114×4.0	D114×4.0
G7	D48×2.9	D48×2.9	D48×2.9	D48×2.9	D48×2.9	D48×2.9	D48×2.9
G8	D76×3.0	D76×3.0	D76×3.0	D76×3.0	D48×2.9	D48×2.9	D48×2.9
G9	D168×5.0	D168×5.0	D168×5.0	D168×5.0	D140×5.0	D140×4.0	D140×4.0
G10	D168×5.0	D140×5.0	D140×5.0	D140×4.0	D140×4.0	D114×4.0	D114×4.0
G11	D114×4.0	D114×4.0	D114×4.0	D114×4.0	D114×4.0	D114×4.0	D89×3.0
G12	D89×3.0	D89×3.0	D89×3.0	D89×3.0	D76×3.0	D76×3.0	D89×3.0
G13	D89×3.0	D76×3.0	D76×3.0	D76×3.0	D76×3.0	D76×3.0	D76×3.0
G14	D76×3.0	D76×3.0	D76×3.0	D76×3.0	D48×2.9	D48×2.9	D48×2.9
G15	D76×3.0	D60×3.0	D60×3.0	D60×3.0	D48×2.9	D48×2.9	D48×2.9
G16	D60×3.0	D60×3.0	D60×3.0	D60×3.0	D60×3.0	D60×3.0	D60×3.0
G17	D48×2.9	D48×2.9	D48×2.9	D48×2.9	D48×2.9	D48×2.9	D48×2.9
G18	D89×3.0	D89×3.0	D89×3.0	D89×3.0	D76×3.0	D76×3.0	D76×3.0
G19	D76×3.0	D76×3.0	D76×3.0	D76×3.0	D76×3.0	D76×3.0	D76×3.0
Weight (kg)	46433.01	45733.71	45004.71	44791.49	40433.67	40051.17	39277.88
$\Delta_{max} / \Delta_{all}$	0.806	0.824	0.837	0.839	0.922	0.936	0.922
$\sigma_{max} / \sigma_{all}$	0.999	0.998	0.999	0.999	-	-	-
f_{app} / f_u	-	-	-	-	0.689	0.704	0.714

to be 30 and the maximum number of iterations for all algorithms is limited to 200. In this study, Δ_{all}=10.5 cm.

In order to determine the best values of internal parameters of ICBO, C^0 and α^0, a sensitivity analysis is carried out for linear and nonlinear optimization cases. Four values of 1.0, 2.0, 3.0 and 4.0 are considered for C^0 and three values of 1.5, 2.0 and 2.5 for α^0. In the case of each combination of the parameters 10 independent optimization runs are implemented and the best results are reported in Table 5.

The best results found by ICBO considering linear and nonlinear behaviors are compared in Table 6 with those of CBO, ECBO and the fully stressed design (FSD) found by SAP2000 (2011) based on AISC-LRFD (2001) considering linear behavior.

Convergence histories of CBO, ECBO and ICBO during the linear and nonlinear optimization processes are shown in Figs. 14 and 15, respectively.

For linear optimization the structural weight of optimal design found by ICBO is 2.06%, and 0.47% lighter than the optimal weights found by CBO and ECBO, respectively. ICBO converges to an optimal design which is 3.54% lighter than that of the FSD. The results indicate that the

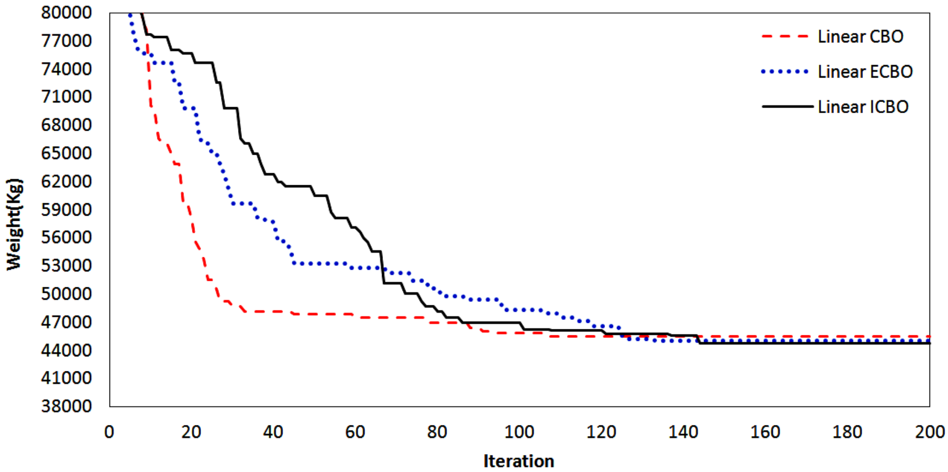

Fig. 14 The convergence history of the linear optimization processes

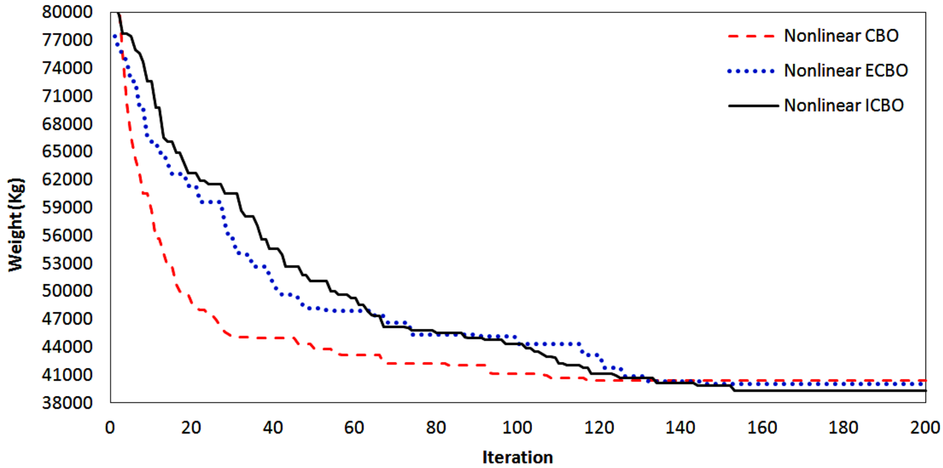

Fig. 15 The convergence history of the nonlinear optimization processes

maximum stress constraint is the active constraint of the linear optimization process. It can be easily observed from Fig. 14 that the convergence rate of ICBO is better than those of CBO and ECBO meta-heuristics.

In the case of nonlinear optimization process, it can be observed that the computational performance of the proposed ICBO is better than that of the CBO and ECBO. The structural weight of nonlinear optimal design found by ICBO is 2.86%, and 1.93% lighter than the optimal weights found by CBO and ECBO, respectively. In addition, the ICBO converges to an optimal design which is 15.41% lighter than that of the FSD. The values of f_{app}/f_u and $\Delta_{max}/\Delta_{all}$ reported in Table 6 demonstrate the feasibility of the nonlinear optimal designs. The values of f_{app}/f_u confirm that the nonlinear optimal designs have acceptable safety against overall instability. In addition, it can be observed that the displacement constraints dominate the nonlinear optimal solutions. Comparison of convergence histories of CBO, ECBO and ICBO indicate that ICBO possesses the best convergence rate.

8. Conclusions

The main aim of this study is to optimize double layer barrel vaults considering non-linear behavior. Prior to optimization process, the best form for double-layer barrel vaults having the least construction cost is determined among 36 models. In order to solve the optimization problem an improved colliding bodies optimization (ICBO) algorithm is proposed. In the framework of ICBO to escape from local optima a simple mechanism is proposed. In addition, the global best body up to current iteration is saved and passed onto the next generation to prevent from losing the fittest solutions. Two optimization processes are achieved using CBO, ECBO, and the proposed ICBO considering linear and nonlinear behaviors and the results are compared with those of fully stressed design (FSD).

The numerical results demonstrate the superiourity of ICBO over the CBO and ECBO in terms of optimal weight and convergence rate for both linear and nonlinear processes. It is observed that in the linear and nonlinear optimization processes the proposed ICBO meta-heuristic converges to an optimal solution which is respectively 3.54% and 15.41% lighter than the FSD while all the optimization constraints are satisfied. The numerical results demonstrate that the best solution found by ICBO in the framework of nonlinear optimization process is 12.31% lighter than the solution found by ICBO during the linear optimization process. This amount of weight reduction is significant from an engineering standpoint and clearly implies on the efficiency of considering nonlinear behavior in optimization process of structural systems.

The convergence histories of the linear and nonlinear optimization processes show that CBO is a meta-heuristic with dominant exploitation characteristic and therefore the algorithm traps in a local optimum. It is clear that, for ICBO, there is a fine balance between the exploration and exploitation and therefore the final solution of the algorithm is better than those of the CBO and ECBO. However, it seems that there is still room for establishing a better balance between exploration and exploitation by hybridizing CBO and ICBO algorithms.

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